Statically Indeterminate Axially Loaded Members

Week 4

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Chapter Objectives

- Determine deformation of axially loaded members.
- Analyze support reactions in statically indeterminate systems.
- Analyze the effects of thermal stress.
- Discuss inelastic deformation and residual stress.

Review of Axially Loaded Members

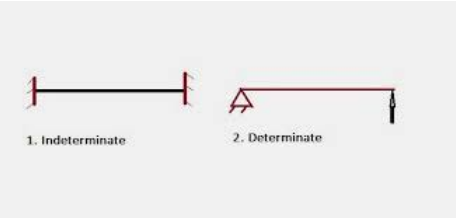
- **Normal stress**: $\sigma = \frac{P}{A}$
- Axial force (P) and its sign convention:
  - Tension: Positive ($P > 0$)
  - Compression: Negative ($P < 0$)
- Cross-sectional area (A)
- Elastic deformation: $\delta = \frac{PL}{AE}$ (for elastic deformation under uniaxial load, this is a simplified form assuming linear elasticity)

Definition of Statically Indeterminate Members

- **Static Indeterminacy**:
  - Equilibrium equations are insufficient to solve for all unknowns.
  - More unknown reactions than available equilibrium equations.
  - Additional equations based on compatibility or material behavior are required for solution. "Compatibility" here refers to the requirement that displacements must be consistent with the boundary conditions of the system.

Examples of Statically Indeterminate Systems

- A bar fixed at both ends.
- A bar with multiple supports.
- A composite structure with axial loading (note that these often involve different materials with distinct moduli of elasticity, requiring careful analysis using both equilibrium and compatibility conditions).



General Procedure for Analysis

1. **Equilibrium**:
   - Draw Free Body Diagrams (FBDs).
   - Write equilibrium equations.
2. **Compatibility**:
   - Analyze how the structure deforms.
3. **Force-Displacement Relations**:
   - Relate forces and deformations using material properties. These relations can involve concepts like Hooke's Law for linear elasticity or more complex relationships in case of non-linear behavior.
4. Solve equations simultaneously.

Compatibility Equations

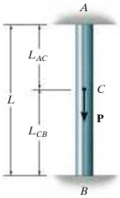
- Displacements or deformations must be consistent with constraints.
- Relate the deformation in each segment of the structure:

$$\delta_{A/B} = \delta_A - \delta_B$$

This equation ensures that the displacements at points A and B match in a way that maintains the structure's integrity.

Example Problem 1 - Bar Fixed at Both Ends

- A bar fixed at both ends with an axial load P.
- Free Body Diagram:



(a)

- Equilibrium: $\sum F_x = 0 \rightarrow F_A + F_B - P = 0$ (where F_A and F_B are the internal forces at the ends of the bar)
- Compatibility: $\delta_{A/B} = 0$

Example Problem 1 - Solution

- Using $\delta = \frac{FL}{AE}$:

$$\delta_{A/B} = \frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0$$

  Here, $\delta_{A/B}$ is the total displacement at the point of interest due to the internal forces.

- Solving for F_A and F_B (ensure L_{AC} and L_{CB} are well-defined in the problem statement):

$$F_A = P\left(\frac{L_{CB}}{L}\right), \quad F_B = P\left(\frac{L_{AC}}{L}\right)$$

Thermal Stress

- Temperature changes cause expansion or contraction.
- If movement is constrained, thermal stress develops.
- Thermal strain: $\epsilon_T = \alpha \Delta T$ where α is the coefficient of thermal expansion and ΔT is the temperature change.
- Thermal stress is only relevant when the material is restrained from free expansion or contraction.

Conclusion

- Statically indeterminate members require additional compatibility equations.
- Thermal stress analysis is essential in many applications.
- Inelastic deformation leads to residual stress, which can arise from plastic deformations that are often irreversible.
- Superposition is a valuable tool for linear elastic problems.