PID Controller Design with Pole Placement Method

Following is the symbolic math script to calculate the gain of the PID controller using the desired closed loop pole locations, also knows as pole placement method.

```
clc, clear, clf;
syms s Kp Ki Kd a b p1 p2 p3 K

% Let us assume a systeme Transfer function without the PID controller
G = K / (s ^ 2 + a * s + b);
% The characteristic equation of the closed loop system with PID controller
G_cs = collect((s ^ 2 + a * s + b) * s + K * (Kd * s ^ 2 + Kp * s + Ki), s)
```

```
G_cs =

s^3 + (a + K*Kd)*s^2 + (b + K*Kp)*s + K*Ki
```

Now let us assume the desired closed loop pole locations are -p1, -p2, and -p3, that make the desired characteristic equation as follows:

```
G_des = (s + p1) * (s + p2) * (s + p3);
G_des = collect(G_des, s)
% Equating the coefficients of the characteristic equation of the closed
loop system with PID controller and the desired characteristic equation, we
```

```
get:

eq1 = coeffs(G_cs, s) == coeffs(G_des, s);

% Solving the above equations, we get the values of Kp, Ki, and Kd as
follows:
sol = solve(eq1, [Kp, Ki, Kd])
Kp_s = simplify(sol.Kp)
Ki_s = simplify(sol.Ki)
Kd_s = simplify(sol.Kd)
```

```
G_des =

s^3 + (p1 + p2 + p3)*s^2 + (p3*(p1 + p2) + p1*p2)*s + p1*p2*p3


sol =

  struct with fields:

    Kp: (p1*p2 - b + p1*p3 + p2*p3)/K
    Ki: (p1*p2*p3)/K
    Kd: (p1 - a + p2 + p3)/K


Kp_s =

(p1*p2 - b + p1*p3 + p2*p3)/K


Ki_s =

(p1*p2*p3)/K


Kd_s =

(p1 - a + p2 + p3)/K
```

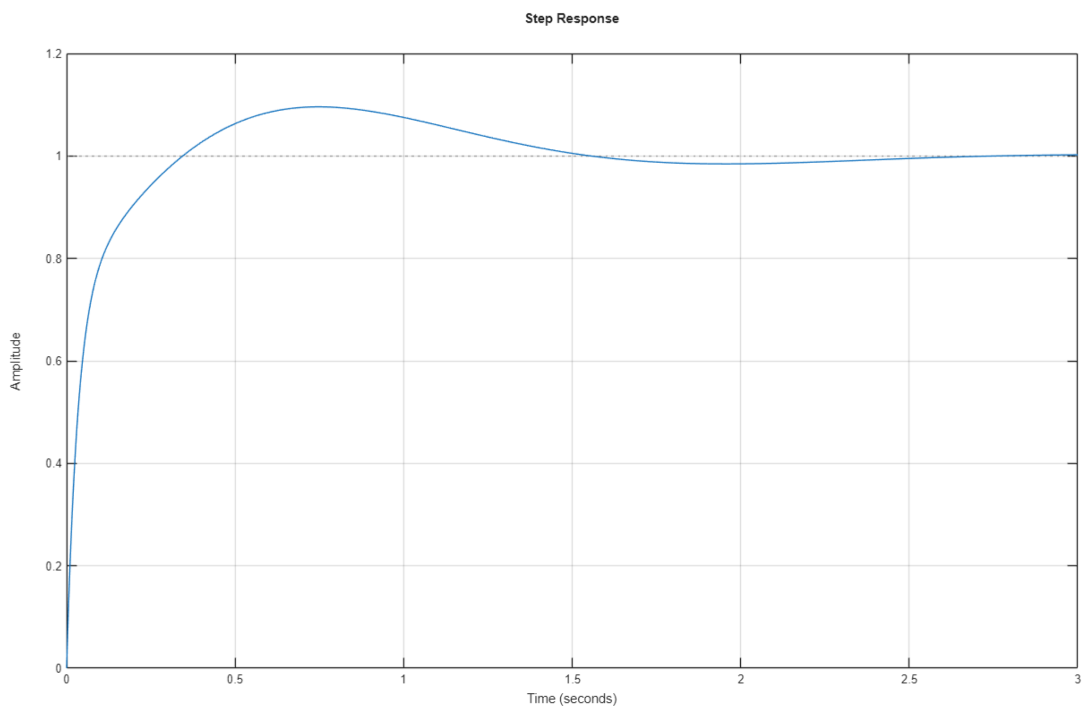
Example: Let us assume a system with the following transfer function and design a PID controller using pole placement method.

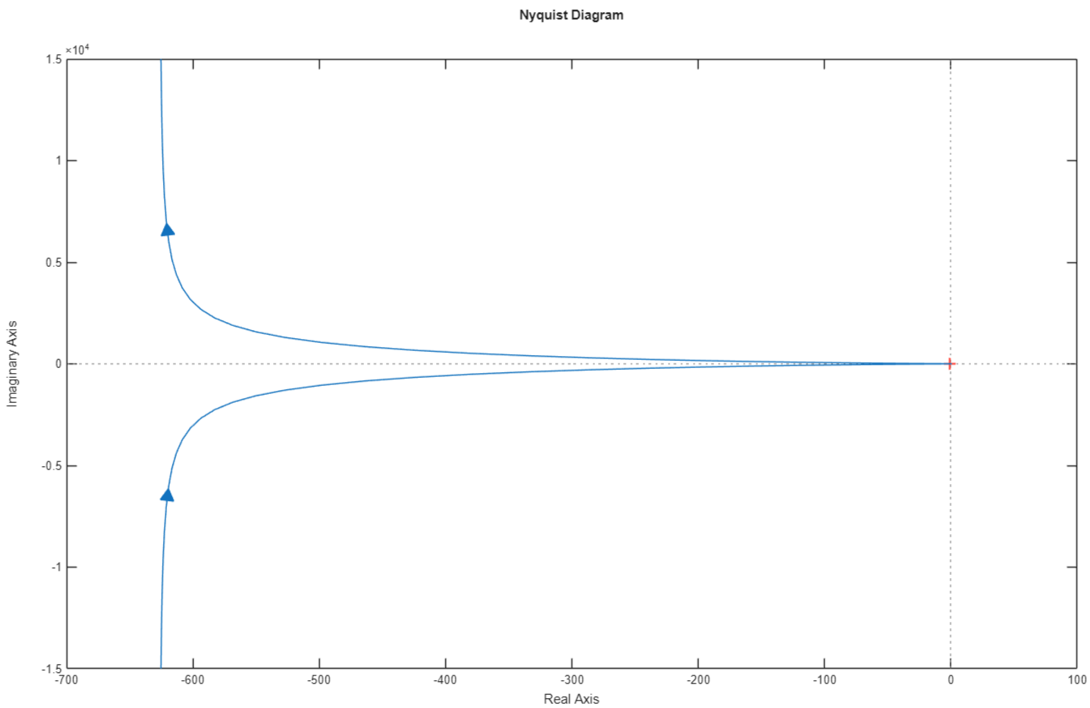
```
clear s Kp Ki Kd a b p1 p2 p3
s = tf('s');
G = 10 / (s ^ 2 + 10 * s + 2)
[num, den] = tfdata(G);
a = den{1}(2);
b = den{1}(3);
[~, ~, K] = zpkdata(G);
zpk(G)

% Desired closed loop pole locations
omega_n = 3;
```

```
p3 = 10 * omega_n;
i = 1;
for eta = 0.5:0.1:1

    p1 = (eta * omega_n + sqrt(1 - eta ^ 2) * omega_n * 1i);
  p2 = (eta * omega_n - sqrt(1 - eta ^ 2) * omega_n * 1i);

  Kp(i, :) = double(subs(Kp_s)); %#ok<*SAGROW>
  Ki(i, :) = double(subs(Ki_s));
  Kd(i, :) = double(subs(Kd_s));
  p1_n(i, :) = p1;
  p2_n(i, :) = p2;
  p3_n(i, :) = p3;
  zeta_n(i, :) = eta;
  C = pid(Kp(i, :), Ki(i, :), Kd(i, :));
```

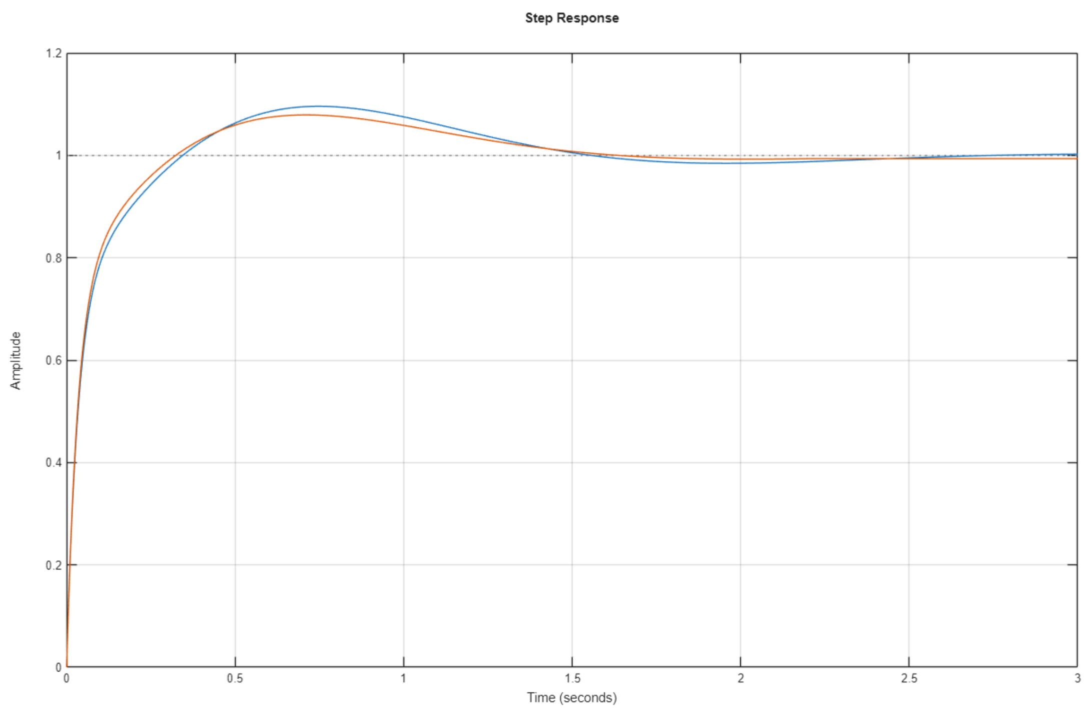
Now let us check the step response of the closed loop system with the designed PID controller.

```
  sys_cl = feedback(C * G, 1);
  figure(1)
  hold on
  step(sys_cl)
  grid on
```

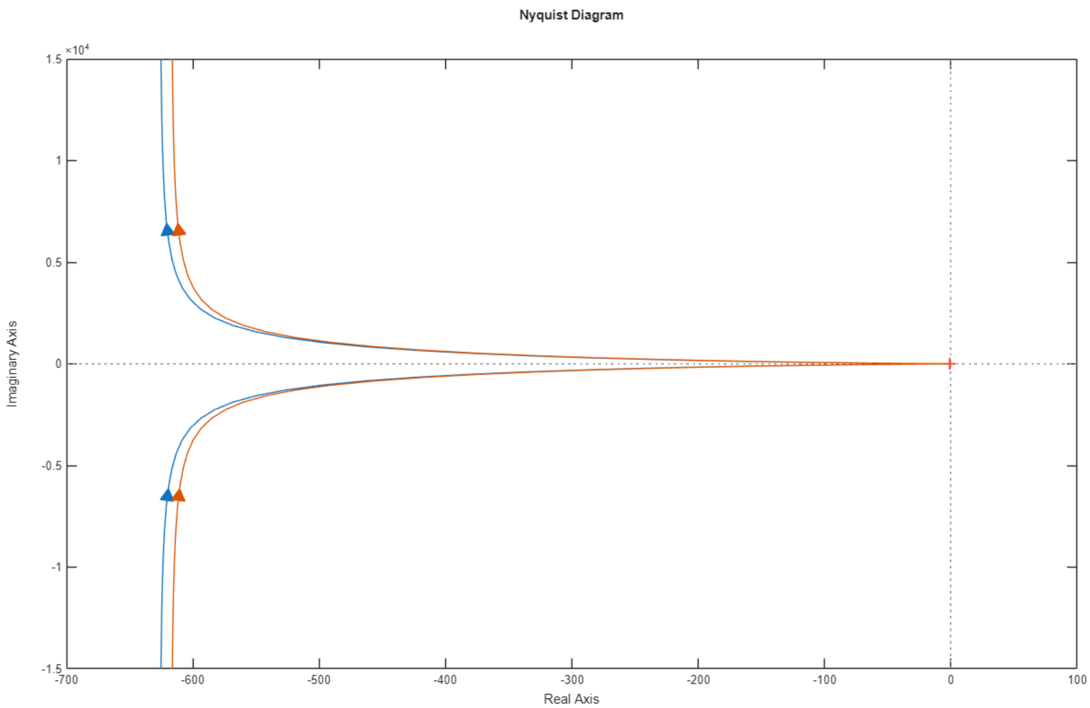


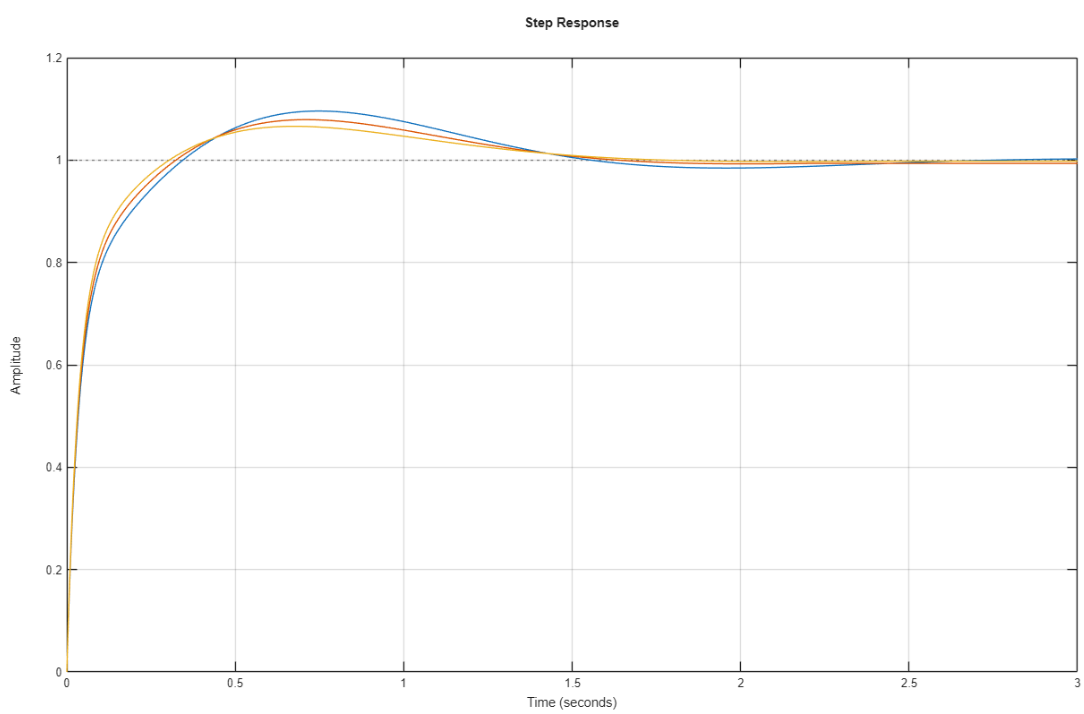


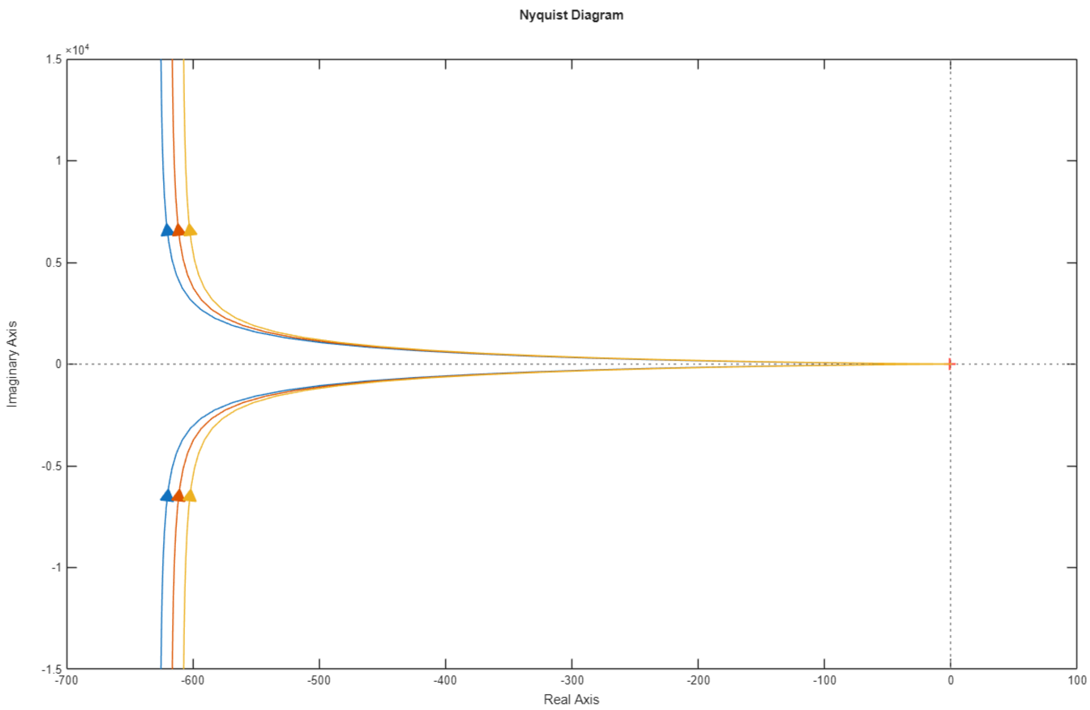
Nyquist Diagram
×10^4
1.5
1
0.5
0
-0.5
-1
-1.5
Imaginary Axis
-700
-600
-500
-400
-300
-200
-100
0
100
Real Axis

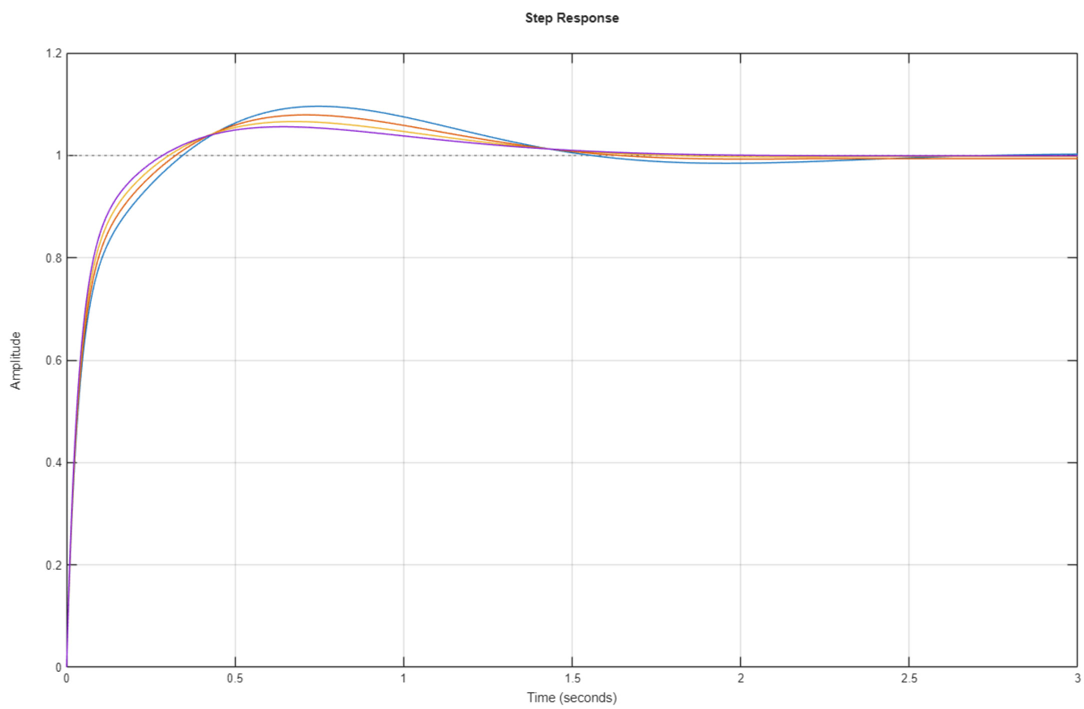


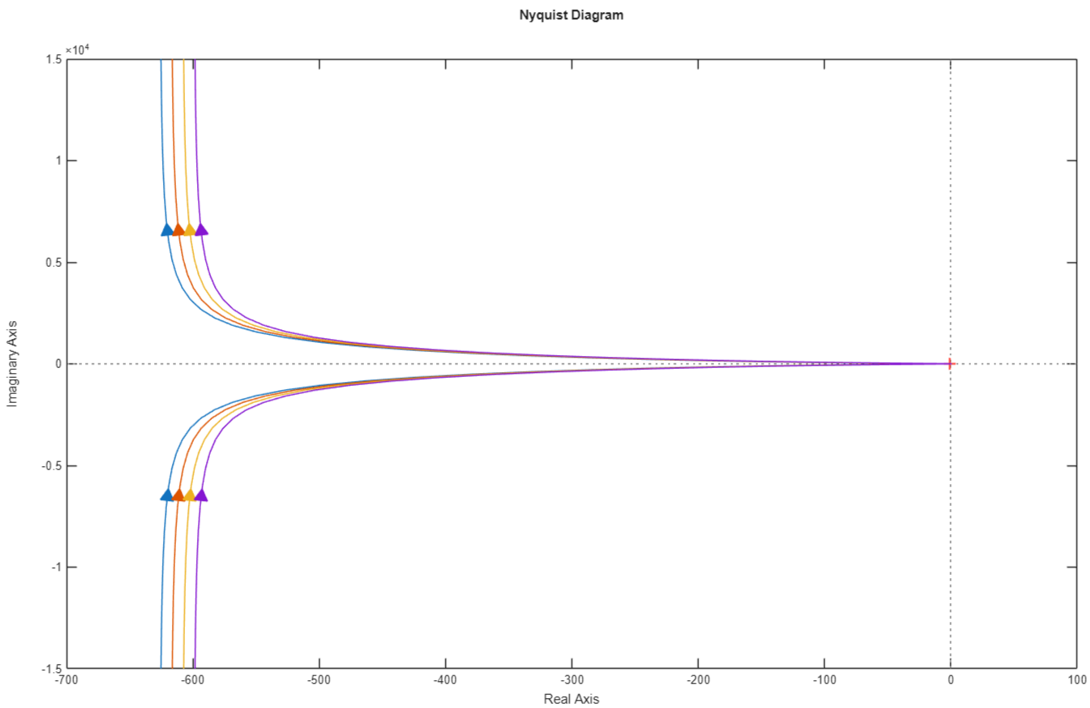
Step Response
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
0.5
1
1.5
2
2.5
3
Time (seconds)

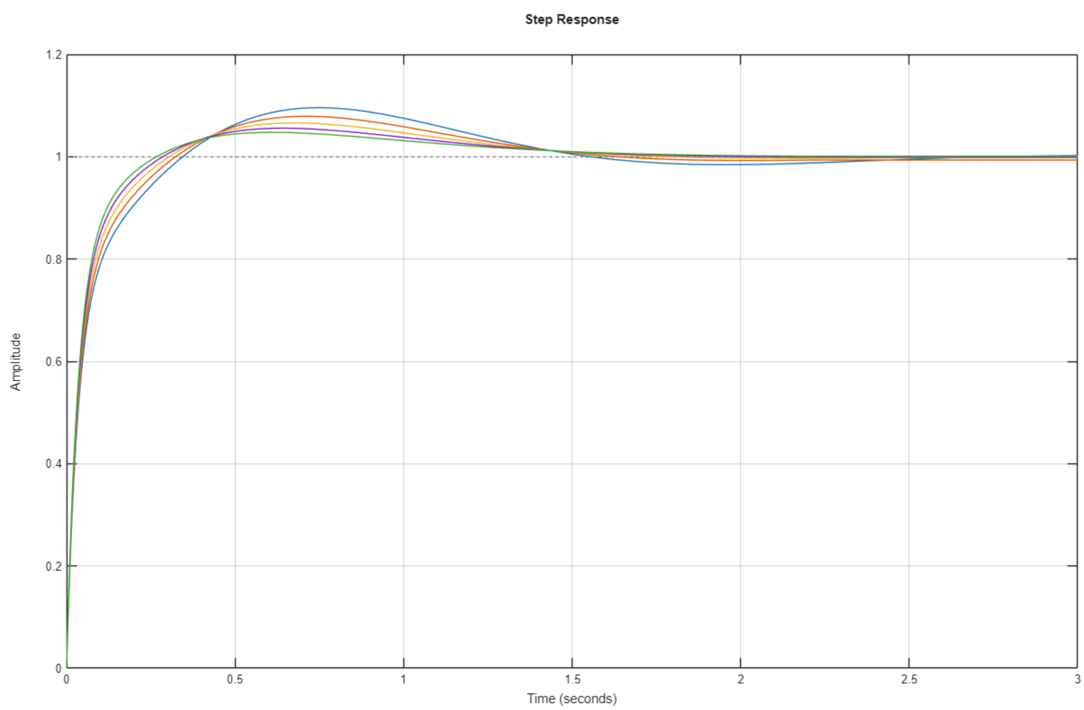

Nyquist Diagram
×10^4
1.5
1
0.5
0
-0.5
-1
-1.5
Imaginary Axis
-700
-600
-500
-400
-300
-200
-100
0
100
Real Axis

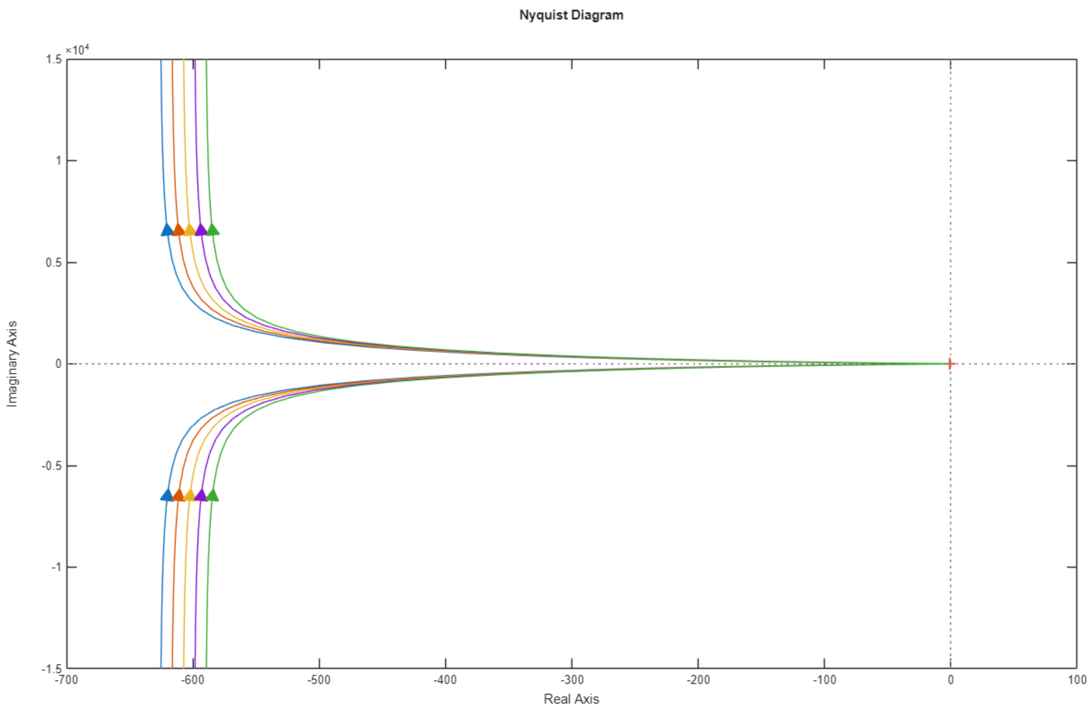

Step Response
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
0.5
1
1.5
2
2.5
3
Time (seconds)

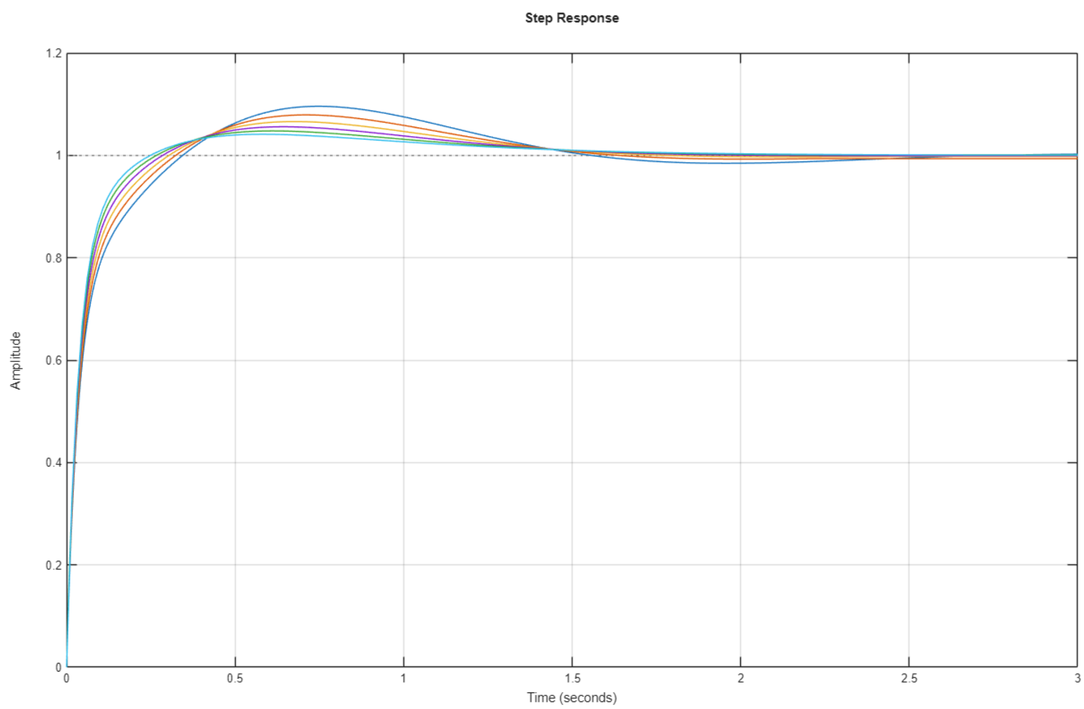

Nyquist Diagram
×10^4
1.5
1
0.5
0
-0.5
-1
-1.5
-700
-600
-500
-400
-300
-200
-100
0
100
Real Axis
Imaginary Axis


Step Response
1.2
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3
Time (seconds)
Amplitude


Nyquist Diagram
×10^4
1.5
1
0.5
0
-0.5
-1
-1.5
Imaginary Axis
-700
-600
-500
-400
-300
-200
-100
0
100
Real Axis


Step Response
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
0.5
1
1.5
2
2.5
3
Time (seconds)



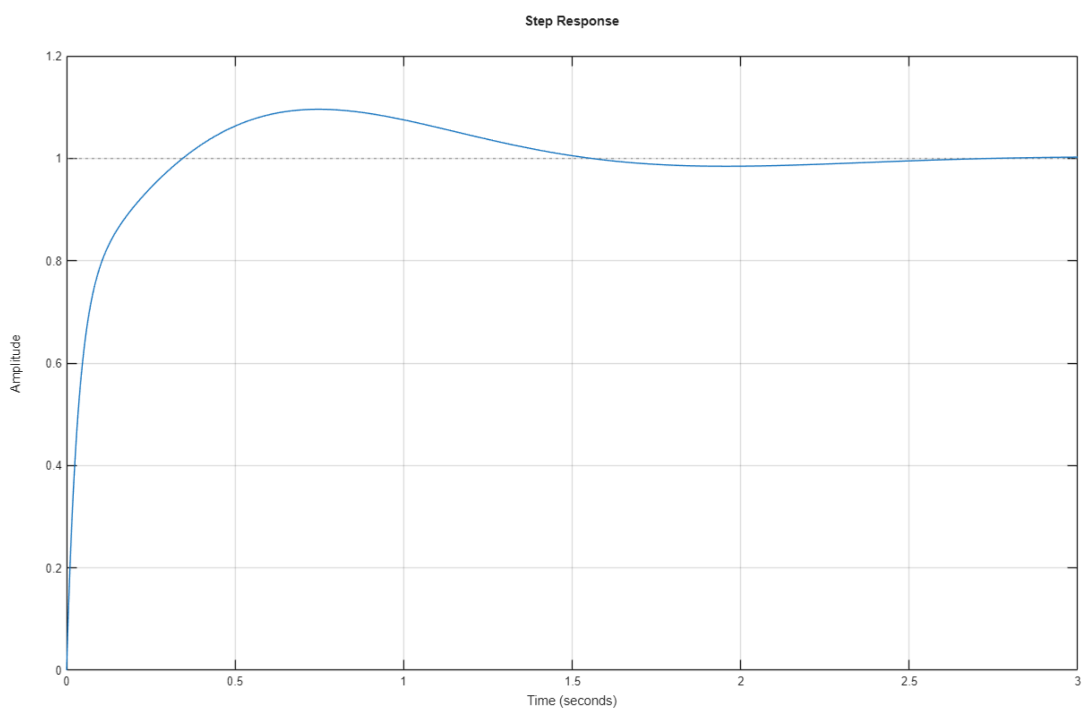


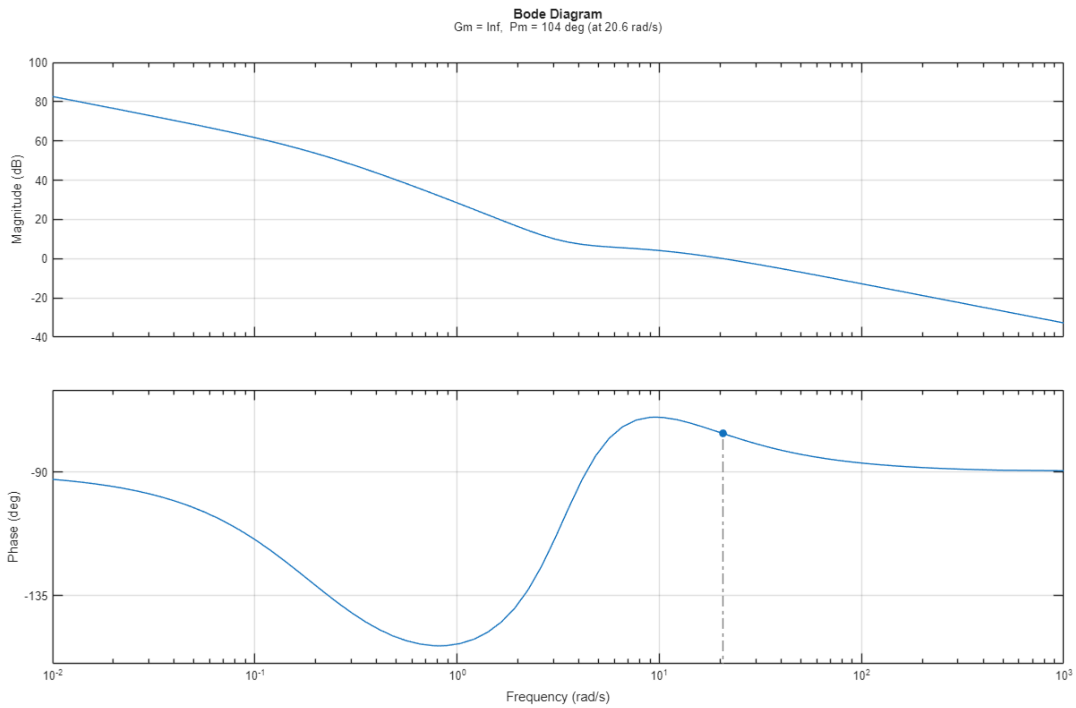
Now let us check the frequency response of the open loop system with the designed PID controller.

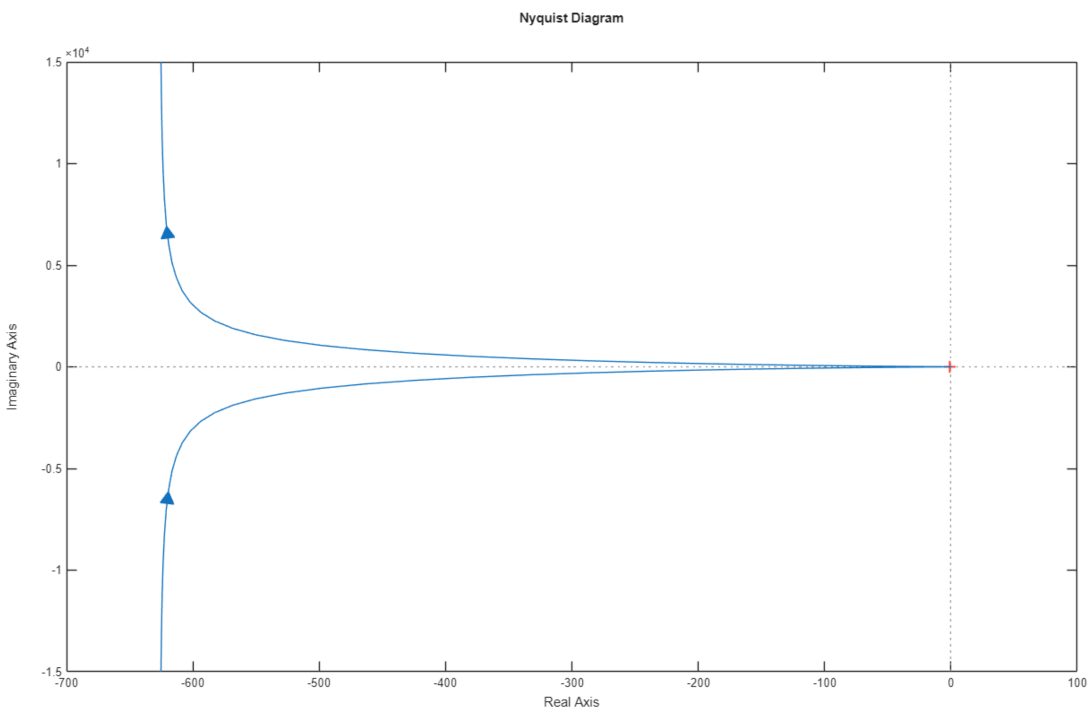
```
figure(2)
hold on
margin(C * G)
```

```
  grid on
  [Gm(i, :), Pm(i, :), Wpc(i, :), Wgc(i, :)] = margin(C * G);
  fprintf('Gain Margin: %.2f dB at Frequency: %.2f rad/s\n', db(Gm(i, :)),
Wpc(i, :));
  fprintf('Phase Margin: %.2f degrees at Frequency: %.2f rad/s\n', Pm(i, :),
Wgc(i, :));
  fprintf('The closed Loop system for zeta =  %f',zeta_n(i,:))
  display(zpk(feedback(G*C,1)))
  figure(3), hold on;
  np = nyquistplot(C * G);
  i = i + 1;
```

```
Gain Margin: Inf dB at Frequency: NaN rad/s
Phase Margin: 104.09 degrees at Frequency: 20.65 rad/s
The closed Loop system for zeta =  0.500000
ans =

  23 (s^2 + 4.217s + 11.74)
  -------------------------
    (s+30) (s^2 + 3s + 9)

Continuous-time zero/pole/gain model.
```

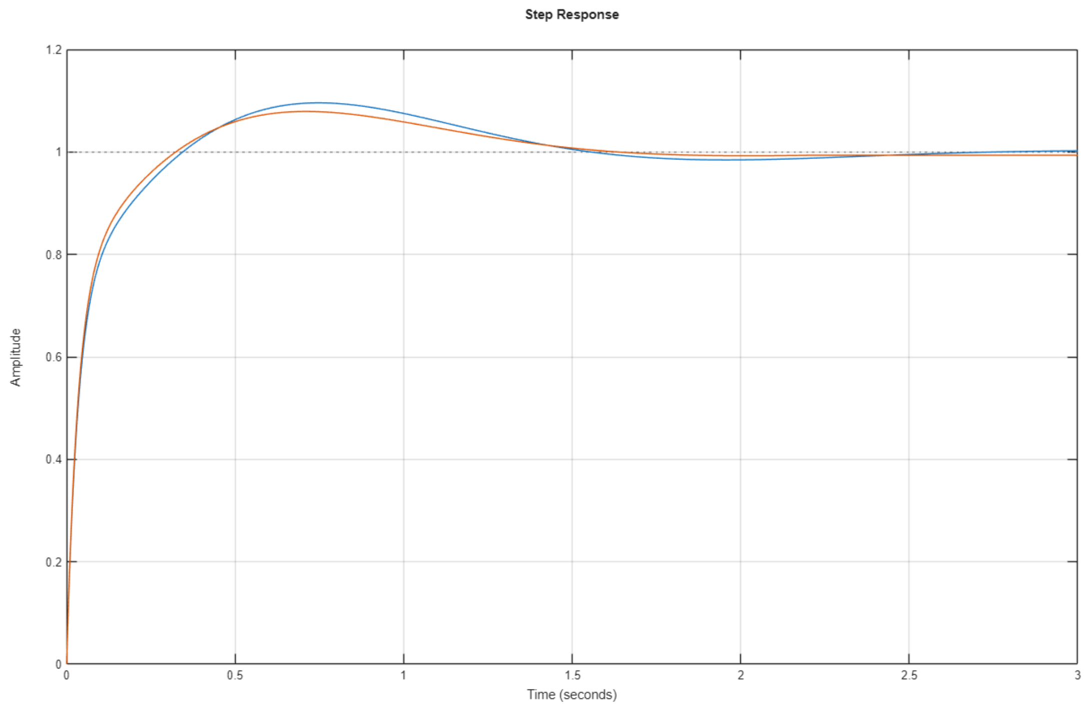


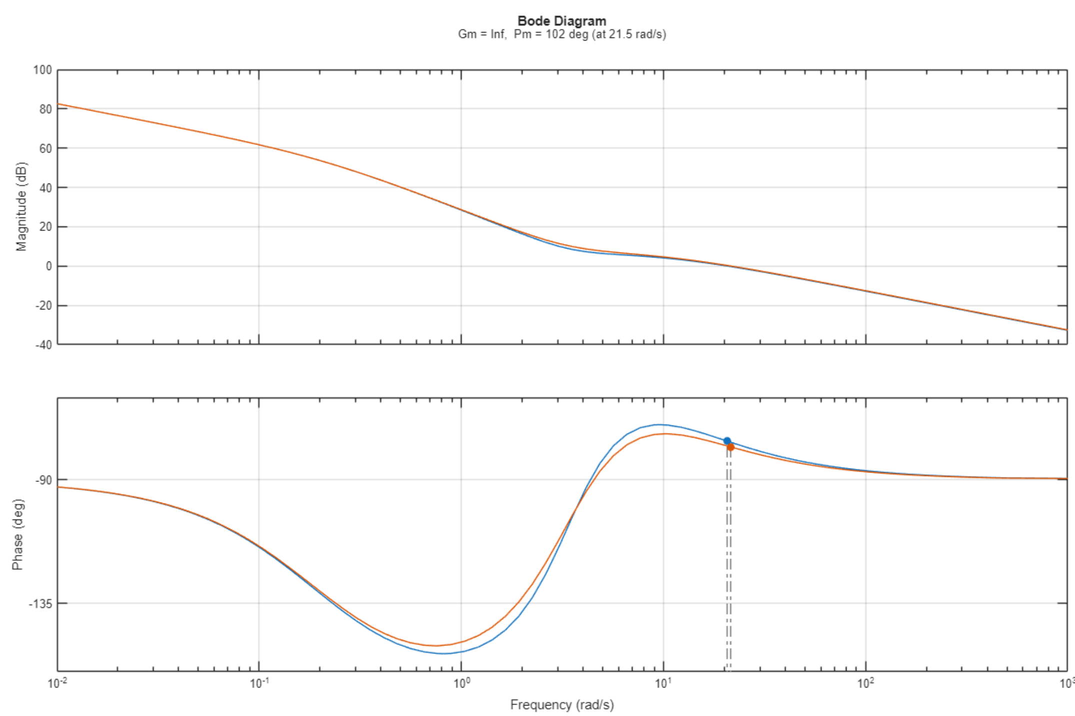


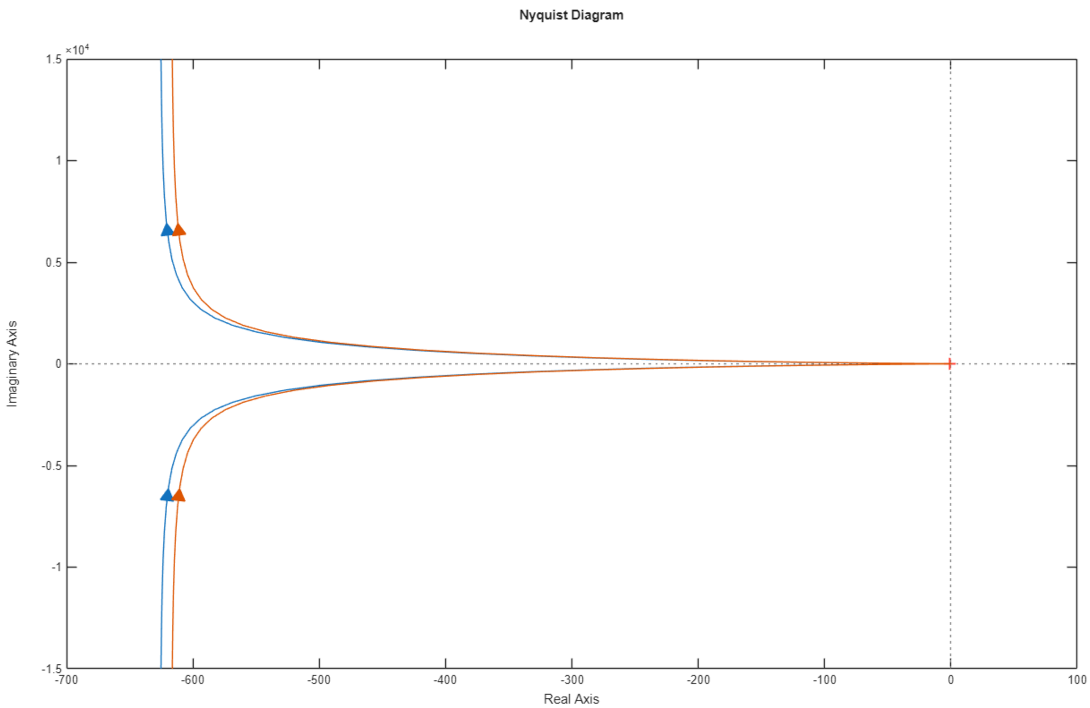



```
Gain Margin: Inf dB at Frequency: NaN rad/s
Phase Margin: 101.96 degrees at Frequency: 21.50 rad/s
The closed Loop system for zeta =  0.600000
ans =
```

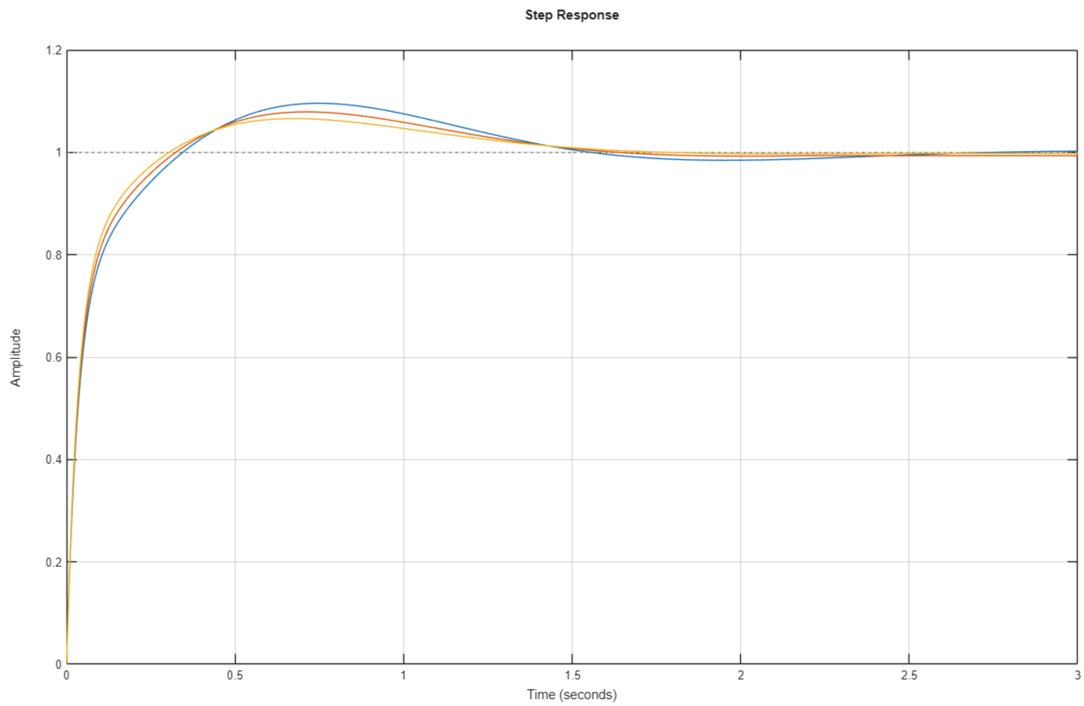
```
  23.6 (s^2 + 4.873s + 11.44)
  ---------------------------
    (s+30) (s^2 + 3.6s + 9)

Continuous-time zero/pole/gain model.
```

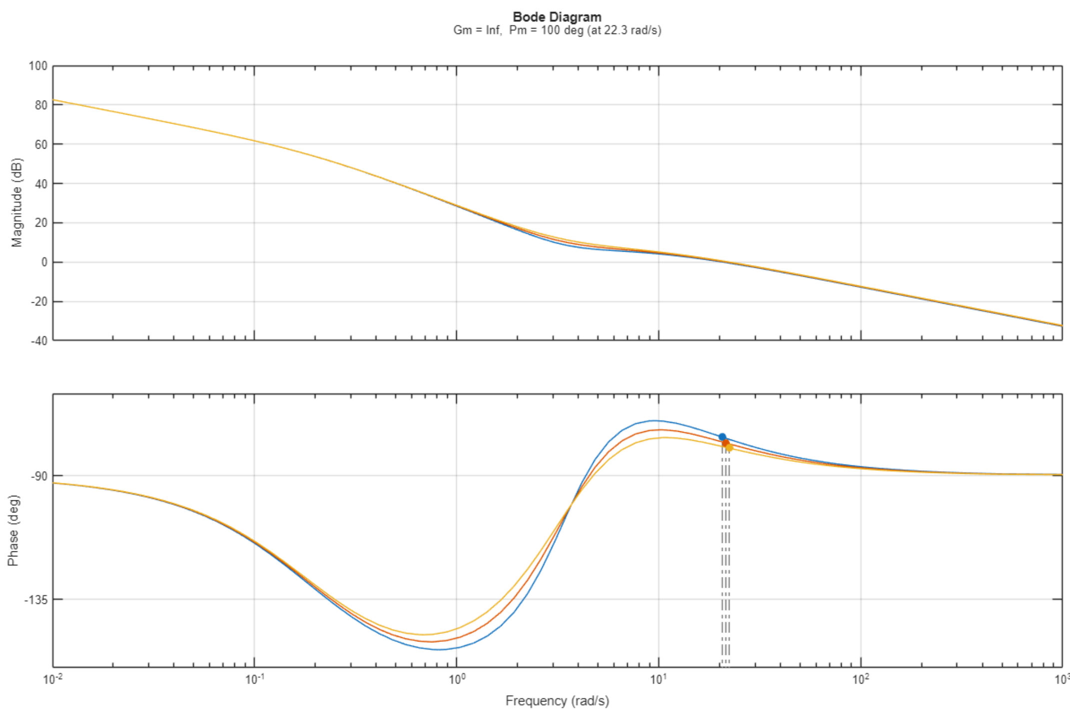


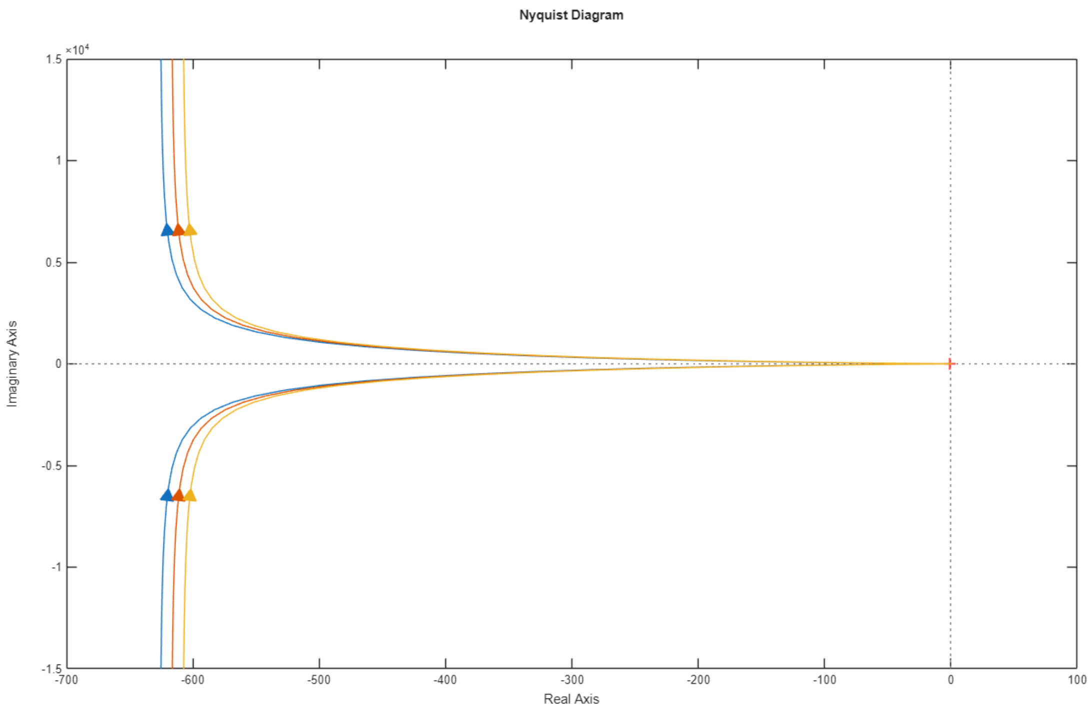




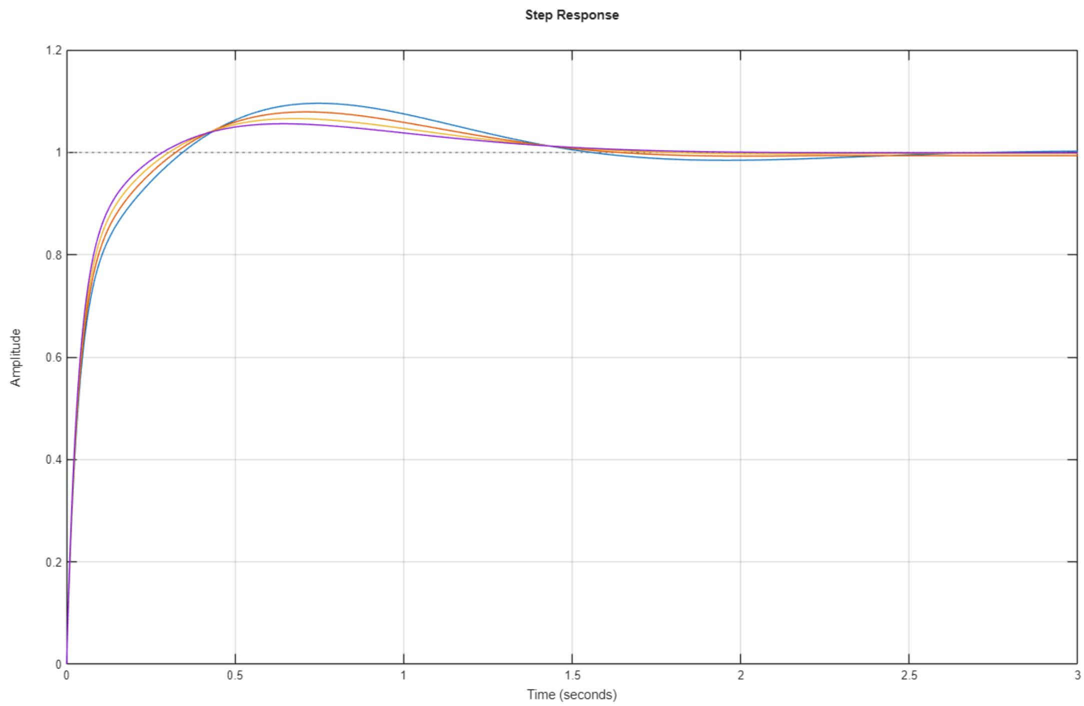

```
Gain Margin: Inf dB at Frequency: NaN rad/s
Phase Margin: 100.08 degrees at Frequency: 22.34 rad/s
The closed Loop system for zeta =  0.700000
ans =

  24.2 (s^2 + 5.496s + 11.16)
  ---------------------------
    (s+30) (s^2 + 4.2s + 9)

Continuous-time zero/pole/gain model.
```

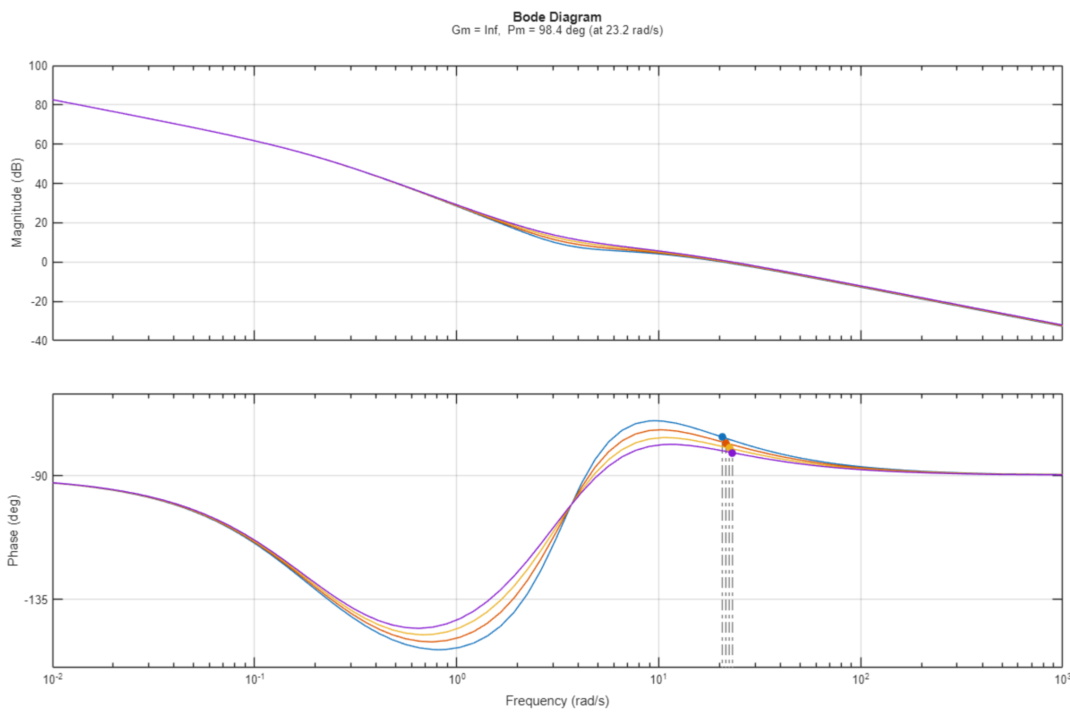

Step Response
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0.5
1.5
2
2.5
3

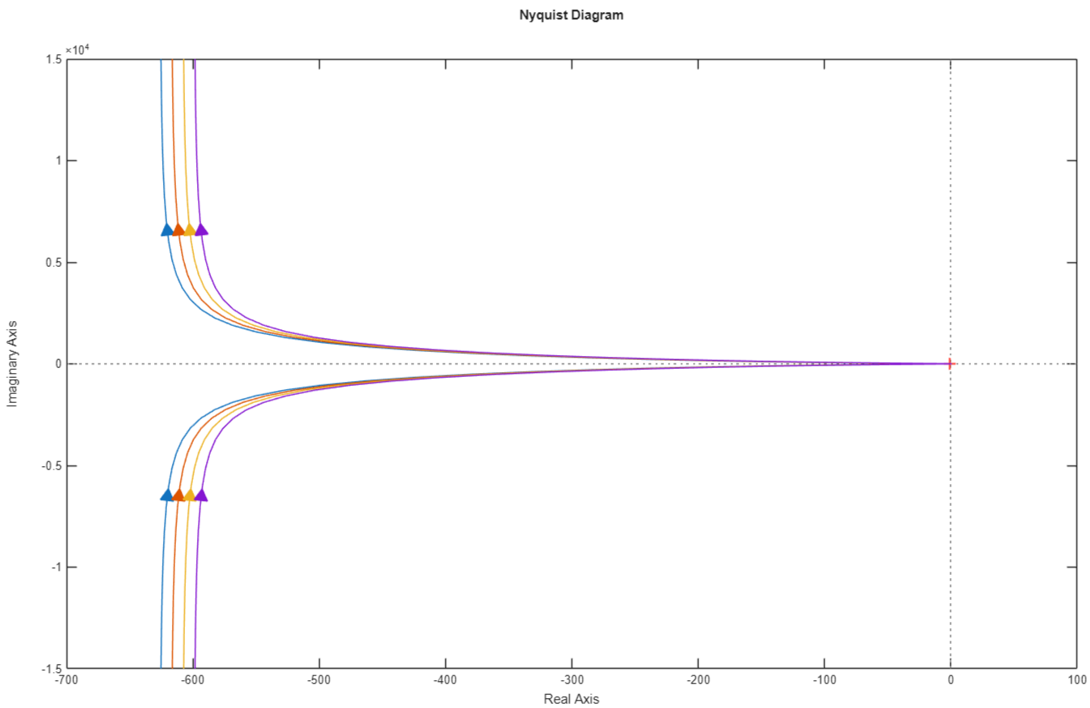

Bode Diagram
Gm = Inf, Pm = 100 deg (at 22.3 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
100
80
60
40
20
0
-20
-40
-90
-135
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}



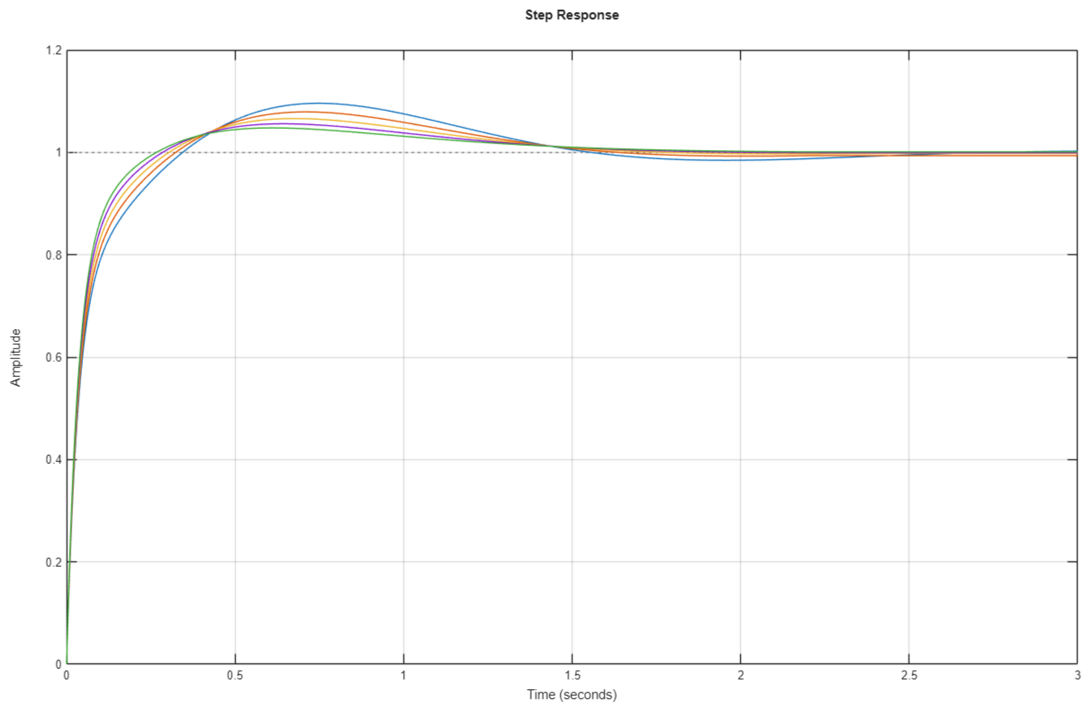

```
Gain Margin: Inf dB at Frequency: NaN rad/s
Phase Margin: 98.41 degrees at Frequency: 23.17 rad/s
The closed Loop system for zeta =  0.800000
ans =

  24.8 (s^2 + 6.089s + 10.89)
  ---------------------------
    (s+30) (s^2 + 4.8s + 9)

Continuous-time zero/pole/gain model.
```

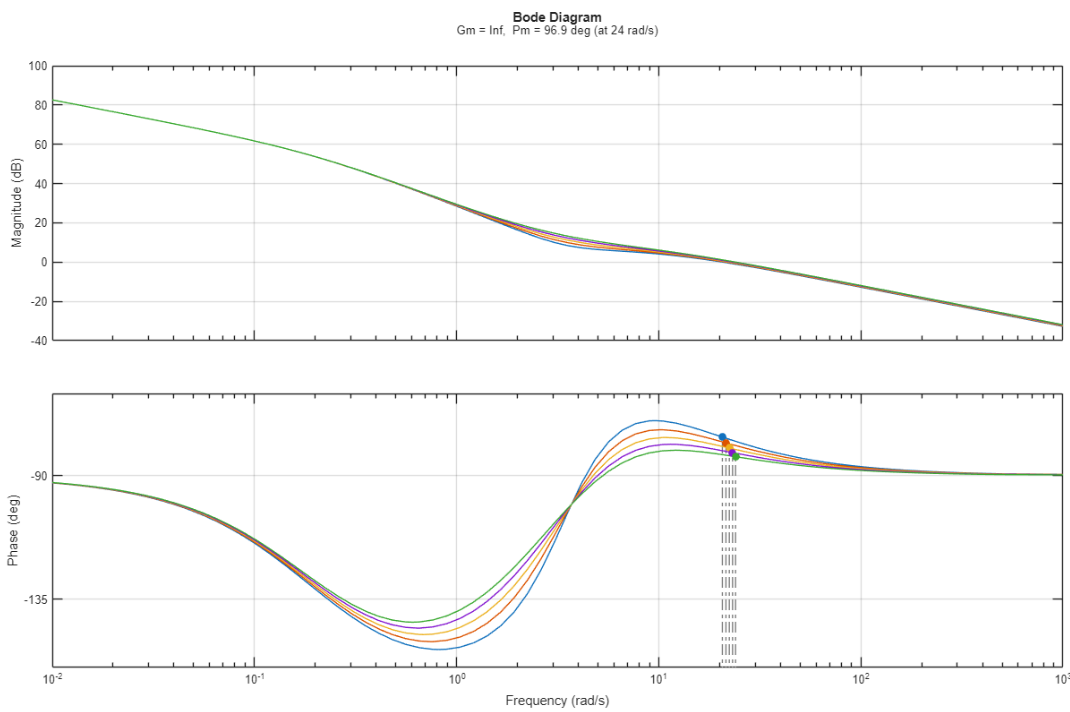

Step Response
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3

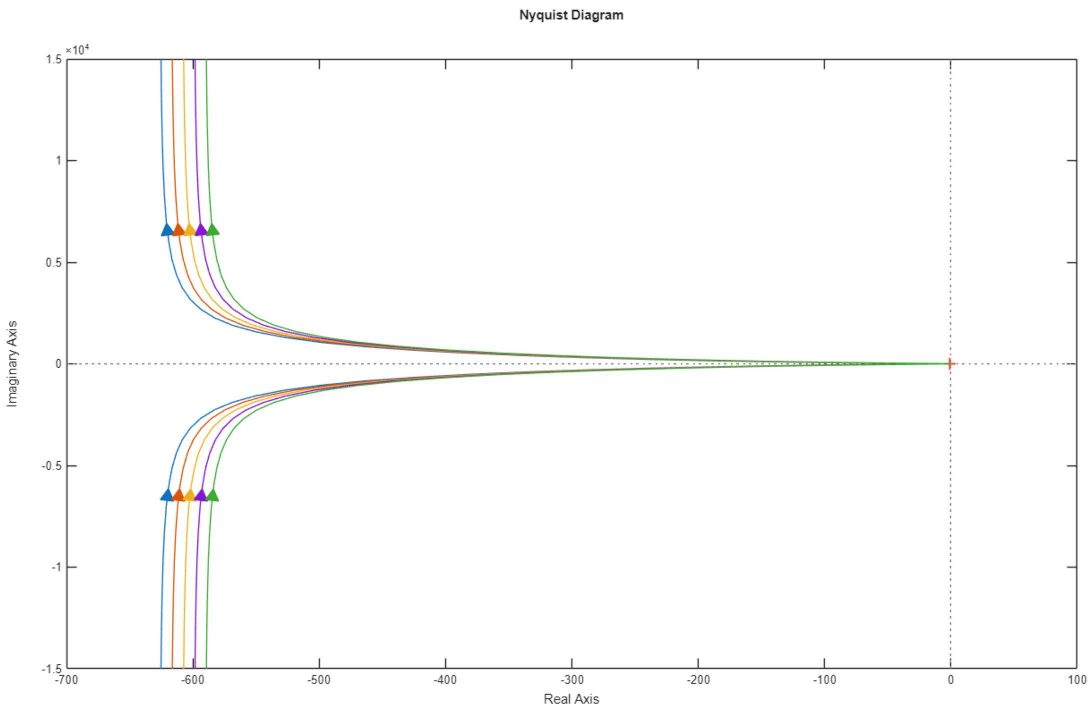

Bode Diagram
Gm = Inf, Pm = 98.4 deg (at 23.2 rad/s)
Magnitude (dB)
100
80
60
40
20
0
-20
-40
Phase (deg)
-90
-135
Frequency (rad/s)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}



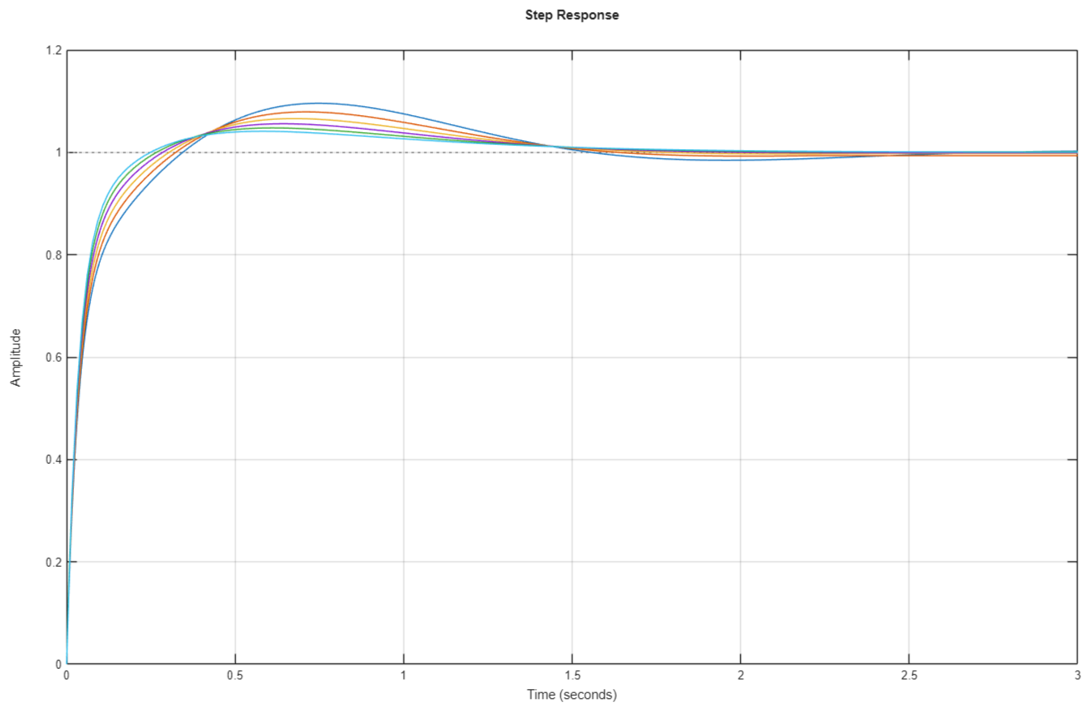

```
Gain Margin: Inf dB at Frequency: NaN rad/s
Phase Margin: 96.92 degrees at Frequency: 23.98 rad/s
The closed Loop system for zeta =  0.900000
ans =

  25.4 (s+3.988) (s+2.665)
  ------------------------
  (s+30) (s^2 + 5.4s + 9)

Continuous-time zero/pole/gain model.
```

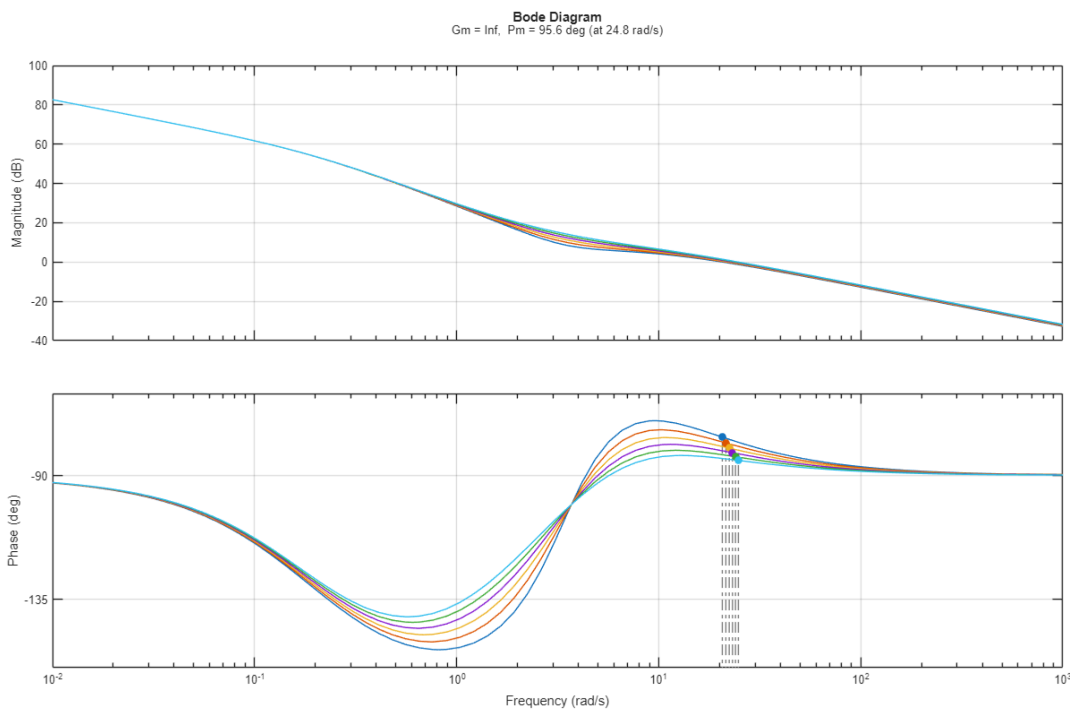

Step Response
Amplitude
Time (seconds)

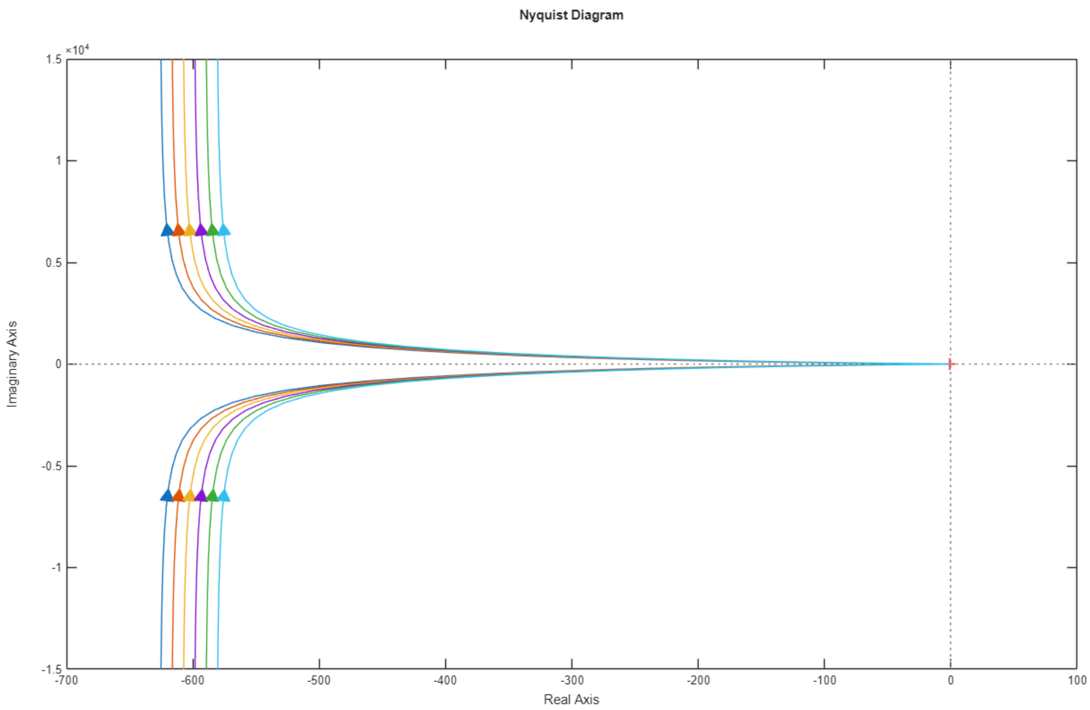

Bode Diagram
Gm = Inf, Pm = 96.9 deg (at 24 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)




```
Gain Margin: Inf dB at Frequency: NaN rad/s
Phase Margin: 95.59 degrees at Frequency: 24.78 rad/s
The closed Loop system for zeta =  1.000000
ans =

  26 (s+5.192) (s+2)
  ------------------
    (s+30) (s+3)^2

Continuous-time zero/pole/gain model.
```


Step Response
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3

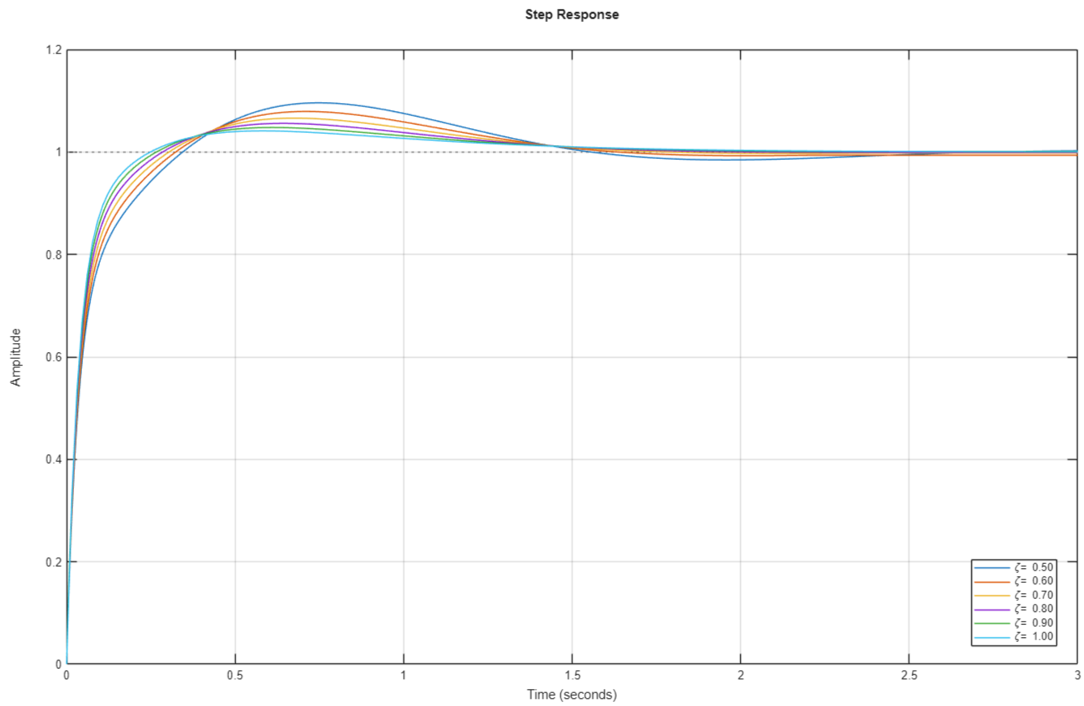

Bode Diagram
Gm = Inf, Pm = 95.6 deg (at 24.8 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
100
80
60
40
20
0
-20
-40
-90
-135
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

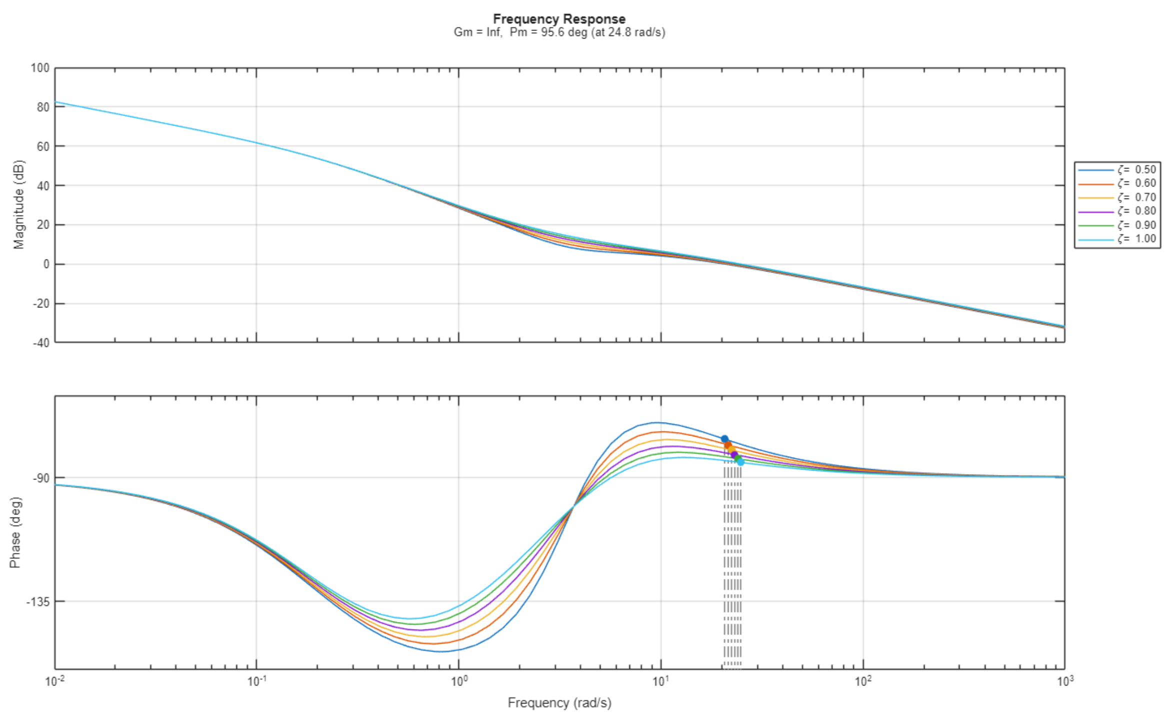


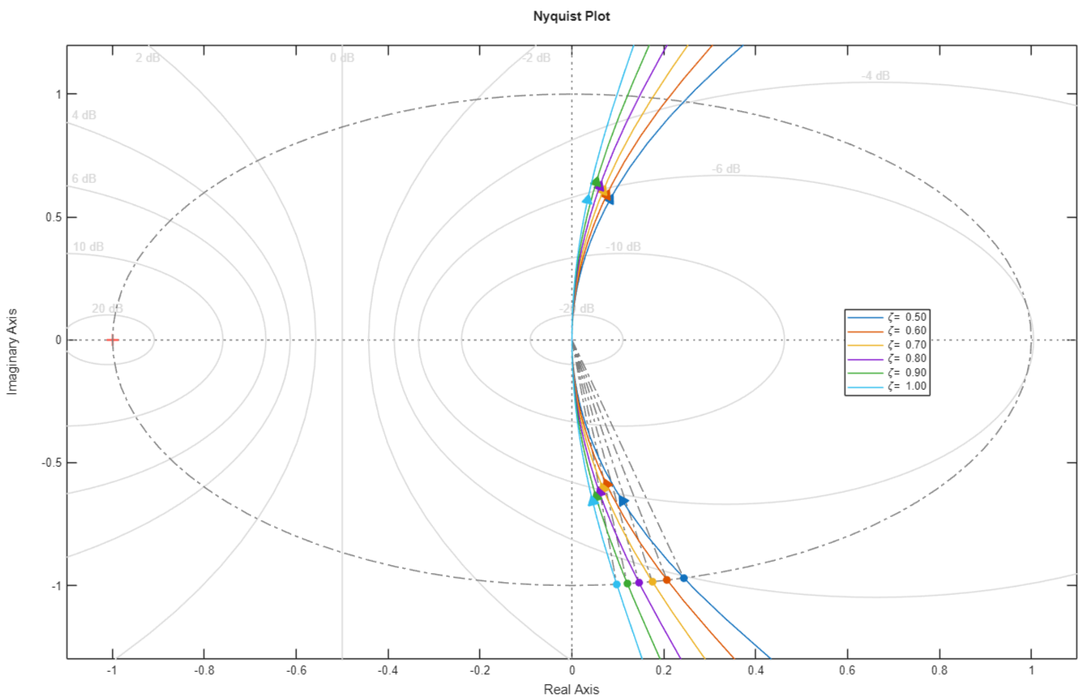

```
end

legendLabels = arrayfun(@(x) sprintf('\\zeta= %.2f', x), zeta_n,
'UniformOutput', false);
figure(1), legend(legendLabels, Location = "best"), hold off;
title('Step Response')
figure(2), legend(legendLabels, Location = "eastoutside"), hold off;
title('Frequency Response');
figure(3), legend(legendLabels, Location = "best"), hold off;
np.XLim = [-1.1, 1.1];
np.YLim = [-1.3, 1.2];
np.Characteristics.AllStabilityMargins.Visible = 'on';
grid("on")
title('Nyquist Plot');
```

```
G =

        10
  --------------
  s^2 + 10 s + 2

Continuous-time transfer function.

ans =

           10
  --------------------
  (s+9.796) (s+0.2042)
```

Continuous-time zero/pole/gain model.








```
tab = table(zeta_n, [p1_n, p2_n, p3_n], Kp, Ki, Kd, Gm, Pm, Wpc, Wgc, ...
  'VariableNames', {'Damping Ratio', 'Desired Closed Loop Poles', 'Kp',
'Ki', 'Kd', 'GM_dB', 'PM_deg', 'w_gc', 'w_pc'})
```

```
tab =

  6×9 table

    Damping Ratio                   Desired Closed Loop Poles
Kp      Ki      Kd      GM_dB     PM_deg     w_gc      w_pc
        _____________   ________________________________________________
____     __     ____     _____     ______     ____     ______

           0.5          1.5+2.5981i          1.5-2.5981i               30+0i
9.7     27       2.3     Inf      104.09      NaN      20.648
           0.6          1.8+2.4i             1.8-2.4i                  30+0i
11.5     27      2.36     Inf      101.96      NaN      21.502
           0.7          2.1+2.1424i          2.1-2.1424i               30+0i
13.3     27      2.42     Inf      100.08      NaN       22.34
           0.8          2.4+1.8i             2.4-1.8i                  30+0i
15.1     27      2.48     Inf      98.408      NaN      23.168
           0.9          2.7+1.3077i          2.7-1.3077i               30+0i
16.9     27      2.54     Inf      96.922      NaN      23.983
             1           3+0i                 3+0i                     30+0i
18.7     27       2.6     Inf      95.592      NaN      24.784
```

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