

Mechanics of Materials: Key Formulas, Notations, and Units

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January 29, 2025

Contents

1	Notation and Units	3
2	Stress and Strain	3
2.1	Normal Stress	3
2.2	Shear Stress	3
2.3	Normal Strain	4
3	Stress-Strain Relationships	4
3.1	Hooke's Law	4
3.2	Shear Modulus	4
3.3	Generalized Hooke's Law	4
4	Torsion	4
4.1	Torsional Stress	4
4.2	Angle of Twist	5
4.3	Thin-Walled Tubes (Average Shear Stress)	5
4.4	Power in Torsion	5
5	Beam Bending	5
5.1	Bending Stress	5
5.2	Unsymmetric Bending	5
5.3	Transverse Shear Stress	6
5.4	Deflection (Curvature)	6

6	Combined Stresses	6
6.1	Principal Stresses	6
6.2	Maximum Shear Stress	6
6.3	Stress Transformation	6
7	Mohr's Circle	7
7.1	Center and Radius	7
8	Energy Methods	7
8.1	Axial Strain Energy	7
8.2	Torsional Strain Energy	7
8.3	Strain Energy for Shear	7
9	Thin-Walled Pressure Vessels	7
9.1	Cylinders	7
9.2	Spheres	8
10	Additional Formulas	8
10.1	Euler's Buckling Formula for Columns	8
10.2	Thermal Expansion Stress	8
10.3	Secant Formula (Advanced)	8
11	Dynamic Effects	8
11.1	Impact Loading	8
11.2	Vibration and Resonance	9
12	Fatigue Analysis	9
12.1	S-N Curve	9
12.2	Miner's Rule for Cumulative Damage	9
13	Advanced Topics in Plasticity	10
13.1	Ramberg-Osgood Equation	10
13.2	Strain Hardening	10
13.3	Plastic Collapse Load	10

1 Notation and Units

Symbol	Description [Units]
P	Axial load [N]
A	Cross-sectional area [m ²]
L	Length of the member [m]
E	Young's modulus [Pa]
G	Shear modulus [Pa]
V	Shear force [N]
T	Torque [Nm]
J	Polar moment of inertia [m ⁴]
M	Bending moment [Nm]
I	Area moment of inertia [m ⁴]
γ	Shear strain [dimensionless]
θ	Angle of twist [rad]
y	Distance from neutral axis [m]
σ_x, σ_y	Normal stresses in x and y directions [Pa]
τ_{xy}	Shear stress in xy plane [Pa]
ν	Poisson's ratio [dimensionless]
α	Coefficient of thermal expansion [K ⁻¹]
σ_z	Normal stress in z direction [Pa]

2 Stress and Strain

2.1 Normal Stress

$$\sigma = \frac{P}{A} \quad (1)$$

This equation relates the normal stress to the applied axial load and the cross-sectional area.

2.2 Shear Stress

$$\tau = \frac{V}{A} \quad (2)$$

Shear stress is calculated as the shear force divided by the area over which it acts.

2.3 Normal Strain

$$\varepsilon = \frac{\Delta L}{L} \quad (3)$$

Normal strain is the ratio of change in length to the original length.

3 Stress-Strain Relationships

3.1 Hooke's Law

$$\sigma = E\varepsilon \quad (4)$$

This linear relationship describes how stress is proportional to strain within the elastic limit.

3.2 Shear Modulus

$$\tau = G\gamma \quad (5)$$

Shear stress is directly proportional to shear strain, with G as the proportionality constant.

3.3 Generalized Hooke's Law

$$\varepsilon_x = \frac{1}{E} (\sigma_x - \nu(\sigma_y + \sigma_z)) \quad (6)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \quad (7)$$

These equations account for the effect of Poisson's ratio in multi-axial stress states.

4 Torsion

4.1 Torsional Stress

$$\tau = \frac{Tr}{J} \quad (8)$$

Here, r is the radial distance from the center of the shaft.

4.2 Angle of Twist

$$\theta = \frac{TL}{GJ} \quad (9)$$

The angle of twist is proportional to the torque and length, inversely to the shear modulus and polar moment of inertia.

4.3 Thin-Walled Tubes (Average Shear Stress)

$$\tau_{\text{avg}} = \frac{T}{2tA_m} \quad (10)$$

where t is the thickness of the tube wall and A_m is the mean area enclosed by the centerline of the tube's cross-section.

4.4 Power in Torsion

$$P = T\omega = 2\pi fT \quad (11)$$

This equation shows how power is related to torque and rotational speed or frequency.

5 Beam Bending

5.1 Bending Stress

$$\sigma = \frac{My}{I} \quad (12)$$

Bending stress varies linearly with the distance from the neutral axis.

5.2 Unsymmetric Bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \quad (13)$$

$$\tan \alpha = \frac{I_z}{I_y} \tan \theta \quad (14)$$

These equations account for bending in two directions, where α is the angle between the neutral axis and the x -axis.

5.3 Transverse Shear Stress

$$\tau = \frac{VQ}{It} \quad (15)$$

Here, Q is the first moment of area about the neutral axis.

5.4 Deflection (Curvature)

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad (16)$$

This differential equation describes how deflection relates to bending moment.

6 Combined Stresses

6.1 Principal Stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (17)$$

Principal stresses are calculated to find the maximum and minimum normal stresses on a plane.

6.2 Maximum Shear Stress

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (18)$$

This gives the maximum shear stress in a plane.

6.3 Stress Transformation

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \quad (19)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \quad (20)$$

These transformations help in analyzing stress on any rotated plane.

7 Mohr's Circle

7.1 Center and Radius

$$\text{Center} = \frac{\sigma_x + \sigma_y}{2}, \quad \text{Radius} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (21)$$

Note: Mohr's Circle is a graphical method used to determine stress states under transformation.

8 Energy Methods

8.1 Axial Strain Energy

$$U = \frac{1}{2} \frac{P^2 L}{EA} \quad (22)$$

Strain energy due to axial load.

8.2 Torsional Strain Energy

$$U = \frac{1}{2} \frac{T^2 L}{GJ} \quad (23)$$

Energy stored due to torsion.

8.3 Strain Energy for Shear

$$U = \int \frac{V^2}{2GA} dx \quad (24)$$

This is the strain energy due to shear forces along the length of a member.

9 Thin-Walled Pressure Vessels

9.1 Cylinders

$$\sigma_1 = \frac{pr}{t} \quad (25)$$

$$\sigma_2 = \frac{pr}{2t} \quad (26)$$

Valid for thin-walled cylinders where the wall thickness is small compared to the radius.

9.2 Spheres

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} \quad (27)$$

For thin-walled spheres, both stresses are equal due to symmetry.

10 Additional Formulas

10.1 Euler's Buckling Formula for Columns

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad (28)$$

where L_e is the effective length of the column, accounting for end conditions.

10.2 Thermal Expansion Stress

$$\sigma = E\alpha\Delta T \quad (29)$$

where α is the coefficient of thermal expansion, and ΔT is the change in temperature.

10.3 Secant Formula (Advanced)

$$\sigma_{\max} = \frac{P}{A} \left(1 + e \frac{r^2}{L^2} \sec \left(\frac{\pi L}{2r} \right) \right) \quad (30)$$

This formula is used for columns with eccentric loading. Here, e is the eccentricity of the load.

11 Dynamic Effects

11.1 Impact Loading

$$\sigma_{\text{impact}} = \sigma_{\text{static}} \cdot \left(1 + \sqrt{1 + \frac{2Eh}{g\sigma_{\text{static}}}} \right) \quad (31)$$

where σ_{impact} is the stress under impact, σ_{static} is the static stress, h is the height from which the load is dropped, g is the acceleration due to gravity, and E is Young's modulus.

11.2 Vibration and Resonance

The natural frequency of a system can be calculated by:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (32)$$

where f_n is the natural frequency, k is the stiffness of the system, and m is the mass.

12 Fatigue Analysis

12.1 S-N Curve

The fatigue life of a material can often be described by the S-N curve:

$$N = C \left(\frac{\sigma_a}{\sigma_f} \right)^{-b} \quad (33)$$

where N is the number of cycles to failure, σ_a is the alternating stress, σ_f is the fatigue strength coefficient, C and b are material constants.

12.2 Miner's Rule for Cumulative Damage

For variable amplitude loading, the damage accumulation can be calculated using:

$$\sum \frac{n_i}{N_i} = D \quad (34)$$

where n_i is the number of cycles at stress level i , N_i is the number of cycles to failure at that stress level, and D is the cumulative damage (failure when $D \geq 1$).

13 Advanced Topics in Plasticity

13.1 Ramberg-Osgood Equation

For materials beyond their elastic limit, the stress-strain relationship can be modeled by:

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{\sigma_0} \right)^n \quad (35)$$

where ϵ is the total strain, σ is the stress, E is Young's modulus, σ_0 is a reference stress (often yield stress), and n is the hardening exponent.

13.2 Strain Hardening

The true stress-true strain curve after yielding can often be approximated by:

$$\sigma = K\epsilon^n \quad (36)$$

where σ is the true stress, ϵ is the true strain, K is the strength coefficient, and n is the strain hardening exponent.

13.3 Plastic Collapse Load

For structures, the plastic collapse load can be determined by:

$$P_p = \sum (A_i \sigma_y) \quad (37)$$

where P_p is the plastic collapse load, A_i are areas of cross-section contributing to plastic deformation, and σ_y is the yield stress.

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