Key Concepts from Hibbeler's Mechanics of Materials

Sections 6.3, 6.4, 7.1, and 7.2

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Outline

6.3 Bending Deformation of a Straight Member

- A **prismatic beam** made of **homogeneous, linear-elastic** material deforms under a bending moment so that its longitudinal fibers stretch or compress.
  (Bottom fibers elongate, top fibers shorten, and there is a surface in between—the neutral surface—that does not change length.)
- Cross sections initially plane remain *plane* after deformation, but rotate relative to one another.
- The **neutral axis** (NA) of the cross section experiences zero normal strain and thus zero normal stress.
- For an initially straight member with symmetric cross section bent about the axis of symmetry, the neutral axis coincides with the centroidal axis.

Reference: See Section 6.3.
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6.4 The Flexure Formula

- Under pure bending, normal stress σ along a cross section varies linearly from the neutral axis to the outer fibers.
- The classic **flexure formula** is

$$\sigma = -\frac{M\,y}{I},$$

  where
  - M is the internal bending moment about the section's neutral axis,
  - y is the distance from the neutral axis,
  - I is the cross-sectional moment of inertia about the neutral axis,
  - and the sign is determined by tension $(+)$ or compression $(-)$.
- The maximum normal stress occurs at the outermost fiber, $\sigma_{\max} = \frac{M\,c}{I}$, where c is the maximum distance from the NA.
- Key assumptions: linear-elastic material, small deformations, plane sections remain plane.

Reference: See Section 6.4.

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7.1 Shear in Straight Members

- A transverse shear force V on a beam produces *transverse shear stress* over the cross section and complementary *longitudinal shear stress* along horizontal planes.
- In practice, if multiple wooden boards are used to form a beam, they must be **bonded or nailed** together to transmit these shear stresses and act as a single unit.
- Cross sections do *not* remain perfectly plane under shear; slight warping occurs. However, for **slender beams**, this warping is generally small and often neglected.

Reference: See Section 7.1.
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7.2 The Shear Formula

- Shear stress distribution in a beam can be approximated under linear-elastic assumptions via:

$$\tau = \frac{V\,Q}{I\,t},$$

  where:
  - V = internal shear force at the section,
  - Q = first moment of the area above (or below) the plane where τ is being found,
  - I = moment of inertia of the *entire* cross section about its neutral axis,
  - t = width of the member at the point of interest.
- $Q = \bar{y}\,A'$ is computed by multiplying the area A' above (or below) the cut by the distance $\bar{y}$ from the NA to the centroid of that area.
- The formula yields an *average* shear stress across width t; more complex sections sometimes require additional consideration.

Reference: See Section 7.2.

Key Equations (Summary)

Bending Stress (Flexure Formula)

$$\sigma = -\frac{M\,y}{I}, \quad \sigma_{\text{max}} = \frac{M\,c}{I}.$$

Shear Stress (Shear Formula)

$$\tau = \frac{V\,Q}{I\,t}, \quad Q = \int_A y\,dA = \bar{y}\,A'.$$

Limitations and Remarks

- Both formulas assume a **linear-elastic** material response.
- Cross sections should not be *extremely* wide (large width-to-depth ratio), or else the shear stress distribution can differ significantly from the idealization.
- Near abrupt changes in geometry (e.g., at fillets or flanges), stress concentrations can occur, so local stresses can exceed those predicted by the simple formulas.
- Warping deformations in shear are neglected for slender beams; for short beams of large depth, warping can be more substantial.

Conclusion

- **Section 6.3** shows how bending curves a beam, introducing tension on one side and compression on the other, with a neutral axis in between.
- **Section 6.4** develops the Flexure Formula $\sigma = -\frac{M\,y}{I}$, relating bending stress to the internal moment and the beam's geometry.
- **Section 7.1** introduces how shear forces lead to both transverse and longitudinal shear in beams.
- **Section 7.2** provides the Shear Formula $\tau = \frac{VQ}{I\,t}$, enabling a (usually adequate) calculation of shear stress in prismatic, linear-elastic beams.

End of Presentation