The normal distribution

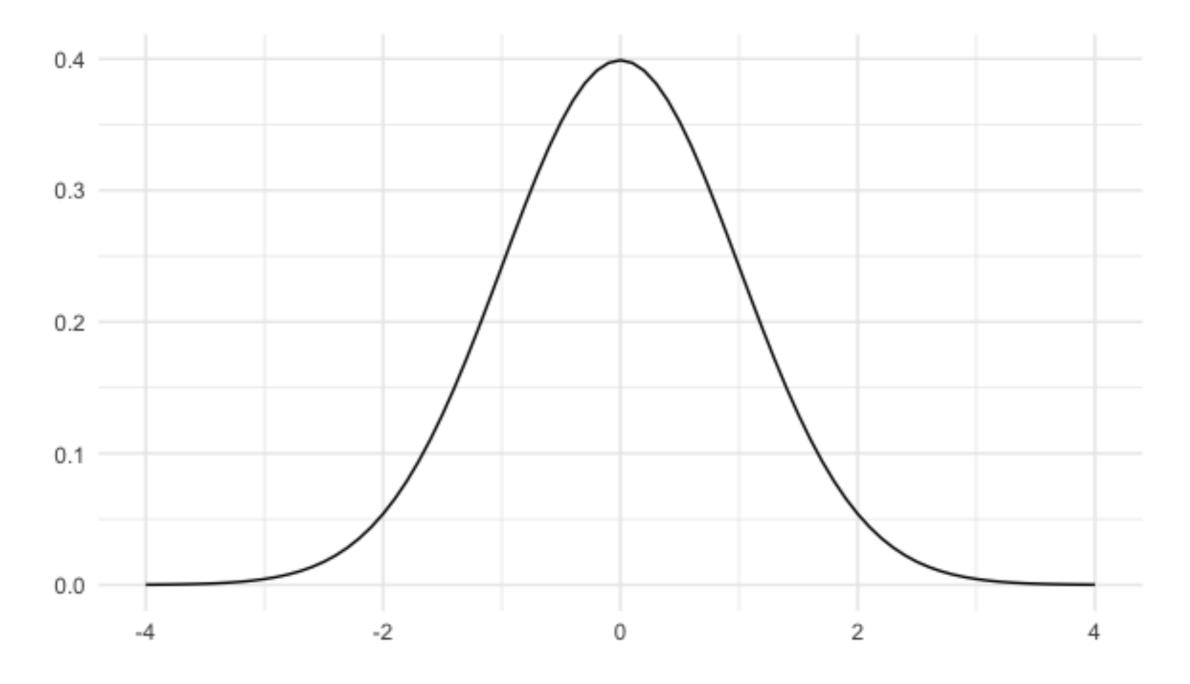
INTRODUCTION TO STATISTICS IN PYTHON



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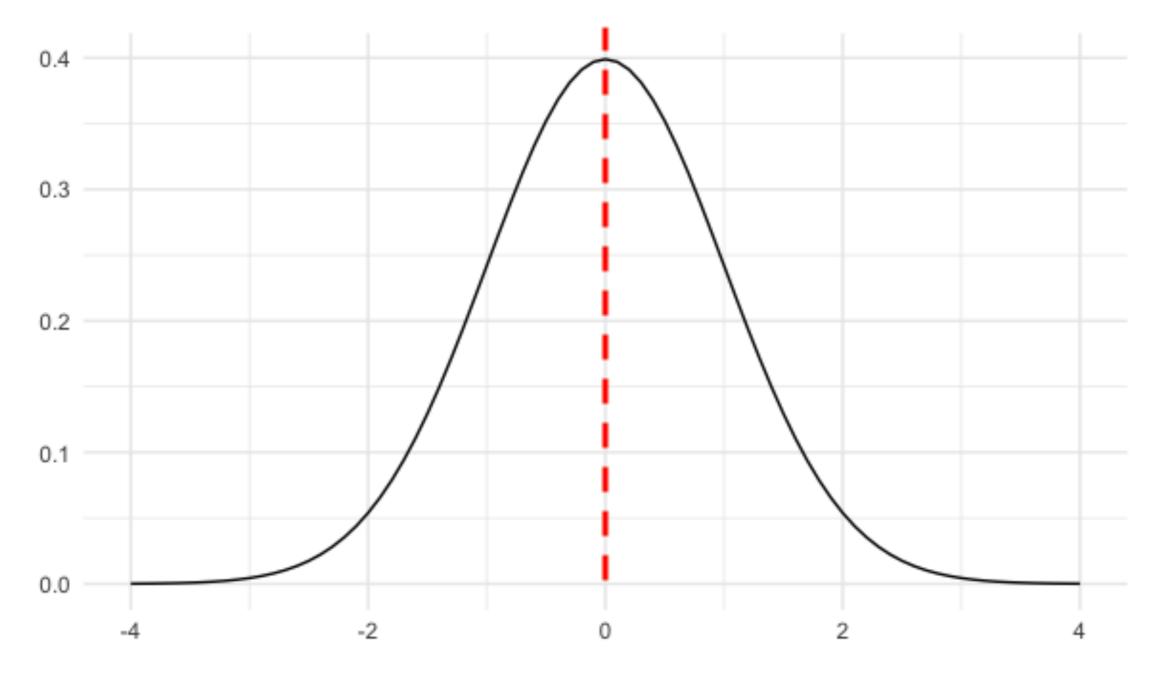


What is the normal distribution?



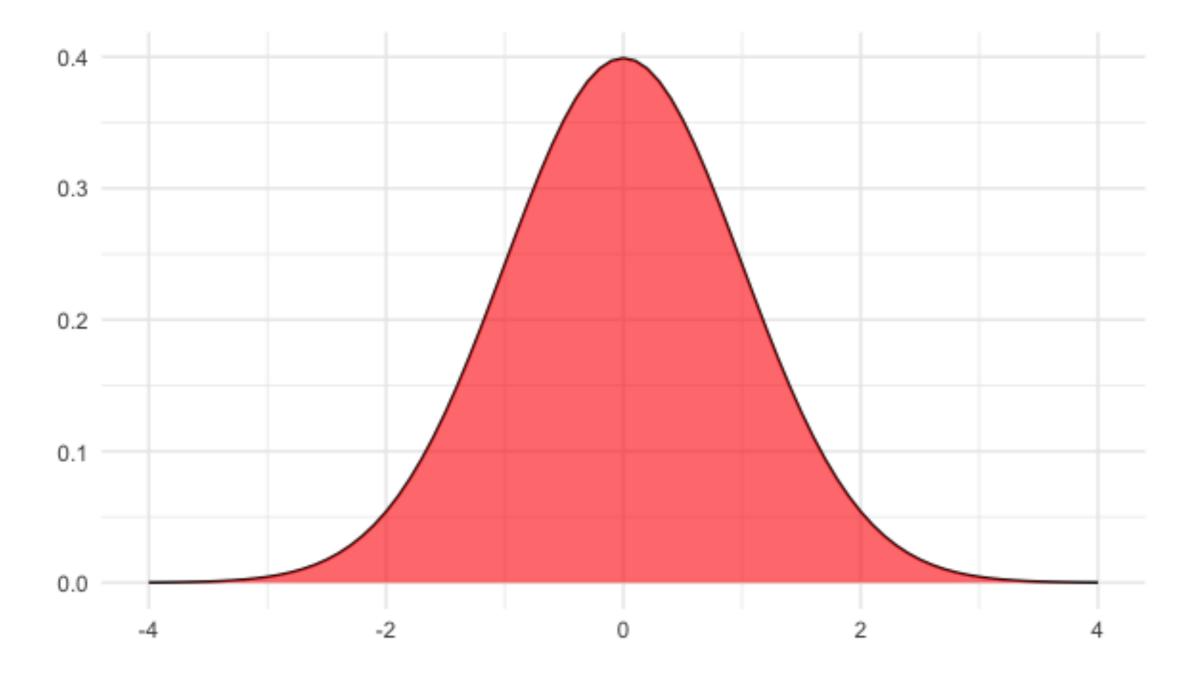


Symmetrical

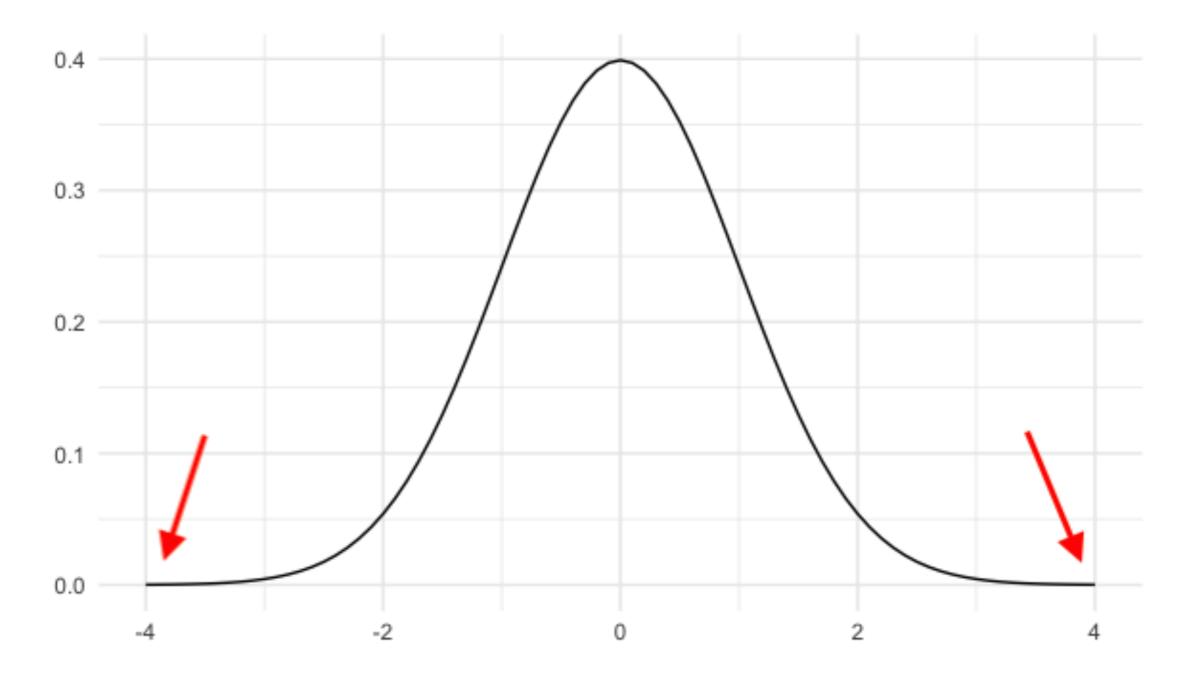




Area = 1



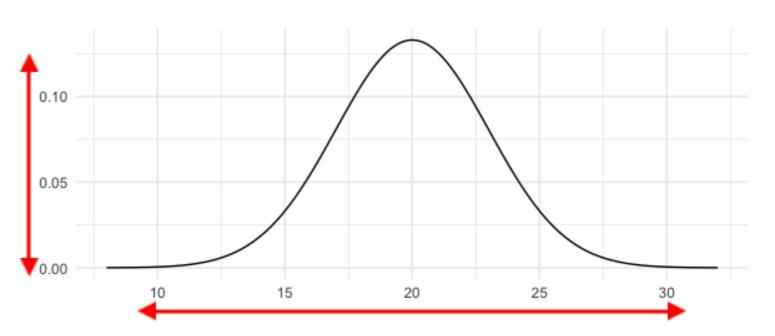
Curve never hits 0



Described by mean and standard deviation

Mean: 20

Standard deviation:



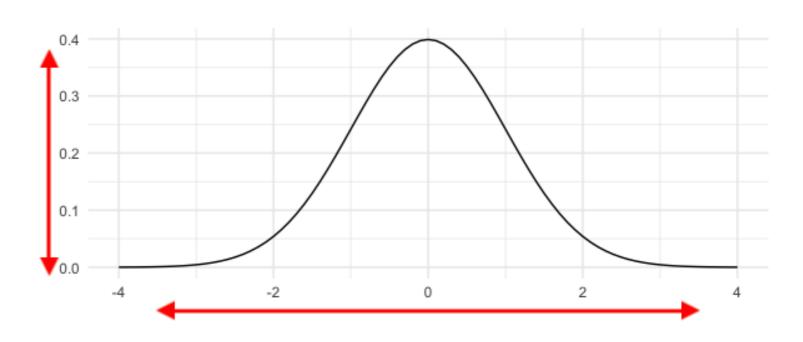
3

Standard normal

distribution

Mean: 0

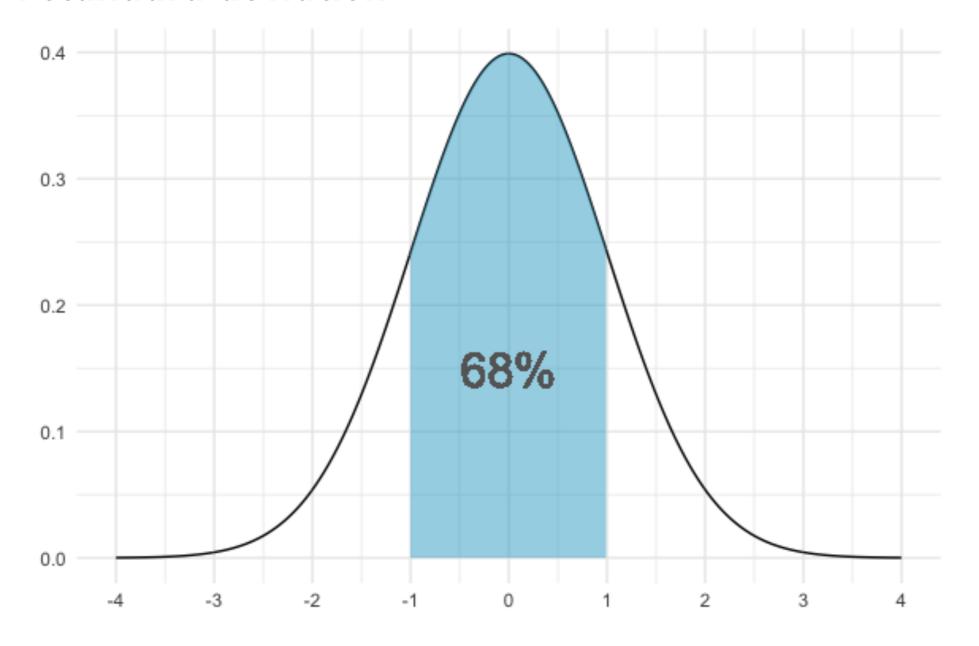
Standard deviation:



1

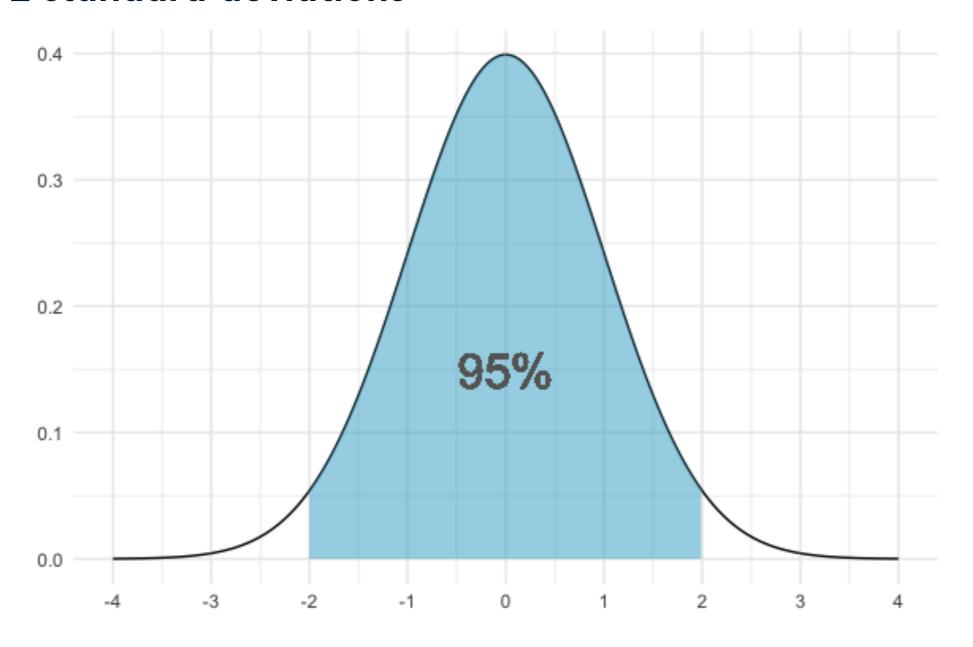
Areas under the normal distribution

68% falls within 1 standard deviation



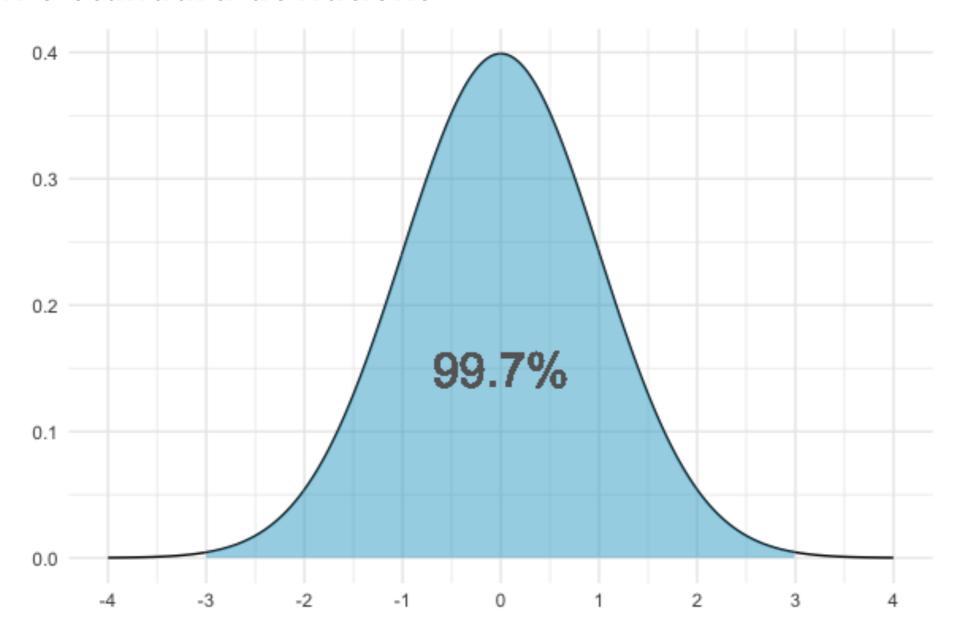
Areas under the normal distribution

95% falls within 2 standard deviations



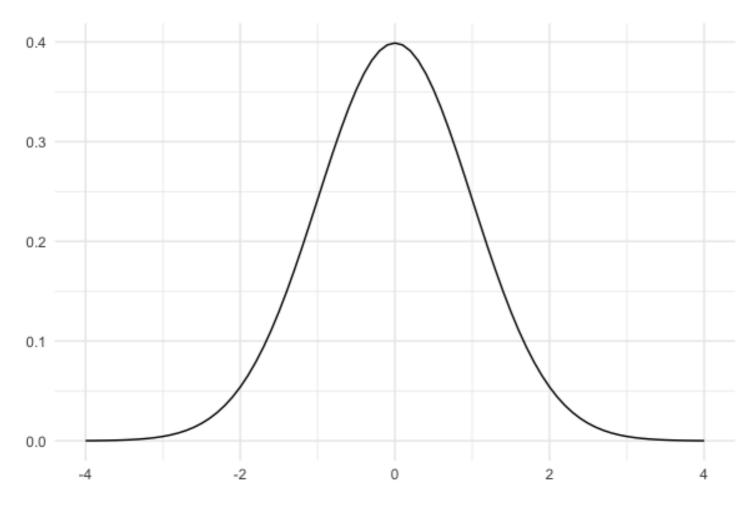
Areas under the normal distribution

99.7% falls within 3 standard deviations

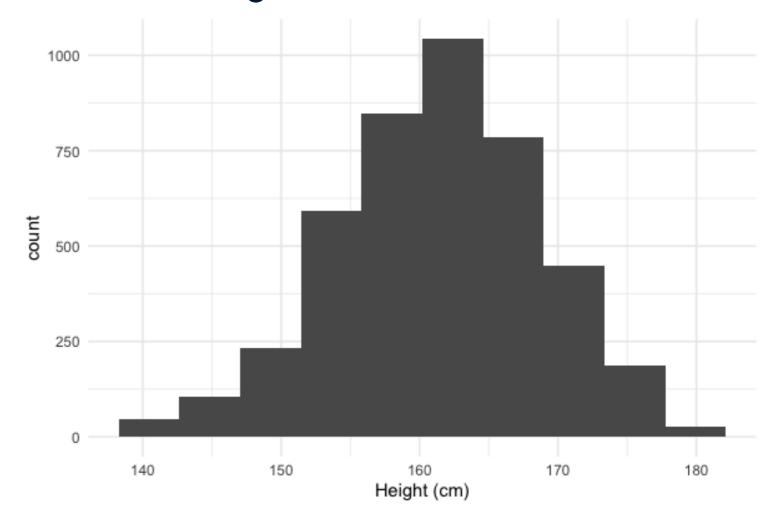


Lots of histograms look normal

Normal distribution



Women's heights from NHANES



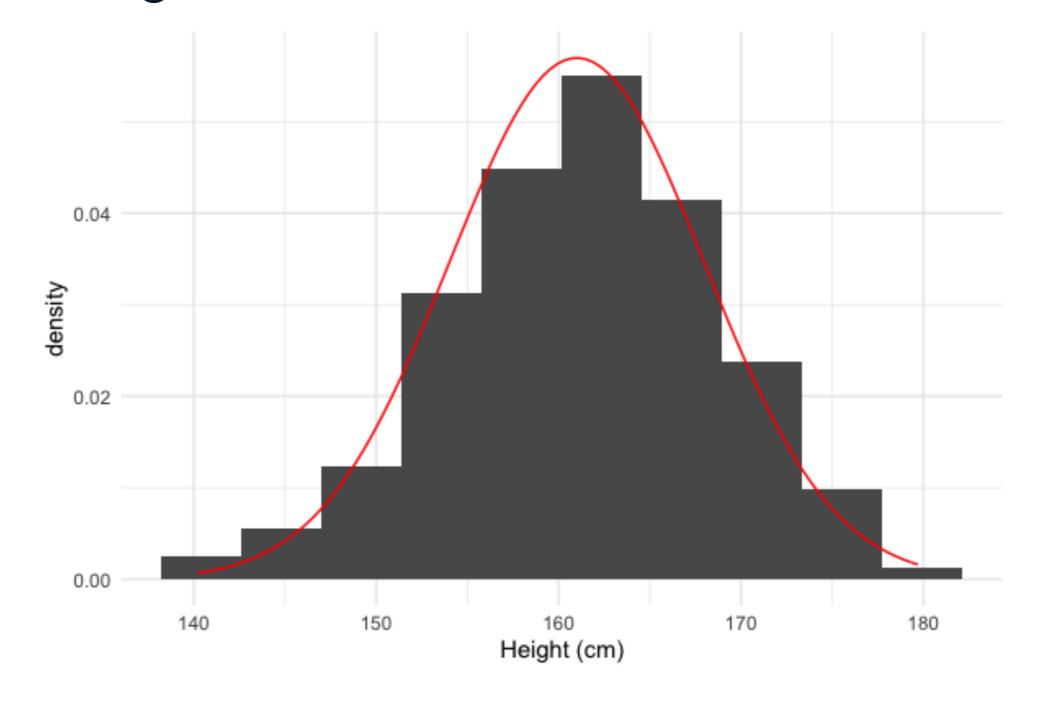
Mean: 161 cm

cm Standard

deviation: 7 cm

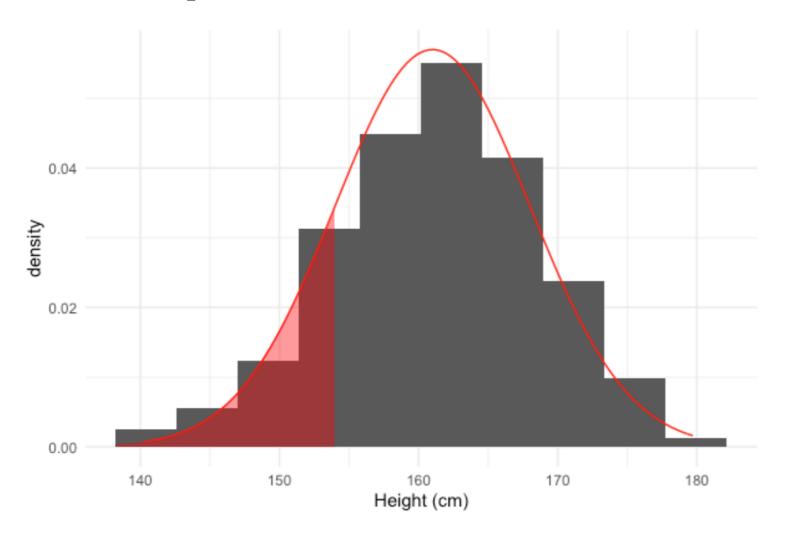


Approximating data with the normal distribution





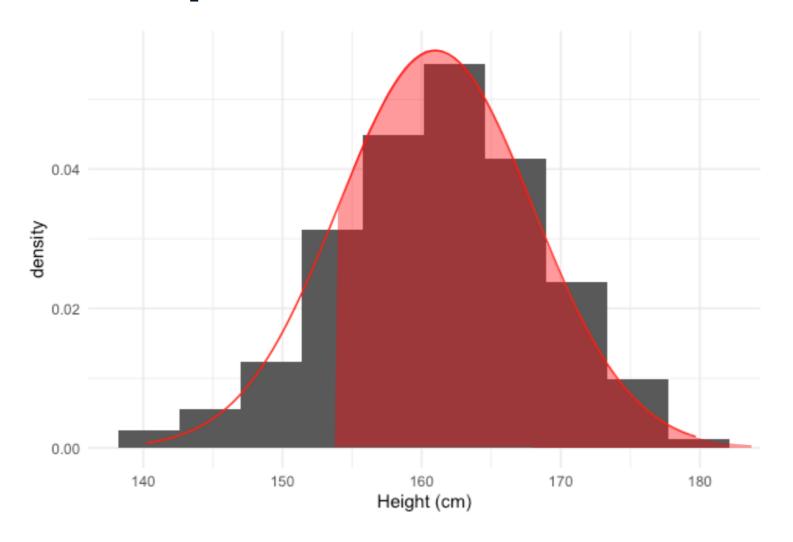
What percent of women are shorter than 154 cm?



16% of women in the survey are shorter than 154 cm

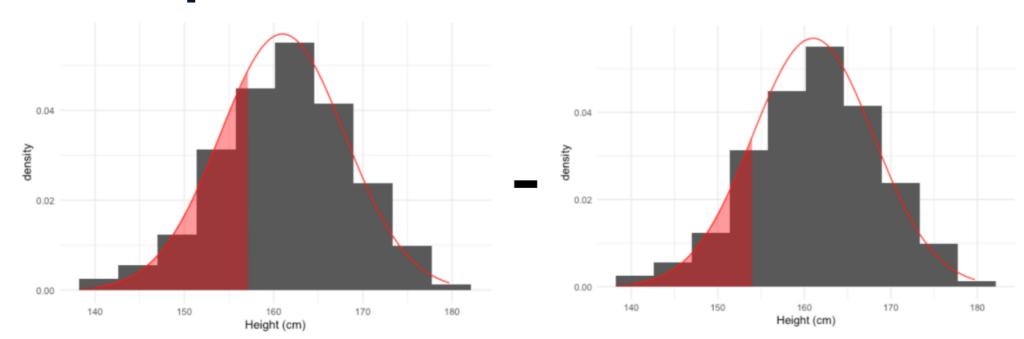
```
from scipy.stats import norm
norm.cdf(154, 161, 7)
```

What percent of women are taller than 154 cm?



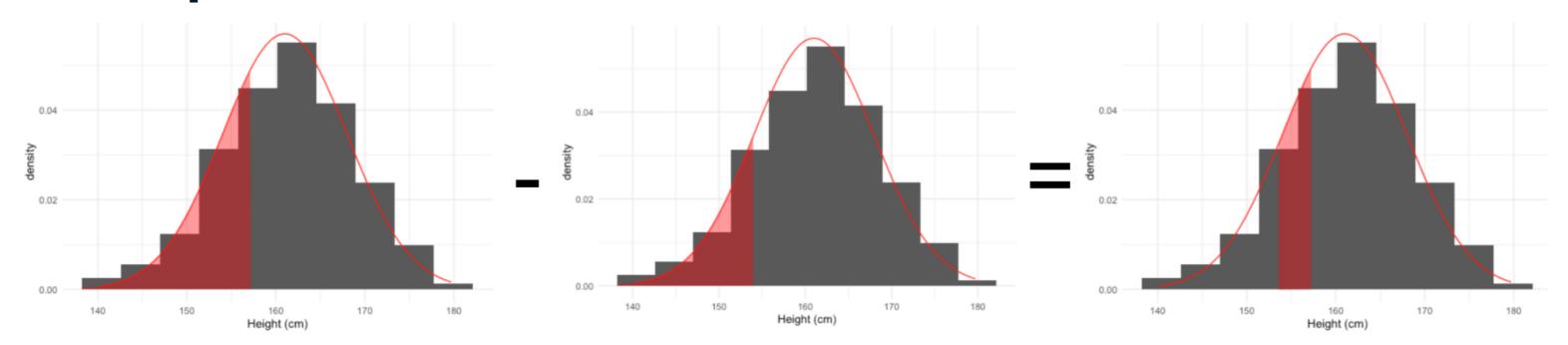
```
from scipy.stats import norm
1 - norm.cdf(154, 161, 7)
```

What percent of women are 154-157 cm?



norm.cdf(157, 161, 7) - norm.cdf(154, 161, 7)

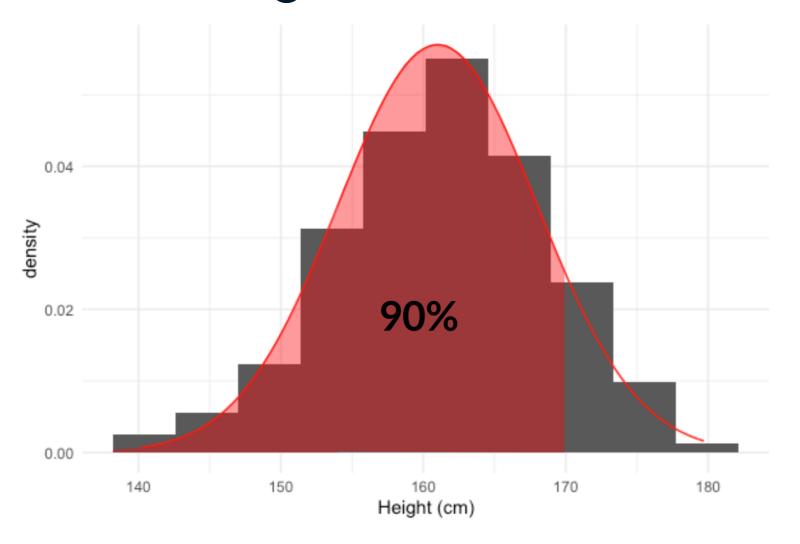
What percent of women are 154-157 cm?



norm.cdf(157, 161, 7) - norm.cdf(154, 161, 7)

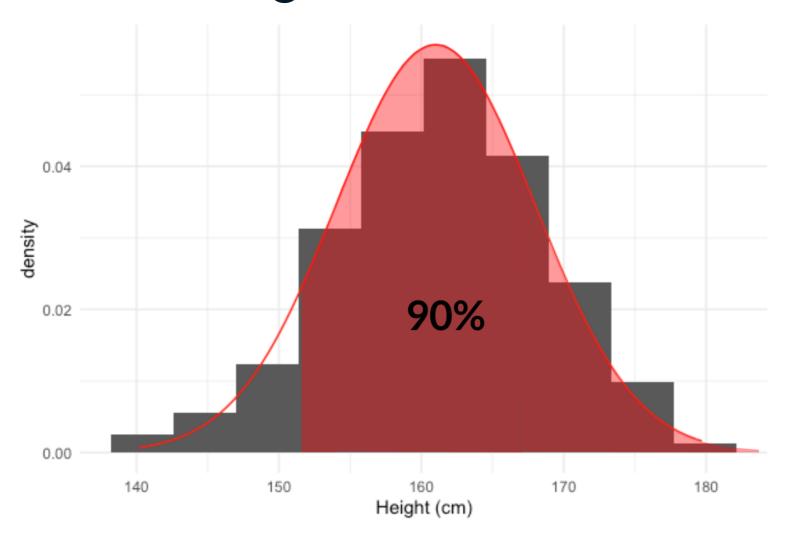


What height are 90% of women shorter than?



norm.ppf(0.9, 161, 7)

What height are 90% of women taller than?



norm.ppf(
$$(1-0.9)$$
, 161 , 7)

Generating random numbers

The central limit theorem

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Rolling the dice 5 times

```
die = pd.Series([1, 2, 3, 4, 5, 6])
# Roll 5 times
samp_5 = die.sample(5, replace=True)
print(samp_5)
```

```
array([3, 1, 4, 1, 1])
```



Rolling the dice 5 times

```
# Roll 5 times and take mean
samp_5 = die.sample(5, replace=True)
np.mean(samp_5)
```

4.4

```
samp_5 = die.sample(5, replace=True)
np.mean(samp_5)
```



Rolling the dice 5 times 10 times

Repeat 10 times:

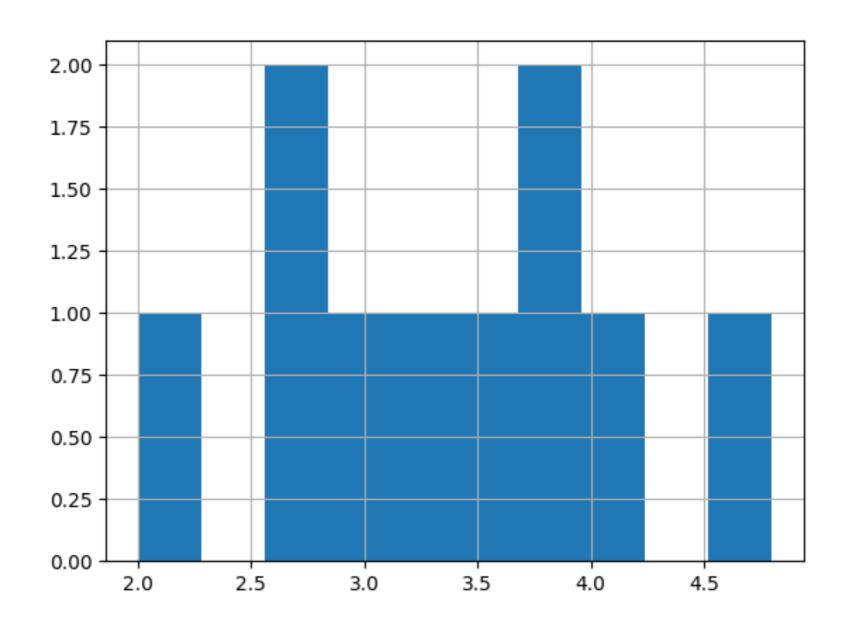
- Roll 5 times
- Take the mean

```
sample_means = []
for i in range(10):
    samp_5 = die.sample(5, replace=True)
    sample_means.append(np.mean(samp_5))
print(sample_means)
```

```
[3.8, 4.0, 3.8, 3.6, 3.2, 4.8, 2.6, 3.0, 2.6, 2.0]
```

Sampling distributions

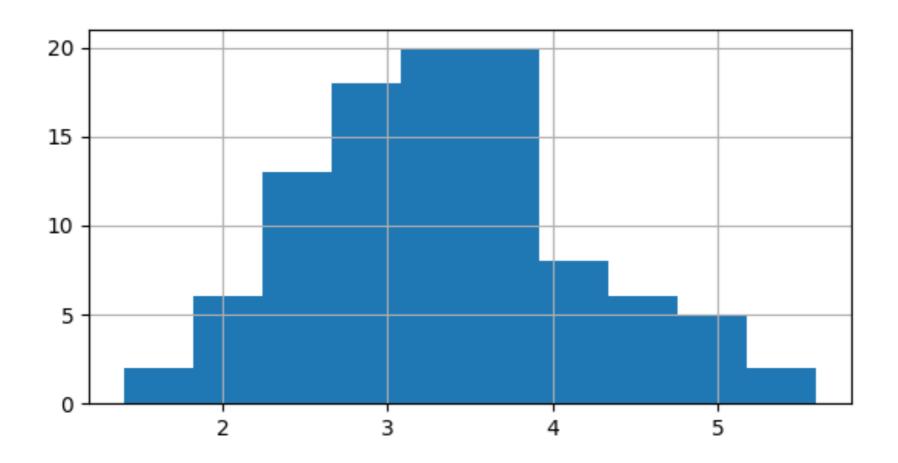
Sampling distribution of the sample mean





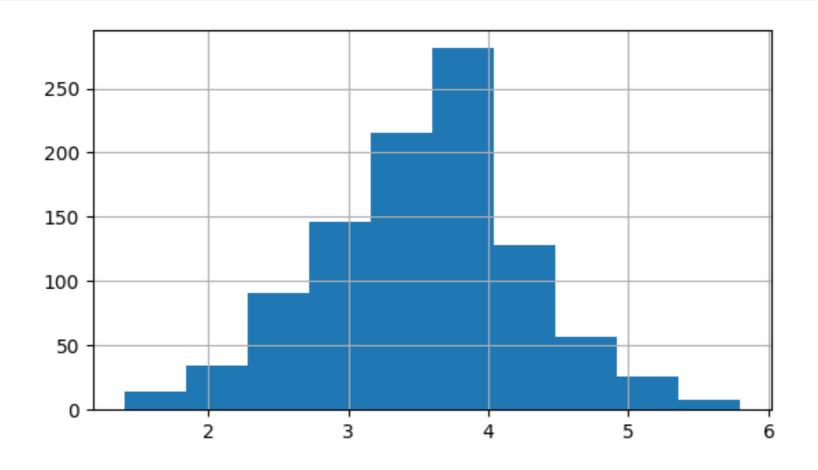
100 sample means

```
sample_means = []
for i in range(100):
    sample_means.append(np.mean(die.sample(5, replace=True)))
```



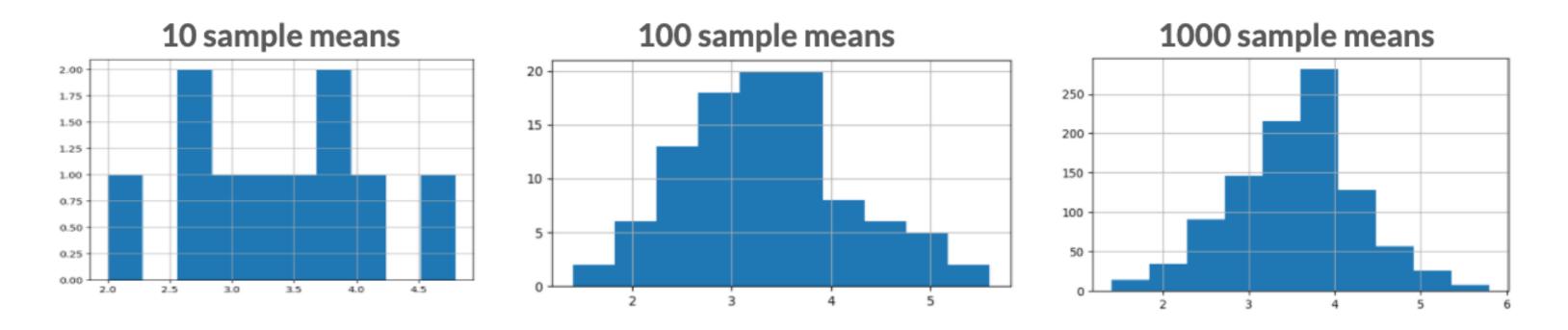
1000 sample means

```
sample_means = []
for i in range(1000):
    sample_means.append(np.mean(die.sample(5, replace=True)))
```



Central limit theorem

The sampling distribution of a statistic becomes closer to the normal distribution as the number of trials increases.

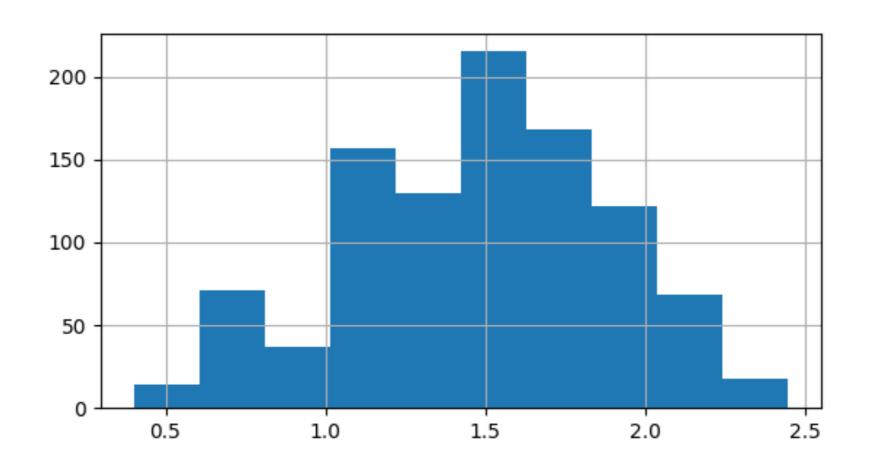




^{*} Samples should be random and independent

Standard deviation and the CLT

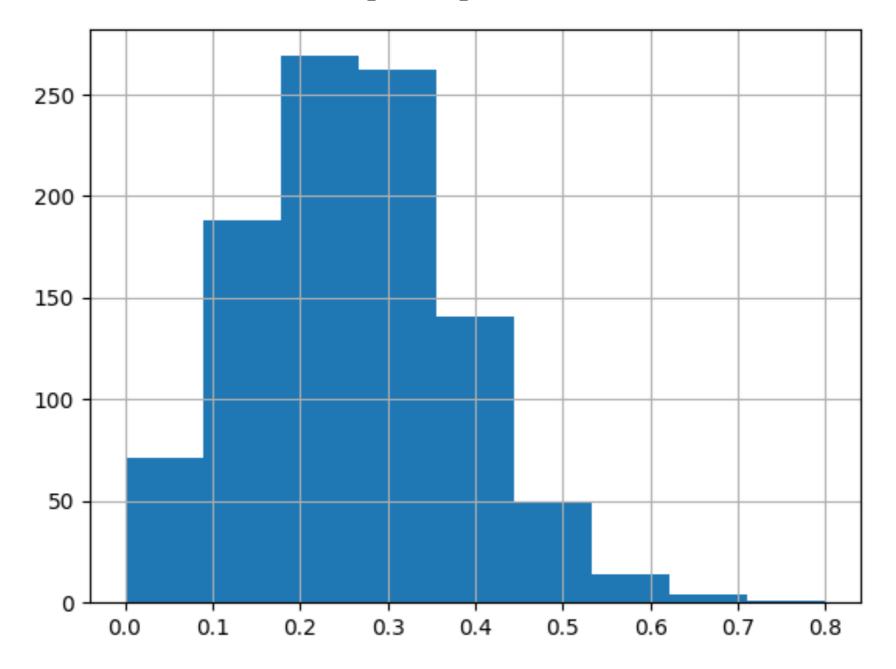
```
sample_sds = []
for i in range(1000):
   sample_sds.append(np.std(die.sample(5, replace=True)))
```



Proportions and the CLT

```
sales_team = pd.Series(["Amir", "Brian", "Claire", "Damian"])
sales_team.sample(10, replace=True)
array(['Claire', 'Damian', 'Brian', 'Damian', 'Damian', 'Amir', 'Amir', 'Amir',
      'Amir', 'Damian'], dtype=object)
sales_team.sample(10, replace=True)
array(['Brian', 'Amir', 'Brian', 'Claire', 'Brian', 'Damian', 'Claire', 'Brian',
      'Claire', 'Claire'], dtype=object)
```

Sampling distribution of proportion



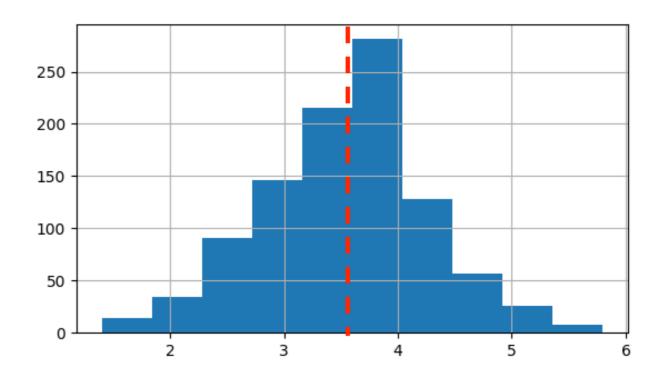


Mean of sampling distribution

```
# Estimate expected value of die
np.mean(sample_means)
```

3.48

```
# Estimate proportion of "Claire"s
np.mean(sample_props)
```



- Estimate characteristics of unknown underlying distribution
- More easily estimate characteristics of large populations

The Poisson distribution

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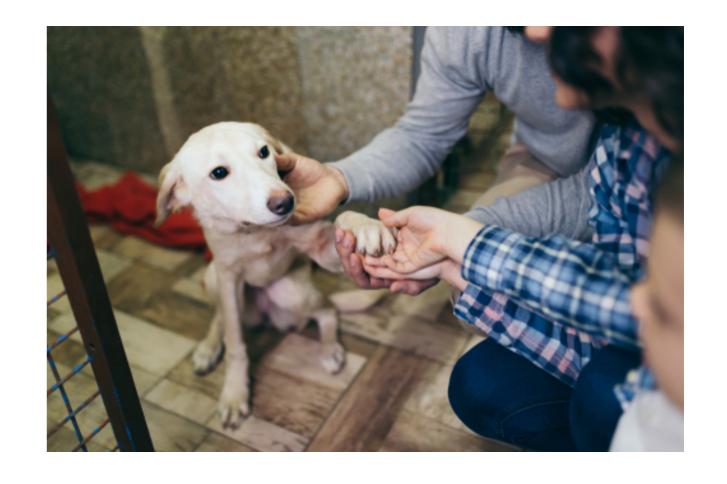


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Poisson processes

- Events appear to happen at a certain rate, but completely at random
- Examples
 - Number of animals adopted from an animal shelter per week
 - Number of people arriving at a restaurant per hour
 - Number of earthquakes in California per year
- Time unit is irrelevant, as long as you use the same unit when talking about the same situation

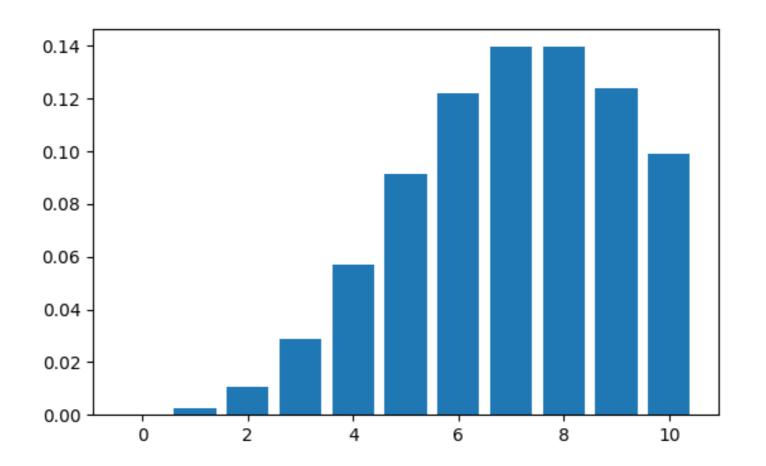


Poisson distribution

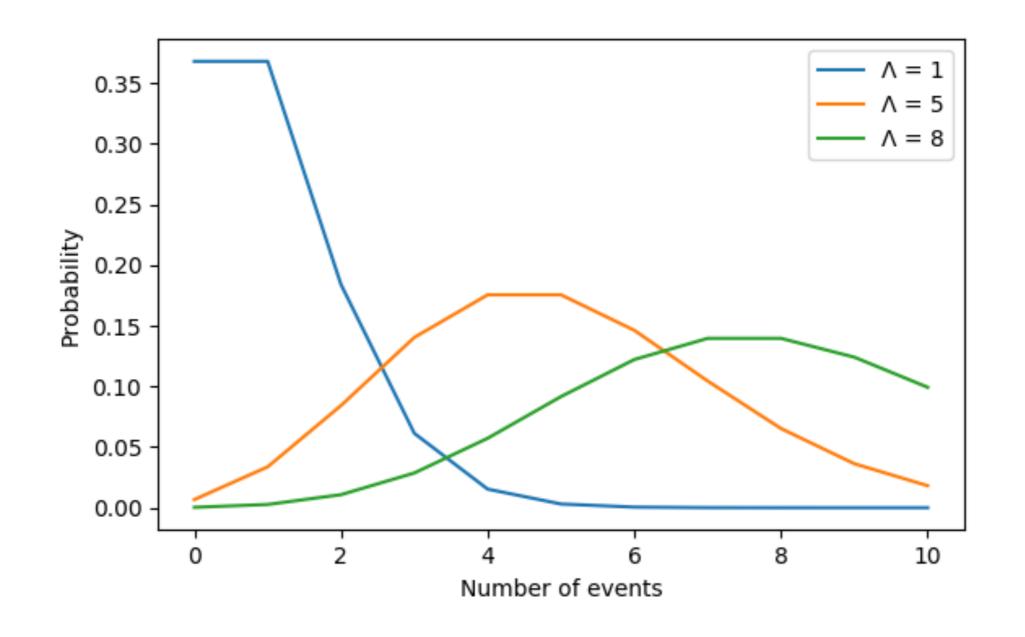
- Probability of some # of events occurring over a fixed period of time
- Examples
 - \circ Probability of \geq 5 animals adopted from an animal shelter per week
 - Probability of 12 people arriving at a restaurant per hour
 - Probability of < 20 earthquakes in California per year

Lambda (λ)

- λ = average number of events per time interval
 - Average number of adoptions per week = 8



Lambda is the distribution's peak





Probability of a single value

If the average number of adoptions per week is 8, what is $P(\# ext{adoptions in a week} = 5)$?

```
from scipy.stats import poisson
poisson.pmf(5, 8)
```

Probability of less than or equal to

If the average number of adoptions per week is 8, what is $P(\# \text{ adoptions in a week} \leq 5)$?

```
from scipy.stats import poisson
poisson.cdf(5, 8)
```

0.1912361

Probability of greater than

If the average number of adoptions per week is 8, what is $P(\# ext{adoptions in a week} > 5)$?

```
1 - poisson.cdf(5, 8)
```

0.8087639

If the average number of adoptions per week is 10, what is P(# adoptions in a week > 5)?

```
1 - poisson.cdf(5, 10)
```

0.932914

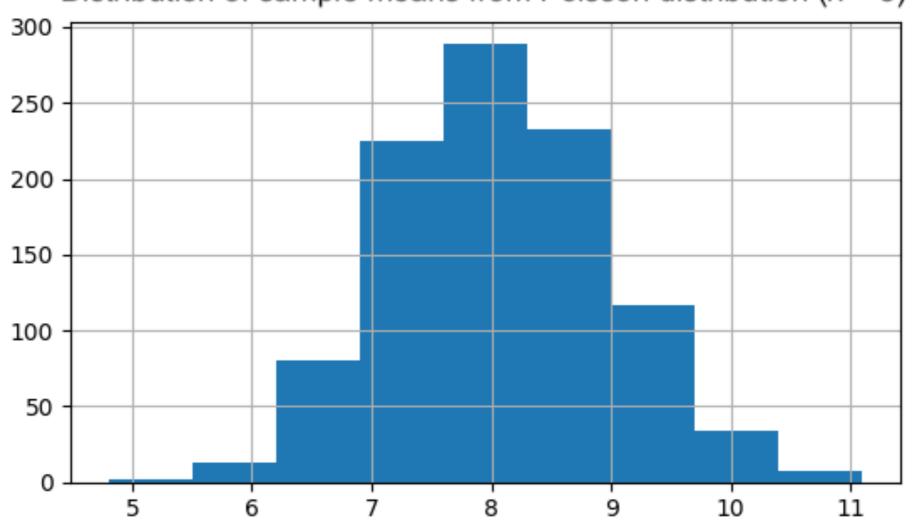
Sampling from a Poisson distribution

```
from scipy.stats import poisson
poisson.rvs(8, size=10)
```

```
array([ 9, 9, 8, 7, 11, 3, 10, 6, 8, 14])
```

The CLT still applies!





More probability distributions

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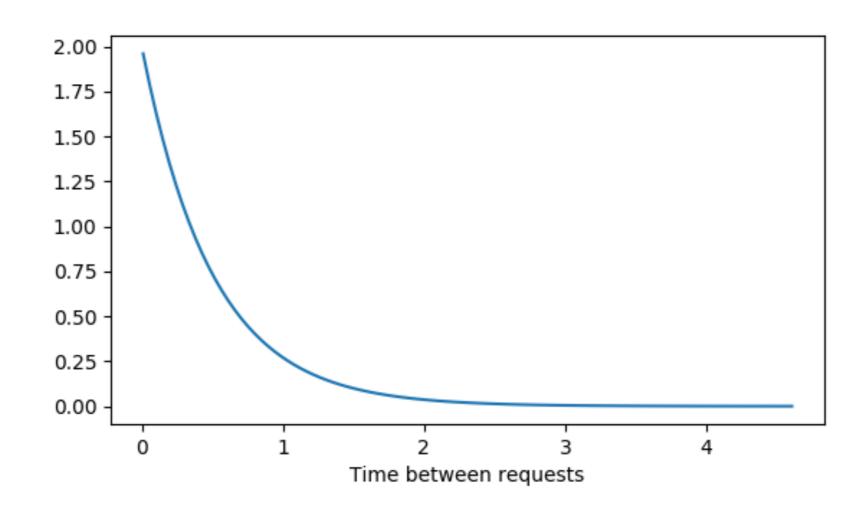


Exponential distribution

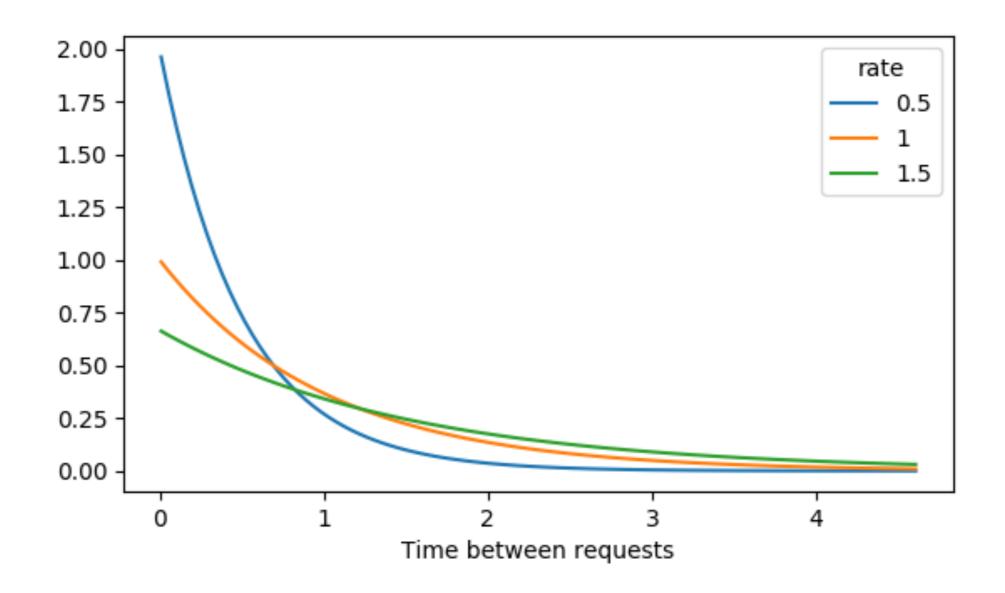
- Probability of time between Poisson events
- Examples
 - Probability of > 1 day between adoptions
 - Probability of < 10 minutes between restaurant arrivals
 - Probability of 6-8 months between earthquakes
- Also uses lambda (rate)
- Continuous (time)

Customer service requests

- On average, one customer service ticket is created every 2 minutes
 - \circ λ = 0.5 customer service tickets created each minute



Lambda in exponential distribution



Expected value of exponential distribution

In terms of rate (Poisson):

• $\lambda = 0.5 \text{ requests per minute}$

In terms of time (exponential):

• $1/\lambda$ = 1 request per 2 minutes

How long until a new request is created?

$$P(\text{wait} < 1 \text{ min}) =$$

from scipy.stats import expon

0.8646647167633873

$$P(\text{wait} > 3 \text{ min}) =$$

$$P(1 \min < \text{wait} < 3 \min) =$$

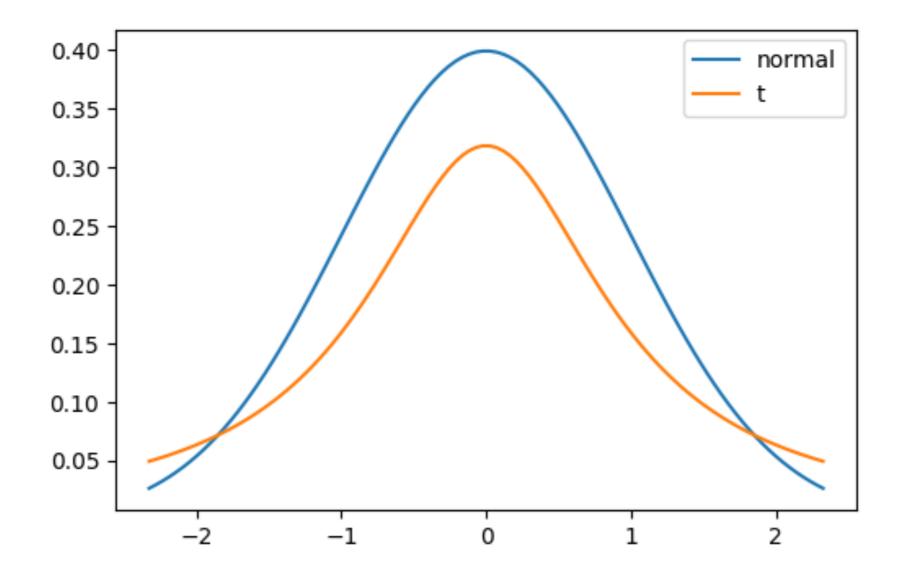
expon.cdf(
$$3$$
, scale= 0.5) - expon.cdf(1 , scale= 0.5)

0.0024787521766663767

0.13285653105994633

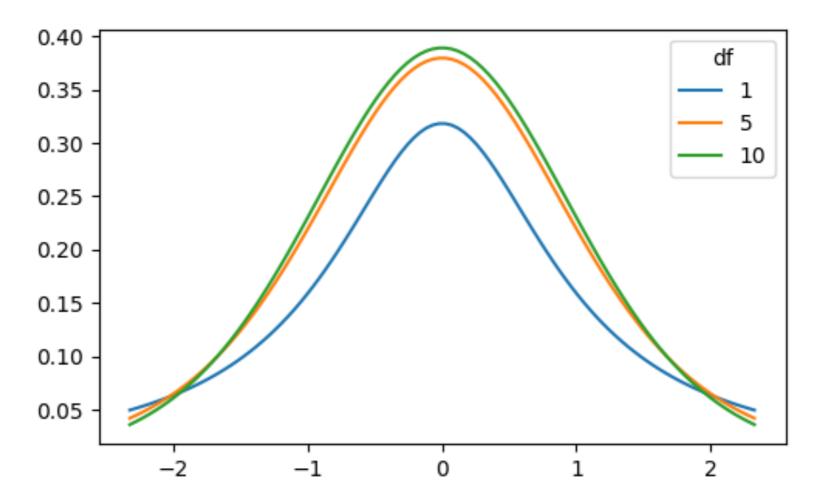
(Student's) t-distribution

• Similar shape as the normal distribution



Degrees of freedom

- Has parameter degrees of freedom (df) which affects the thickness of the tails
 - Lower df = thicker tails, higher standard deviation
 - Higher df = closer to normal distribution



Log-normal distribution

- Variable whose logarithm is normally distributed
- Examples:
 - Length of chess games
 - Adult blood pressure
 - Number of hospitalizations in the 2003
 SARS outbreak

