Mechanics and Materials I - Lecture #2

Plane Stress Analysis

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January 15, 2025

Plane Stress

- **Plane Stress:** When stresses in one direction are negligible compared to the other two.
- Assumptions:
    - $\sigma_z = \tau_{xz} = \tau_{yz} = 0$

Equations for Plane Stress Transformation

Transformation Equations

$$\sigma'_x = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$
$$\sigma'_y = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta$$
$$\tau'_{xy} = (\sigma_y - \sigma_x) \sin\theta \cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

Principal Stress

Principal Stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- σ_1 is the maximum principal stress
- σ_2 is the minimum principal stress

Maximum In-plane Shear Stress

Maximum In-plane Shear Stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Mohr's Circle

- Graphical method for stress transformations.
- Center of Mohr's Circle:

$$\sigma_c = \frac{\sigma_x + \sigma_y}{2}$$

- Radius:

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Absolute Maximum Shear Stress

Absolute Maximum Shear Stress

$$\tau_{abs\_max} = \max\left(|\tau_{max}|, \frac{|\sigma_1 - \sigma_3|}{2}\right)$$

where σ_3 is the out-of-plane principal stress (if considered).

Example: Transformation

Given

$$\sigma_x = 100 \text{ MPa}, \ \sigma_y = 50 \text{ MPa}, \ \tau_{xy} = 30 \text{ MPa}, \ \theta = 45^\circ$$

Calculate

$$\begin{aligned}
\sigma'_x &= 100 \cos^2 45^\circ + 50 \sin^2 45^\circ + 2 \cdot 30 \cdot \sin 45^\circ \cos 45^\circ \\
\sigma'_y &= \text{(Similarly calculated)} \\
\tau'_{xy} &= \text{(Similarly calculated)}
\end{aligned}$$