05.12.2016.

Solving Linear Equation - LMS

$$\boxed{X' = XA}$$

Solve for A in terms of the least mean square. i.e. find A which minimizes:

$$\|X' - XA\|^2$$

Take derivative wrt A and equate to 0:

$$-2X^T(X' - XA) = 0 \longrightarrow X^TX' = X^TXA$$

$$(X^TX)^{-1}X^TX' = (X^TX)^{-1}X^TXA$$

$$\underbrace{(X^TX)^{-1}X^T}_{pinv(X)}X' = A$$

pinv=pseudo inverse

$$\boxed{pinv(X) \cdot X' = A}$$

X' - נקבעת לפי A.

Global Transformations – Global Warping

Affine Warping:

Given 3 paired points:

$$\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_1 & y_1 & 0 & 1 \\ x_2 & y_2 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_2 & y_2 & 0 & 1 \\ x_3 & y_3 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_3 & y_3 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$$

Global Transformations – Global Warping

solution:

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \mathrm{pinv}\left(\begin{bmatrix} x_1 & y_1 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_1 & y_1 & 0 & 1 \\ x_2 & y_2 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_2 & y_2 & 0 & 1 \\ x_3 & y_3 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_3 & y_3 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \end{bmatrix}$$

$$\mathrm{pinv}(X) = (X^T X)^{-1} X^T$$

Global Transformations – Global Warping

Affine Warping:

Given 3 paired points:

$$\begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_1 & y_1 & 0 & 1 \\ x_2 & y_2 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_2 & y_2 & 0 & 1 \\ x_3 & y_3 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_3 & y_3 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$$

Global Transformations – Global Warping

What about Projective Transformations?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneity must be preserved!

$$\boxed{x' = \frac{ax + by + e}{gx + hy + 1} \;\; ; \;\; y' = \frac{cx + dy + f}{gx + hy + 1}}$$

Global Transformations – Global Warping

Rearrange:

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$$

Affine Warping:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = A \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Global Transformations – Global Warping

Projective Transformation:

Given 4 paired points:

$$\begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \\ x_4' \\ y_4' \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 0 & 0 & 1 & 0 & -x_1x_1' & -y_1x_1' \\ 0 & 0 & x_1 & y_1 & 0 & 1 & -x_1y_1' & -y_1y_1' \\ x_2 & y_2 & 0 & 0 & 1 & 0 & -x_2x_2' & -y_2x_2' \\ 0 & 0 & x_2 & y_2 & 0 & 1 & -x_2y_2' & -y_2y_2' \\ x_3 & y_3 & 0 & 0 & 1 & 0 & -x_3x_3' & -y_3x_3' \\ 0 & 0 & x_3 & y_3 & 0 & 1 & -x_3y_3' & -y_3y_3' \\ x_4 & y_4 & 0 & 0 & 1 & 0 & -x_4x_4' & -y_4x_4' \\ 0 & 0 & x_4 & y_4 & 0 & 1 & -x_4y_4' & -y_4y_4' \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix}$$

Global Transformations – Global Warping

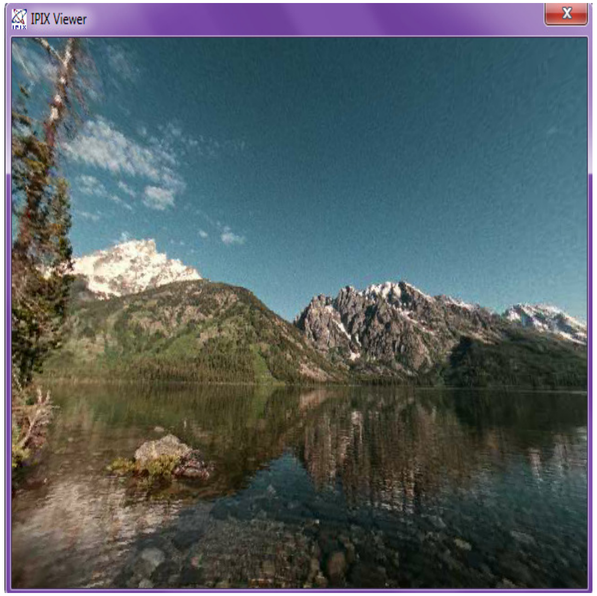
What about Projective Transformations?

$$\boxed{x' = \frac{ax+by+e}{gx+hy+1} \quad ; \quad y' = \frac{cx+dy+f}{gx+hy+1}}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x & y & 0 & 0 & 1 & 0 & -xx' & -yx' \\ 0 & 0 & x & y & 0 & 1 & -xy' & -yy' \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix}$$

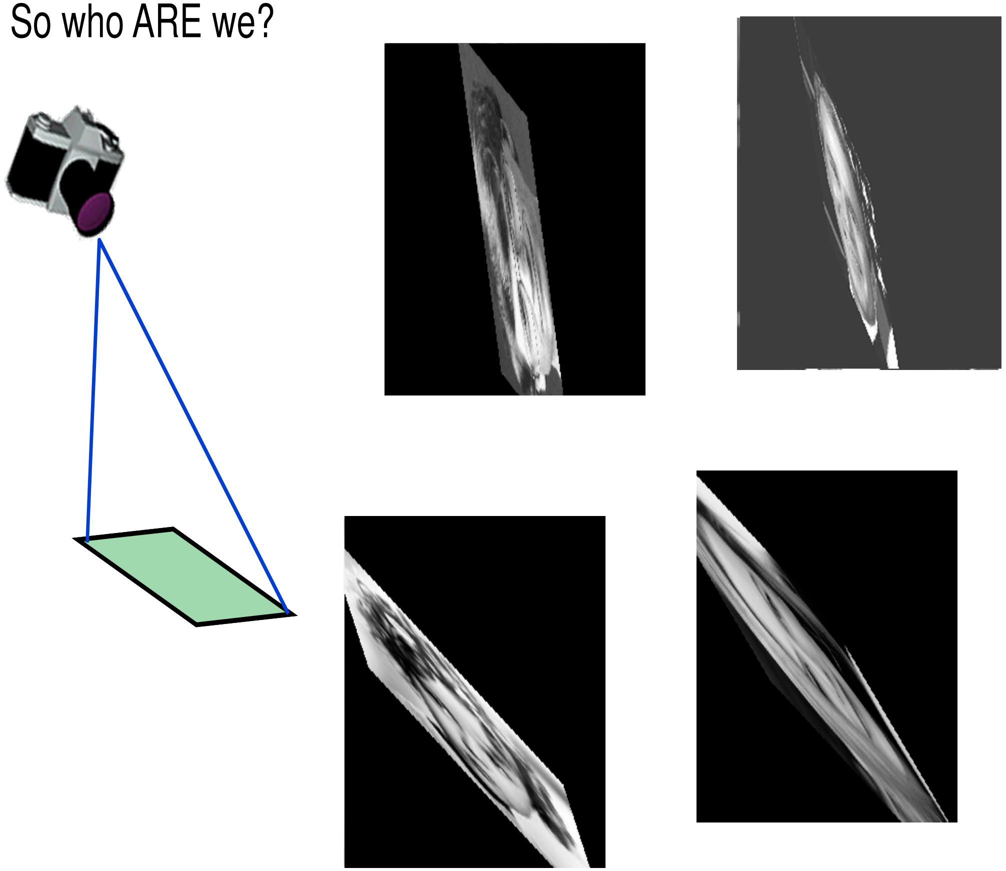
DEMOS

IPIX
Viewer



Global Transformations – Image Rectification

So who ARE we?



DEMOS

Julian Beever Street Art

DEMOS

Julian Beever Street Art

DEMOS

Geometric
Warping
Art

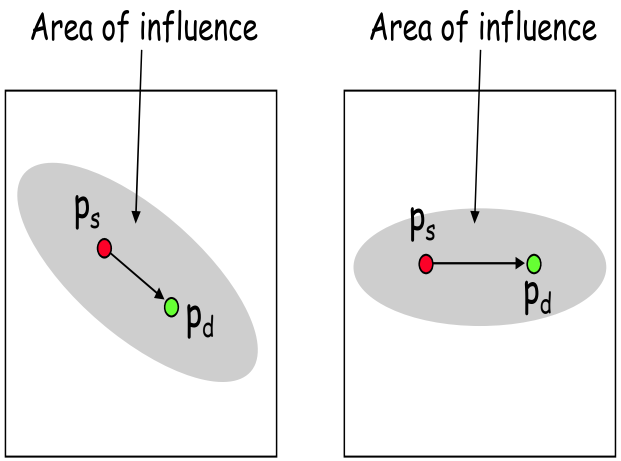
George V Avenue
in Paris

DEMOS

3D perspective

Flagellation by Pietro della Francesca (*1416-92, Italian Renaissance period*)
Animation by Criminisi et al., ICCV 99

Local Transformations – Image Warping



p_s = source point
p_d = destination point

Local Transformations – Image Warping

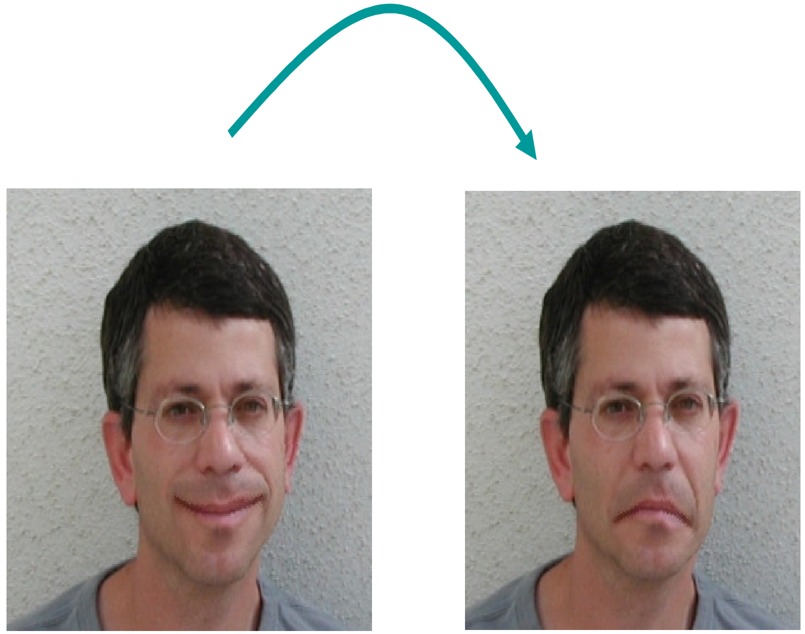
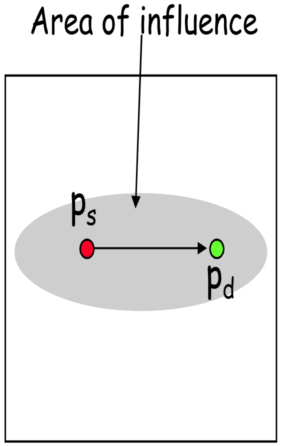
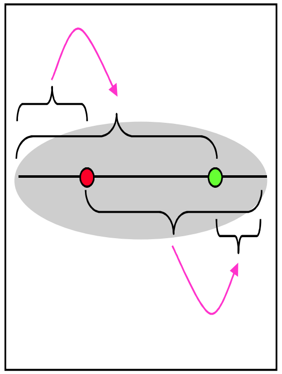


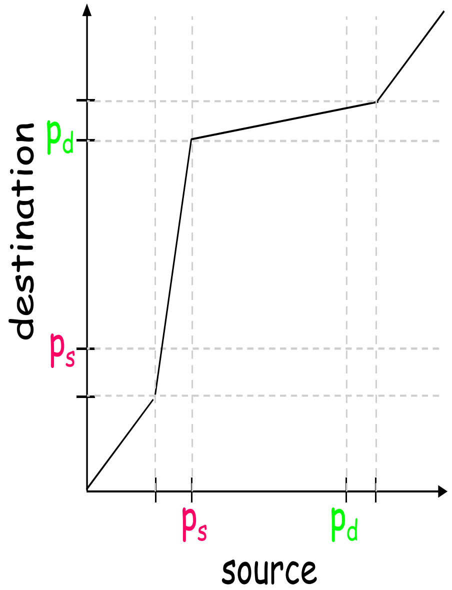
Image Morphing (Image Metamorphosis)

Local Transformations – Image Warping



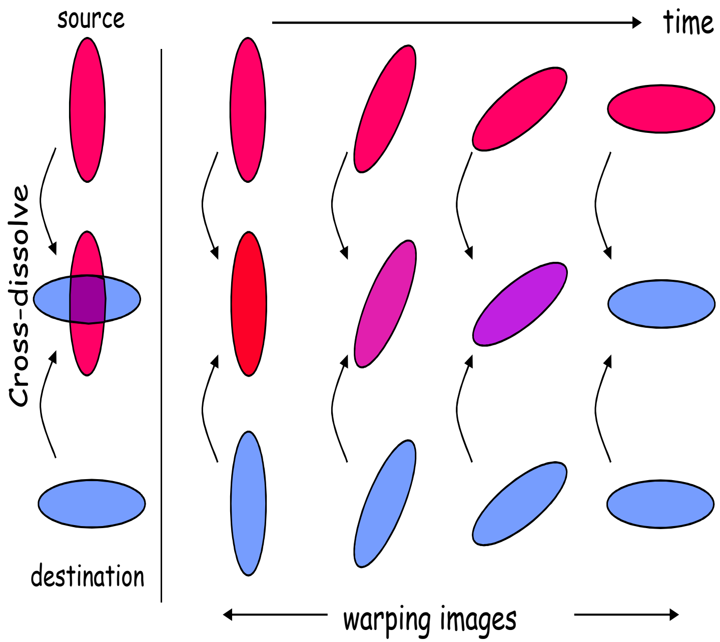
p_s = source point
p_d = destination point





Warping + Cross Dissolve

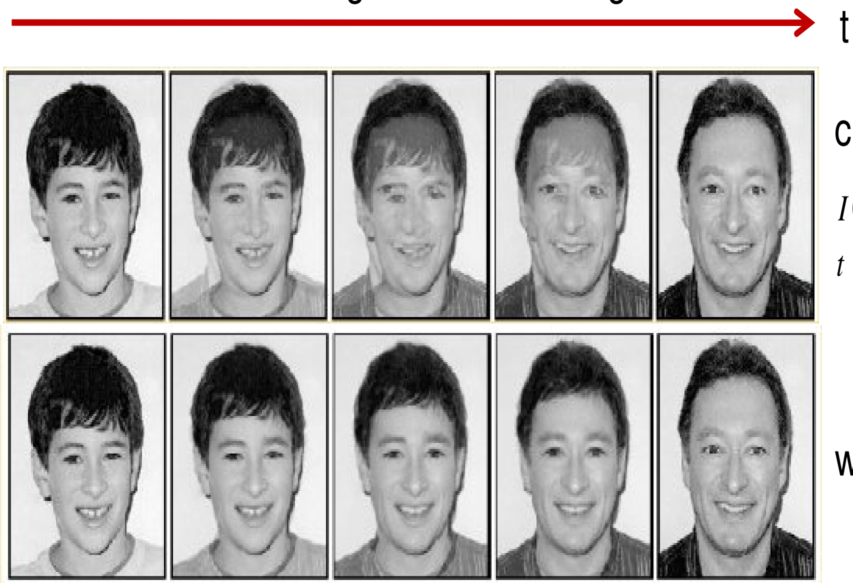
- Warp source image towards intermediate image.
- Warp destination image towards intermediate image.
- Cross-dissolve the two images by taking the weighted average at each pixel.



Cross Dissolve (pixel operations)

Source Image

Destination Image



cross dissolve

$$I(t) = (1-t)\cdot S + t\cdot T$$

$$t \in [0,1]$$

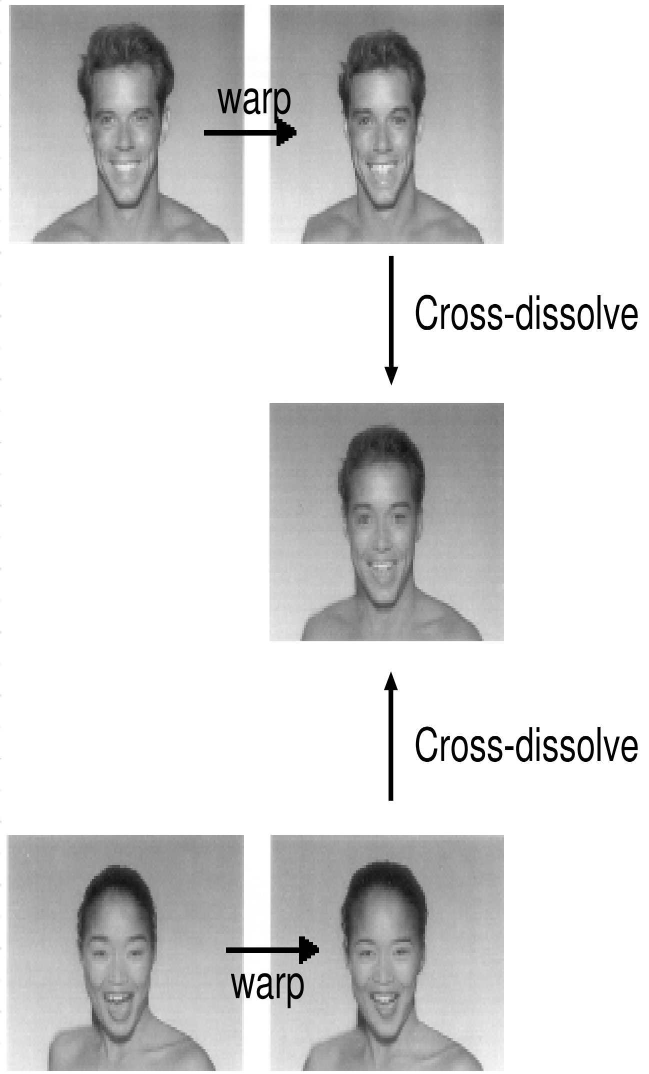
warp + dissolve

Image Metamorphosis

- Let S,T be the source and the target images
- Let G(p) be the transformation from S towards T, where G(0)=I (the identity)
- Let $t\in[0,1]$ the time step to be synthesized

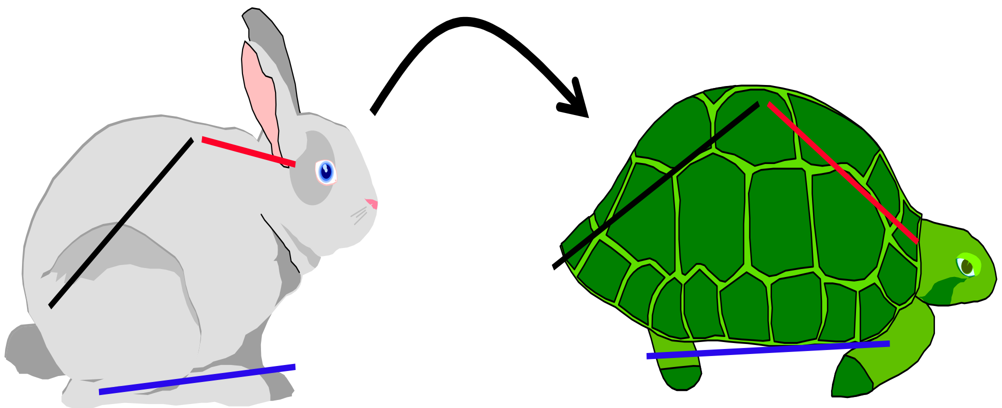
Algorithm:

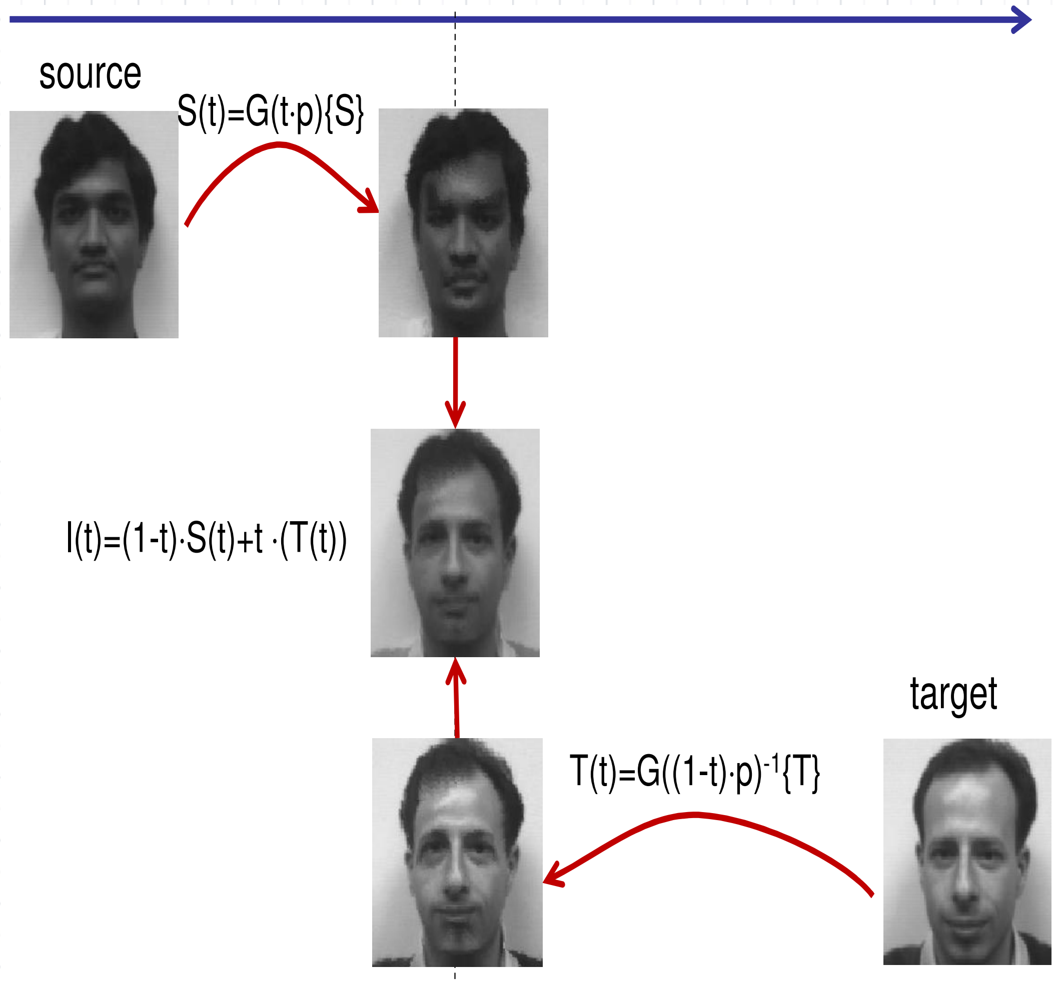
1. Warp S towards T: $S(t)=G(t\cdot p)\{S\}$
2. Warp T toward S: $T(t)=G((1-t)\cdot p)^{-1}\{T\}$
3. Cross dissolve: $I(t)=(1-t)\cdot S(t)+t\cdot T(t)$

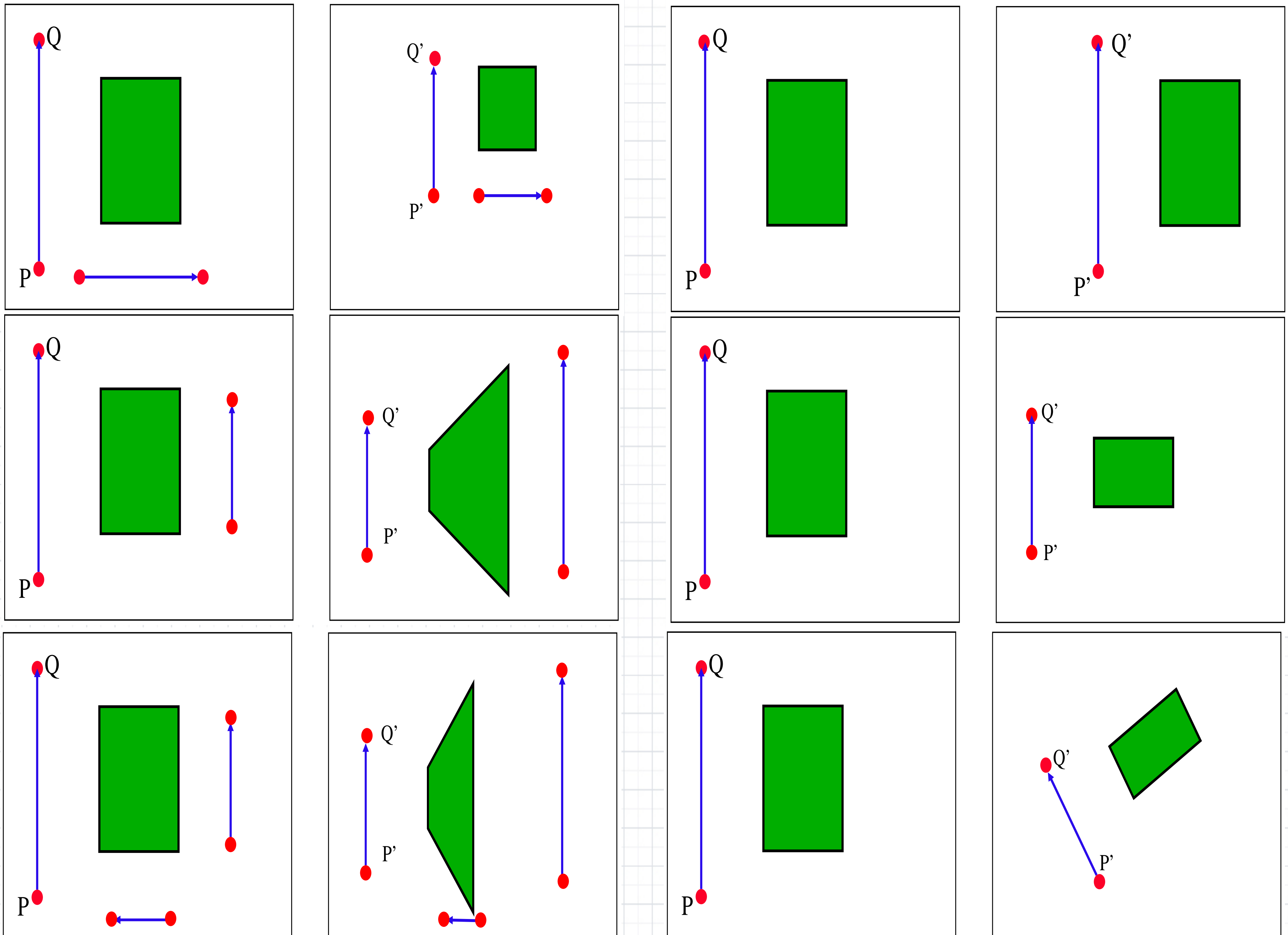


Feature Based Morphing

- Morph one shape into another shape
- Use local features to define the geometric warping






Q
P
Q'
P'
Q
P
Q'
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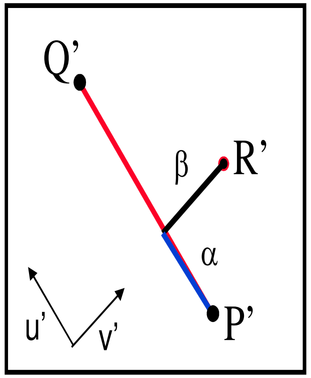


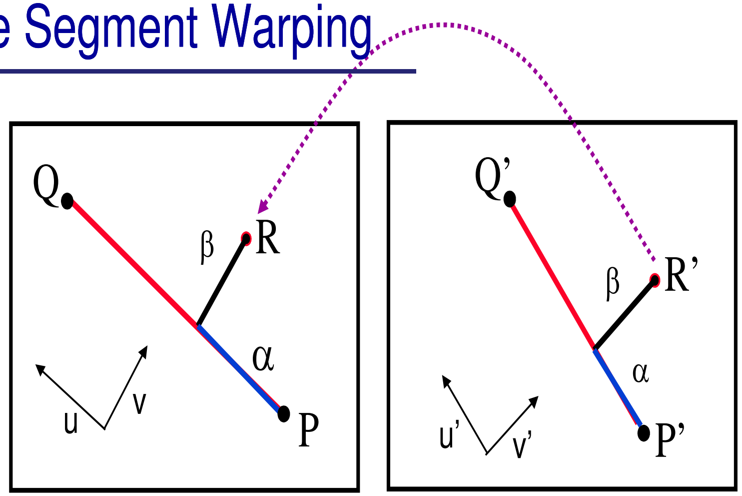
- The point R' is mapped into (α,β) :

$$\alpha = \frac{(R' - P')\cdot u'}{\left\| Q' - P' \right\|} \quad ; \quad \beta = (R' - P')\cdot v'$$

where

$$R' = P' + \alpha \left\| Q' - P' \right\| u' + \beta v'$$

One Segment Warping

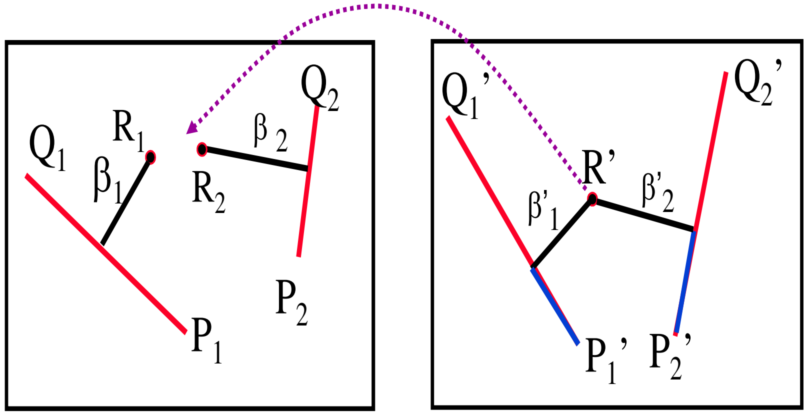


Source Image Dest Image

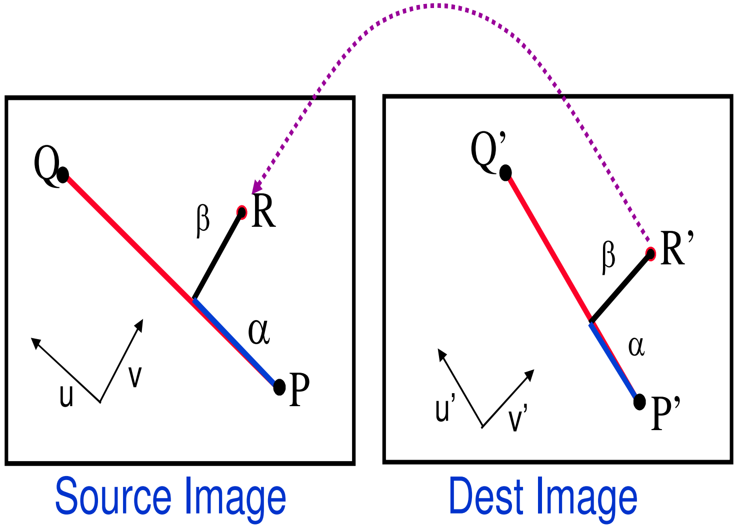
- $\alpha \in [0,1]$ is the **relative** position along the segment (P',Q').
- β is the **actual** perpendicular distance to the segment.
- **(u',v')** is the local coordinates of the segment (P',Q'):
  - **u'** is a unit vector parallel to Q'-P'
  - **v'** is the unit vector perpendicular to Q'-P'

$$u' = \frac{(Q' - P')}{\left\| Q' - P' \right\|} \qquad v' = \perp u' = \begin{pmatrix} u'_y \\ -u'_x \end{pmatrix}$$

Multiple Segment Warping



- In multiple segment warping the point R' is influenced by multiple segments.
- The influence **strength** of each segments is proportional to:
  - Segment length
  - The distance from the point R'



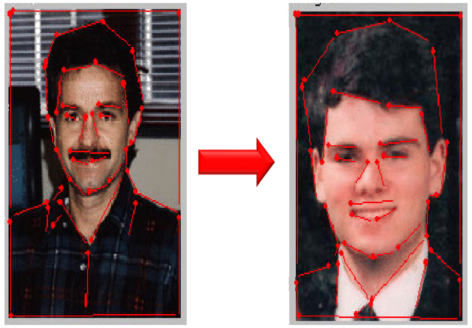
Inverse Mapping:

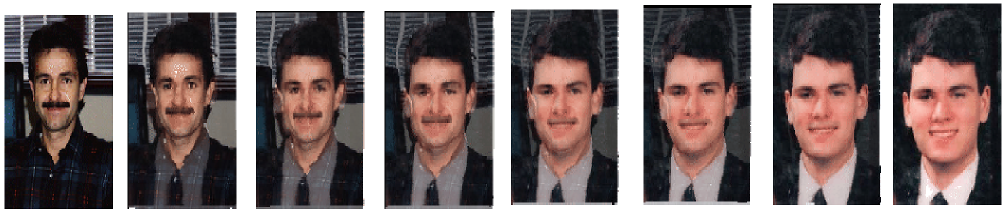
$$R(\alpha,\beta) = P + \alpha\|Q-P\|u + \beta v$$

where **(u,v)** is the local coordinates of the segment (P,Q):

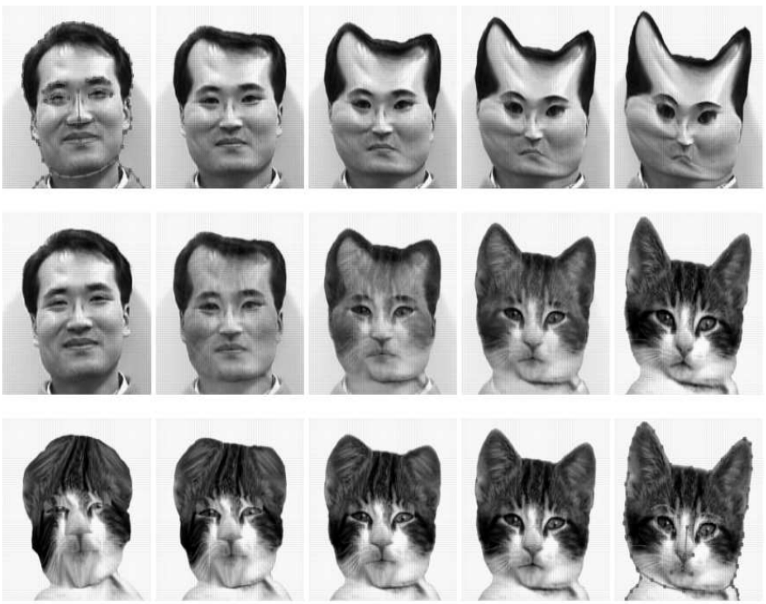
$$u = \frac{(Q-P)}{\|Q-P\|} \qquad v = \perp u = \begin{pmatrix} u_y \\ -u_x \end{pmatrix}$$

Example:





Another Example:



- The influence of each segments is:

$$W_i = \left(\frac{|Q_i - P_i|^p}{a + \beta_i} \right)^b$$

- The value $p \in [0,1]$ controls the influence of the line length.
- The value a is a small number avoiding division by zero.
- The value b determines how the relative weight diminish as the β increases
- The final mapping is:

$$R = \frac{\sum_k W_k R_k}{\sum_k W_k}$$