# Mechanics of Materials: Key Formulas, Notations, and Units

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## Contents

# 1 Notation and Units

| Symbol | Description [Units] |
|---|---|
| $P$ | Axial load [N] |
| $A$ | Cross-sectional area [m$^2$] |
| $L$ | Length of the member [m] |
| $E$ | Young's modulus [Pa] |
| $G$ | Shear modulus [Pa] |
| $V$ | Shear force [N] |
| $T$ | Torque [Nm] |
| $J$ | Polar moment of inertia [m$^4$] |
| $M$ | Bending moment [Nm] |
| $I$ | Area moment of inertia [m$^4$] |
| $\gamma$ | Shear strain [dimensionless] |
| $\theta$ | Angle of twist [rad] |
| $y$ | Distance from neutral axis [m] |
| $\sigma_x, \sigma_y$ | Normal stresses in $x$ and $y$ directions [Pa] |
| $\tau_{xy}$ | Shear stress in $xy$ plane [Pa] |
| $\nu$ | Poisson's ratio [dimensionless] |
| $\alpha$ | Coefficient of thermal expansion [K$^{-1}$] |
| $\sigma_z$ | Normal stress in $z$ direction [Pa] |

# 2 Stress and Strain

## 2.1 Normal Stress

$$\sigma = \frac{P}{A} \tag{1}$$

This equation relates the normal stress to the applied axial load and the cross-sectional area.

## 2.2 Shear Stress

$$\tau = \frac{V}{A} \tag{2}$$

Shear stress is calculated as the shear force divided by the area over which it acts.

### 2.3 Normal Strain

$$\varepsilon = \frac{\Delta L}{L} \tag{3}$$

Normal strain is the ratio of change in length to the original length.

## 3 Stress-Strain Relationships

### 3.1 Hooke's Law

$$\sigma = E\varepsilon \tag{4}$$

This linear relationship describes how stress is proportional to strain within the elastic limit.

### 3.2 Shear Modulus

$$\tau = G\gamma \tag{5}$$

Shear stress is directly proportional to shear strain, with $G$ as the proportionality constant.

### 3.3 Generalized Hooke's Law

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right) \tag{6}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \tag{7}$$

These equations account for the effect of Poisson's ratio in multi-axial stress states.

## 4 Torsion

### 4.1 Torsional Stress

$$\tau = \frac{Tr}{J} \tag{8}$$

Here, $r$ is the radial distance from the center of the shaft.

### 4.2 Angle of Twist

$$\theta = \frac{TL}{GJ} \tag{9}$$

The angle of twist is proportional to the torque and length, inversely to the shear modulus and polar moment of inertia.

### 4.3 Thin-Walled Tubes (Average Shear Stress)

$$\tau_{\text{avg}} = \frac{T}{2tA_m} \tag{10}$$

where $t$ is the thickness of the tube wall and $A_m$ is the mean area enclosed by the centerline of the tube's cross-section.

### 4.4 Power in Torsion

$$P = T\omega = 2\pi f T \tag{11}$$

This equation shows how power is related to torque and rotational speed or frequency.

## 5 Beam Bending

### 5.1 Bending Stress

$$\sigma = \frac{My}{I} \tag{12}$$

Bending stress varies linearly with the distance from the neutral axis.

### 5.2 Unsymmetric Bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \tag{13}$$

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta \tag{14}$$

These equations account for bending in two directions, where $\alpha$ is the angle between the neutral axis and the $x$-axis.

### 5.3 Transverse Shear Stress

$$\tau = \frac{VQ}{It} \tag{15}$$

Here, $Q$ is the first moment of area about the neutral axis.

### 5.4 Deflection (Curvature)

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{16}$$

This differential equation describes how deflection relates to bending moment.

## 6 Combined Stresses

### 6.1 Principal Stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{17}$$

Principal stresses are calculated to find the maximum and minimum normal stresses on a plane.

### 6.2 Maximum Shear Stress

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{18}$$

This gives the maximum shear stress in a plane.

### 6.3 Stress Transformation

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \tag{19}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \tag{20}$$

These transformations help in analyzing stress on any rotated plane.

# 7 Mohr's Circle

## 7.1 Center and Radius

$$\text{Center} = \frac{\sigma_x + \sigma_y}{2}, \quad \text{Radius} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{21}$$

*Note: Mohr's Circle is a graphical method used to determine stress states under transformation.*

# 8 Energy Methods

## 8.1 Axial Strain Energy

$$U = \frac{1}{2}\frac{P^2 L}{EA} \tag{22}$$

Strain energy due to axial load.

## 8.2 Torsional Strain Energy

$$U = \frac{1}{2}\frac{T^2 L}{GJ} \tag{23}$$

Energy stored due to torsion.

## 8.3 Strain Energy for Shear

$$U = \int \frac{V^2}{2GA} dx \tag{24}$$

This is the strain energy due to shear forces along the length of a member.

# 9 Thin-Walled Pressure Vessels

## 9.1 Cylinders

$$\sigma_1 = \frac{pr}{t} \tag{25}$$

$$\sigma_2 = \frac{pr}{2t} \tag{26}$$

Valid for thin-walled cylinders where the wall thickness is small compared to the radius.

### 9.2 Spheres

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} \tag{27}$$

For thin-walled spheres, both stresses are equal due to symmetry.

## 10 Additional Formulas

### 10.1 Euler's Buckling Formula for Columns

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \tag{28}$$

where $L_e$ is the effective length of the column, accounting for end conditions.

### 10.2 Thermal Expansion Stress

$$\sigma = E\alpha\Delta T \tag{29}$$

where $\alpha$ is the coefficient of thermal expansion, and $\Delta T$ is the change in temperature.

### 10.3 Secant Formula (Advanced)

$$\sigma_{\max} = \frac{P}{A}\left(1 + e\frac{r^2}{L^2}\sec\left(\frac{\pi L}{2r}\right)\right) \tag{30}$$

This formula is used for columns with eccentric loading. Here, $e$ is the eccentricity of the load.

## 11 Dynamic Effects

### 11.1 Impact Loading

$$\sigma_{\text{impact}} = \sigma_{\text{static}} \cdot \left(1 + \sqrt{1 + \frac{2Eh}{g\sigma_{\text{static}}}}\right) \tag{31}$$

where $\sigma_{\text{impact}}$ is the stress under impact, $\sigma_{\text{static}}$ is the static stress, $h$ is the height from which the load is dropped, $g$ is the acceleration due to gravity, and $E$ is Young's modulus.

### 11.2 Vibration and Resonance

The natural frequency of a system can be calculated by:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{32}$$

where $f_n$ is the natural frequency, $k$ is the stiffness of the system, and $m$ is the mass.

## 12 Fatigue Analysis

### 12.1 S-N Curve

The fatigue life of a material can often be described by the S-N curve:

$$N = C\left(\frac{\sigma_a}{\sigma_f}\right)^{-b} \tag{33}$$

where $N$ is the number of cycles to failure, $\sigma_a$ is the alternating stress, $\sigma_f$ is the fatigue strength coefficient, $C$ and $b$ are material constants.

### 12.2 Miner's Rule for Cumulative Damage

For variable amplitude loading, the damage accumulation can be calculated using:

$$\sum \frac{n_i}{N_i} = D \tag{34}$$

where $n_i$ is the number of cycles at stress level $i$, $N_i$ is the number of cycles to failure at that stress level, and $D$ is the cumulative damage (failure when $D \geq 1$).

## 13 Advanced Topics in Plasticity

### 13.1 Ramberg-Osgood Equation

For materials beyond their elastic limit, the stress-strain relationship can be modeled by:

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{\sigma_0}\right)^n \tag{35}$$

where $\epsilon$ is the total strain, $\sigma$ is the stress, $E$ is Young's modulus, $\sigma_0$ is a reference stress (often yield stress), and $n$ is the hardening exponent.

### 13.2 Strain Hardening

The true stress-true strain curve after yielding can often be approximated by:

$$\sigma = K\epsilon^n \tag{36}$$

where $\sigma$ is the true stress, $\epsilon$ is the true strain, $K$ is the strength coefficient, and $n$ is the strain hardening exponent.

### 13.3 Plastic Collapse Load

For structures, the plastic collapse load can be determined by:

$$P_p = \sum(A_i\sigma_y) \tag{37}$$

where $P_p$ is the plastic collapse load, $A_i$ are areas of cross-section contributing to plastic deformation, and $\sigma_y$ is the yield stress.

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