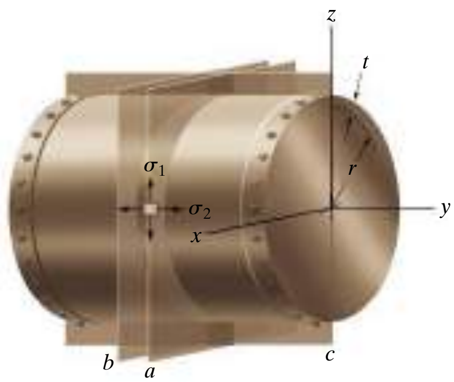
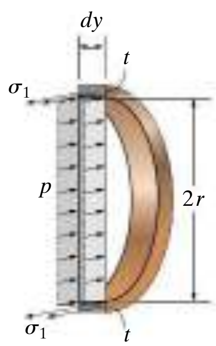
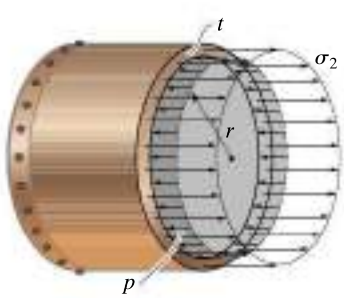


(a)



(b)



(c)

Fig. 8–1

Cylindrical Vessels. Consider the cylindrical vessel in Fig. 8–1*a*, having a wall thickness *t*, inner radius *r*, and subjected to a gauge pressure *p* that developed within the vessel by a contained gas. Due to this loading, a small element of the vessel that is sufficiently removed from the ends and oriented as shown in Fig. 8–1*a*, is subjected to normal stresses σ_1 in the ***circumferential or hoop direction*** and σ_2 in the ***longitudinal or axial direction***.

The hoop stress can be determined by considering the vessel to be sectioned by planes *a*, *b*, and *c*. A free-body diagram of the back segment along with the contained gas is shown in Fig. 8–1*b*. Here only the loadings in the *x* direction are shown. These loadings are developed by the uniform hoop stress σ_1, acting on the vessel's wall, and the pressure acting on the vertical face of the gas. For equilibrium in the *x* direction, we require

$$\Sigma F_x = 0; \qquad 2[\sigma_1(t\,dy)] - p(2r\,dy) = 0$$

$$\boxed{\sigma_1 = \frac{pr}{t}} \tag{8–1}$$

The longitudinal stress can be determined by considering the left portion of section *b* of the cylinder, Fig. 8–1*a*. As shown in Fig. 8–1*c*, σ_2 acts uniformly throughout the wall, and *p* acts on the section of the contained gas. Since the mean radius is approximately equal to the vessel's inner radius, equilibrium in the *y* direction requires

$$\Sigma F_y = 0; \qquad \sigma_2(2\pi rt) - p(\pi r^2) = 0$$

$$\boxed{\sigma_2 = \frac{pr}{2t}} \tag{8–2}$$

In the above equations,

σ_1, σ_2 = the normal stress in the hoop and longitudinal directions, respectively. Each is assumed to be *constant* throughout the wall of the cylinder, and each subjects the material to tension.
p = the internal gauge pressure developed by the contained gas
r = the inner radius of the cylinder
t = the thickness of the wall ($r/t \geq 10$)

8

By comparison, note that the hoop or circumferential stress is twice as large as the longitudinal or axial stress. Consequently, when fabricating cylindrical pressure vessels from rolled-formed plates, the longitudinal joints must be designed to carry twice as much stress as the circumferential joints.

This thin-walled pipe was subjected to an excessive gas pressure that caused it to rupture in the circumferential or hoop direction. The stress in this direction is twice that in the axial direction as noted by Eqs. 8–1 and 8–2.

Spherical Vessels.

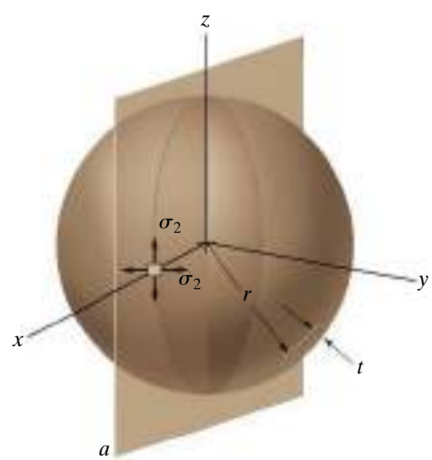
We can analyze a spherical pressure vessel in a similar manner. To do this, consider the vessel to have a wall thickness t, inner radius r, and subjected to an internal gauge pressure p, Fig. 8–2*a*. If the vessel is sectioned in half, the resulting free-body diagram is shown in Fig. 8–2*b*. Like the cylinder, equilibrium in the y direction requires

$$\Sigma F_y = 0; \qquad \sigma_2(2\pi r t) - p(\pi r^2) = 0$$

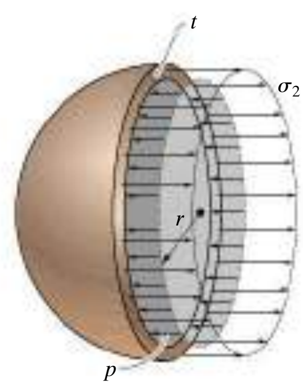
$$\boxed{\sigma_2 = \frac{pr}{2t}} \tag{8–3}$$

This is the *same result* as that obtained for the longitudinal stress in the cylindrical pressure vessel. Furthermore, from the analysis, this stress will be the same *regardless* of the orientation of the hemispheric free-body diagram. Consequently, a small element of the material is subjected to the state of stress shown in Fig. 8–2*a*.

The above analysis indicates that an element of material taken from either a cylindrical or a spherical pressure vessel is subjected to ***biaxial stress***, i.e., normal stress existing in only two directions. Actually, the pressure also subjects the material to a ***radial stress***, σ_3, which acts along a radial line. This stress has a maximum value equal to the pressure p at the interior wall and it decreases through the wall to zero at the exterior surface of the vessel, since the gauge pressure there is zero. For thin-walled vessels, however, we will *ignore* this radial-stress component, since our limiting assumption of $r/t = 10$ results in σ_2 and σ_1 being, respectively, 5 and 10 times *higher* than the maximum radial stress, $(\sigma_3)_{max} = p$. Finally, if the vessel is subjected to an *external pressure*, the compressive stress developed within the thin wall may cause the vessel to become unstable, and collapse may occur by buckling rather than causing the material to fracture.



(a)



(b)

Fig. 8–2

8

EXAMPLE 8.1

A cylindrical pressure vessel has an inner diameter of 4 ft and a thickness of $\frac{1}{2}$ in. Determine the maximum internal pressure it can sustain so that neither its circumferential nor its longitudinal stress component exceeds 20 ksi. Under the same conditions, what is the maximum internal pressure that a similar-size spherical vessel can sustain?

SOLUTION

Cylindrical Pressure Vessel. The maximum stress occurs in the circumferential direction. From Eq. 8–1 we have

$$\sigma_1 = \frac{pr}{t}; \qquad 20 \text{ kip/in}^2 = \frac{p(24 \text{ in.})}{\frac{1}{2} \text{ in.}}$$

$$p = 417 \text{ psi} \qquad \textit{Ans.}$$

Note that when this pressure is reached, from Eq. 8–2, the stress in the longitudinal direction will be $\sigma_2 = \frac{1}{2}(20 \text{ ksi}) = 10$ ksi. Furthermore, the *maximum stress* in the *radial direction* occurs on the material at the inner wall of the vessel and is $(\sigma_3)_{max} = p = 417$ psi. This value is 48 times smaller than the circumferential stress (20 ksi), and as stated earlier, its effects will be neglected.

Spherical Vessel. Here the maximum stress occurs in any two perpendicular directions on an element of the vessel, Fig. 8–2*a*. From Eq. 8–3, we have

$$\sigma_2 = \frac{pr}{2t}; \qquad 20 \text{ kip/in}^2 = \frac{p(24 \text{ in.})}{2\left(\frac{1}{2} \text{ in.}\right)}$$

$$p = 833 \text{ psi} \qquad \textit{Ans.}$$

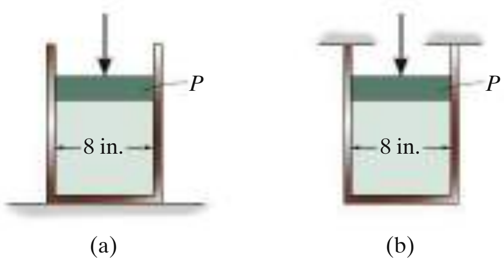
NOTE: Although it is more difficult to fabricate, the spherical pressure vessel will carry twice as much internal pressure as a cylindrical vessel.

PROBLEMS

8–1. A spherical gas tank has an inner radius of $r = 1.5$ m. If it is subjected to an internal pressure of $p = 300$ kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

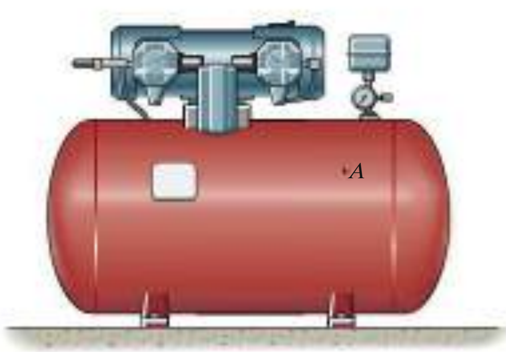
8–2. A pressurized spherical tank is to be made of 0.5-in.-thick steel. If it is subjected to an internal pressure of $p = 200$ psi, determine its outer radius if the maximum normal stress is not to exceed 15 ksi.

8–3. The thin-walled cylinder can be supported in one of two ways as shown. Determine the state of stress in the wall of the cylinder for both cases if the piston P causes the internal pressure to be 65 psi. The wall has a thickness of 0.25 in. and the inner diameter of the cylinder is 8 in.



Prob. 8–3

***8–4.** The tank of the air compressor is subjected to an internal pressure of 90 psi. If the internal diameter of the tank is 22 in., and the wall thickness is 0.25 in., determine the stress components acting at point A. Draw a volume element of the material at this point, and show the results on the element.



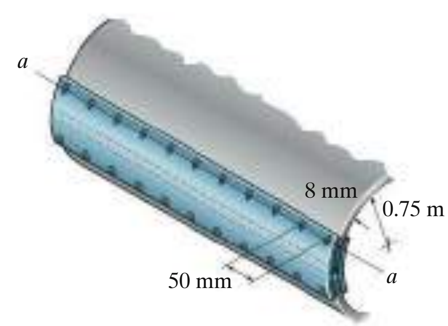
Prob. 8–4

8–5. The open-ended polyvinyl chloride pipe has an inner diameter of 4 in. and thickness of 0.2 in. If it carries flowing water at 60 psi pressure, determine the state of stress in the walls of the pipe.

8–6. If the flow of water within the pipe in Prob. 8–5 is stopped due to the closing of a valve, determine the state of stress in the walls of the pipe. Neglect the weight of the water. Assume the supports only exert vertical forces on the pipe.

Probs. 8–5/6

8–7. A boiler is constructed of 8-mm thick steel plates that are fastened together at their ends using a butt joint consisting of two 8-mm cover plates and rivets having a diameter of 10 mm and spaced 50 mm apart as shown. If the steam pressure in the boiler is 1.35 MPa, determine (a) the circumferential stress in the boiler's plate apart from the seam, (b) the circumferential stress in the outer cover plate along the rivet line a–a, and (c) the shear stress in the rivets.



Prob. 8–7

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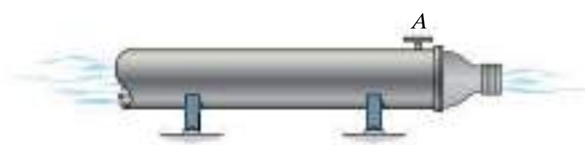
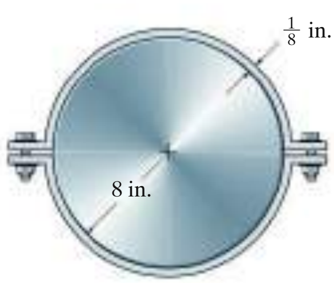
***8–8.** The steel water pipe has an inner diameter of 12 in. and wall thickness 0.25 in. If the valve A is opened and the flowing water is under a gauge pressure of 250 psi, determine the longitudinal and hoop stress developed in the wall of the pipe.

8–9. The steel water pipe has an inner diameter of 12 in. and wall thickness 0.25 in. If the valve A is closed and the water pressure is 300 psi, determine the longitudinal and hoop stress developed in the wall of the pipe. Draw the state of stress on a volume element located on the wall.



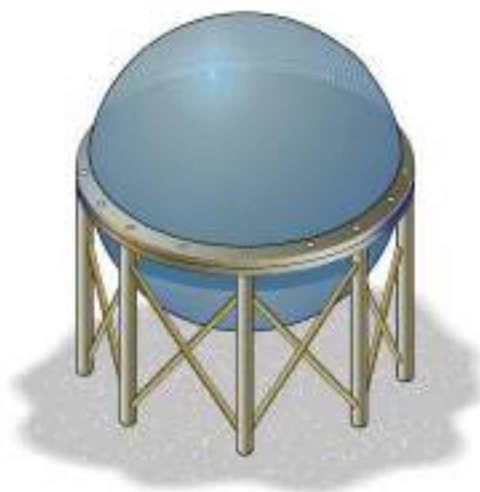
Probs. 8–8/9

8–10. The A-36-steel band is 2 in. wide and is secured around the smooth rigid cylinder. If the bolts are tightened so that the tension in them is 400 lb, determine the normal stress in the band, the pressure exerted on the cylinder, and the distance half the band stretches.



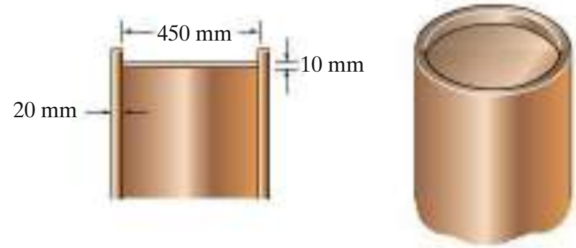
Prob. 8–10

8–11. Two hemispheres having an inner radius of 2 ft and wall thickness of 0.25 in. are fitted together, and the inside gauge pressure is reduced to -10 psi. If the coefficient of static friction is $\mu_s = 0.5$ between the hemispheres, determine (a) the torque T needed to initiate the rotation of the top hemisphere relative to the bottom one, (b) the vertical force needed to pull the top hemisphere off the bottom one, and (c) the horizontal force needed to slide the top hemisphere off the bottom one.



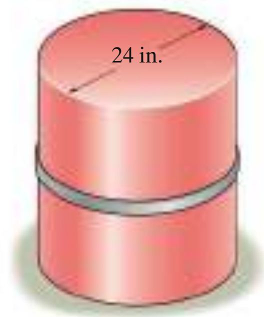
Prob. 8–11

***8–12.** A pressure-vessel head is fabricated by gluing the circular plate to the end of the vessel as shown. If the vessel sustains an internal pressure of 450 kPa, determine the average shear stress in the glue and the state of stress in the wall of the vessel.



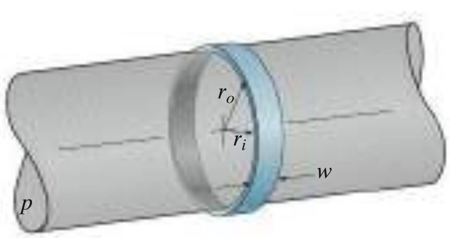
Prob. 8–12

8–13. An A-36-steel hoop has an inner diameter of 23.99 in., thickness of 0.25 in., and width of 1 in. If it and the 24-in.-diameter rigid cylinder have a temperature of 65° F, determine the temperature to which the hoop should be heated in order for it to just slip over the cylinder. What is the pressure the hoop exerts on the cylinder, and the tensile stress in the ring when it cools back down to 65° F?



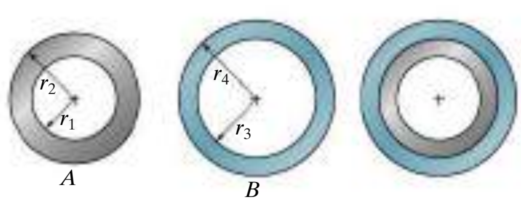
Prob. 8–13

8–14. The ring, having the dimensions shown, is placed over a flexible membrane which is pumped up with a pressure p. Determine the change in the internal radius of the ring after this pressure is applied. The modulus of elasticity for the ring is E.



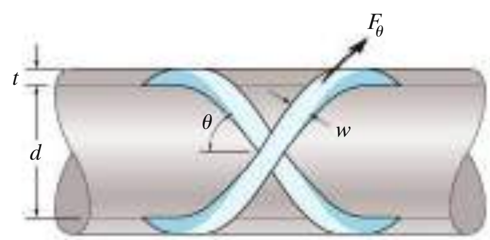
Prob. 8–14

8–15. The inner ring A has an inner radius r_1 and outer radius r_2. Before heating, the outer ring B has an inner radius r_3 and an outer radius r_4, and $r_2 > r_3$. If the outer ring is heated and then fitted over the inner ring, determine the pressure between the two rings when ring B reaches the temperature of the inner ring. The material has a modulus of elasticity of E and a coefficient of thermal expansion of α.



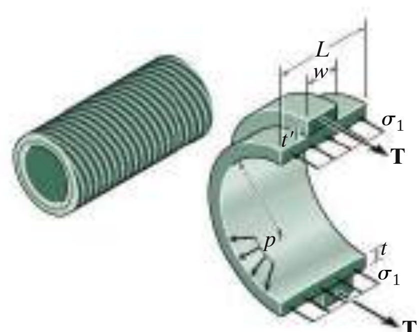
Prob. 8–15

***8–16.** A closed-ended pressure vessel is fabricated by cross winding glass filaments over a mandrel, so that the wall thickness t of the vessel is composed entirely of filament and an expoxy binder as shown in the figure. Consider a segment of the vessel of width w and wrapped at an angle θ. If the vessel is subjected to an internal pressure p, show that the force in the segment is $F_\theta = \sigma_0 wt$, where σ_0 is the stress in the filaments. Also, show that the stresses in the hoop and longitudinal directions are $\sigma_h = \sigma_0 \sin^2\theta$ and $\sigma_1 = \sigma_0 \cos^2\theta$, respectively. At what angle θ (optimum winding angle) would the filaments have to be would so that the hoop and longitudinal stresses are equivalent?



Prob. 8–16

8–17. In order to increase the strength of the pressure vessel, filament winding of the same material is wrapped around the circumference of the vessel as shown. If the pretension in the filament is T and the vessel is subjected to an internal pressure p, determine the hoop stresses in the filament and in the wall of the vessel. Use the free-body diagram shown, and assume the filament winding has a thickness t' and width w for a corresponding length L of the vessel.



Prob. 8–17

8

This chimney is subjected to the combined loading of wind and weight. It is important to investigate the tensile stress in the chimney since masonry is weak in tension.

8.2 State of Stress Caused by Combined Loadings

In previous chapters we developed methods for determining the stress distributions in a member subjected to either an internal axial force, a shear force, a bending moment, or a torsional moment. Most often, however, the cross section of a member is subjected to *several* of these loadings *simultaneously*. When this occurs, the method of superposition can be used to determine the *resultant* stress distribution. Recall from Sec. 4.3 that the principle of superposition can be used for this purpose provided a *linear relationship* exists between the *stress* and the *loads*. Also, the geometry of the member should *not* undergo *significant change* when the loads are applied. These conditions are necessary in order to ensure that the stress produced by one load is not related to the stress produced by any other load.

Procedure for Analysis

The following procedure provides a general means for establishing the normal and shear stress components at a point in a member when the member is subjected to several different types of loadings simultaneously. It is assumed that the material is homogeneous and behaves in a linear elastic manner. Also, Saint-Venant's principle requires that the point where the stress is to be determined is far removed from any discontinuities in the cross section or points of applied load.

Internal Loading.

- Section the member perpendicular to its axis at the point where the stress is to be determined and obtain the resultant internal normal and shear force components and the bending and torsional moment components.
- The force components should act through the *centroid* of the cross section, and the moment components should be computed about *centroidal axes*, which represent the principal axes of inertia for the cross section.

Stress Components.

- Determine the stress component associated with *each* internal loading. For each case, represent the effect either as a distribution of stress acting over the entire cross-sectional area, or show the stress on an element of the material located at a specified point on the cross section.

Normal Force.

- The internal normal force is developed by a uniform normal-stress distribution determined from $\sigma = P/A$.

Shear Force.

- The internal shear force in a member is developed by a shear-stress distribution determined from the shear formula, $\tau = VQ/It$. Special care, however, must be exercised when applying this equation, as noted in Sec. 7.2.

Bending Moment.

- For *straight members* the internal bending moment is developed by a normal-stress distribution that varies linearly from zero at the neutral axis to a maximum at the outer boundary of the member. This stress distribution is determined from the flexure formula, $\sigma = -My/I$. If the member is *curved*, the stress distribution is nonlinear and is determined from $\sigma = My/[Ae(R - y)]$.

Torsional Moment.

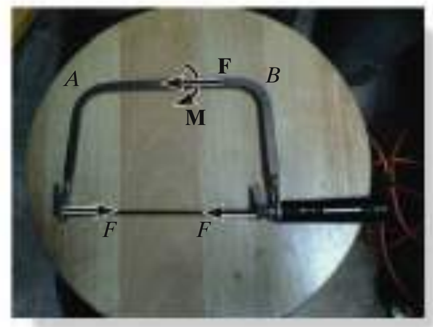
- For circular shafts and tubes the internal torsional moment is developed by a shear-stress distribution that varies linearly from zero at the central axis of the shaft to a maximum at the shaft's outer boundary. This stress distribution is determined from the torsional formula, $\tau = T\rho/J$.

Thin-Walled Pressure Vessels.

- If the vessel is a thin-walled cylinder, the internal pressure p will cause a biaxial state of stress in the material such that the hoop or circumferential stress component is $\sigma_1 = pr/t$ and the longitudinal stress component is $\sigma_2 = pr/2t$. If the vessel is a thin-walled sphere, then the biaxial state of stress is represented by two equivalent components, each having a magnitude of $\sigma_2 = pr/2t$.

Superposition.

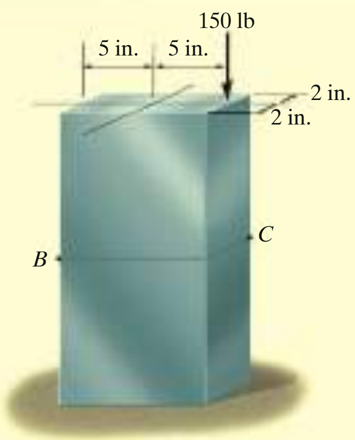
- Once the normal and shear stress components for each loading have been calculated, use the principle of superposition and determine the resultant normal and shear stress components.
- Represent the results on an element of material located at the point, or show the results as a distribution of stress acting over the member's cross-sectional area.



When a pretension force F is developed in the blade of this coping saw, it will produce both a compressive force F and bending moment M in the region AB of the frame. The material must therefore resist the normal stress produced by both of these loadings.

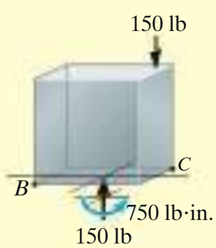
Problems in this section, which involve combined loadings, serve as a basic *review* of the application of the stress equations mentioned above. A thorough understanding of how these equations are applied, as indicated in the previous chapters, is necessary if one is to successfully solve the problems at the end of this section. The following examples should be carefully studied before proceeding to solve the problems.

EXAMPLE 8.2



(a)

Fig. 8–3



(b)

A force of 150 lb is applied to the edge of the member shown in Fig. 8–3*a*. Neglect the weight of the member and determine the state of stress at points *B* and *C*.

SOLUTION

Internal Loadings. The member is sectioned through *B* and *C*. For equilibrium at the section there must be an axial force of 150 lb acting through the *centroid* and a bending moment of 750 lb · in. about the centroidal principal axis, Fig. 8–3*b*.

Stress Components.

Normal Force. The uniform normal-stress distribution due to the normal force is shown in Fig. 8–3*c*. Here

$$\sigma = \frac{P}{A} = \frac{150 \text{ lb}}{(10 \text{ in.})(4 \text{ in})} = 3.75 \text{ psi}$$

Bending Moment. The normal-stress distribution due to the bending moment is shown in Fig. 8–3*d*. The maximum stress is

$$\sigma_{\max} = \frac{Mc}{I} = \frac{750 \text{ lb} \cdot \text{in. } (5 \text{ in.})}{\frac{1}{12}(4 \text{ in.})\,(10 \text{ in.})^3} = 11.25 \text{ psi}$$

Superposition. If the above normal-stress distributions are added algebraically, the resultant stress distribution is shown in Fig. 8–3*e*.

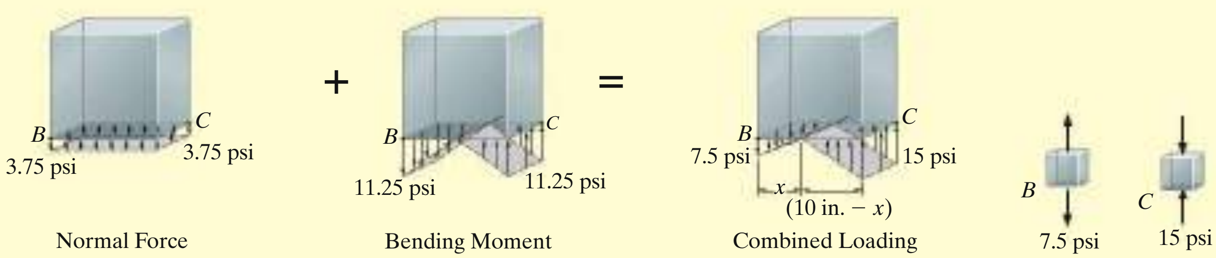
Elements of material at *B* and *C* are subjected only to normal or *uniaxial stress* as shown in Fig. 8–3*f* and 8–3*g*. Hence,

$$\sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -3.75 \text{ psi} + 11.25 \text{ psi} = 7.5 \text{ psi} \quad \text{(tension)} \qquad \textit{Ans.}$$

$$\sigma_C = -\frac{P}{A} - \frac{Mc}{I} = -3.75 \text{ psi} - 11.25 \text{ psi} = -15 \text{ psi} \quad \text{(compression)} \qquad \textit{Ans.}$$

Although it is not needed here, the location of the line of zero stress can be determined by proportional triangles; i.e.,

$$\frac{7.5 \text{ psi}}{x} = \frac{15 \text{ psi}}{10 \text{ in.} - x}; \quad x = 3.33 \text{ in.}$$



Normal Force (c) Bending Moment (d) Combined Loading (e) (f) (g)

EXAMPLE 8.3

The tank in Fig. 8–4*a* has an inner radius of 24 in. and a thickness of 0.5 in. It is filled to the top with water having a specific weight of $\gamma_w = 62.4\ \text{lb/ft}^3$. If it is made of steel having a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$, determine the state of stress at point A. The tank is open at the top.

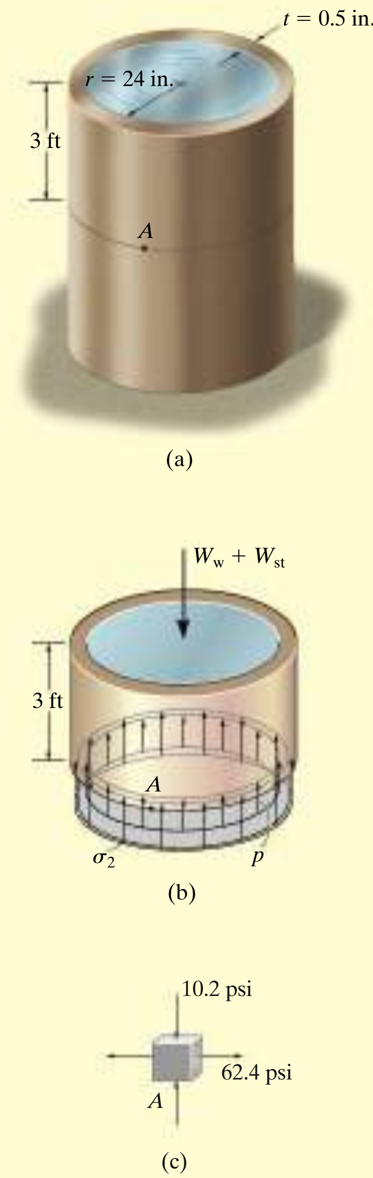


Fig. 8–4

SOLUTION

Internal Loadings. The free-body diagram of the section of both the tank and the water above point A is shown in Fig. 8–4*b*. Notice that the weight of the water is supported by the water surface just *below* the section, *not* by the walls of the tank. In the vertical direction, the walls simply hold up the weight of the tank. This weight is

$$W_{st} = \gamma_{st} V_{st} = (490\ \text{lb/ft}^3)\left[\pi\left(\frac{24.5}{12}\ \text{ft}\right)^2 - \pi\left(\frac{24}{12}\ \text{ft}\right)^2\right](3\ \text{ft})$$
$$= 777.7\ \text{lb}$$

The stress in the circumferential direction is developed by the water pressure at level A. To obtain this (gauge) pressure we must use $p = \gamma_w z$, which gives the pressure at a point located a depth z in the water. Consequently, the pressure on the tank at level A is

$$p = \gamma_w z = (62.4\ \text{lb/ft}^3)(3\ \text{ft}) = 187.2\ \text{lb/ft}^2 = 1.30\ \text{psi}$$

Stress Components.

Circumferential Stress. Since $r/t = 24\ \text{in.}/0.5\ \text{in.} = 48 > 10$, the tank is a thin-walled vessel. Applying Eq. 8–1, using the inner radius $r = 24$ in., we have

$$\sigma_1 = \frac{pr}{t} = \frac{1.30\ \text{lb/in}^2\ (24\ \text{in.})}{0.5\ \text{in.}} = 62.4\ \text{psi} \qquad \textit{Ans.}$$

Longitudinal Stress. Since the weight of the tank is supported uniformly by the walls, we have

$$\sigma_2 = \frac{W_{st}}{A_{st}} = \frac{777.7\ \text{lb}}{\pi[(24.5\ \text{in.})^2 - (24\ \text{in.})^2]} = 10.2\ \text{psi} \qquad \textit{Ans.}$$

NOTE: Eq. 8–2, $\sigma_2 = pr/2t$, does *not apply* here, since the tank is open at the top and therefore, as stated previously, the water cannot develop a loading on the walls in the longitudinal direction.

Point A is therefore subjected to the biaxial stress shown in Fig. 8–4*c*.

EXAMPLE 8.4

The member shown in Fig. 8–5*a* has a rectangular cross section. Determine the state of stress that the loading produces at point *C*.

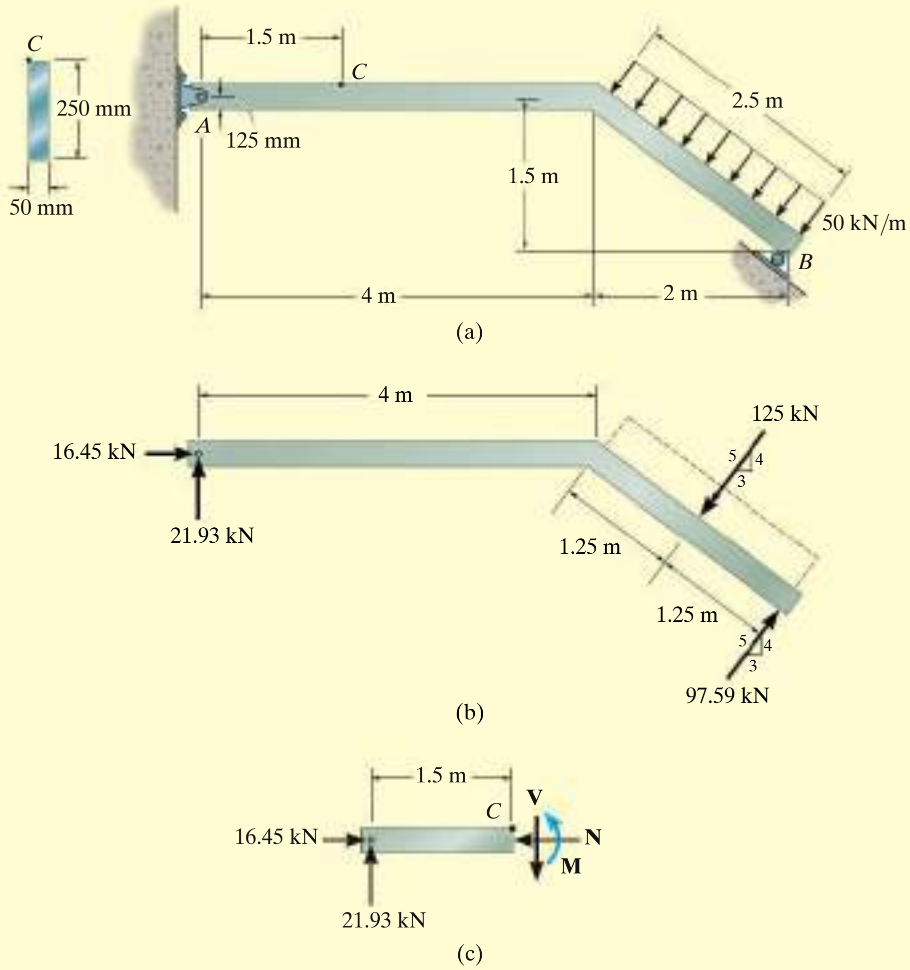


Fig. 8–5

SOLUTION

Internal Loadings. The support reactions on the member have been determined and are shown in Fig. 8–5*b*. If the left segment *AC* of the member is considered, Fig. 8–5*c*, the resultant internal loadings at the section consist of a normal force, a shear force, and a bending moment. Solving,

$$N = 16.45 \text{ kN} \qquad V = 21.93 \text{ kN} \qquad M = 32.89 \text{ kN} \cdot \text{m}$$

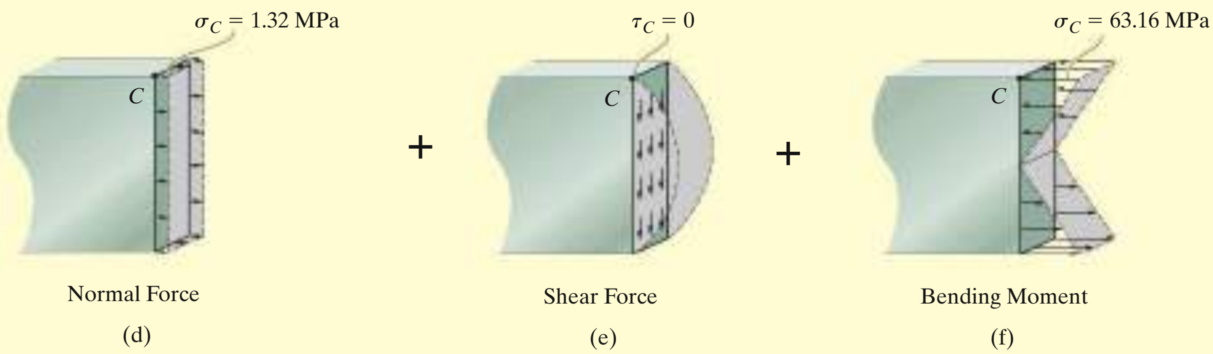


Fig. 8–5 (cont.)

Stress Components.

Normal Force. The uniform normal-stress distribution acting over the cross section is produced by the normal force, Fig. 8–5*d*. At point *C*,

$$\sigma_C = \frac{P}{A} = \frac{16.45(10^3)\ \text{N}}{(0.050\ \text{m})\ (0.250\ \text{m})} = 1.32\ \text{MPa}$$

Shear Force. Here the area $A' = 0$, since point C is located at the top of the member. Thus $Q = \bar{y}'A' = 0$ and for C, Fig. 8–5*e*, the shear stress

$$\tau_C = 0$$

Bending Moment. Point C is located at $y = c = 0.125$ m from the neutral axis, so the normal stress at C, Fig. 8–5*f*, is

$$\sigma_C = \frac{Mc}{I} = \frac{(32.89(10^3)\ \text{N}\cdot\text{m})(0.125\ \text{m})}{\left[\frac{1}{12}(0.050\ \text{m})\ (0.250\ \text{m}^3)\right]} = 63.16\ \text{MPa}$$

Superposition. The shear stress is zero. Adding the normal stresses determined above gives a compressive stress at C having a value of

$$\sigma_C = 1.32\ \text{MPa} + 63.16\ \text{MPa} = 64.5\ \text{MPa} \qquad \textit{Ans.}$$

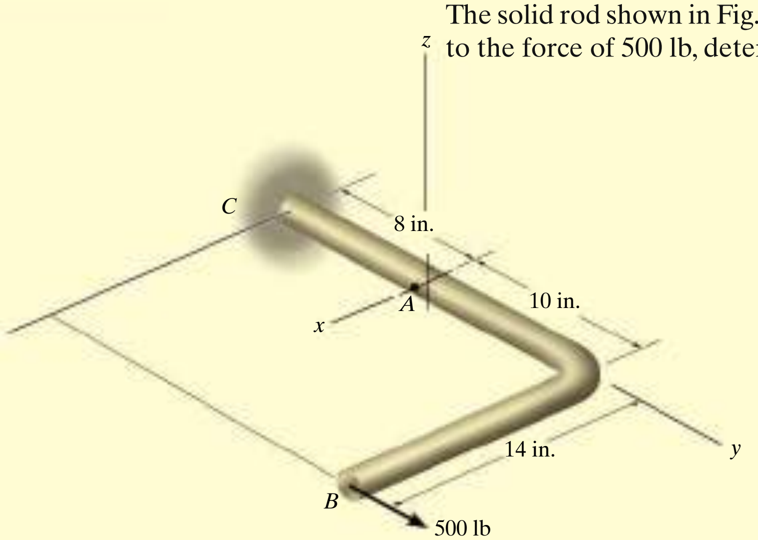
(g)

This result, acting on an element at C, is shown in Fig. 8–5*g*.

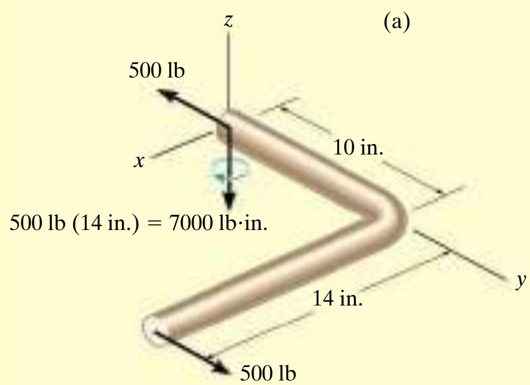
8

EXAMPLE 8.5

The solid rod shown in Fig. 8–6*a* has a radius of 0.75 in. If it is subjected to the force of 500 lb, determine the state of stress at point *A*.



(a)



(b)

SOLUTION

Internal Loadings. The rod is sectioned through point *A*. Using the free-body diagram of segment *AB*, Fig. 8–6*b*, the resultant internal loadings are determined from the equations of equilibrium. Verify these results. In order to better "visualize" the stress distributions due to these loadings, we can consider the *equal but opposite resultants* acting on segment *AC*, Fig. 8–6*c*.

Stress Components.

Normal Force. The normal-stress distribution is shown in Fig. 8–6*d*. For point *A*, we have

$$(\sigma_A)_y = \frac{P}{A} = \frac{500 \text{ lb}}{\pi(0.75 \text{ in.})^2} = 283 \text{ psi} = 0.283 \text{ ksi}$$

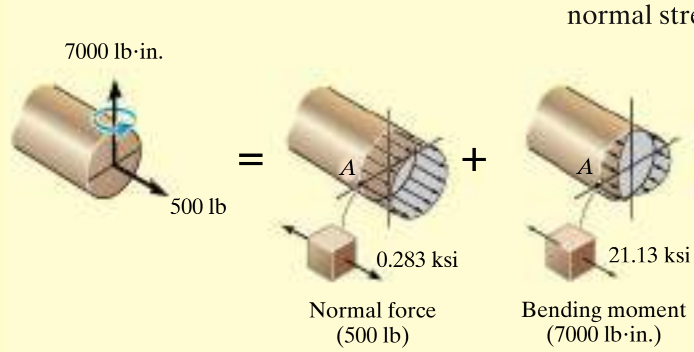
Bending Moment. For the moment, $c = 0.75$ in., so the normal stress at point *A*, Fig. 8–6*e*, is

$$(\sigma_A)_y = \frac{Mc}{I} = \frac{7000 \text{ lb}\cdot\text{in.}(0.75 \text{ in.})}{\left[\frac{1}{4}\pi(0.75 \text{ in.})^4\right]}$$

$$= 21{,}126 \text{ psi} = 21.13 \text{ ksi}$$

Superposition. When the above results are superimposed, it is seen that an element of material at *A* is subjected to the normal stress

$$(\sigma_A)_y = 0.283 \text{ ksi} + 21.13 \text{ ksi} = 21.4 \text{ ksi} \qquad \textit{Ans.}$$



Normal force (500 lb) | Bending moment (7000 lb·in.)

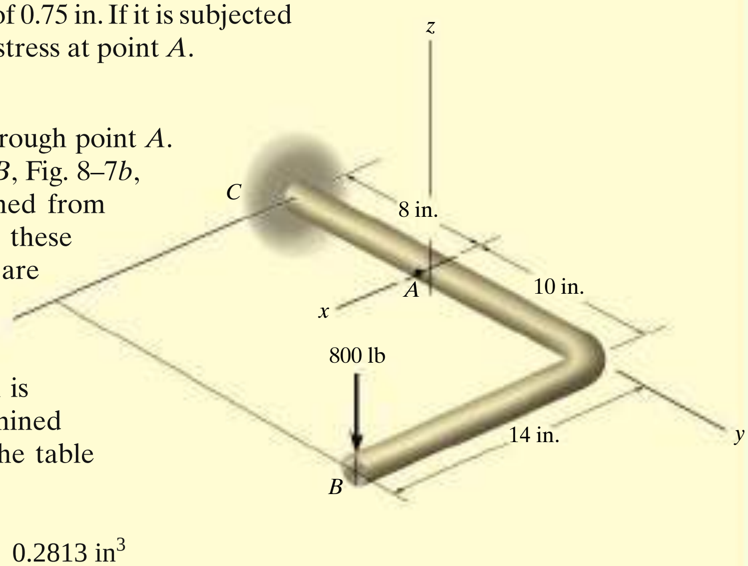
(c) (d) (e)

Fig. 8–6

8

EXAMPLE 8.6

The solid rod shown in Fig. 8–7*a* has a radius of 0.75 in. If it is subjected to the force of 800 lb, determine the state of stress at point *A*.



(a)

SOLUTION

Internal Loadings. The rod is sectioned through point *A*. Using the free-body diagram of segment *AB*, Fig. 8–7*b*, the resultant internal loadings are determined from the six equations of equilibrium. Verify these results. The *equal but opposite resultants* are shown acting on segment *AC*, Fig. 8–7*c*.

Stress Components.

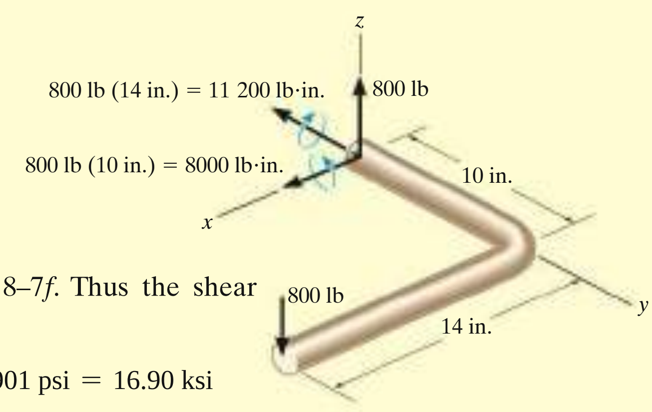
Shear Force. The shear-stress distribution is shown in Fig. 8–7*d*. For point *A*, *Q* is determined from the shaded *semi-circular* area. Using the table on the inside front cover, we have

$$Q = \bar{y}'A' = \frac{4(0.75\text{ in.})}{3\pi}\left[\frac{1}{2}\pi(0.75\text{ in.})^2\right] = 0.2813\text{ in}^3$$

so that

$$(\tau_{yz})_A = \frac{VQ}{It} = \frac{800\text{ lb}(0.2813\text{ in}^3)}{\left[\frac{1}{4}\pi(0.75\text{ in.})^4\right]2(0.75\text{ in.})}$$

$$= 604\text{ psi} = 0.604\text{ ksi}$$



(b)

Fig. 8–7

Bending Moment. Since point *A* lies on the neutral axis, Fig. 8–7*e*, the normal stress is

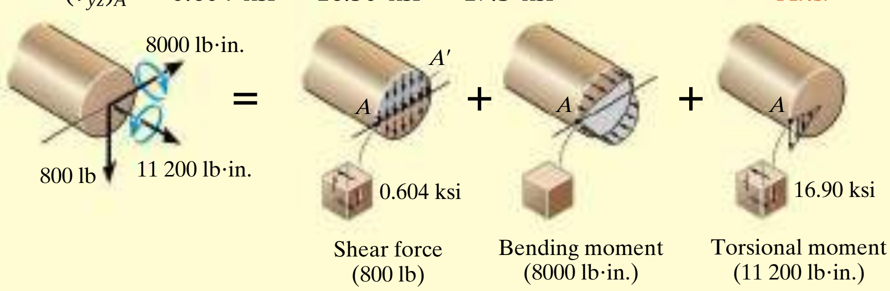
$$\sigma_A = 0$$

Torque. At point *A*, $\rho_A = c = 0.75$ in., Fig. 8–7*f*. Thus the shear stress is

$$(\tau_{yz})_A = \frac{Tc}{J} = \frac{11\,200\text{ lb}\cdot\text{in.}(0.75\text{ in.})}{\left[\frac{1}{2}\pi(0.75\text{ in.})^4\right]} = 16\,901\text{ psi} = 16.90\text{ ksi}$$

Superposition. Here the element of material at *A* is subjected only to a shear stress component, where

$$(\tau_{yz})_A = 0.604\text{ ksi} + 16.90\text{ ksi} = 17.5\text{ ksi} \qquad \textit{Ans.}$$

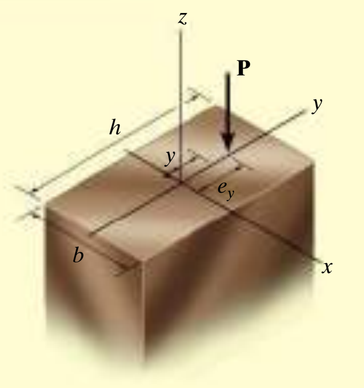


(c) (d) (e) (f)

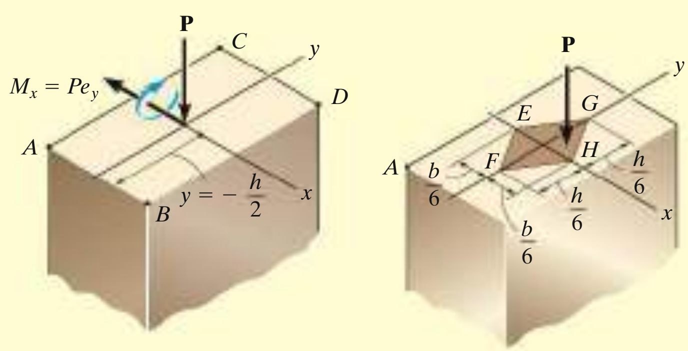
8

EXAMPLE 8.7

A rectangular block has a negligible weight and is subjected to a vertical force **P**, Fig. 8–8*a*. (*a*) Determine the range of values for the eccentricity e_y of the load along the y axis so that it does not cause any tensile stress in the block. (*b*) Specify the region on the cross section where **P** may be applied without causing a tensile stress in the block.



(a)



(b) (c)

Fig. 8–8

SOLUTION

Part (a). When **P** is moved to the centroid of the cross section, Fig. 8–8*b*, it is necessary to add a couple moment $M_x = Pe_y$ in order to maintain a statically equivalent loading. The combined normal stress at any coordinate location y on the cross section caused by these two loadings is

$$\sigma = -\frac{P}{A} - \frac{(Pe_y)y}{I_x} = -\frac{P}{A}\left(1 + \frac{Ae_y y}{I_x}\right)$$

Here the negative sign indicates compressive stress. For positive e_y, Fig. 8–7*a*, the *smallest* compressive stress will occur along edge AB, where $y = -h/2$, Fig. 8–8*b*. (By inspection, **P** causes compression there, but $\mathbf{M}_x$ causes tension.) Hence,

$$\sigma_{\min} = -\frac{P}{A}\left(1 - \frac{Ae_y h}{2I_x}\right)$$

This stress will remain negative, i.e., compressive, provided the term in parentheses is positive; i.e.,

$$1 > \frac{Ae_y h}{2I_x}$$

Since $A = bh$ and $I_x = \frac{1}{12}bh^3$, then

$$1 > \frac{6e_y}{h} \quad \text{or} \quad e_y < \frac{1}{6}h \qquad \textit{Ans.}$$

In other words, if $-\frac{1}{6}h \leq e_y \leq \frac{1}{6}h$, the stress in the block along edge AB or CD will be zero or remain *compressive*.

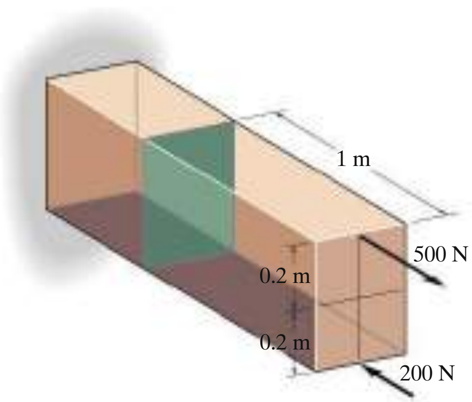
NOTE: This is sometimes referred to as the "*middle-third rule.*" It is very important to keep this rule in mind when loading columns or arches having a rectangular cross section and made of material such as stone or concrete, which can support little or no tensile stress. We can extend this analysis in the same way by placing **P** along the x axis in Fig. 8–8. The result will produce a shaded parallelogram shown in Fig. 8–8*c*. This region is referred to as the core or kern of the section. When **P** is applied within the kern, the normal stress at the corners of the cross section will be compressive.

Here is an example of where combined axial and bending stress can occur.

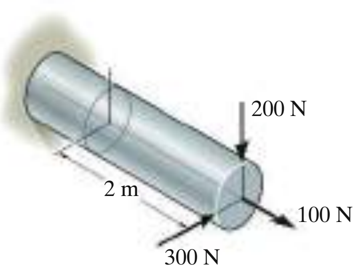
8

PRELIMINARY PROBLEMS

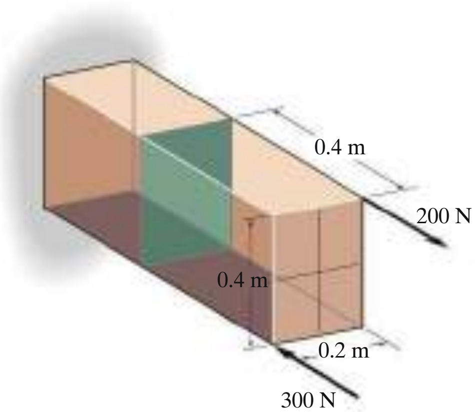
P8–1. In each case, determine the internal loadings that act on the indicated section. Show the results on the left segment.



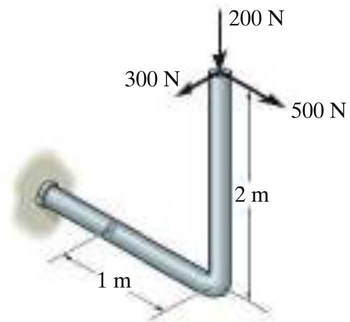
(a)



(b)



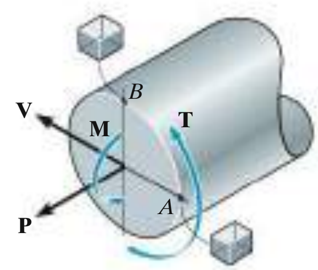
(c)



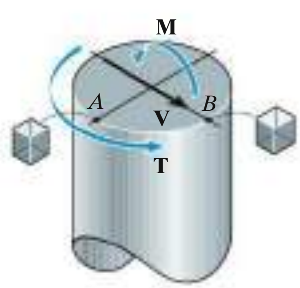
(d)

P8–1

P8–2. The internal loadings act on the section. Show the stress that each of these loads produce on differential elements located at point A and point B.



(a)

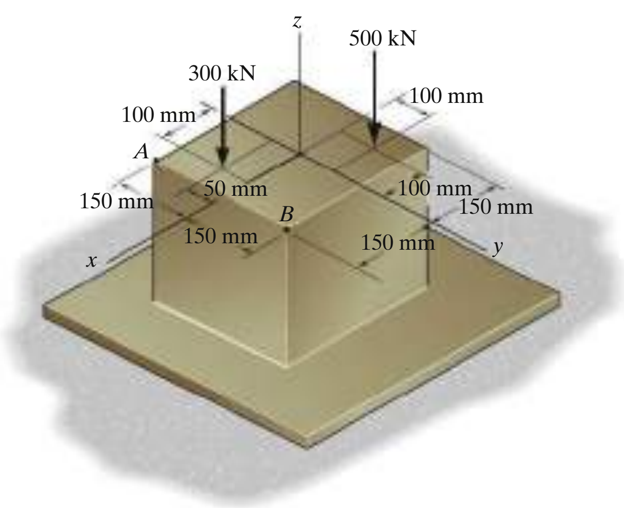


(b)

P8–2

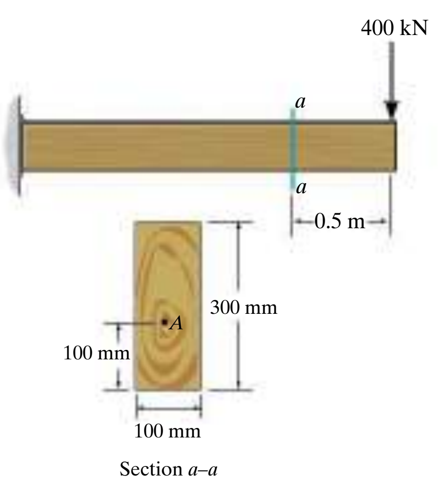
FUNDAMENTAL PROBLEMS

F8–1. Determine the normal stress developed at corners A and B of the column.



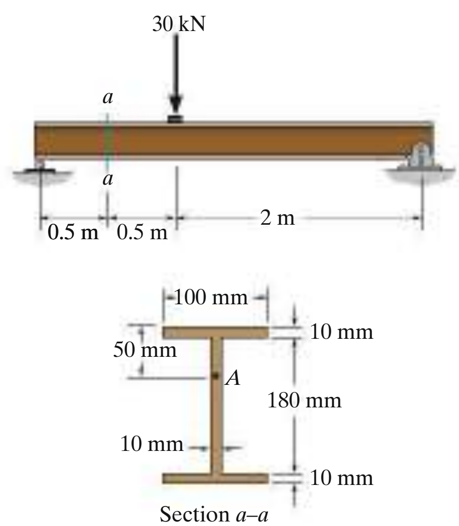
F8–1

F8–2. Determine the state of stress at point A on the cross section at section a–a of the cantilever beam. Show the results in a differential element at the point.



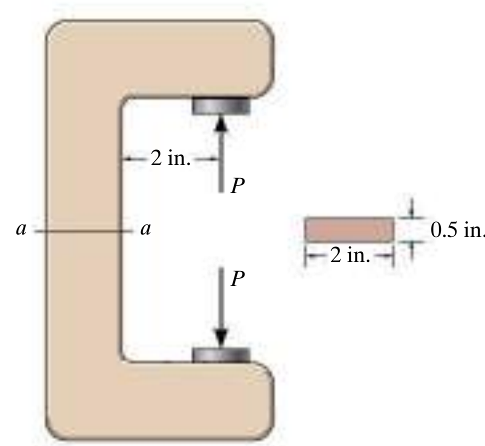
F8–2

F8–3. Determine the state of stress at point A on the cross section of the beam at section a–a. Show the results in a differential element at the point.



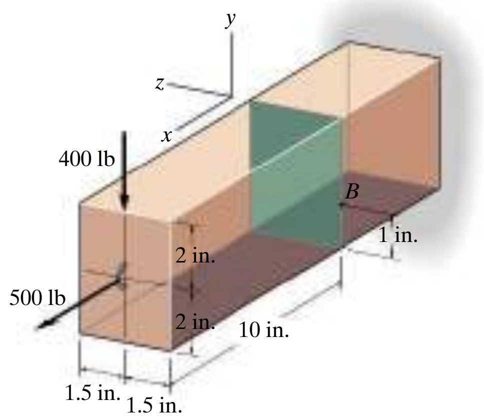
F8–3

F8–4. Determine the magnitude of the load P that will cause a maximum normal stress of $\sigma_{max} = 30$ ksi in the link along section a–a.



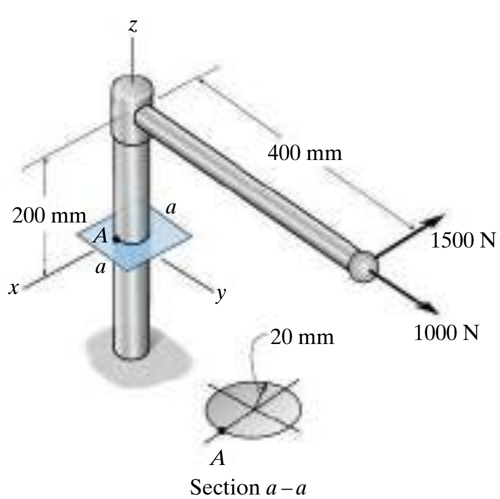
F8–4

F8–5. The beam has a rectangular cross section and is subjected to the loading shown. Determine the state of stress at point B. Show the results in a differential element at the point.



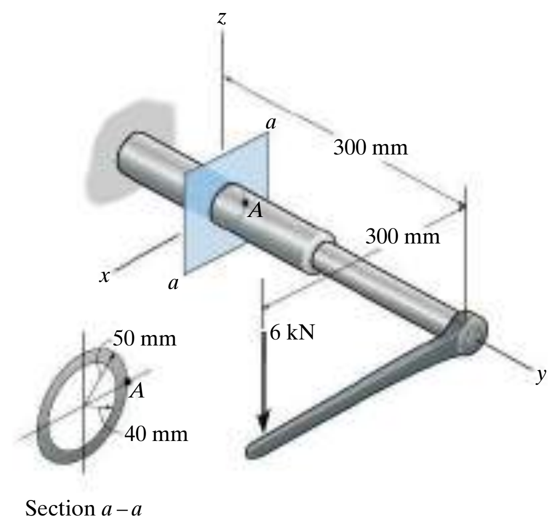
F8–5

F8–6. Determine the state of stress at point A on the cross section of the pipe assembly at section a–a. Show the results in a differential element at the point.



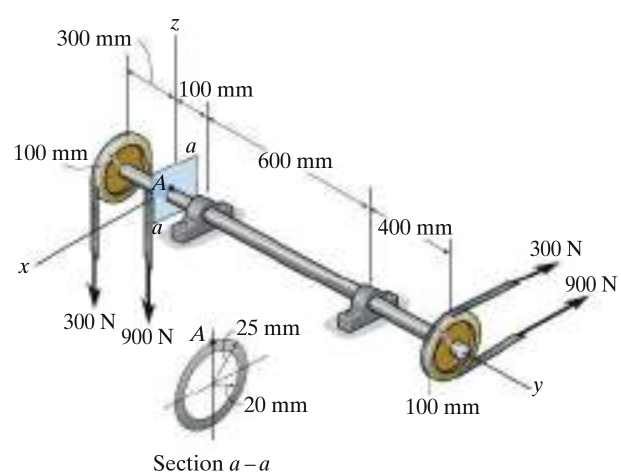
F8–6

F8–7. Determine the state of stress at point A on the cross section of the pipe at section a–a. Show the results in a differential element at the point.



F8–7

F8–8. Determine the state of stress at point A on the cross section of the shaft at section a–a. Show the results in a differential element at the point.

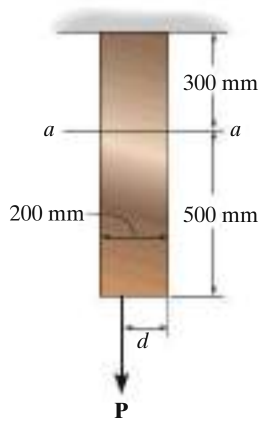


F8–8

8

PROBLEMS

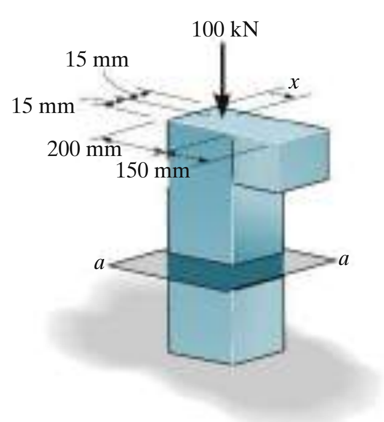
8–18. The vertical force **P** acts on the bottom of the plate having a negligible weight. Determine the shortest distance d to the edge of the plate at which it can be applied so that it produces no compressive stresses on the plate at section a–a. The plate has a thickness of 10 mm and **P** acts along the center line of this thickness.



Prob. 8–18

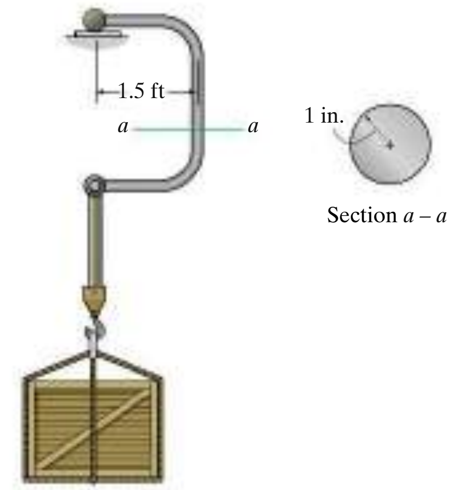
8–19. Determine the maximum and minimum normal stress in the bracket at section a–a when the load is applied at $x = 0$.

***8–20.** Determine the maximum and minimum normal stress in the bracket at section a–a when the load is applied at $x = 300$ mm.



Probs. 8–19/20

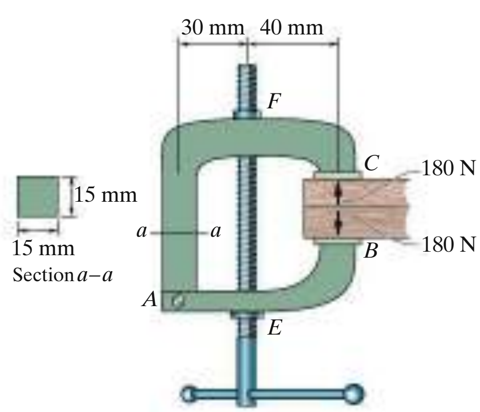
8–21. If the load has a weight of 600 lb, determine the maximum normal stress developed on the cross section of the supporting member at section a–a. Also, plot the normal stress distribution over the cross-section.



Prob. 8–21

8–22. The clamp is made from members AB and AC, which are pin connected at A. If it exerts a compressive force at C and B of 180 N, determine the maximum compressive stress in the clamp at section a–a. The screw EF is subjected only to a tensile force along its axis.

8–23. The clamp is made from members AB and AC, which are pin connected at A. If it exerts a compressive force at C and B of 180 N, sketch the stress distribution acting over section a–a. The screw EF is subjected only to a tensile force along its axis.

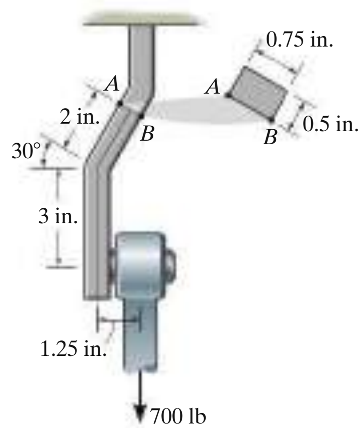


Probs. 8–22/23

8

***8–24.** The bearing pin supports the load of 700 lb. Determine the stress components in the support member at point *A*. The support is 0.5 in. thick.

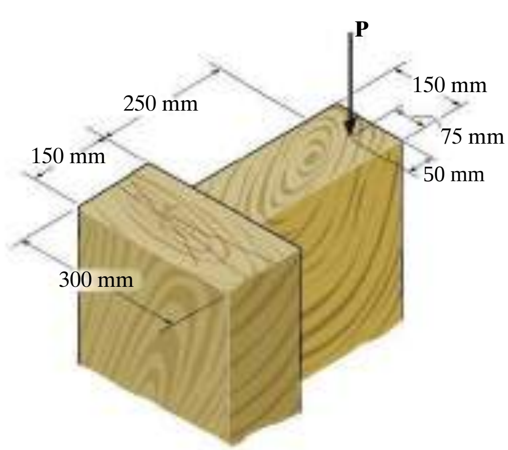
8–25. The bearing pin supports the load of 700 lb. Determine the stress components in the support member at point *B*. The support is 0.5 in. thick.



Probs. 8–24/25

8–26. The column is built up by gluing the two identical boards together. Determine the maximum normal stress developed on the cross section when the eccentric force of $P = 50$ kN is applied.

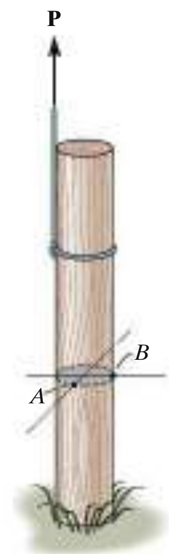
8–27. The column is built up by gluing the two identical boards together. If the wood has an allowable normal stress of $\sigma_{allow} = 6$ MPa, determine the maximum allowable eccentric force **P** that can be applied to the column.



Probs. 8–26/27

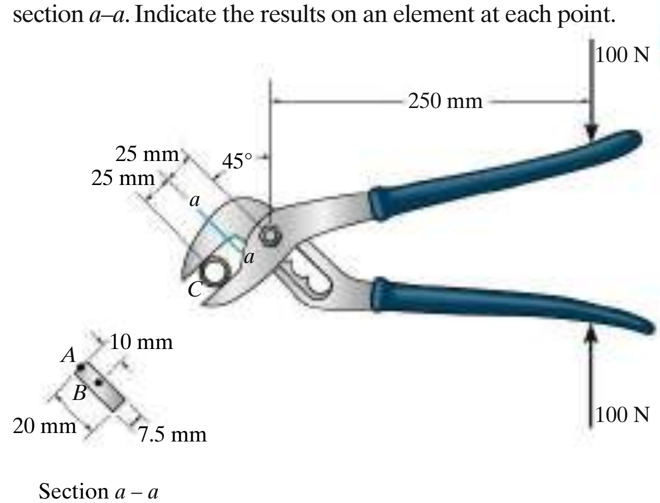
***8–28.** The cylindrical post, having a diameter of 40 mm, is being pulled from the ground using a sling of negligible thickness. If the rope is subjected to a vertical force of $P = 500$ N, determine the normal stress at points *A* and *B*. Show the results on a volume element located at each of these points.

8–29. Determine the maximum load *P* that can be applied to the sling having a negligible thickness so that the normal stress in the post does not exceed $\sigma_{allow} = 30$ MPa. The post has a diameter of 50 mm.



Probs. 8–28/29

8–30. The rib-joint pliers are used to grip the smooth pipe *C*. If the force of 100 N is applied to the handles, determine the state of stress at points *A* and *B* on the cross section of the jaw at section *a–a*. Indicate the results on an element at each point.

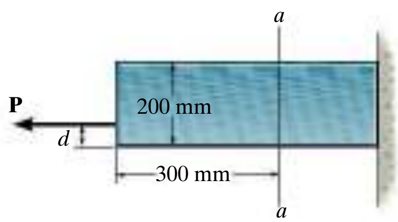


Prob. 8–30

8

8–31. Determine the smallest distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses in the plate at section a–a. The plate has a thickness of 20 mm and **P** acts along the centerline of this thickness.

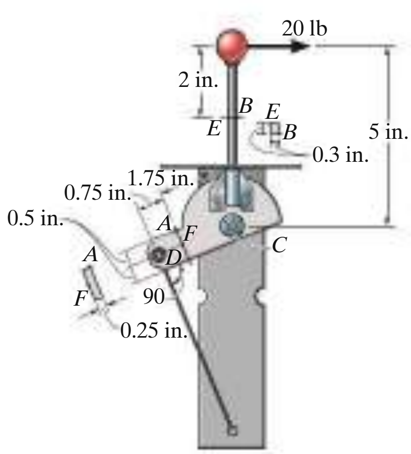
***8–32.** The horizontal force of $P = 80$ kN acts at the end of the plate. The plate has a thickness of 10 mm and **P** acts along the centerline of this thickness such that $d = 50$ mm. Plot the distribution of normal stress acting along section a–a.



Probs. 8–31/32

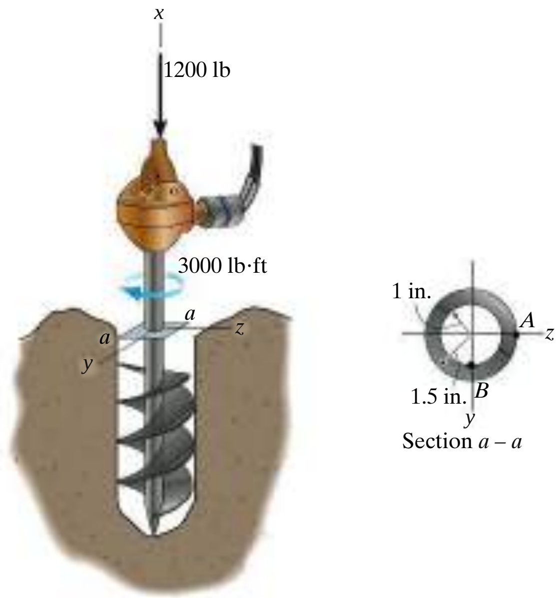
8–33. The control lever is subjected to a horizontal force of 20 lb on the handle. Determine the state of stress at points A and B. Sketch the results on differential elements located at each of these points. The assembly is pin-connected at C and attached to a cable at D.

8–34. The control lever is subjected to a horizontal force of 20 lb on the handle. Determine the state of stress at points E and F. Sketch the results on differential elements located at each of these points. The assembly is pin connected at C and attached to a cable at D.



Probs. 8–33/34

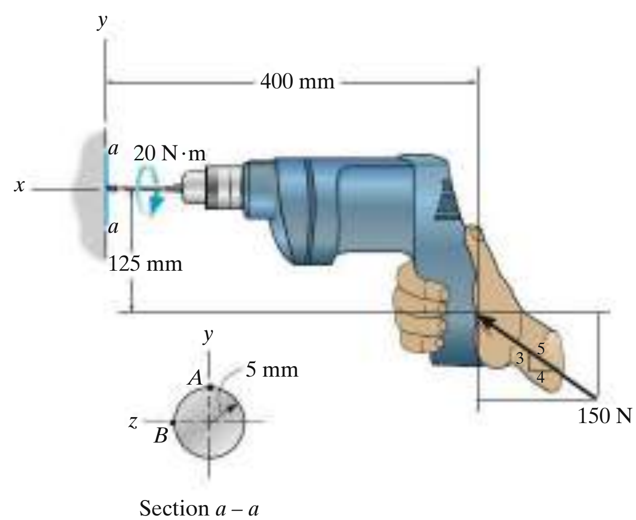
8–35. The tubular shaft of the soil auger is subjected to the axial force and torque shown. If the auger is rotating at a constant rate, determine the state of stress at points A and B on the cross section of the shaft at section a–a.



Prob. 8–35

***8–36.** The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point A on the cross section of drill bit at section a–a.

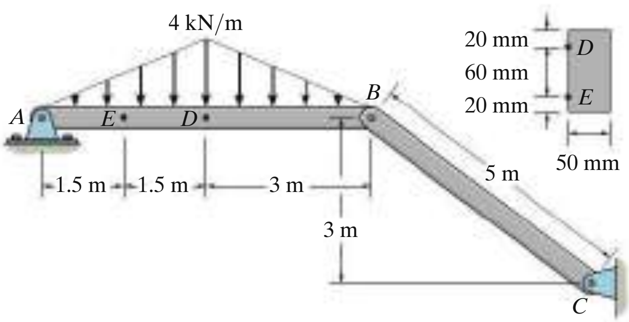
8–37. The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point B on the cross section of drill bit, in back, at section a–a.



Probs. 8–36/37

8–38. The frame supports the distributed load shown. Determine the state of stress acting at point D. Show the results on a differential element at this point.

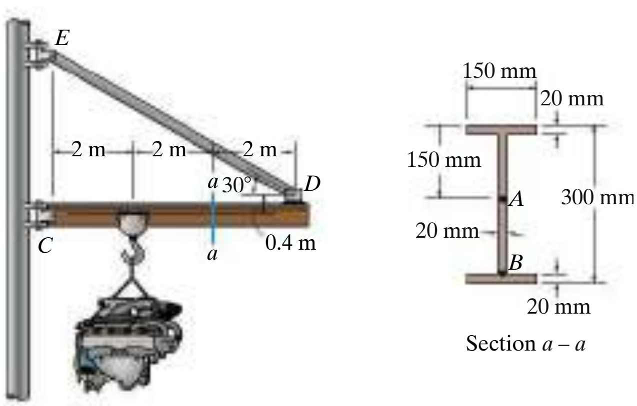
8–39. The frame supports the distributed load shown. Determine the state of stress acting at point E. Show the results on a differential element at this point.



Probs. 8–38/39

***8–40.** The 500-kg engine is suspended from the jib crane at the position shown. Determine the state of stress at point A on the cross section of the boom at section a–a.

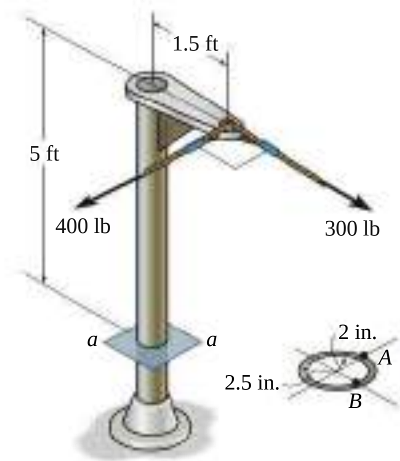
8–41. The 500-kg engine is suspended from the jib crane at the position shown. Determine the state of stress at point B on the cross section of the boom at section a–a. Point B is just above the bottom flange.



Probs. 8–40/41

8–42. Determine the state of stress at point A on the cross section of the post at section a–a. Indicate the results on a differential element at the point.

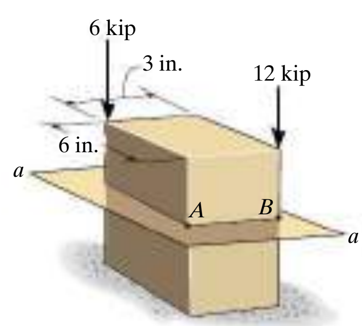
8–43. Determine the state of stress at point B on the cross section of the post at section a–a. Indicate the results on a differential element at the point.



Probs. 8–42/43

***8–44.** Determine the normal stress developed at points A and B. Neglect the weight of the block.

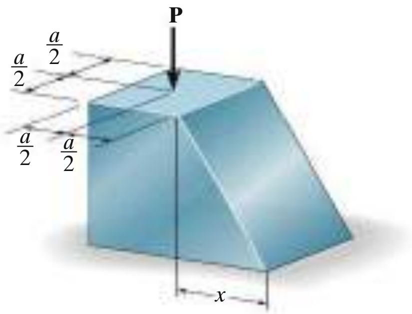
8–45. Sketch the normal stress distribution acting over the cross section at section a–a. Neglect the weight of the block.



Probs. 8–44/45

8

8–46. The support is subjected to the compressive load **P**. Determine the absolute maximum possible and minimum possible normal stress acting in the material, for $x \geq 0$.



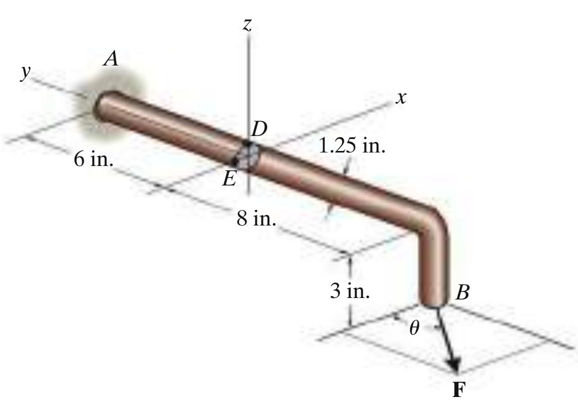
Prob. 8–46

8–47. The bent shaft is fixed in the wall at A. If a force **F** is applied at B, determine the stress components at points D and E. Show the results on a differential element located at each of these points. Take $F = 12$ lb and $\theta = 0°$.

***8–48.** The bent shaft is fixed in the wall at A. If a force **F** is applied at B, determine the stress components at points D and E. Show the results on a differential element located at each of these points. Take $F = 12$ lb and $\theta = 90°$.

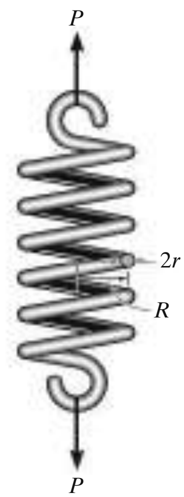
8–49. The bent shaft is fixed in the wall at A. If a force **F** is applied at B, determine the stress components at points D and E. Show the results on a volume element located at each of these points. Take $F = 12$ lb and $\theta = 45°$.

8



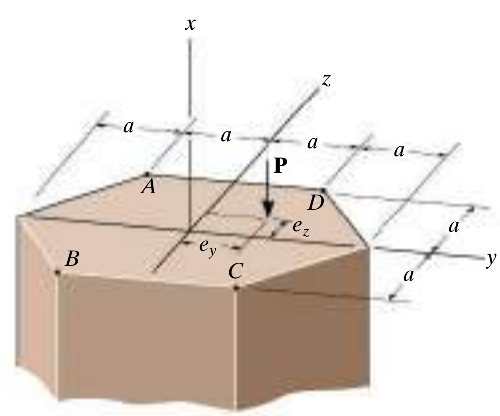
Probs. 8–47/48/49

8–50. The coiled spring is subjected to a force P. If we assume the shear stress caused by the shear force at any vertical section of the coil wire to be uniform, show that the maximum shear stress in the coil is $\tau_{\max} = P/A + PRr/J$, where J is the polar moment of inertia of the coil wire and A is its cross-sectional area.



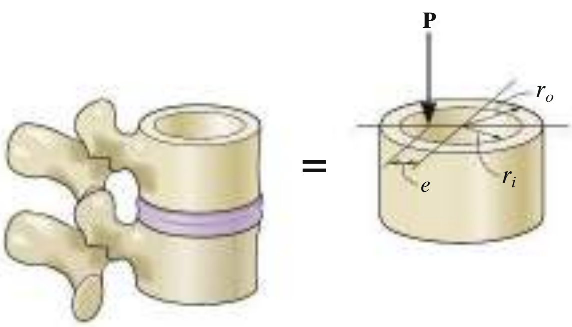
Prob. 8–50

8–51. A post having the dimensions shown is subjected to the bearing load **P**. Specify the region to which this load can be applied without causing tensile stress to be developed at points A, B, C, and D.



Prob. 8–51

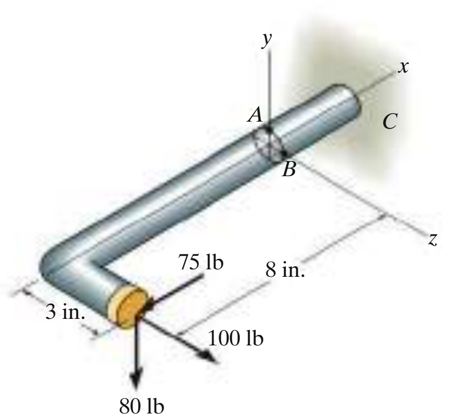
***8–52.** The vertebra of the spinal column can support a maximum compressive stress of σ_{max}, before undergoing a compression fracture. Determine the smallest force P that can be applied to a vertebra, if we assume this load is applied at an eccentric distance e from the centerline of the bone, and the bone remains elastic. Model the vertebra as a hollow cylinder with an inner radius r_i and outer radius r_o.



Prob. 8–52

8–53. The 1-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point A, and show the results on a differential element located at this point.

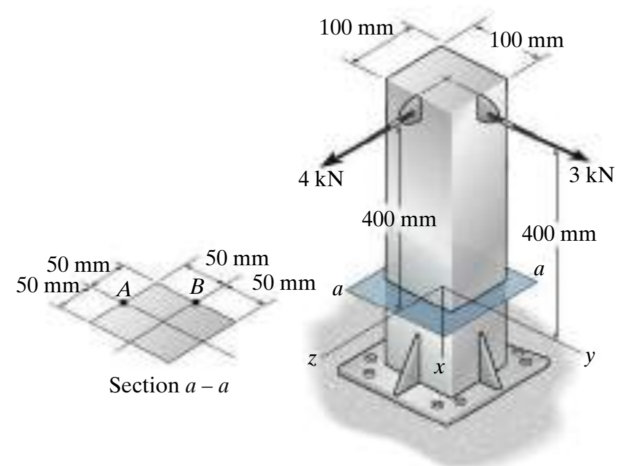
8–54. The 1-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point B, and show the results on a differential element located at this point.



Probs. 8–53/54

8–55. Determine the state of stress at point A on the cross section of the post at section a–a. Indicate the results on a differential element at the point.

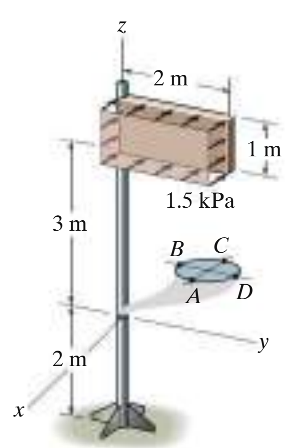
***8–56.** Determine the state of stress at point B on the cross section of the post at section a–a. Indicate the results on a differential element at the point.



Probs. 8–55/56

8–57. The sign is subjected to the uniform wind loading. Determine the stress components at points A and B on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.

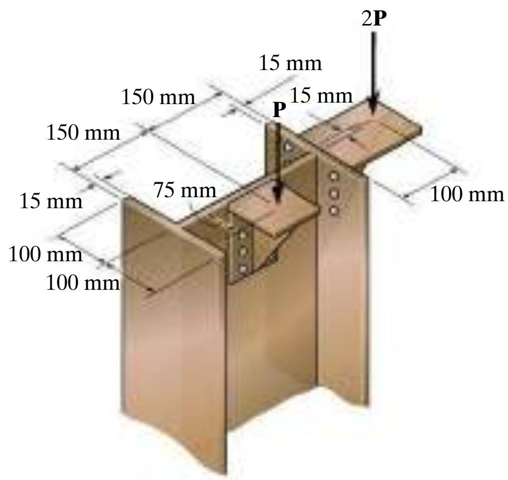
8–58. The sign is subjected to the uniform wind loading. Determine the stress components at points C and D on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.



Probs. 8–57/58

8–59. If $P = 60$ kN, determine the maximum normal stress developed on the cross section of the column.

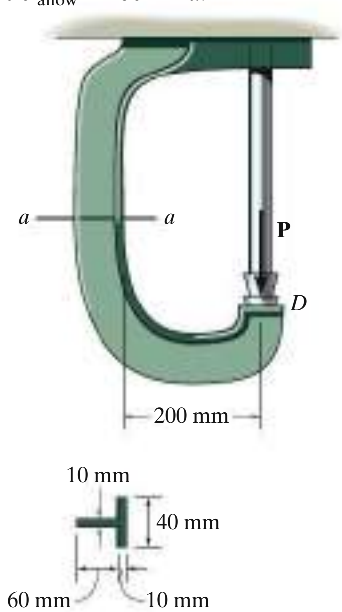
***8–60.** Determine the maximum allowable force **P**, if the column is made from material having an allowable normal stress of $\sigma_{\text{allow}} = 100$ MPa.



Probs. 8–59/60

8–61. The C-frame is used in a riveting machine. If the force at the ram on the clamp at D is $P = 8$ kN, sketch the stress distribution acting over the section a–a.

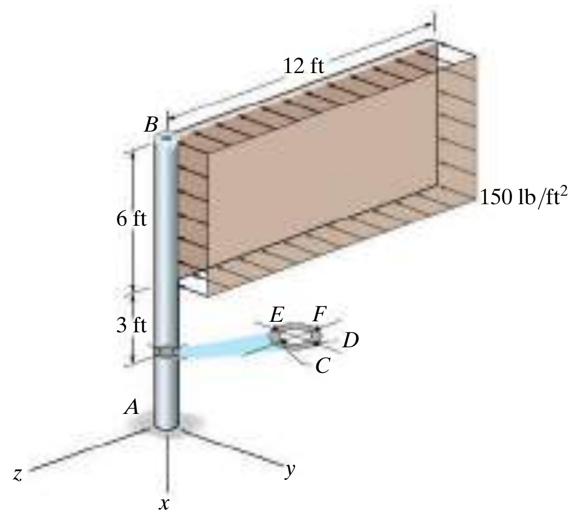
8–62. Determine the maximum ram force P that can be applied to the clamp at D if the allowable normal stress for the material is $\sigma_{\text{allow}} = 180$ MPa.



Probs. 8–61/62

8–63. The uniform sign has a weight of 1500 lb and is supported by the pipe AB, which has an inner radius of 2.75 in. and an outer radius of 3.00 in. If the face of the sign is subjected to a uniform wind pressure of $p = 150\ \text{lb/ft}^2$, determine the state of stress at points C and D. Show the results on a differential volume element located at each of these points. Neglect the thickness of the sign, and assume that it is supported along the outside edge of the pipe.

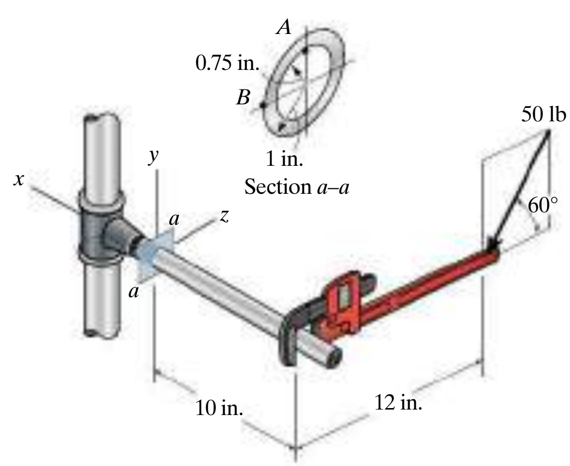
***8–64.** Solve Prob. 8–63 for points E and F.



Probs. 8–63/64

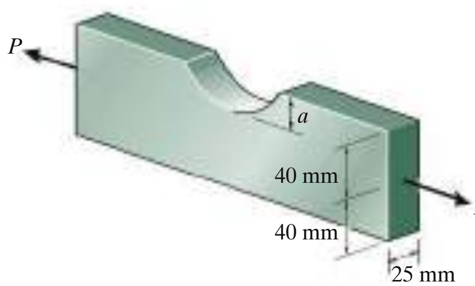
8–65. Determine the state of stress at point A on the cross section of the pipe at section a–a.

8–66. Determine the state of stress at point B on the cross section of the pipe at section a–a.



Probs. 8–65/66

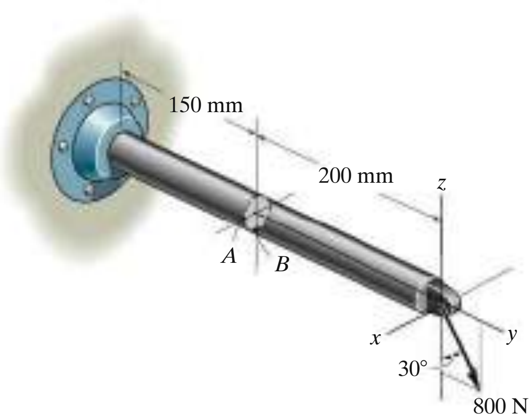
8–67. The metal link is subjected to the axial force of $P = 7$ kN. Its original cross section is to be altered by cutting a circular groove into one side. Determine the distance a the groove can penetrate into the cross section so that the tensile stress does not exceed $\sigma_{allow} = 175$ MPa. Offer a better way to remove this depth of material from the cross section and calculate the tensile stress for this case. Neglect the effects of stress concentration.



Prob. 8–67

*__8–68.__ The bar has a diameter of 40 mm. If it is subjected to a force of 800 N as shown, determine the stress components that act at point A and show the results on a volume element located at this point.

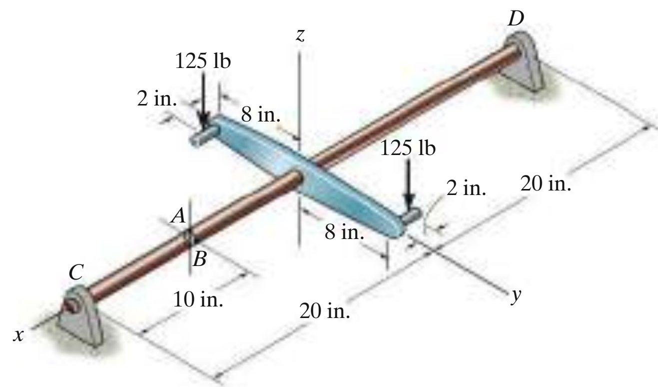
8–69. Solve Prob. 8–68 for point B.



Probs. 8–68/69

8–70. The $\frac{3}{4}$-in.-diameter shaft is subjected to the loading shown. Determine the stress components at point A. Sketch the results on a volume element located at this point. The journal bearing at C can exert only force components $\mathbf{C}_y$ and $\mathbf{C}_z$ on the shaft, and the thrust bearing at D can exert force components $\mathbf{D}_x$, $\mathbf{D}_y$, and $\mathbf{D}_z$ on the shaft.

8–71. Solve Prob. 8–70 for the stress components at point B.

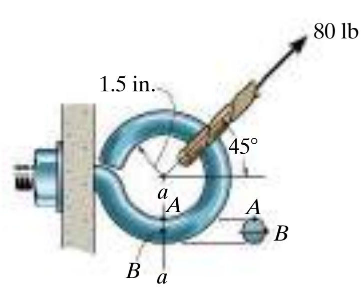


Probs. 8–70/71

*__8–72.__ The hook is subjected to the force of 80 lb. Determine the state of stress at point A at section a–a. The cross section is circular and has a diameter of 0.5 in. Use the curved-beam formula to compute the bending stress.

8–73. The hook is subjected to the force of 80 lb. Determine the state of stress at point B at section a–a. The cross section has a diameter of 0.5 in. Use the curved-beam formula to compute the bending stress.

8



Probs. 8–72/73

CHAPTER REVIEW

A pressure vessel is considered to have a thin wall provided $r/t \geq 10$. For a thin-walled cylindrical vessel, the circumferential or hoop stress is

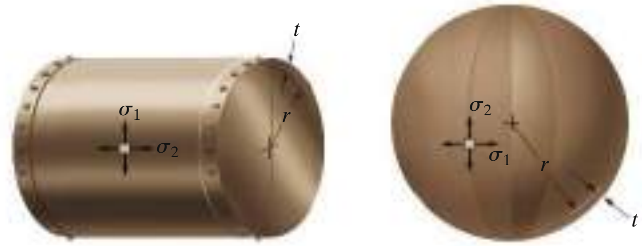
$$\sigma_1 = \frac{pr}{t}$$

This stress is twice as great as the longitudinal stress,

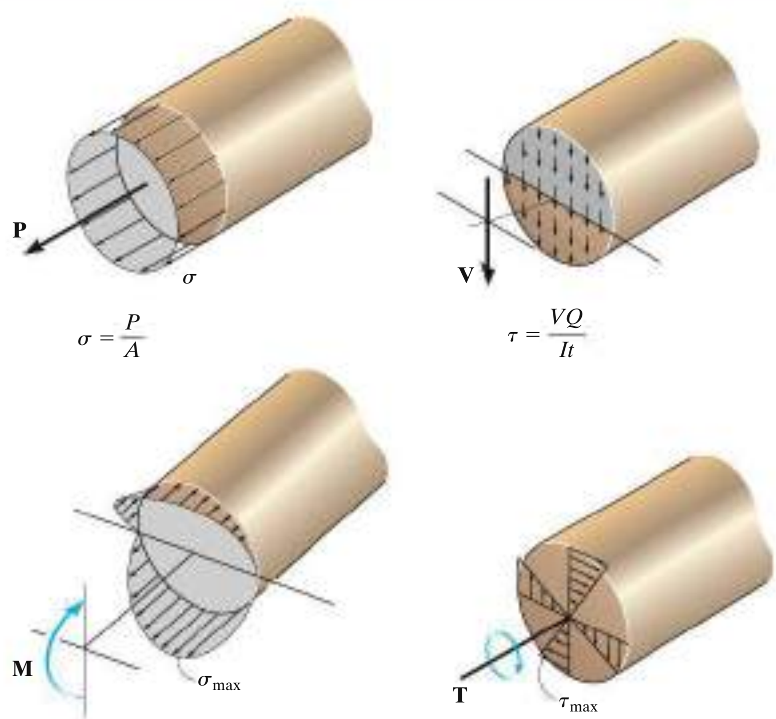
$$\sigma_2 = \frac{pr}{2t}$$

Thin-walled spherical vessels have the same stress within their walls in all directions. It is

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$



Superposition of stress components can be used to determine the normal and shear stress at a point in a member subjected to a combined loading. To do this, it is first necessary to determine the resultant axial and shear forces and the internal resultant torsional and bending moments at the section where the point is located. Then the normal and shear stress resultant components at the point are determined by algebraically adding the normal and shear stress components of each loading.



$$\sigma = \frac{P}{A} \qquad \tau = \frac{VQ}{It}$$

$$\sigma = -\frac{My}{I} \qquad \tau = \frac{T\rho}{J}$$

8