What are the chances?

INTRODUCTION TO STATISTICS IN PYTHON

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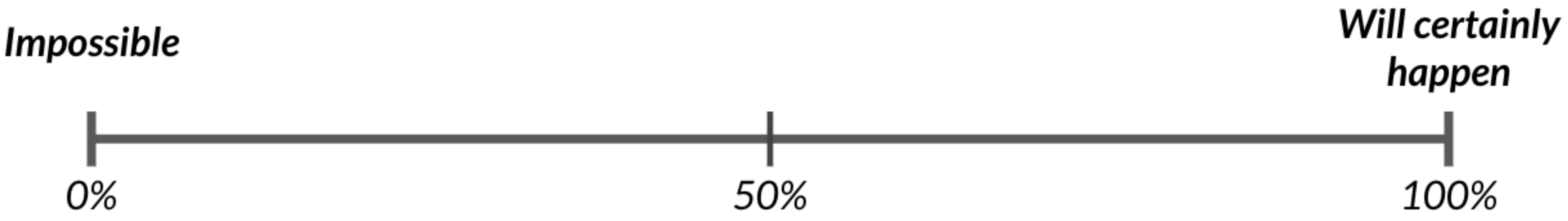
Measuring chance

What's the probability of an event?

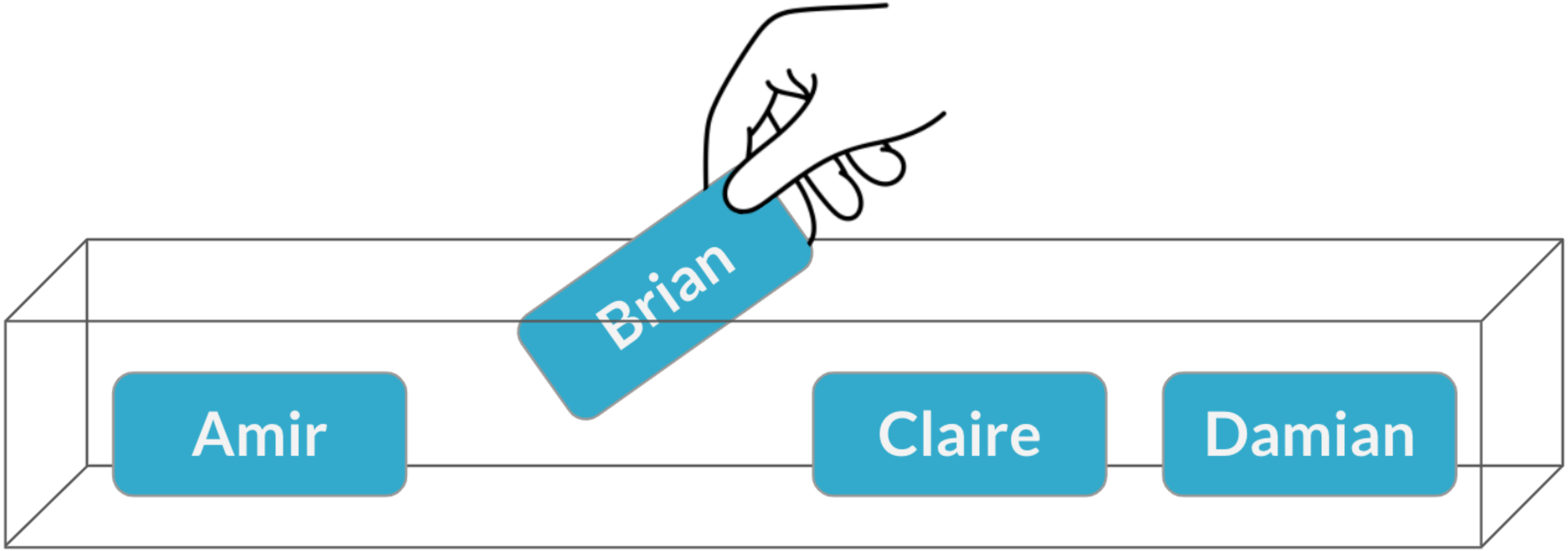
$$P(\text{event}) = \frac{\#\text{ ways event can happen}}{\text{total }\#\text{ of possible outcomes}}$$

Example: a coin flip

$$P(\text{heads}) = \frac{1\text{ way to get heads}}{2\text{ possible outcomes}} = \frac{1}{2} = 50\%$$



Assigning salespeople



$$P(\text{Brian}) = \frac{1}{4} = 25\%$$

Setting a random seed

```
np.random.seed(10)
sales_counts.sample()
```

```
    name  n_sales
1  Brian      128
```

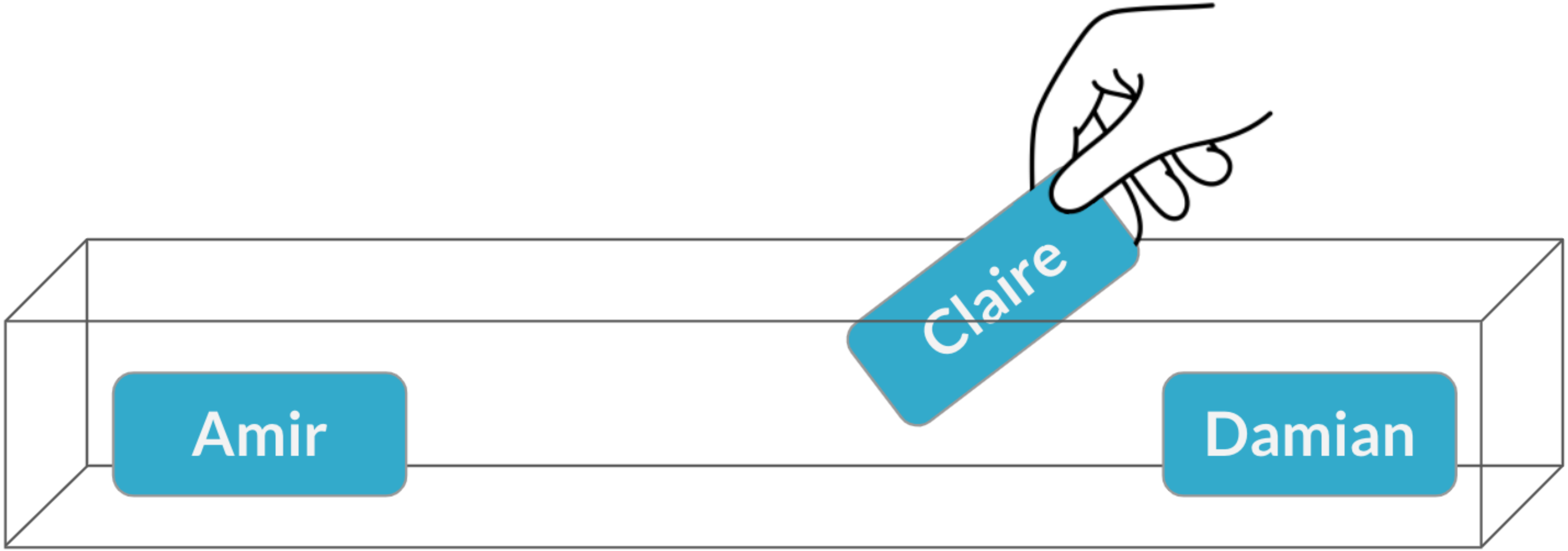
```
np.random.seed(10)
sales_counts.sample()
```

```
    name  n_sales
1  Brian      128
```

```
np.random.seed(10)
sales_counts.sample()
```

```
    name  n_sales
1  Brian      128
```

A second meeting



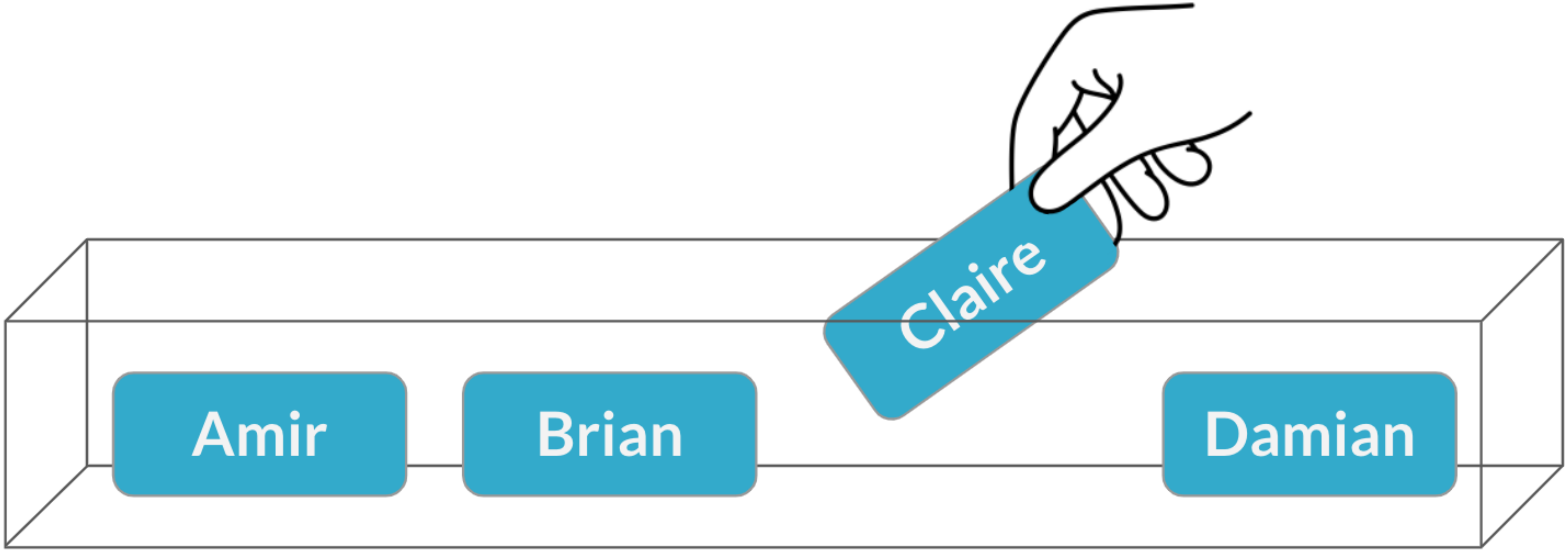
$$P(\text{Claire}) = \frac{1}{3} = 33\%$$

Sampling twice in Python

```
sales_counts.sample(2)
```

```
     name  n_sales
1   Brian      128
2  Claire       75
```

Sampling with replacement



$$P(\text{Claire}) = \frac{1}{4} = 25\%$$

Sampling with/without replacement in Python

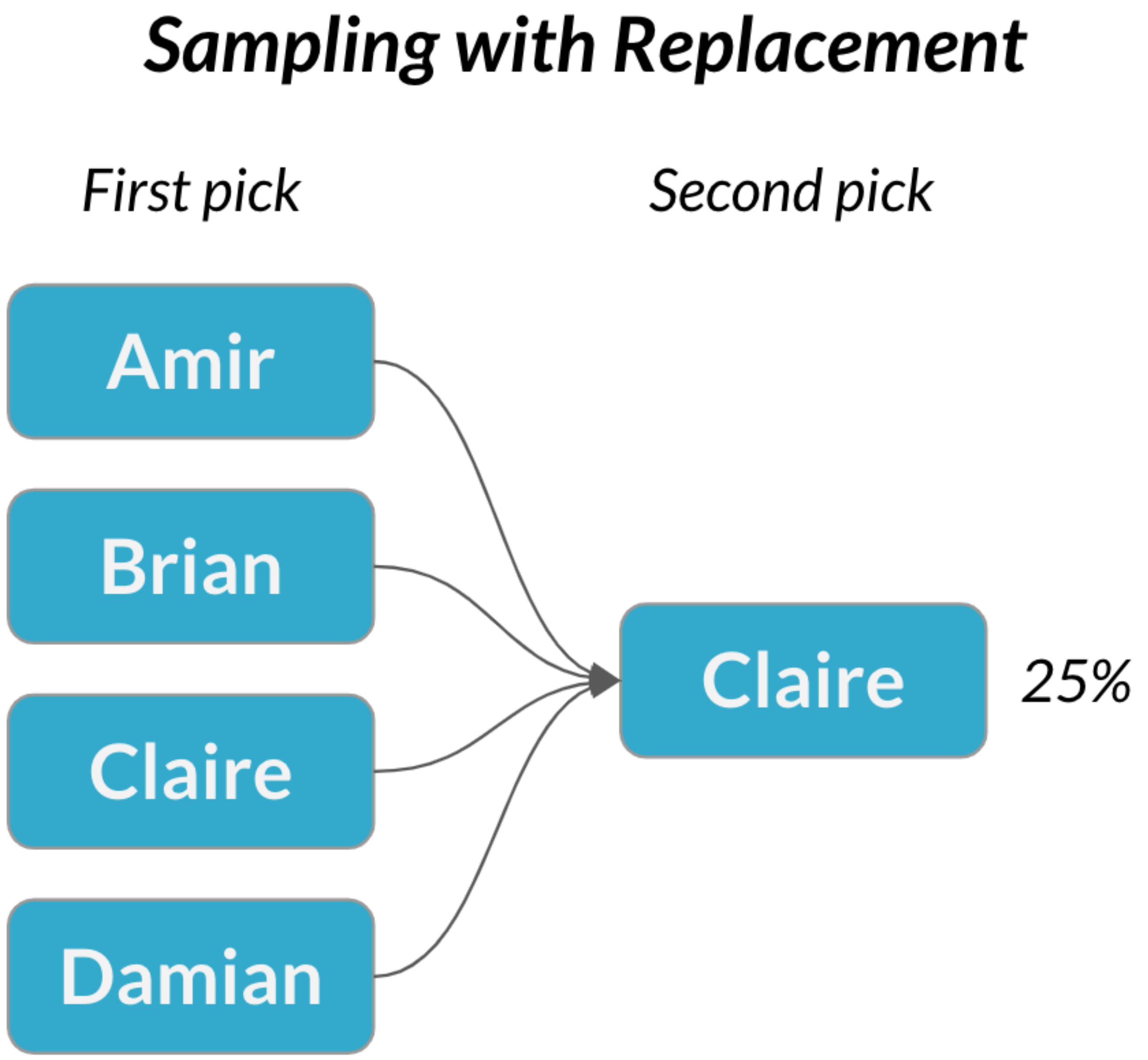
```
sales_counts.sample(5, replace = True)
```

```
     name  n_sales
1   Brian      128
2  Claire       75
1   Brian      128
3  Damian       69
0    Amir      178
```

Independent events

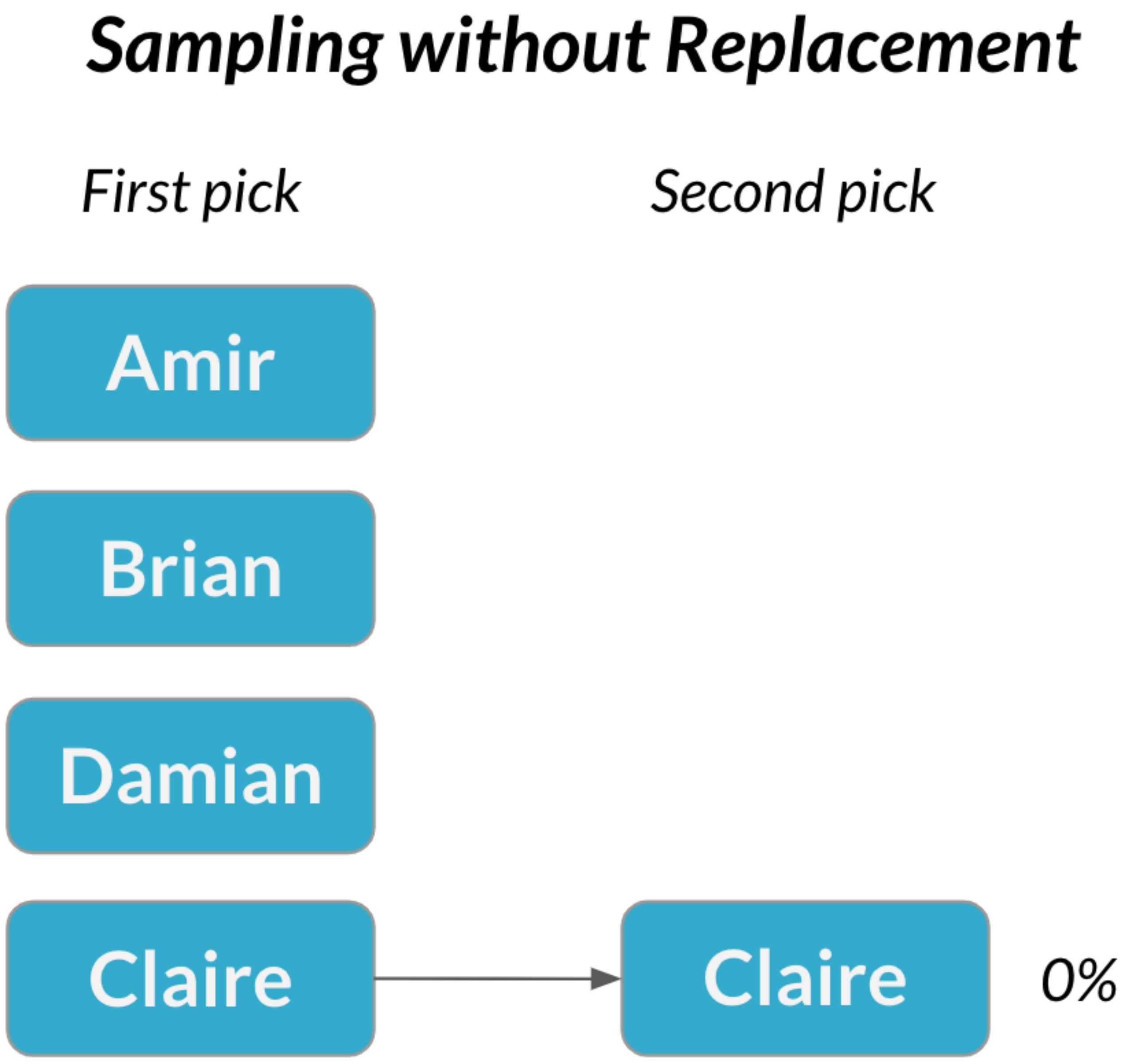
*Two events are **independent** if the probability of the second event **isn't** affected by the outcome of the first event.*

Sampling with replacement = each pick is independent



Dependent events

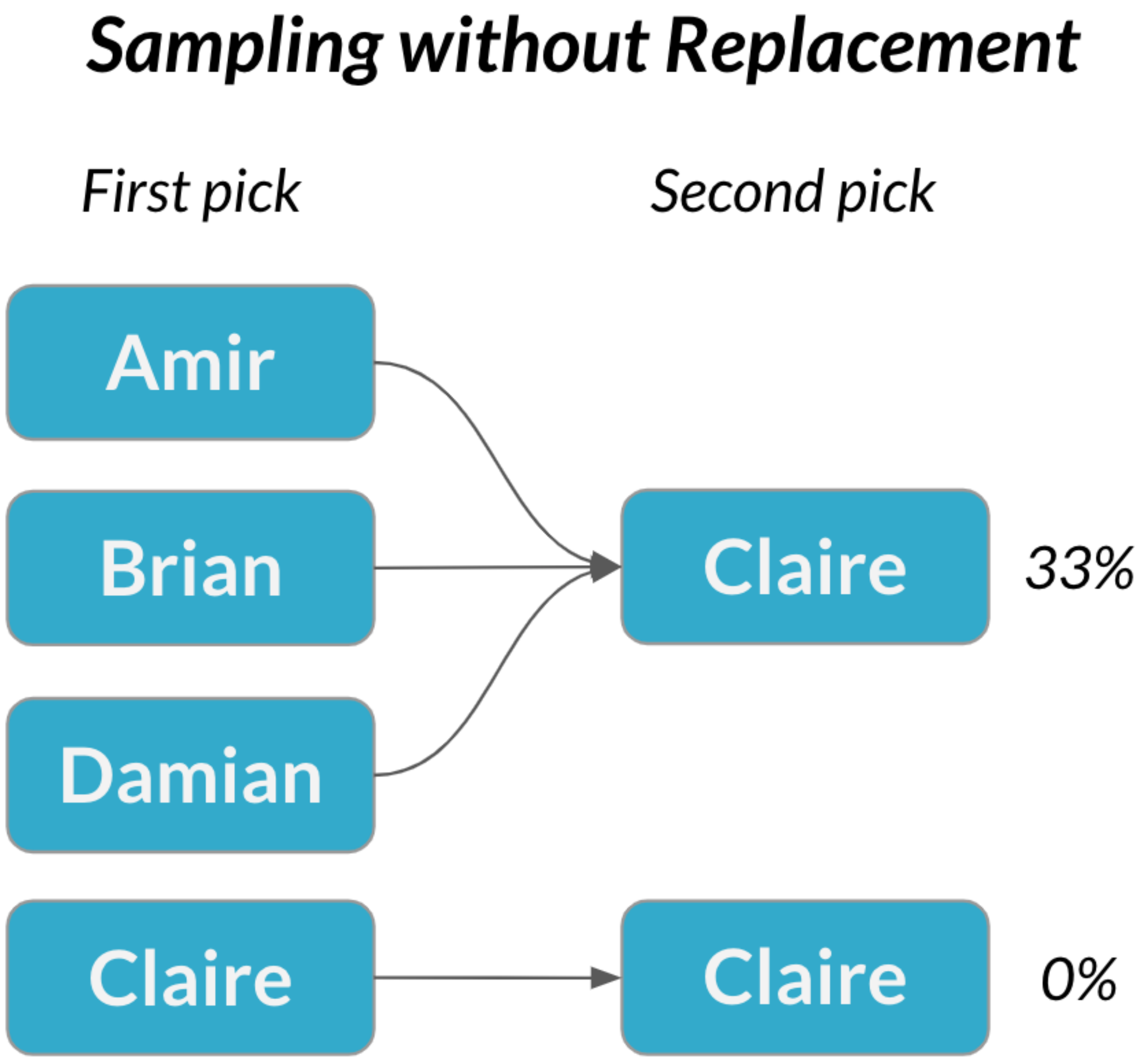
*Two events are **dependent** if the probability of the second event **is** affected by the outcome of the first event.*



Dependent events

*Two events are **dependent** if the probability of the second event **is** affected by the outcome of the first event.*

Sampling without replacement = each pick is dependent



Discrete distributions

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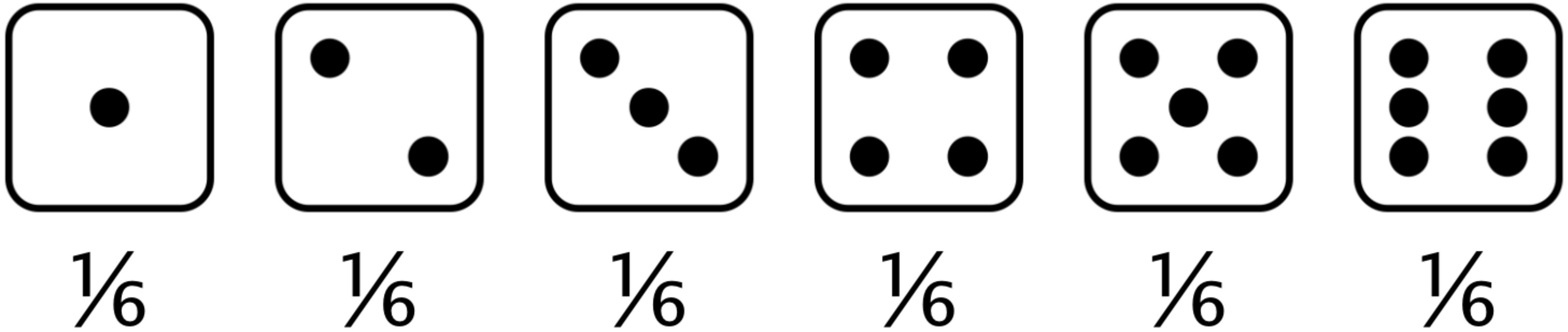
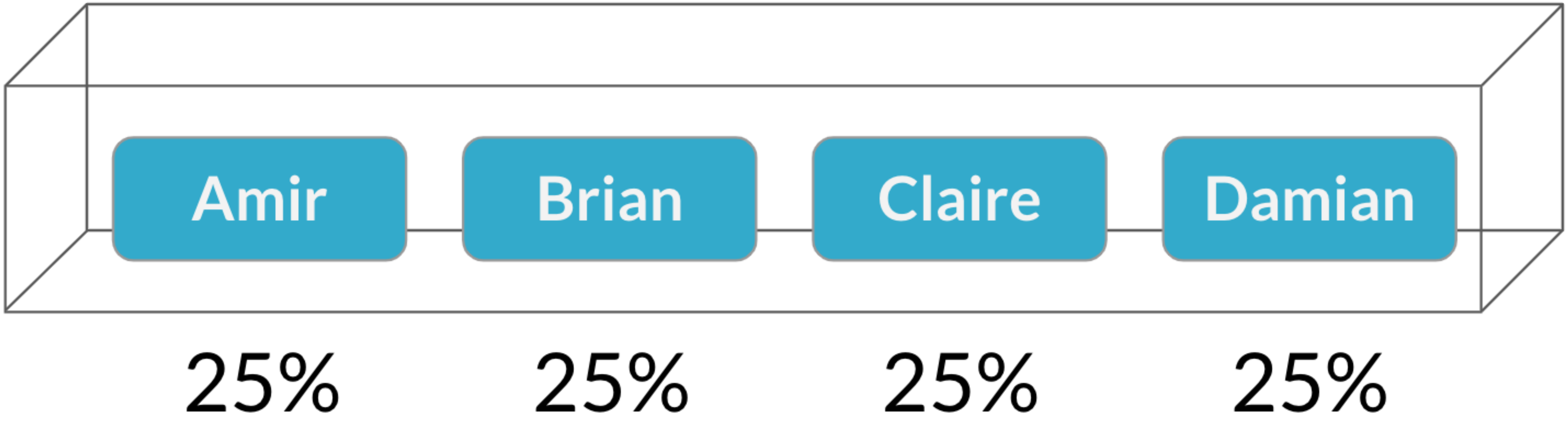
Rolling the dice

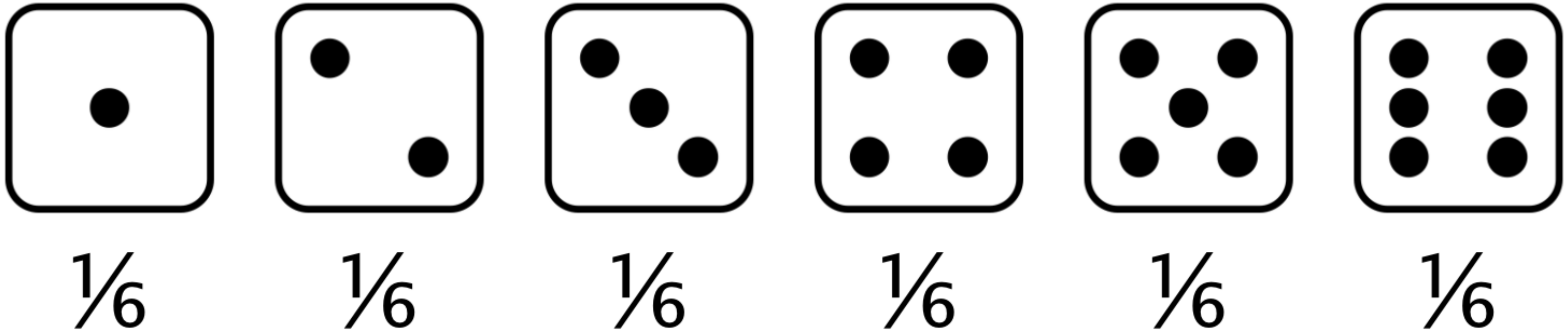


Choosing salespeople



Probability distribution

Describes the probability of each possible outcome in a scenario

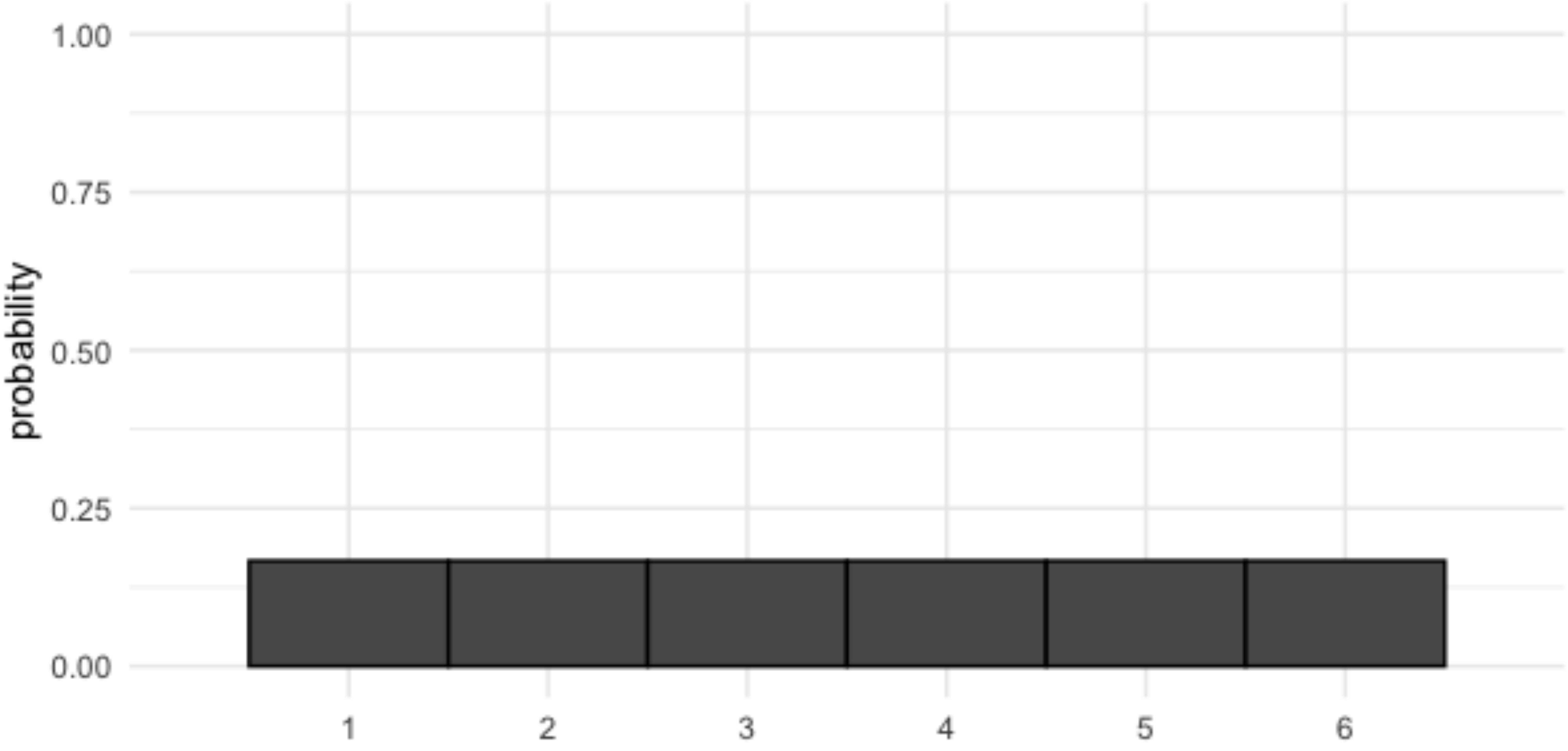


Expected value: mean of a probability distribution

Expected value of a fair die roll =

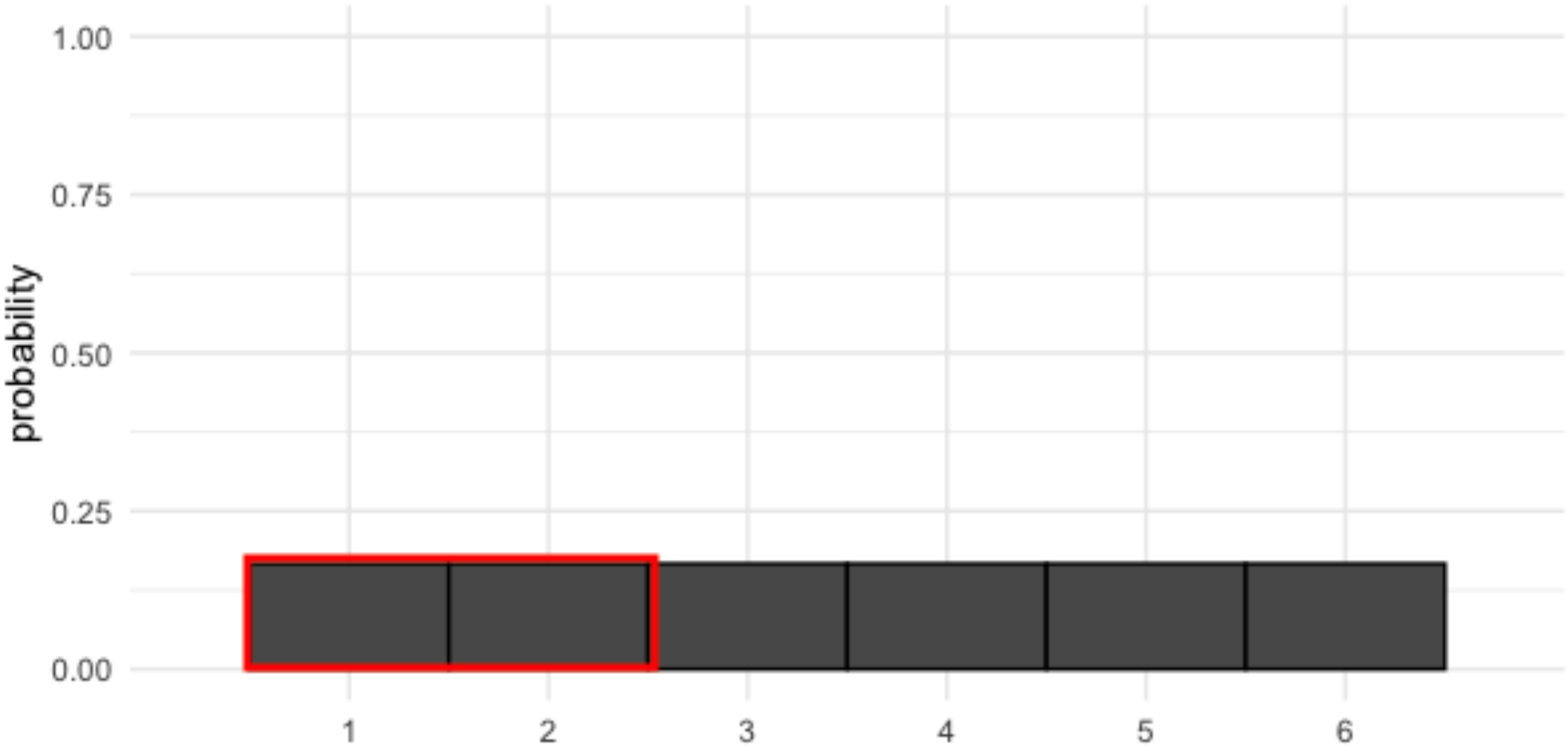
$$(1 \times \frac{1}{6}) + (2 \times \frac{1}{6}) + (3 \times \frac{1}{6}) + (4 \times \frac{1}{6}) + (5 \times \frac{1}{6}) + (6 \times \frac{1}{6}) = 3.5$$

Visualizing a probability distribution



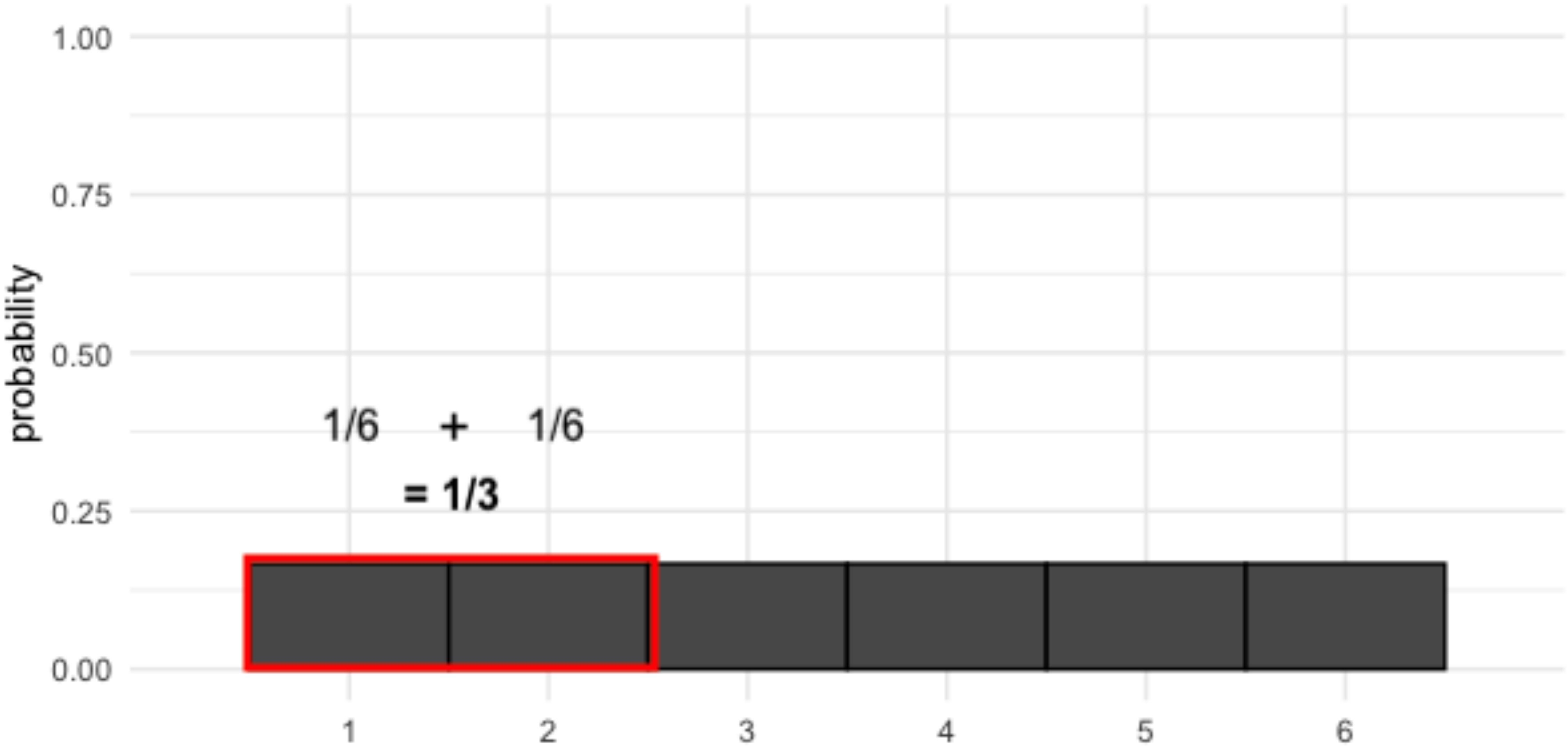
Probability = area

$$P(\text{die roll}) \leq 2 = ?$$

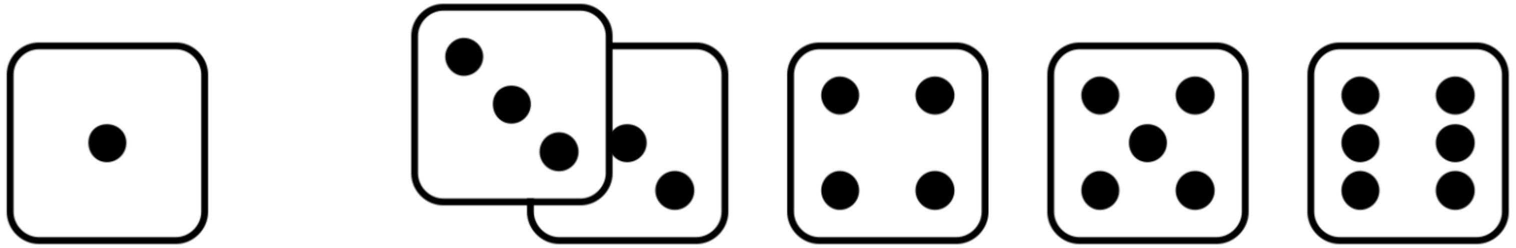


Probability = area

$$P(\text{die roll}) \leq 2 = 1/3$$



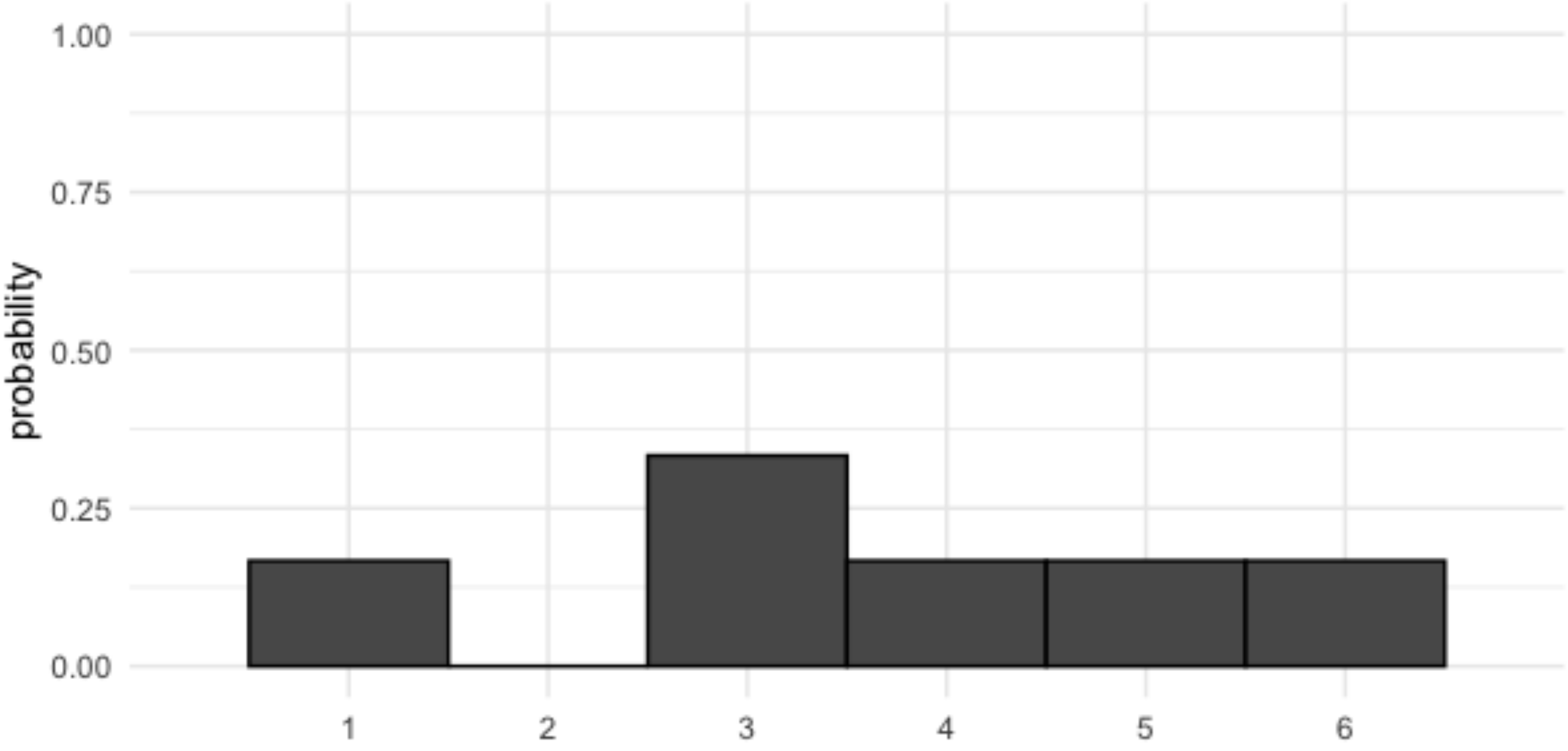
Uneven die



Expected value of uneven die roll =

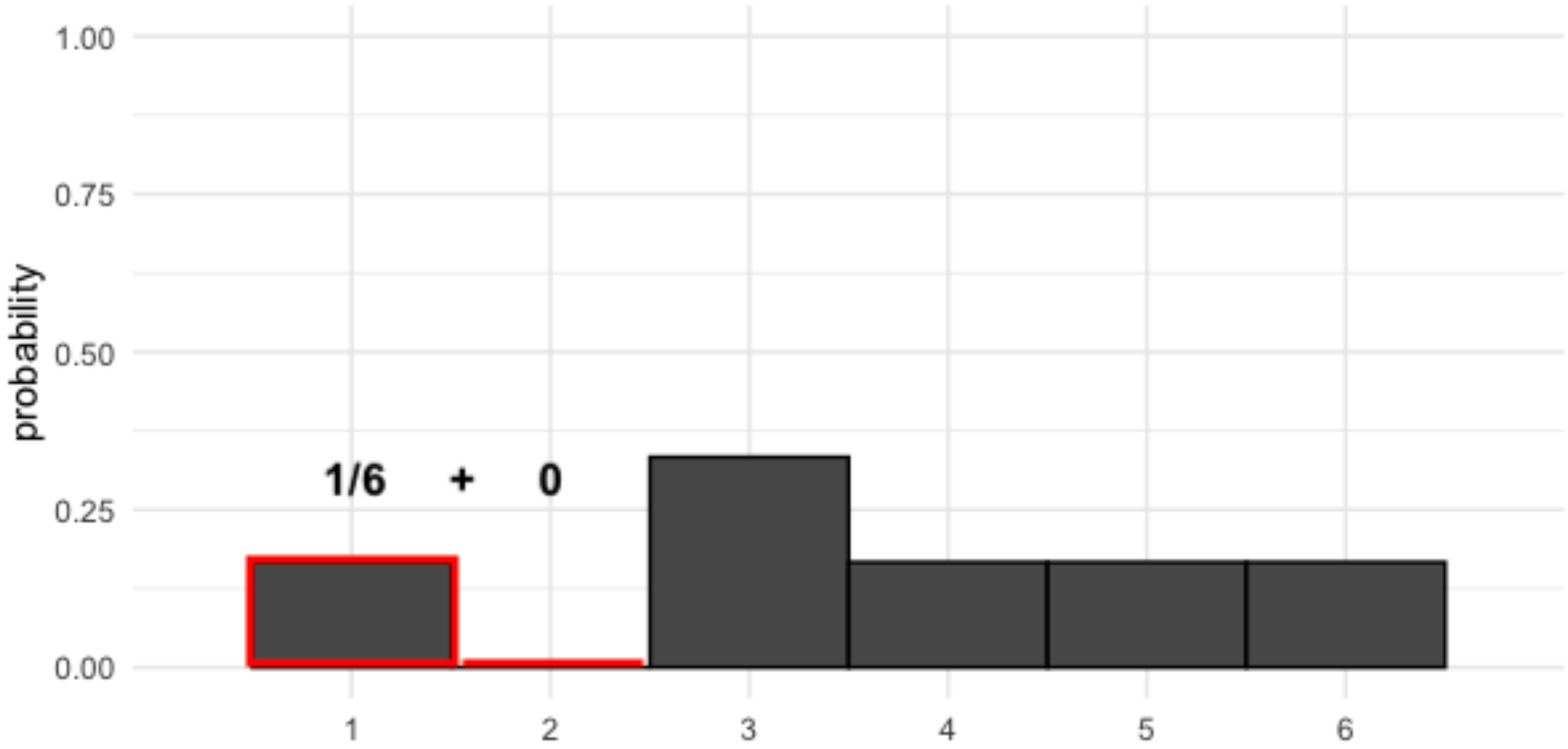
$$(1 \times \tfrac{1}{6}) + (2 \times 0) + (3 \times \tfrac{1}{3}) + (4 \times \tfrac{1}{6}) + (5 \times \tfrac{1}{6}) + (6 \times \tfrac{1}{6}) = 3.67$$

Visualizing uneven probabilities



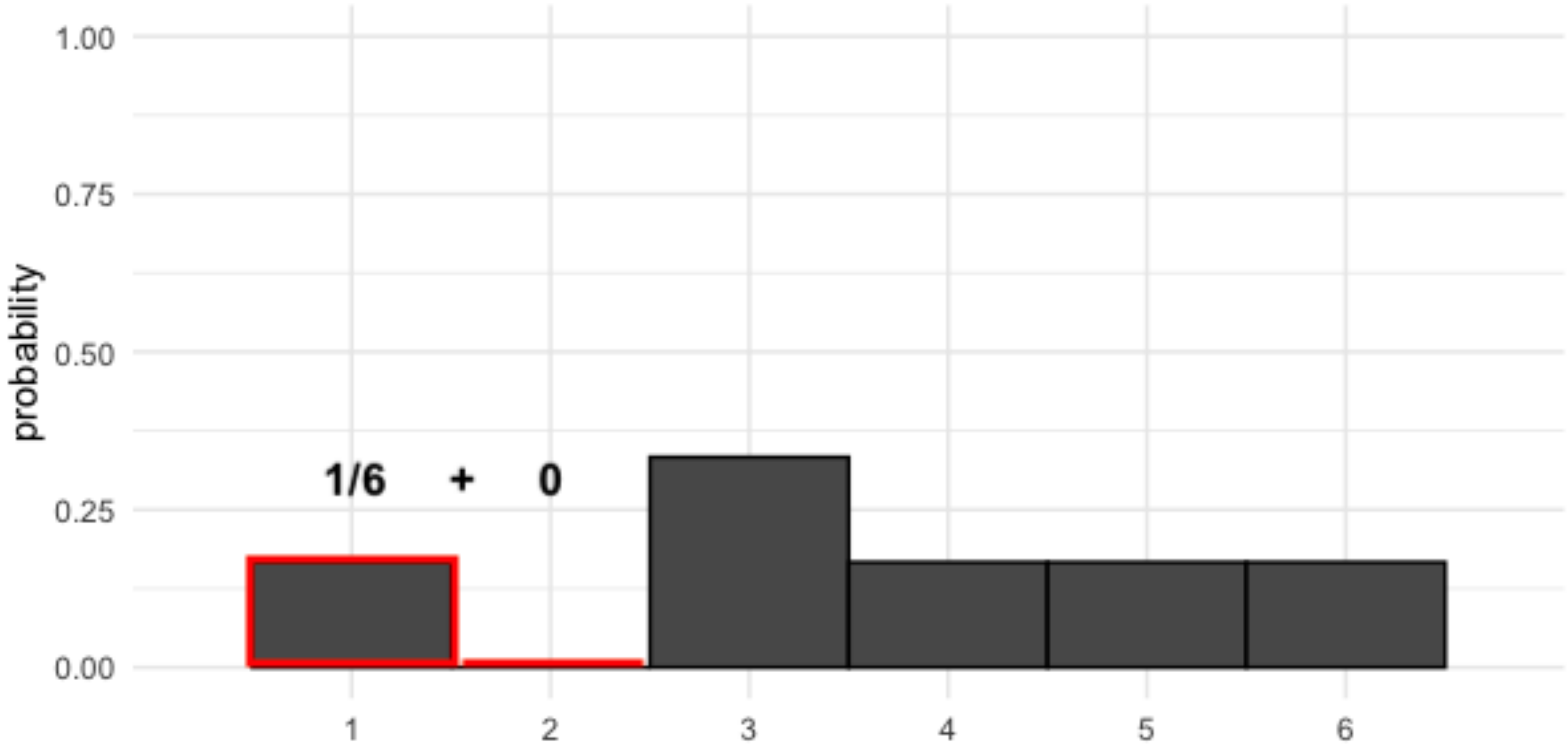
Adding areas

$$P(\text{uneven die roll}) \leq 2 = ?$$



Adding areas

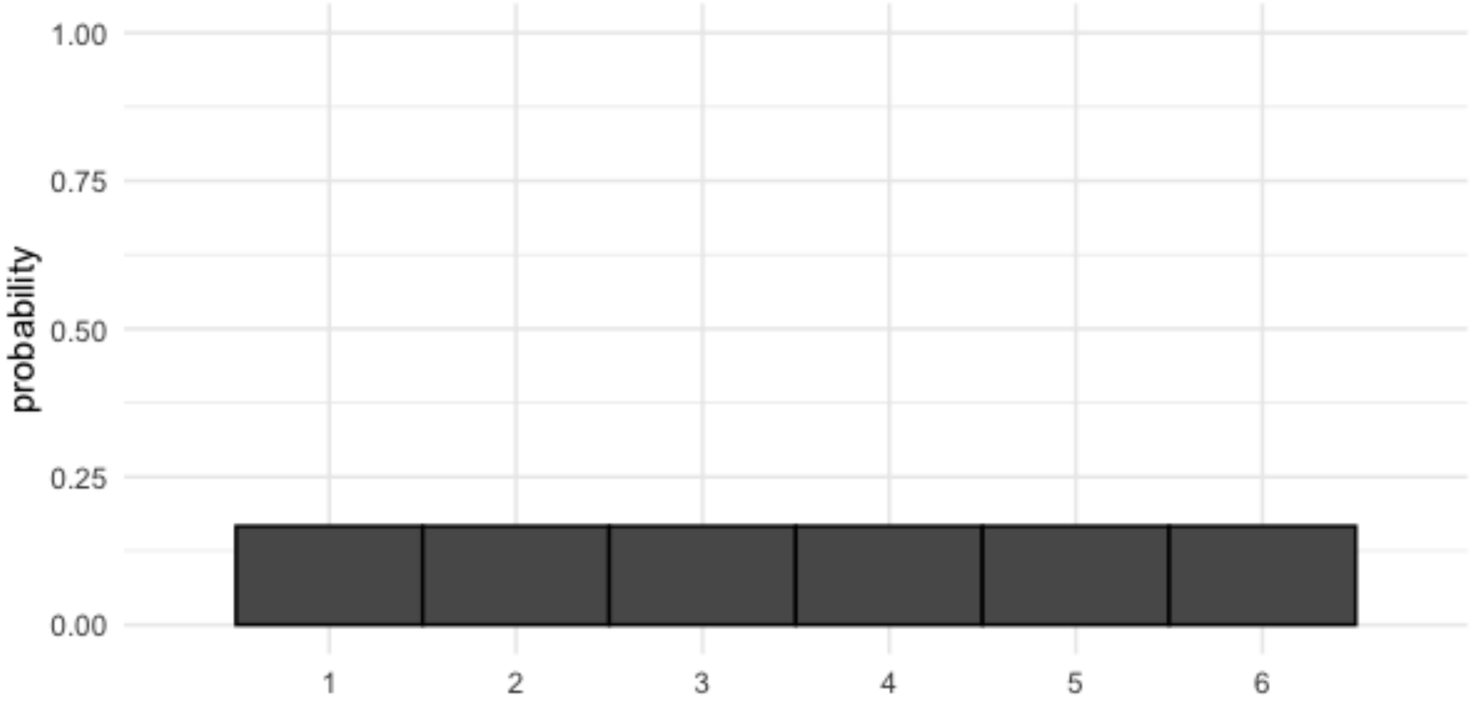
$$P(\text{uneven die roll}) \leq 2 = 1/6$$



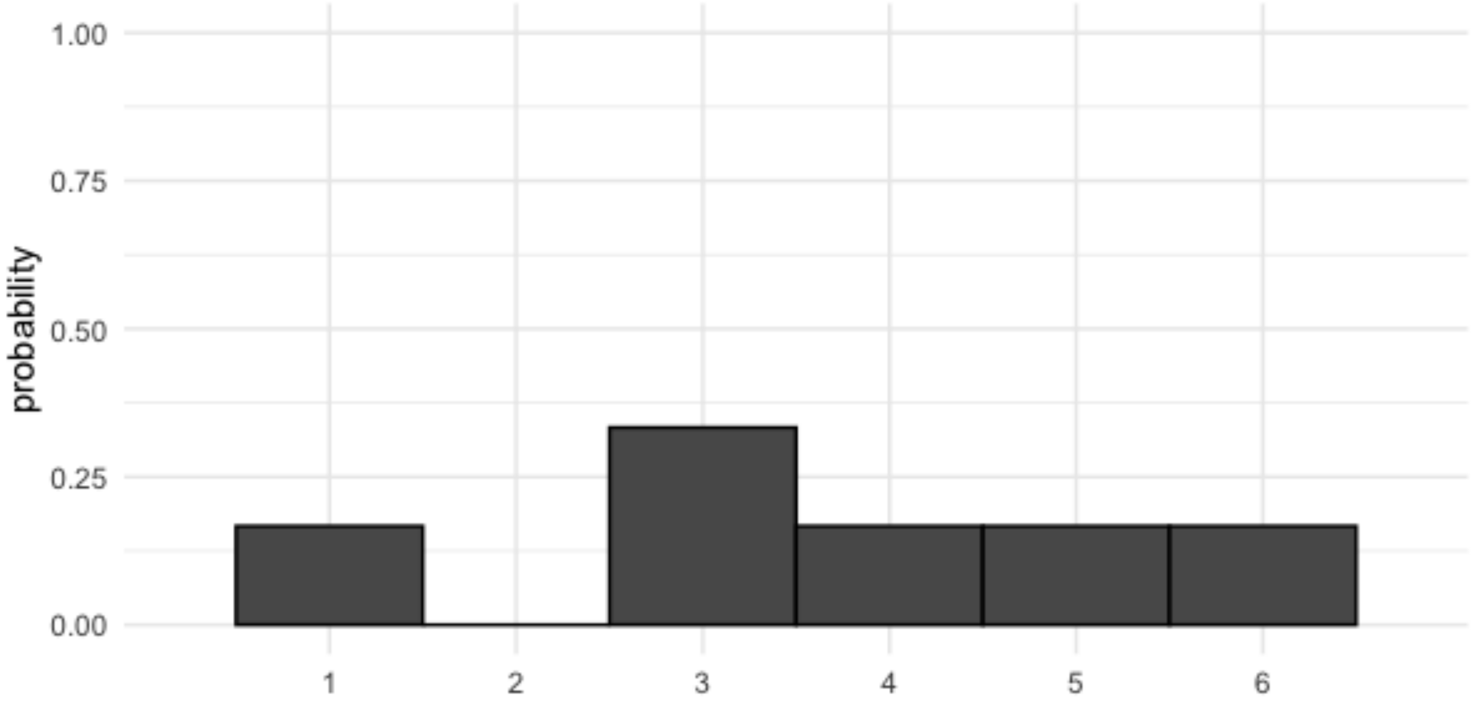
Discrete probability distributions

Describe probabilities for discrete outcomes

Fair die



Uneven die



Discrete uniform distribution

Sampling from discrete distributions

```
print(die)
```

```
   number      prob
0       1  0.166667
1       2  0.166667
2       3  0.166667
3       4  0.166667
4       5  0.166667
5       6  0.166667
```

```
np.mean(die['number'])
```

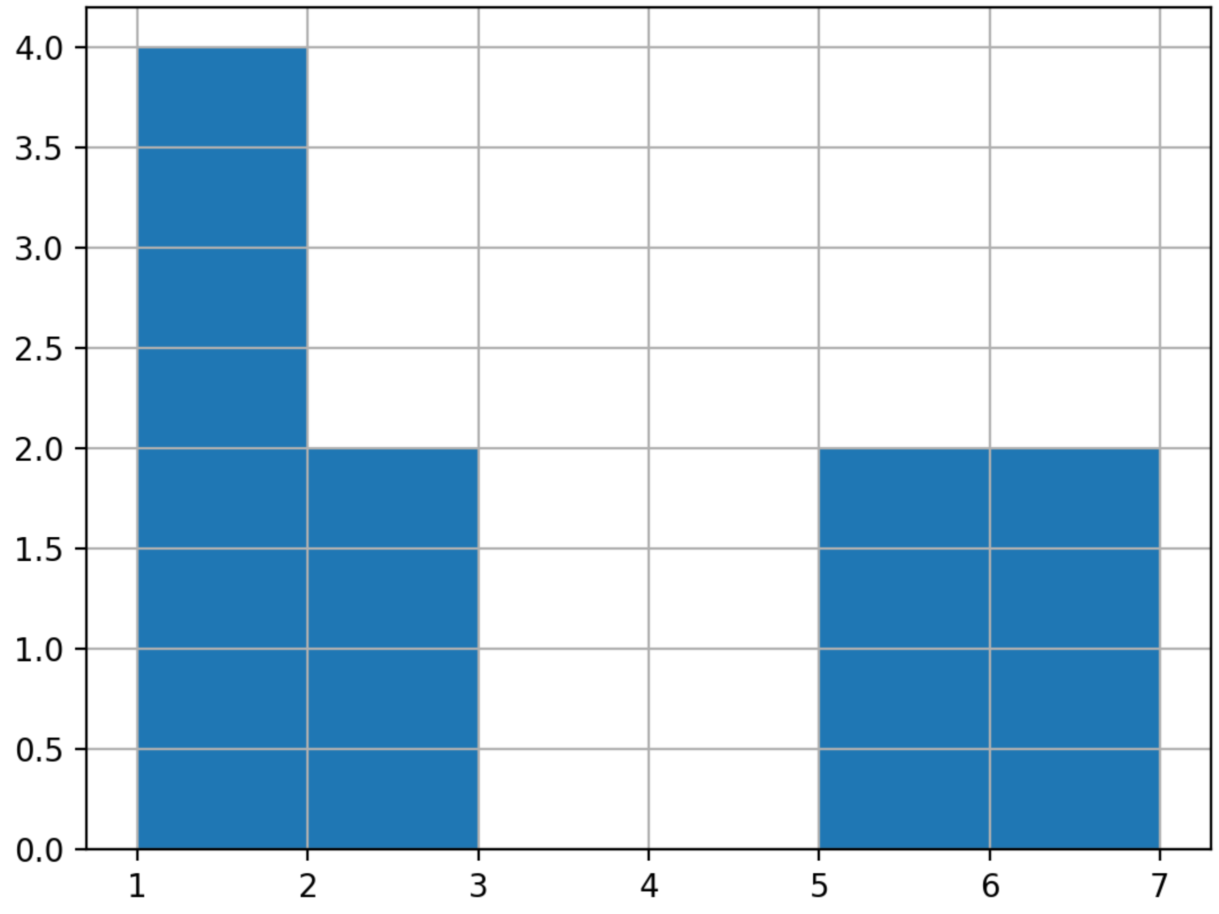
```
3.5
```

```
rolls_10 = die.sample(10, replace = True)
rolls_10
```

```
   number      prob
0       1  0.166667
0       1  0.166667
4       5  0.166667
1       2  0.166667
0       1  0.166667
0       1  0.166667
5       6  0.166667
5       6  0.166667
...
```

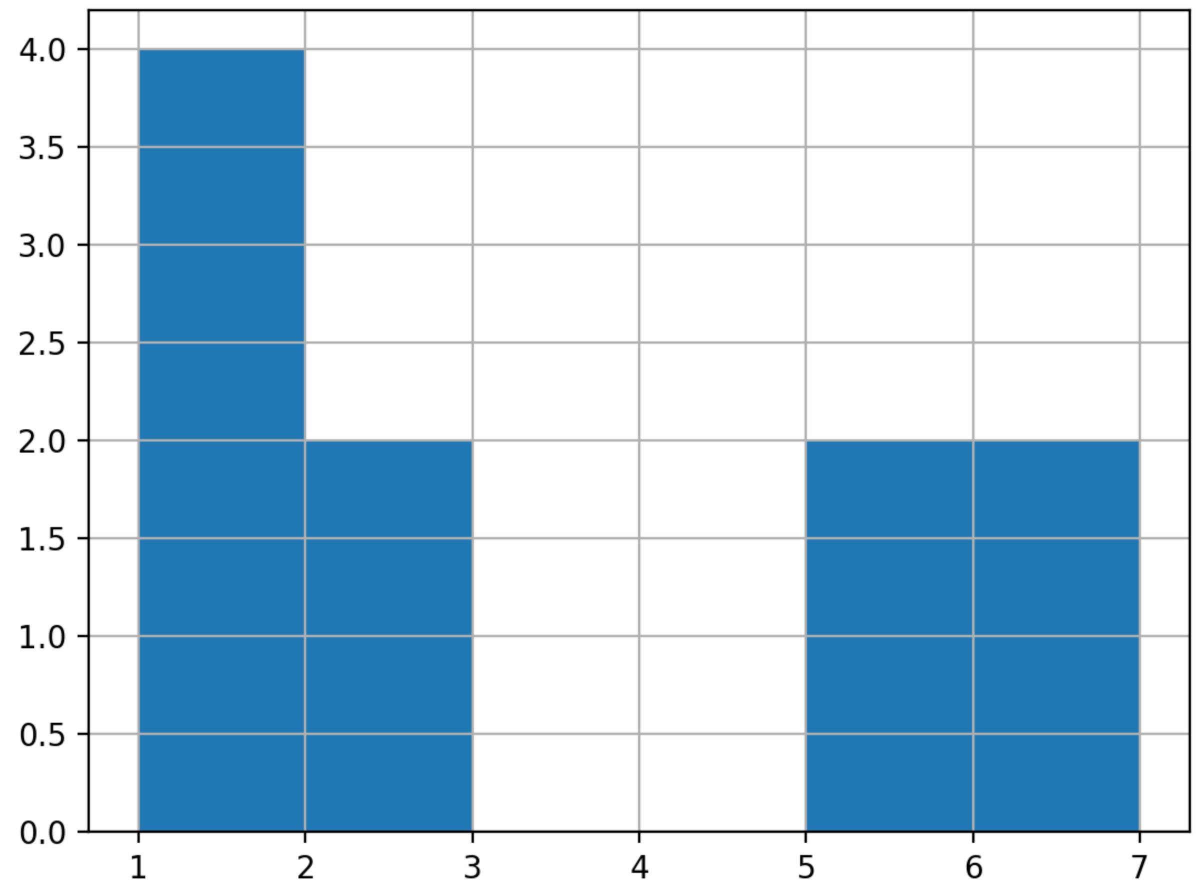
Visualizing a sample

```
rolls_10['number'].hist(bins=np.linspace(1,7,7))
plt.show()
```



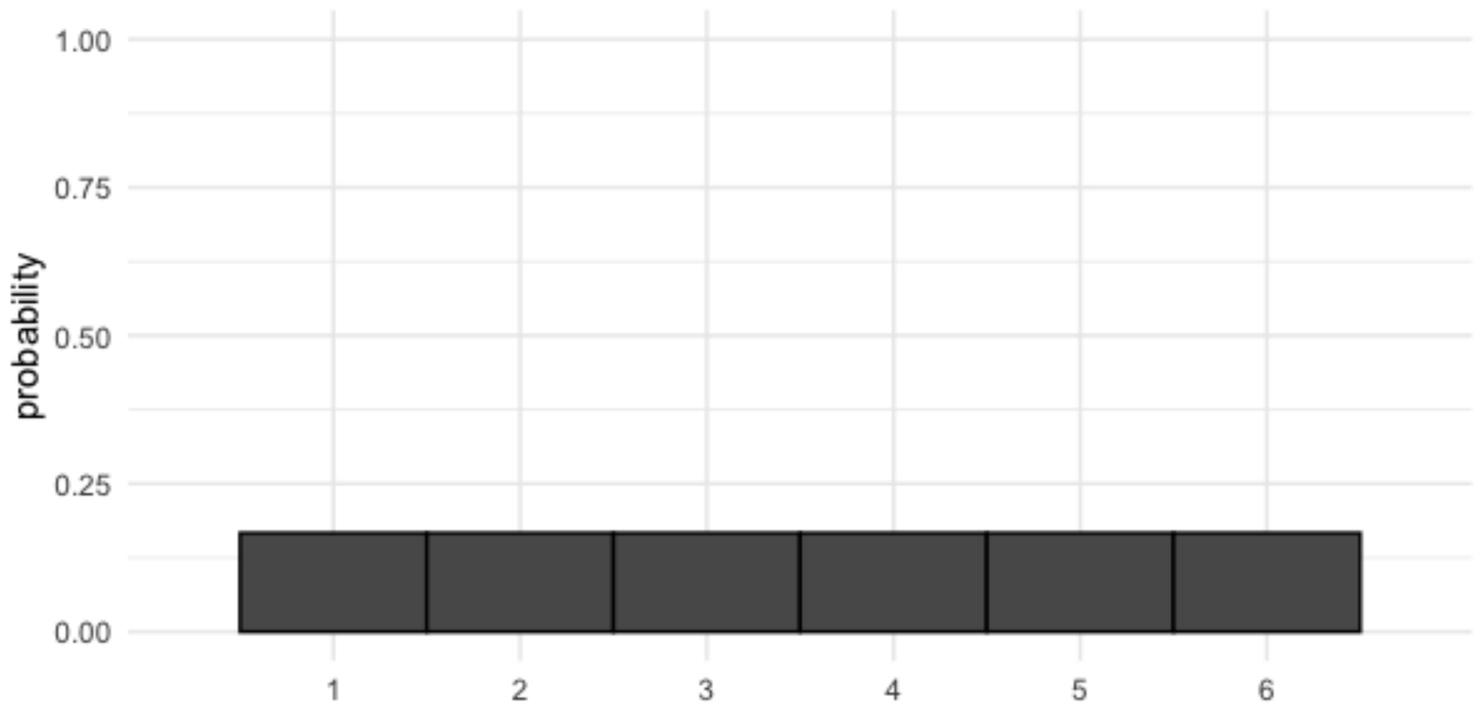
Sample distribution vs. theoretical distribution

Sample of 10 rolls



`np.mean(rolls_10['number'])` = `3.0`

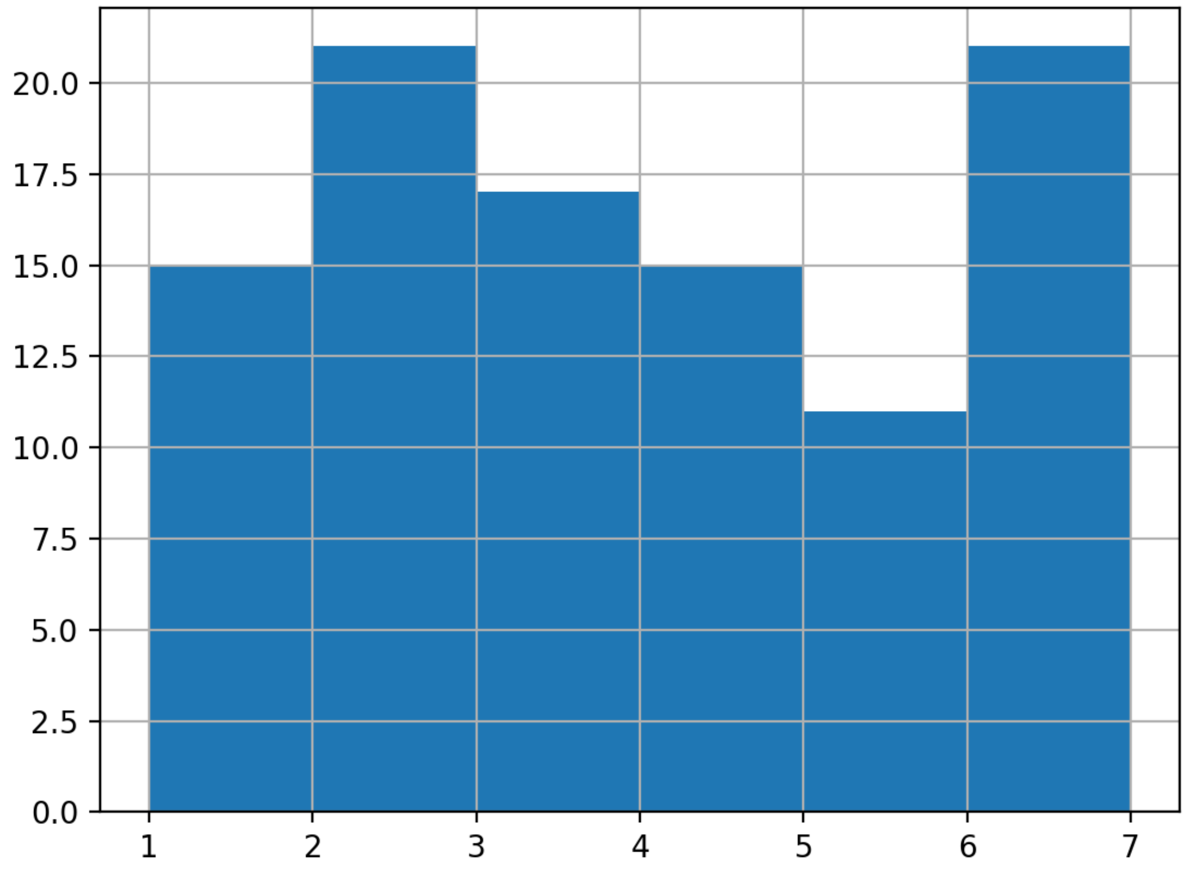
Theoretical probability distribution



`mean(die['number'])` = `3.5`

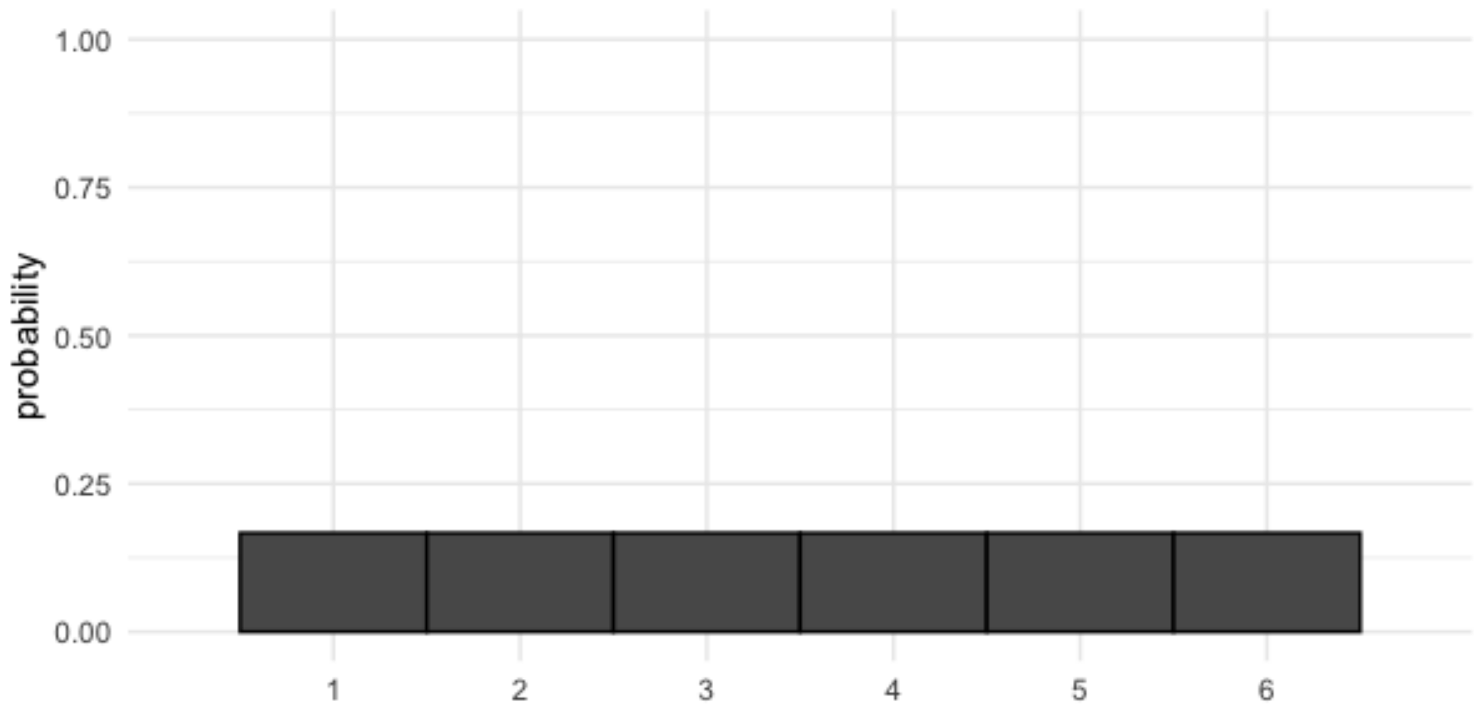
A bigger sample

Sample of 100 rolls



`np.mean(rolls_100['number'])` = `3.4`

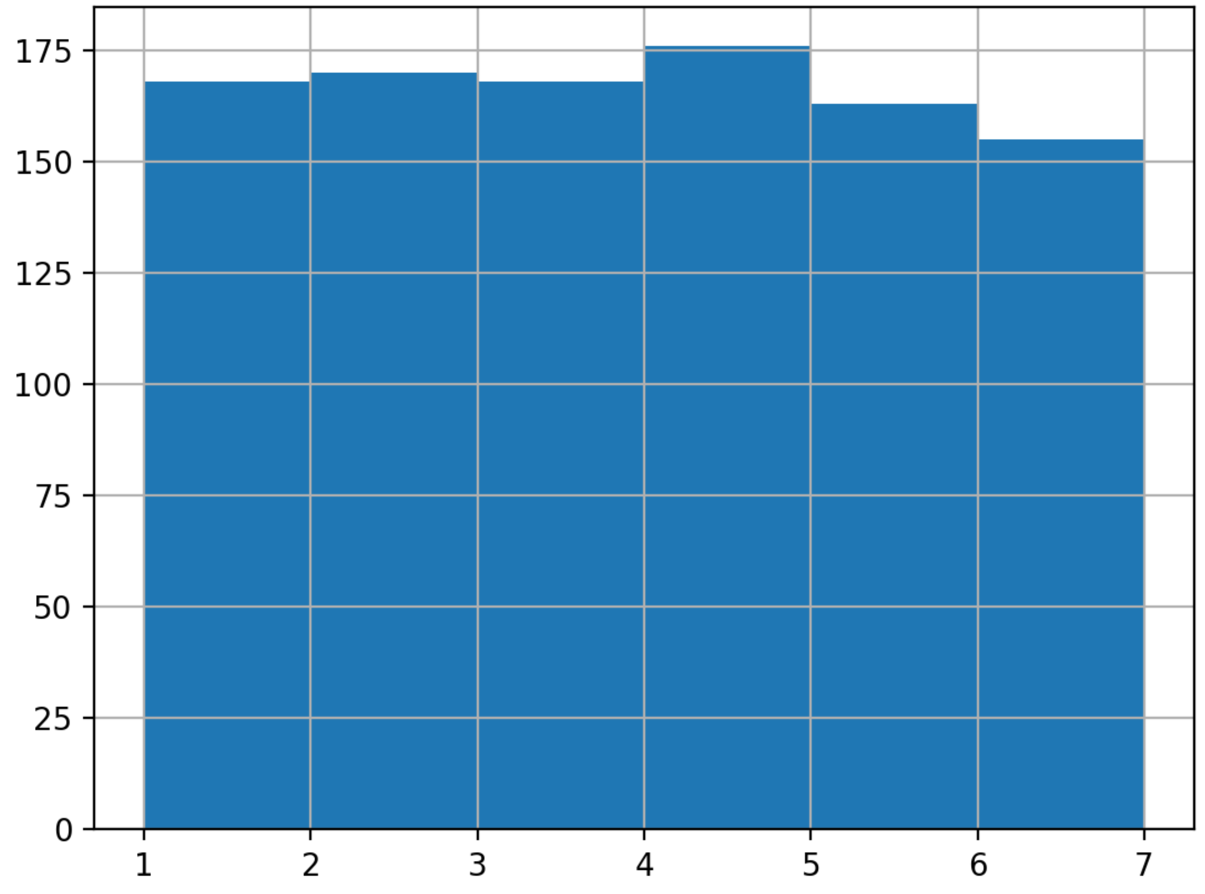
Theoretical probability distribution



`mean(die['number'])` = `3.5`

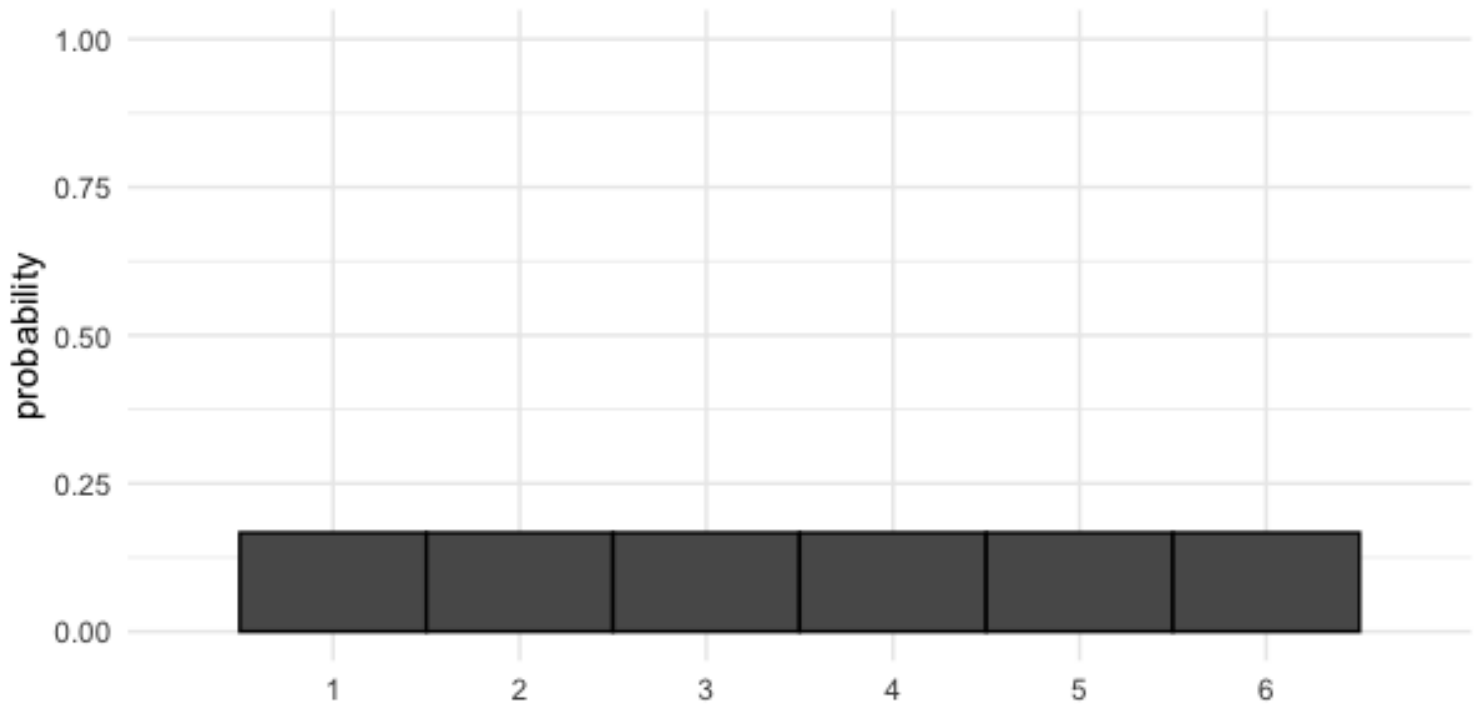
An even bigger sample

Sample of 1000 rolls



`np.mean(rolls_1000['number'])` = `3.48`

Theoretical probability distribution



`mean(die['number'])` = `3.5`

Law of large numbers

As the size of your sample increases, the sample mean will approach the expected value.

Sample size	Mean
10	3.00
100	3.40
1000	3.48

Continuous distributions

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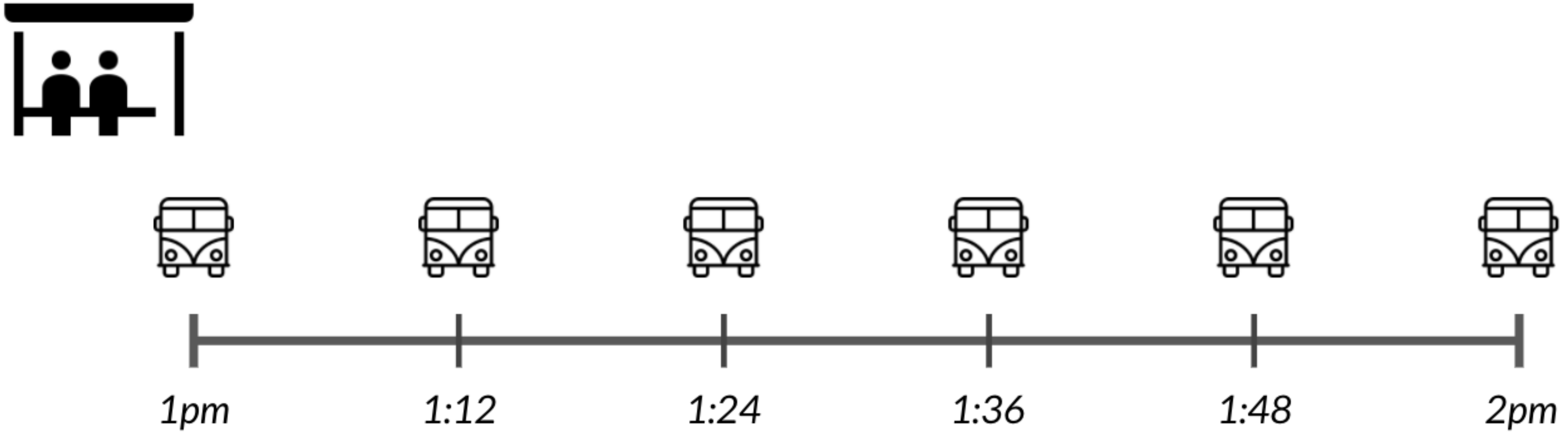
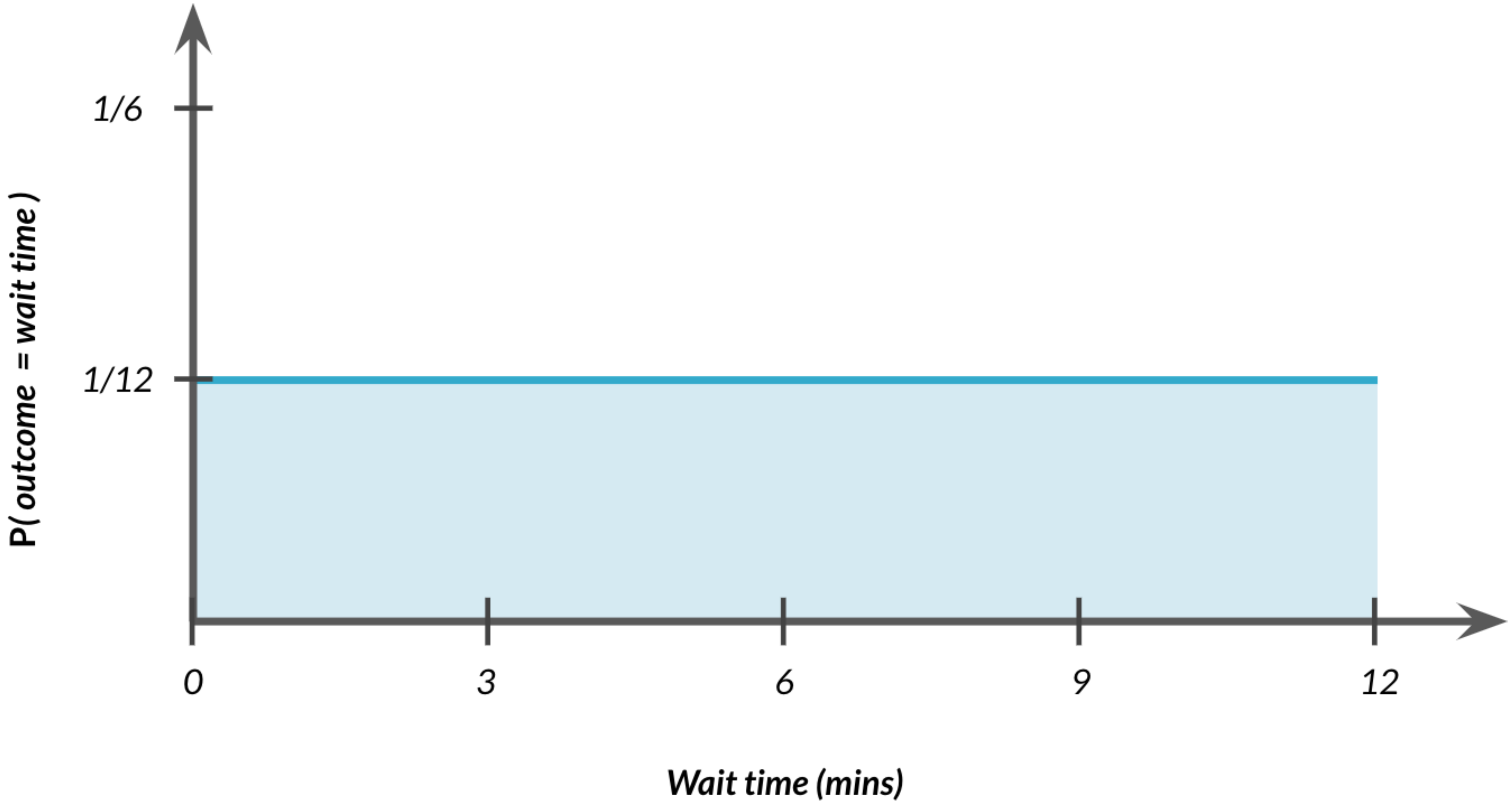
Waiting for the bus

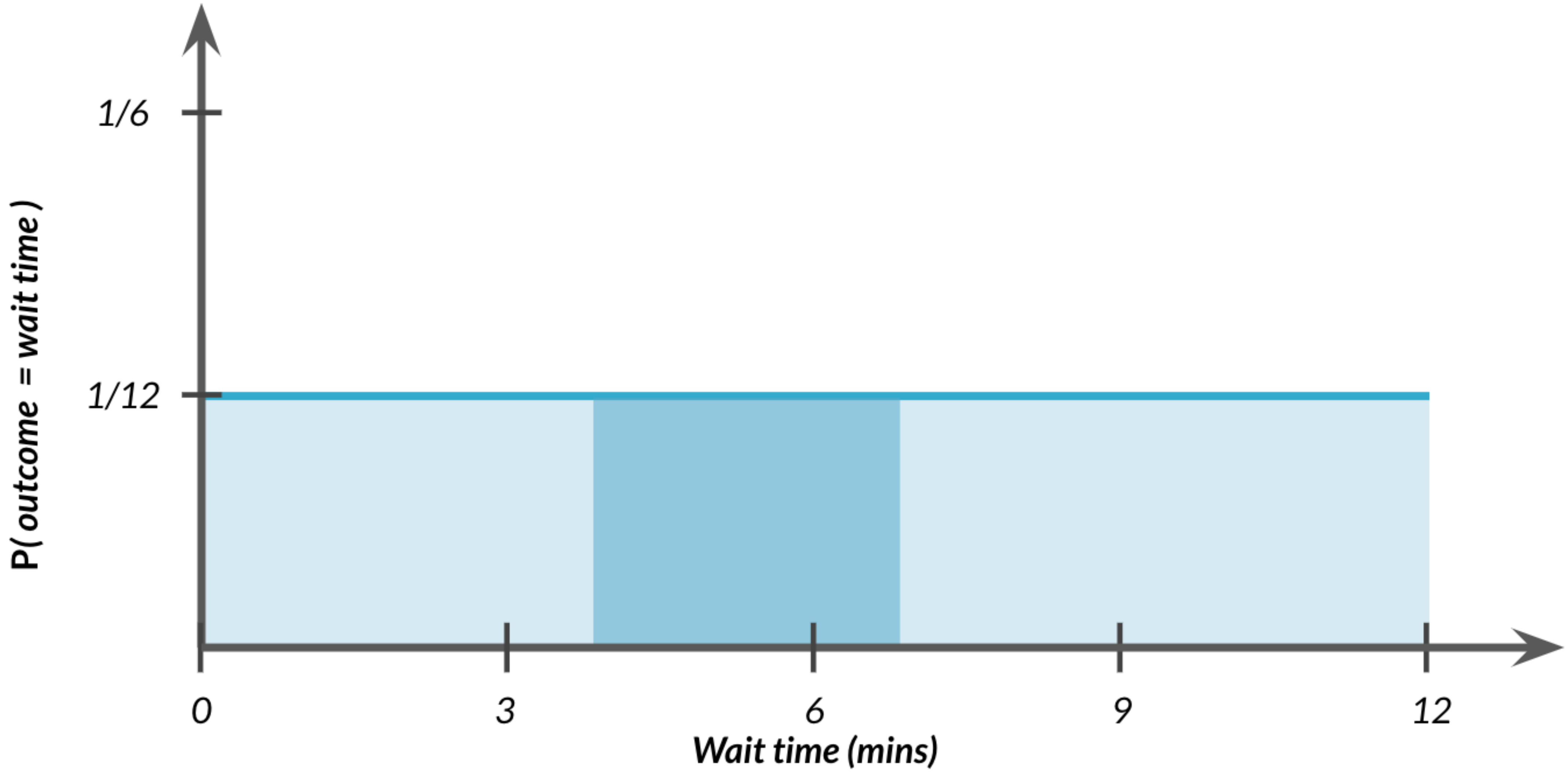


Continuous uniform distribution



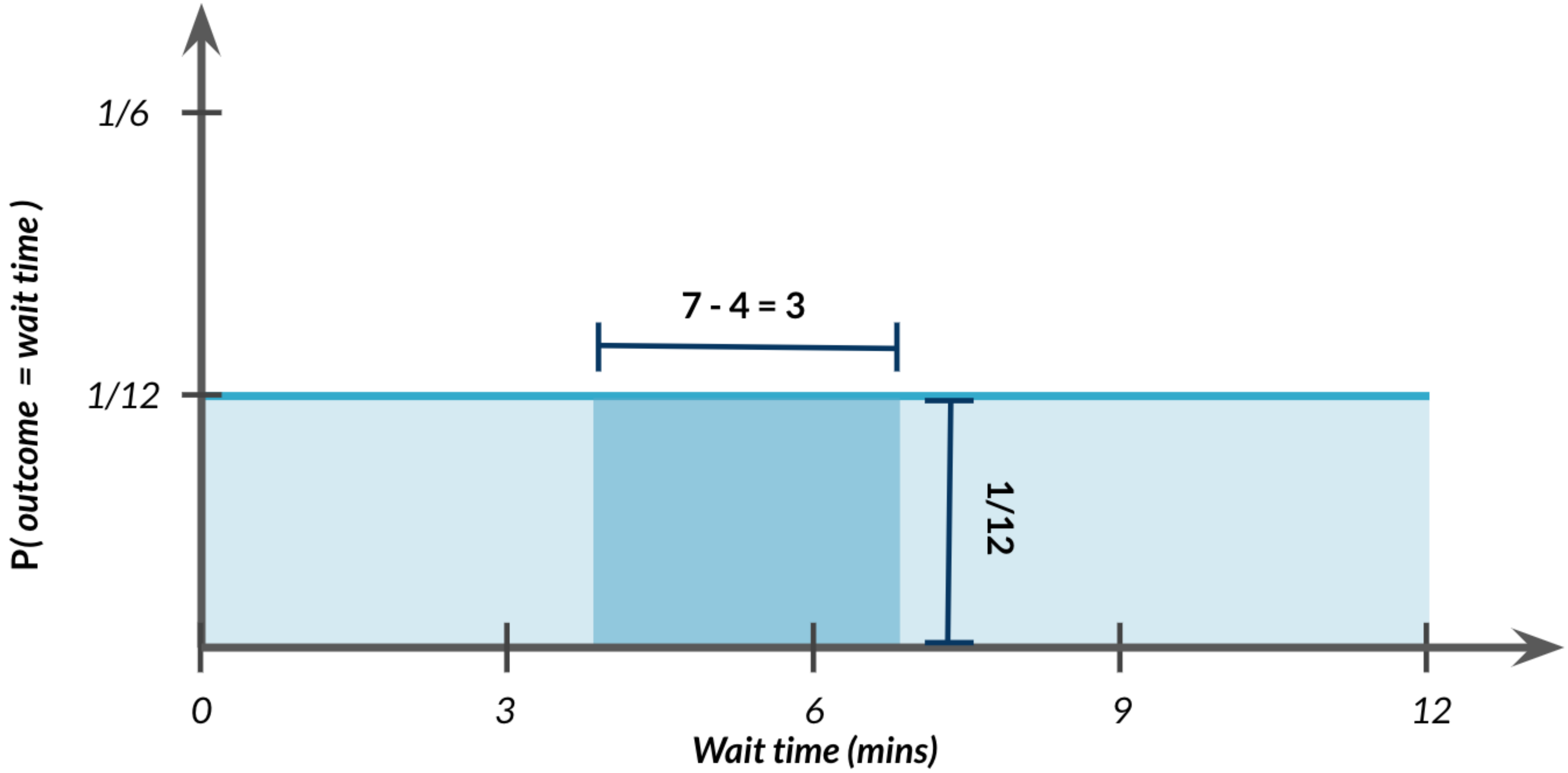
Probability still = area

$$P(4 \leq \text{wait time} \leq 7) = \ ?$$



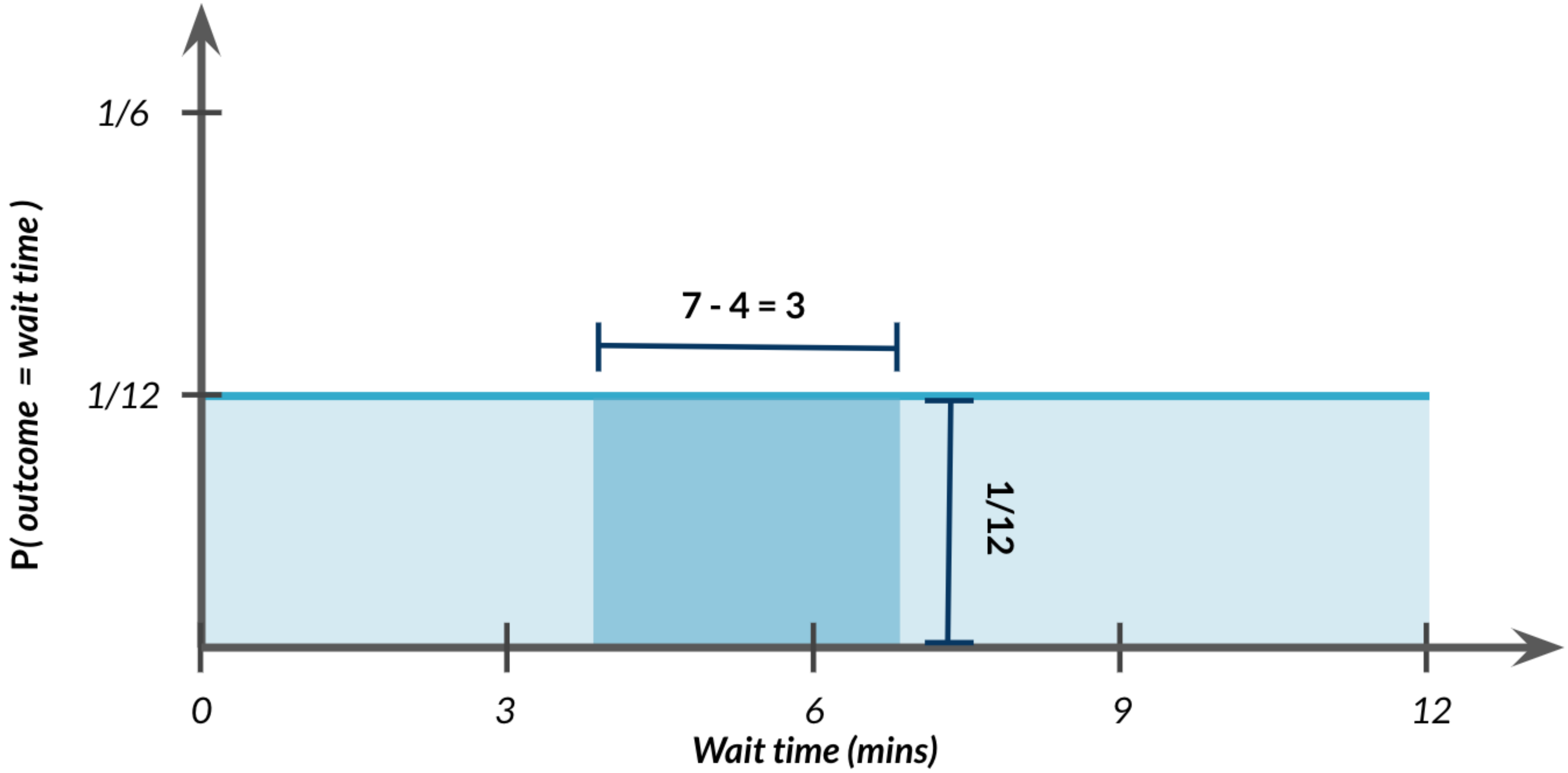
Probability still = area

$$P(4 \leq \text{wait time} \leq 7) = \ ?$$



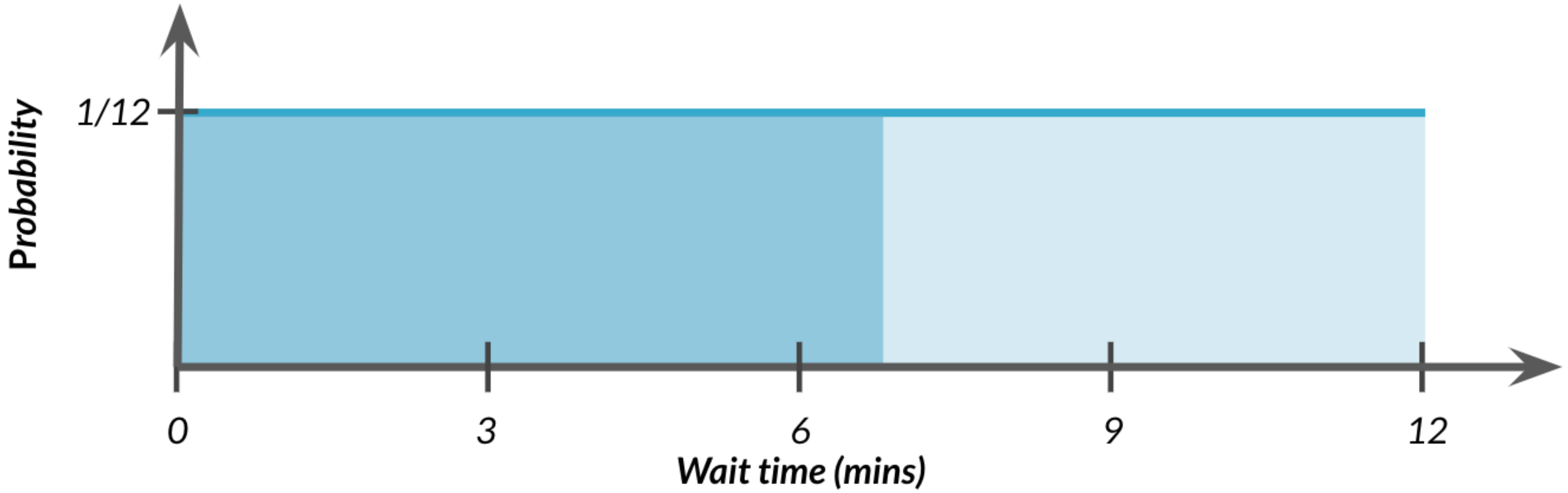
Probability still = area

$$P(4 \le \text{wait time} \le 7) = 3 \times 1/12 = 3/12$$



Uniform distribution in Python

$$P(\text{wait time} \leq 7)$$

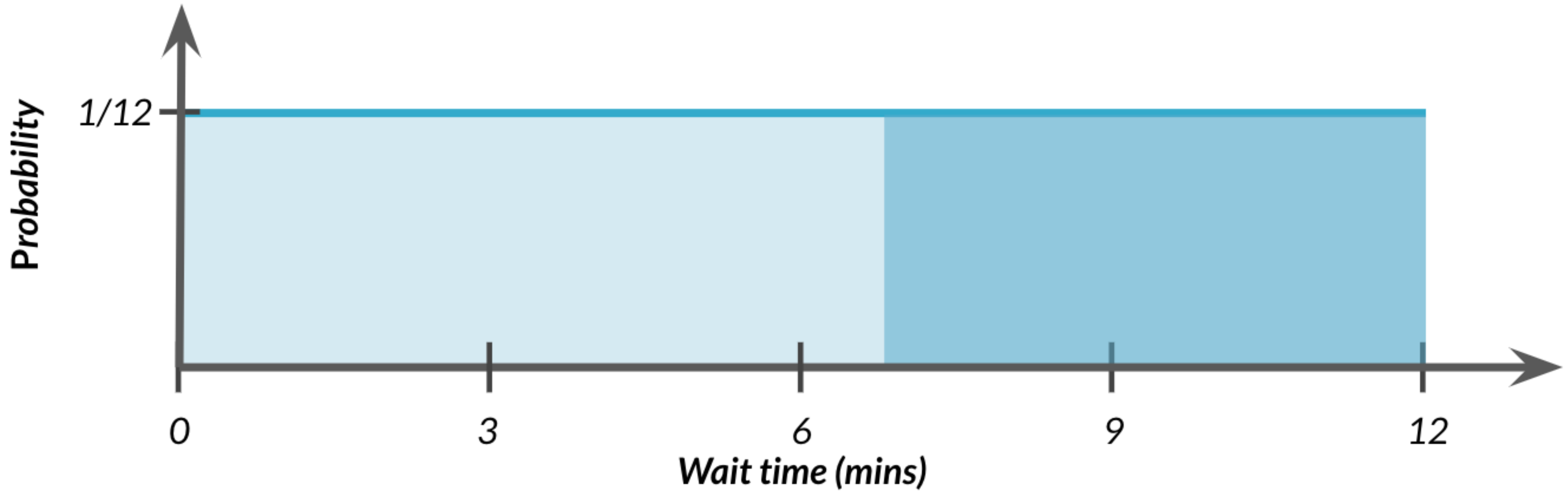



```python
from scipy.stats import uniform
uniform.cdf(7, 0, 12)
```

```
0.5833333
```

"Greater than" probabilities

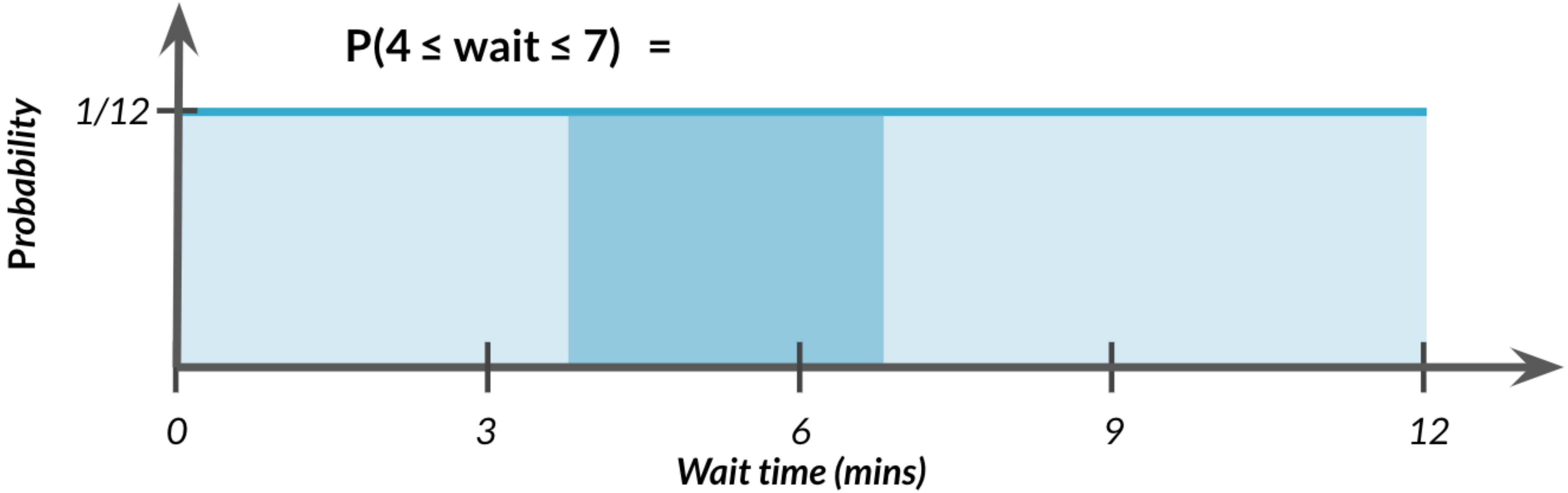
$$P(\text{wait time} \geq 7) = 1 - P(\text{wait time} \leq 7)$$



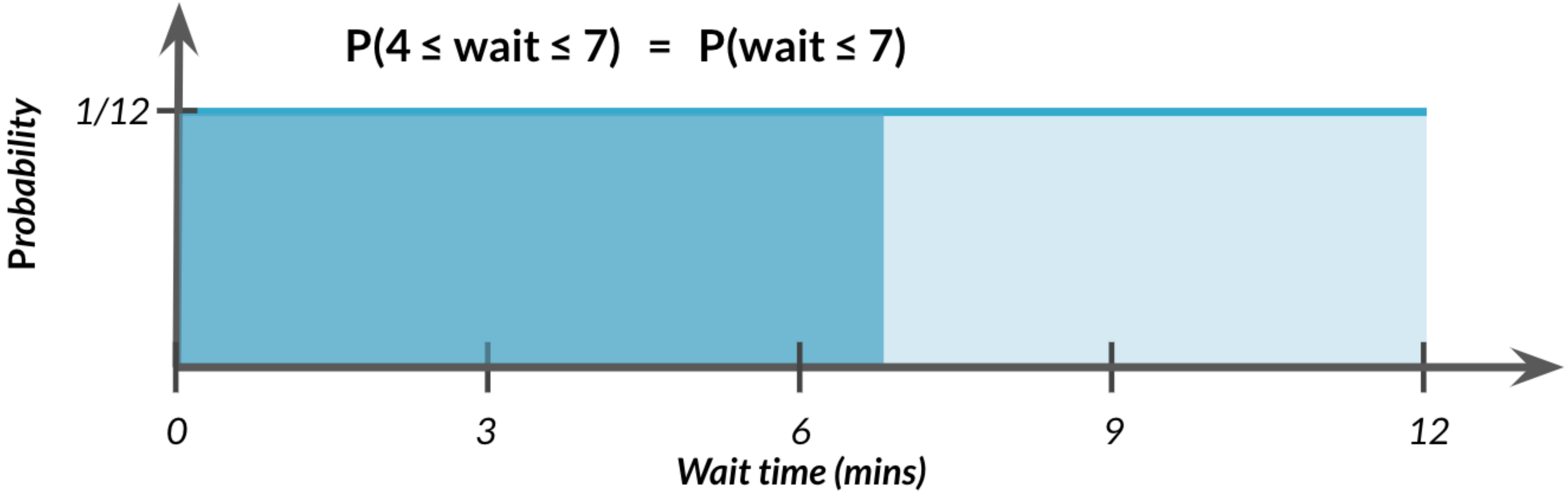

```python
from scipy.stats import uniform
1 - uniform.cdf(7, 0, 12)
```

```
0.4166667
```

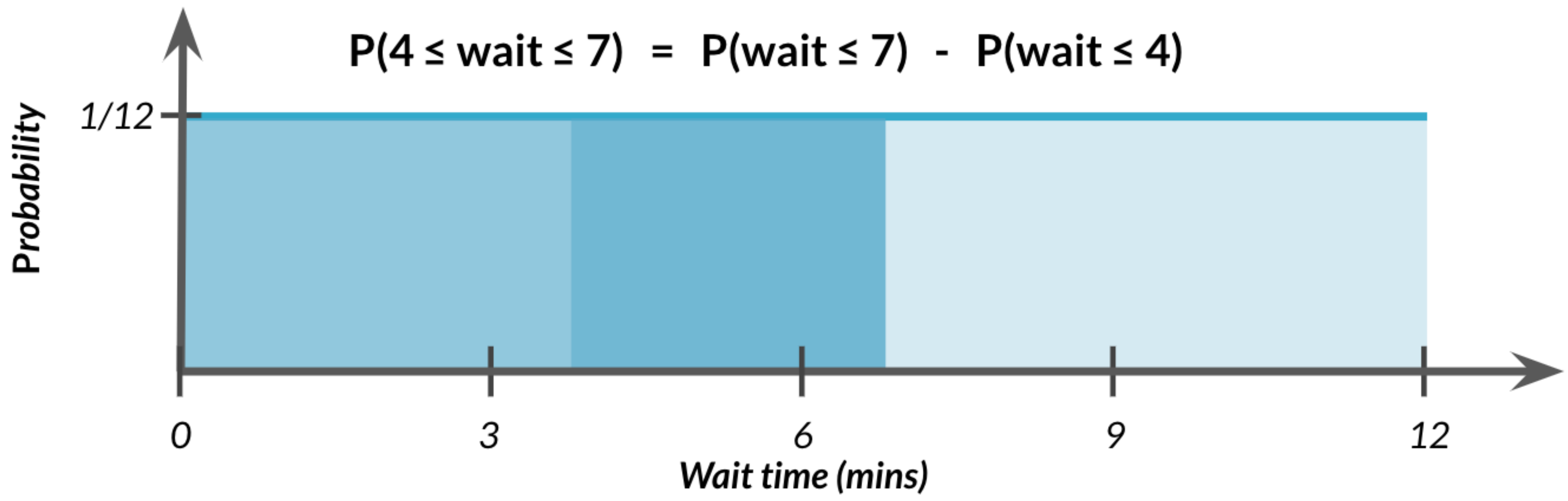
$$P(4 \leq \text{wait time} \leq 7)$$



$$P(4 \leq \text{wait time} \leq 7)$$



$$P(4 \leq \text{wait time} \leq 7)$$

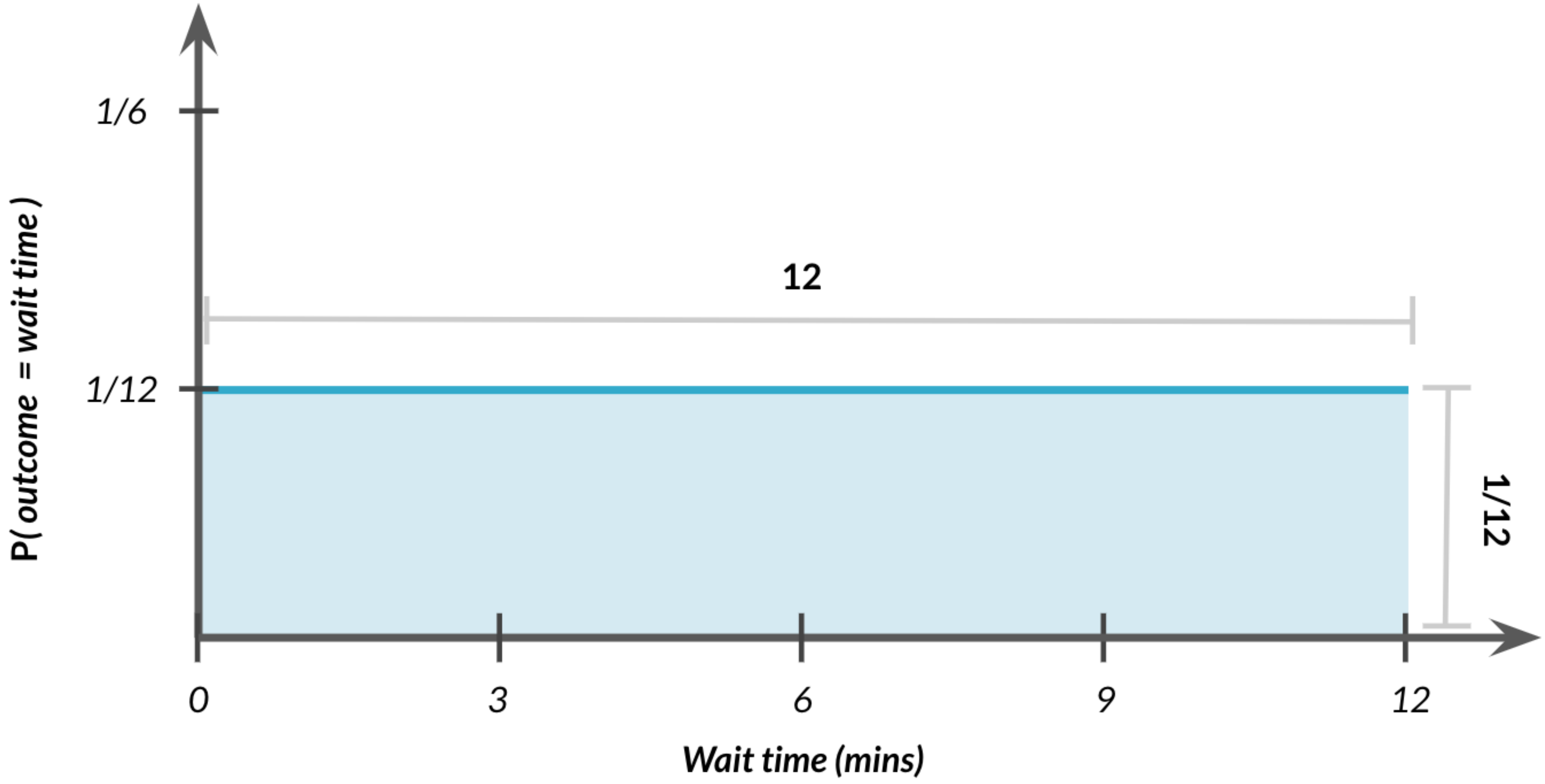



```
from scipy.stats import uniform
uniform.cdf(7, 0, 12) - uniform.cdf(4, 0, 12)
```

```
0.25
```

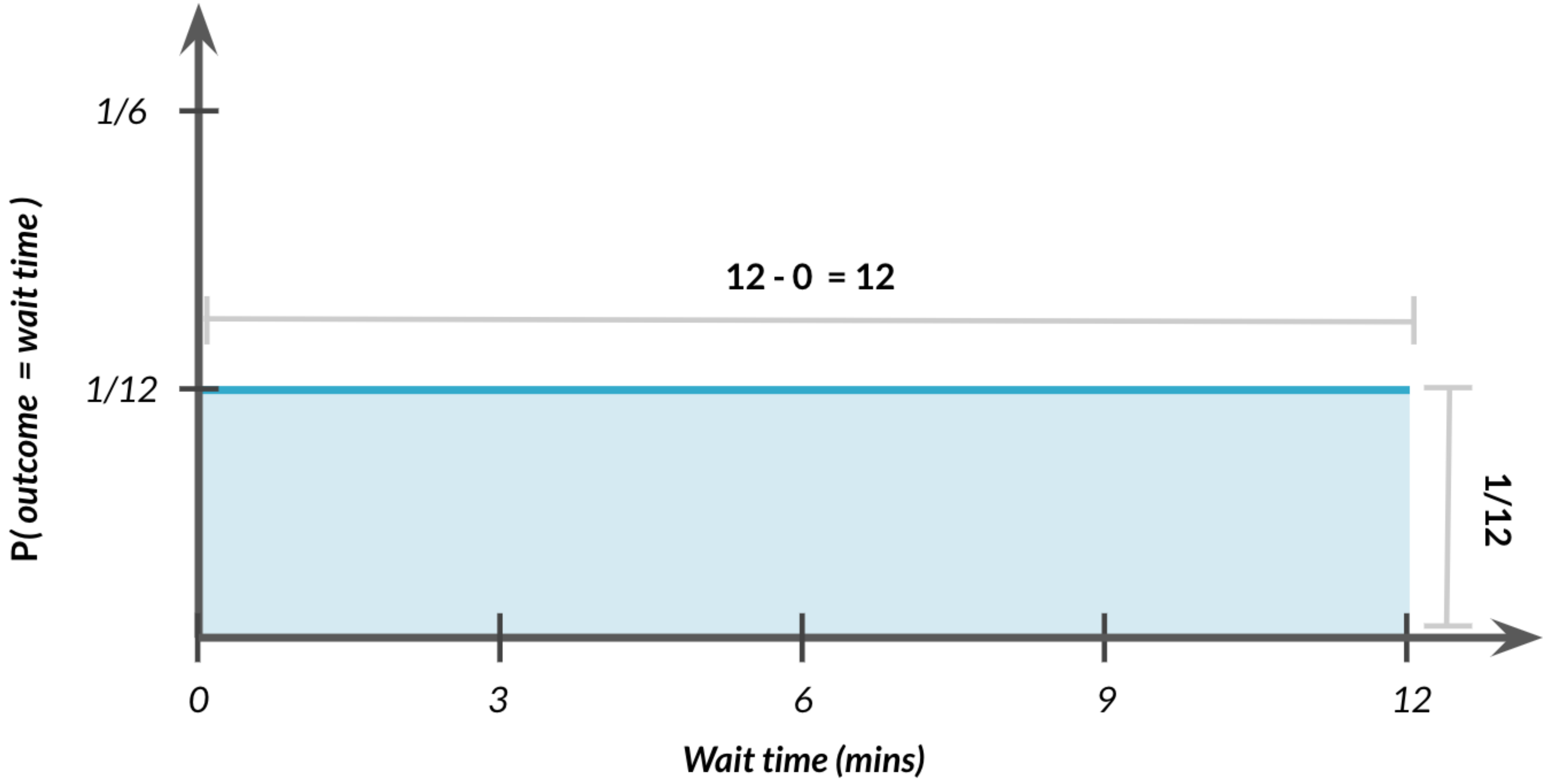
Total area = 1

$$P(0 \leq \text{wait time} \leq 12) = \ ?$$



Total area = 1

$$P(0 \leq \text{outcome} \leq 12) = 12 \times 1/12 = 1$$

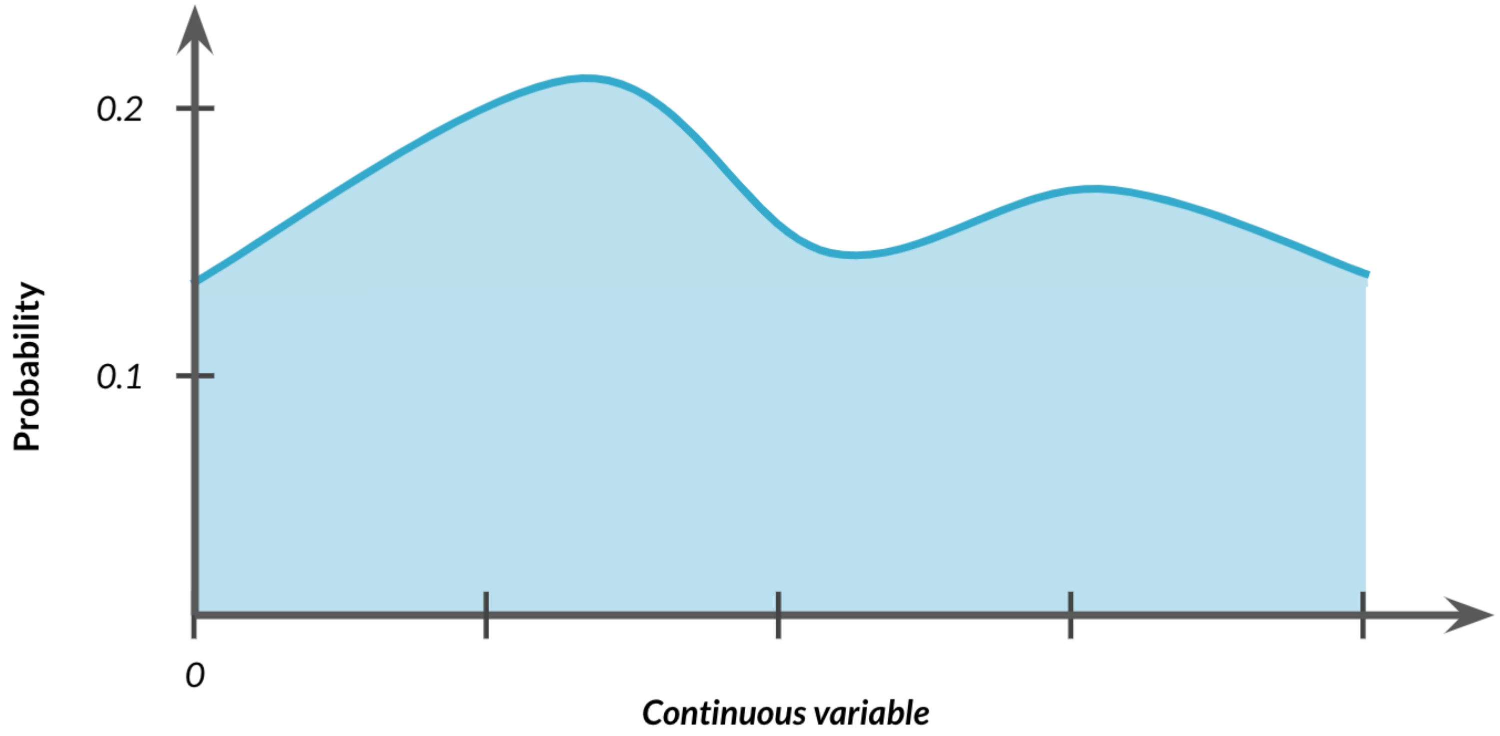


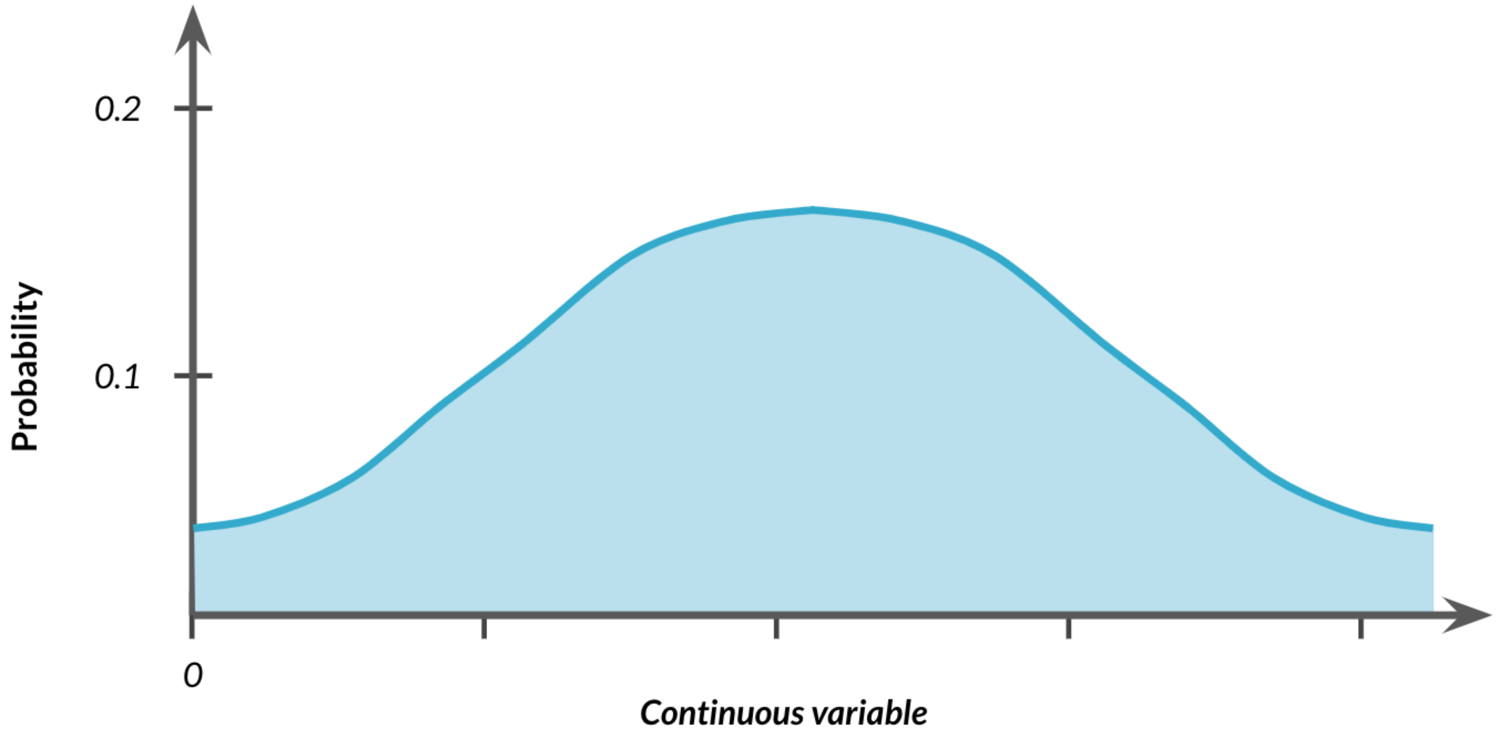
Generating random numbers according to uniform distribution

```
from scipy.stats import uniform
uniform.rvs(0, 5, size=10)
```

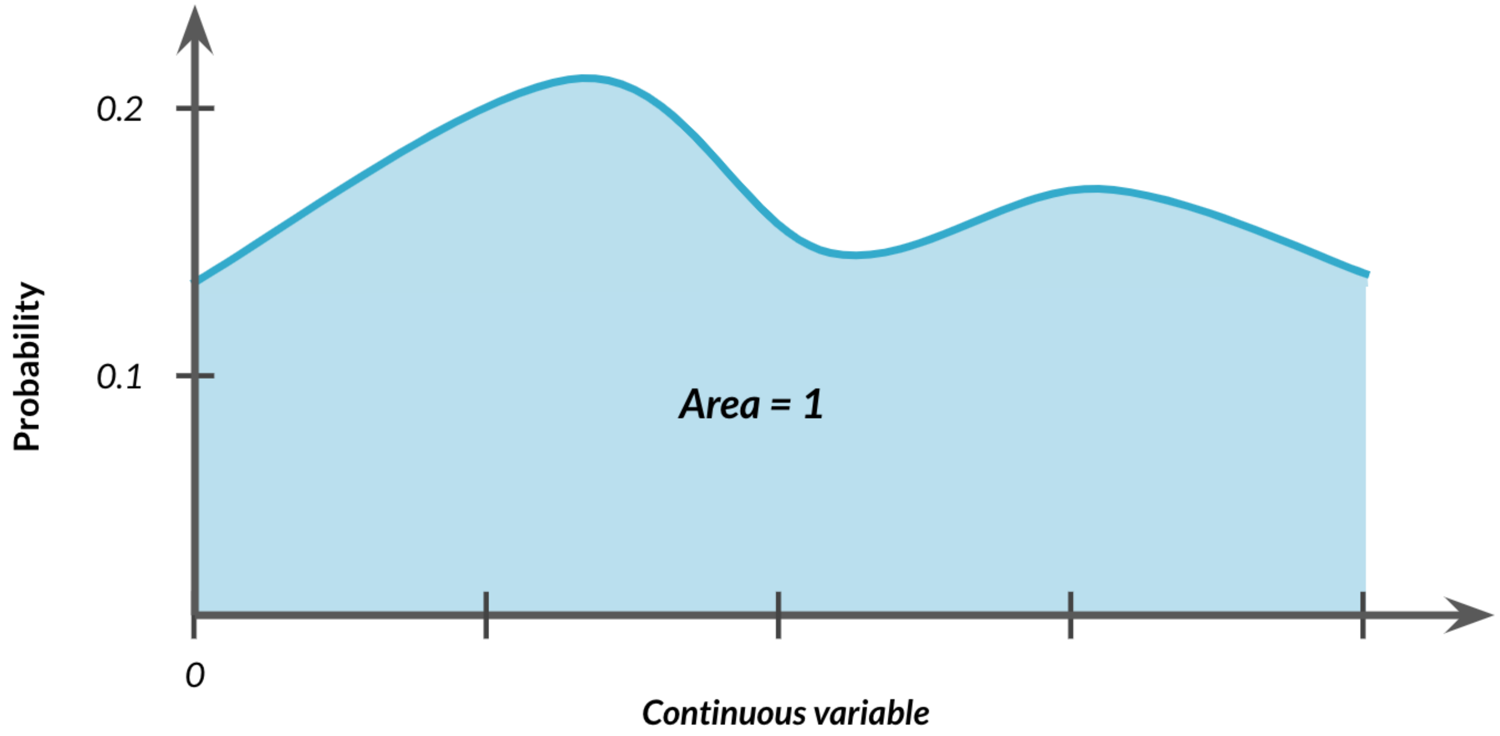
```
array([1.89740094, 4.70673196, 0.33224683, 1.0137103 , 2.31641255,
       3.49969897, 0.29688598, 0.92057234, 4.71086658, 1.56815855])
```

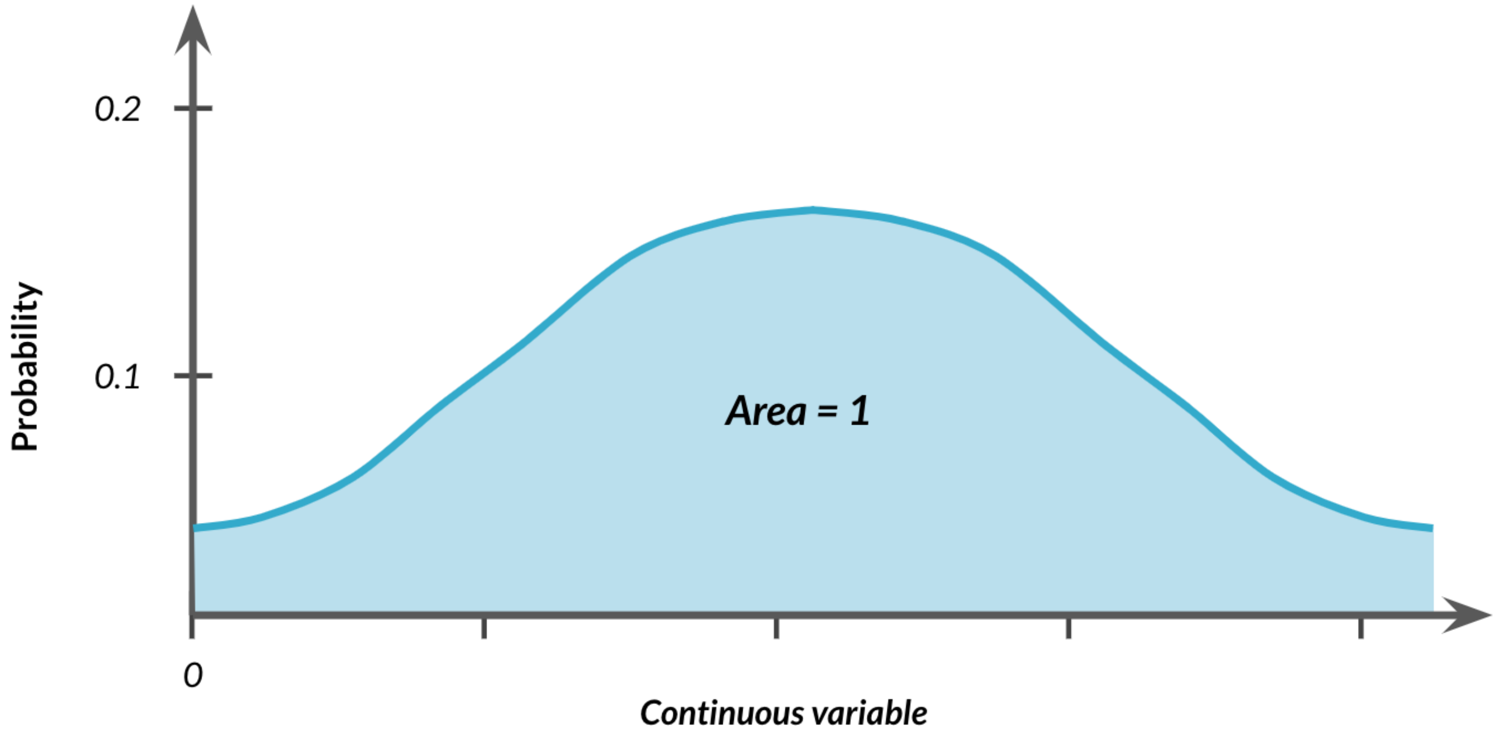
Other continuous distributions





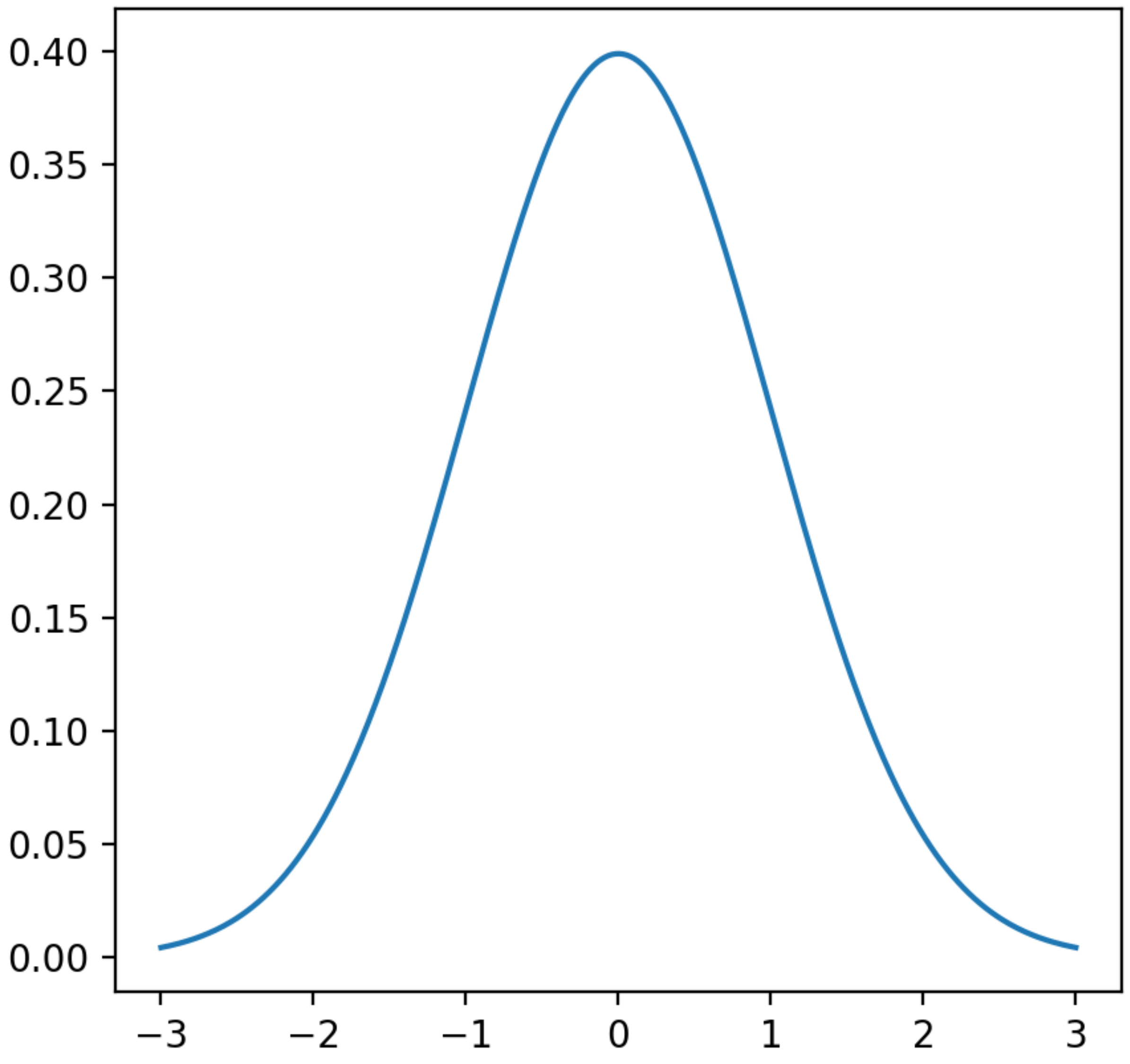
Other continuous distributions



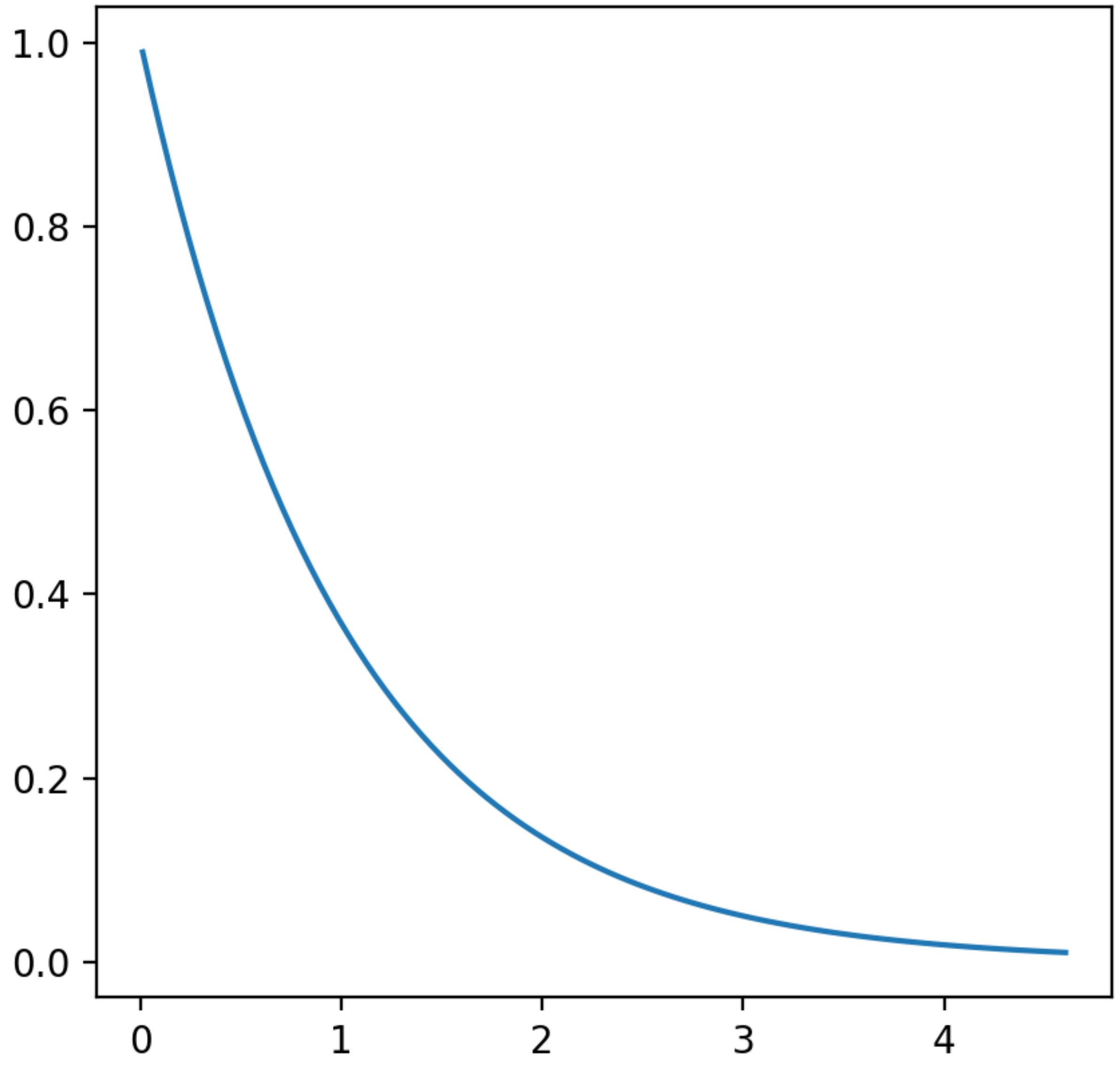


Other special types of distributions

Normal distribution



Exponential distribution



The binomial distribution

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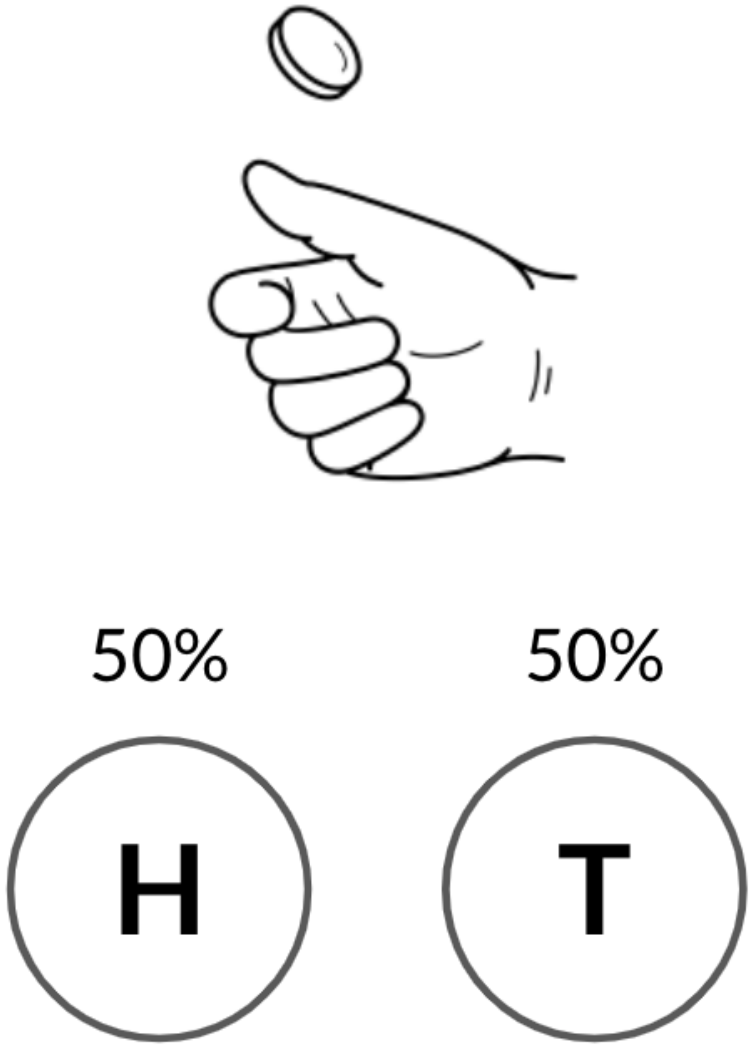
Coin flipping



Binary outcomes

H	T
1	0
Success	Failure
Win	Loss

A single flip

`binom.rvs(# of coins, probability of heads/success, size=# of trials)`

`1` = head, `0` = tails

```
from scipy.stats import binom
binom.rvs(1, 0.5, size=1)
```

```
array([1])
```

One flip many times

```
binom.rvs(1, 0.5, size=8)
```

```
array([0, 1, 1, 0, 1, 0, 1, 1])
```

```
binom.rvs(1, 0.5, size = 8)
```

Flip 1 coin with 50% chance of success 8 times

Many flips one time

```
binom.rvs(8, 0.5, size=1)
```

```
array([5])
```

```
binom.rvs(8, 0.5, size = 1)
```

Flip 8 coins with 50% chance of success 1 time

Many flips many times

```
binom.rvs(3, 0.5, size=10)
```

```
array([0, 3, 2, 1, 3, 0, 2, 2, 0, 0])
```

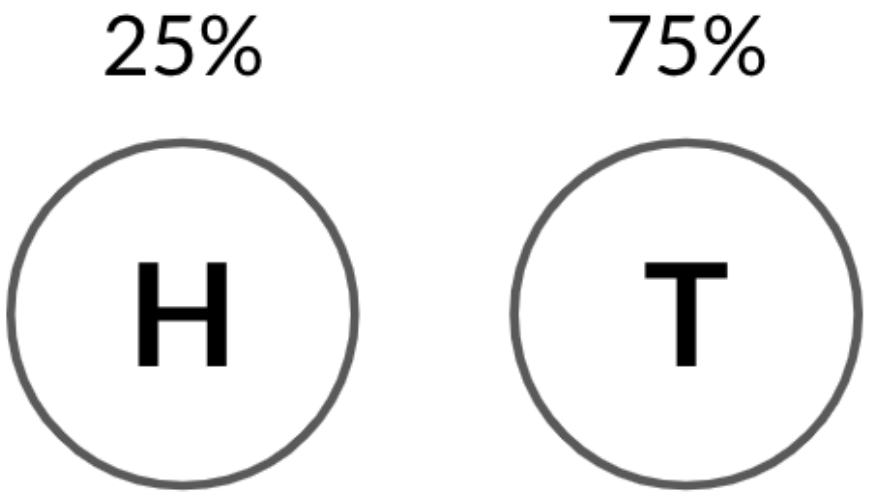
```
binom.rvs(3, 0.5, size = 10)
```

Flip 3 coins with 50% chance of success 10 times

Other probabilities

```
binom.rvs(3, 0.25, size=10)
```

```
array([1, 1, 1, 1, 0, 0, 2, 0, 1, 0])
```



Binomial distribution

Probability distribution of the number of successes in a sequence of independent trials

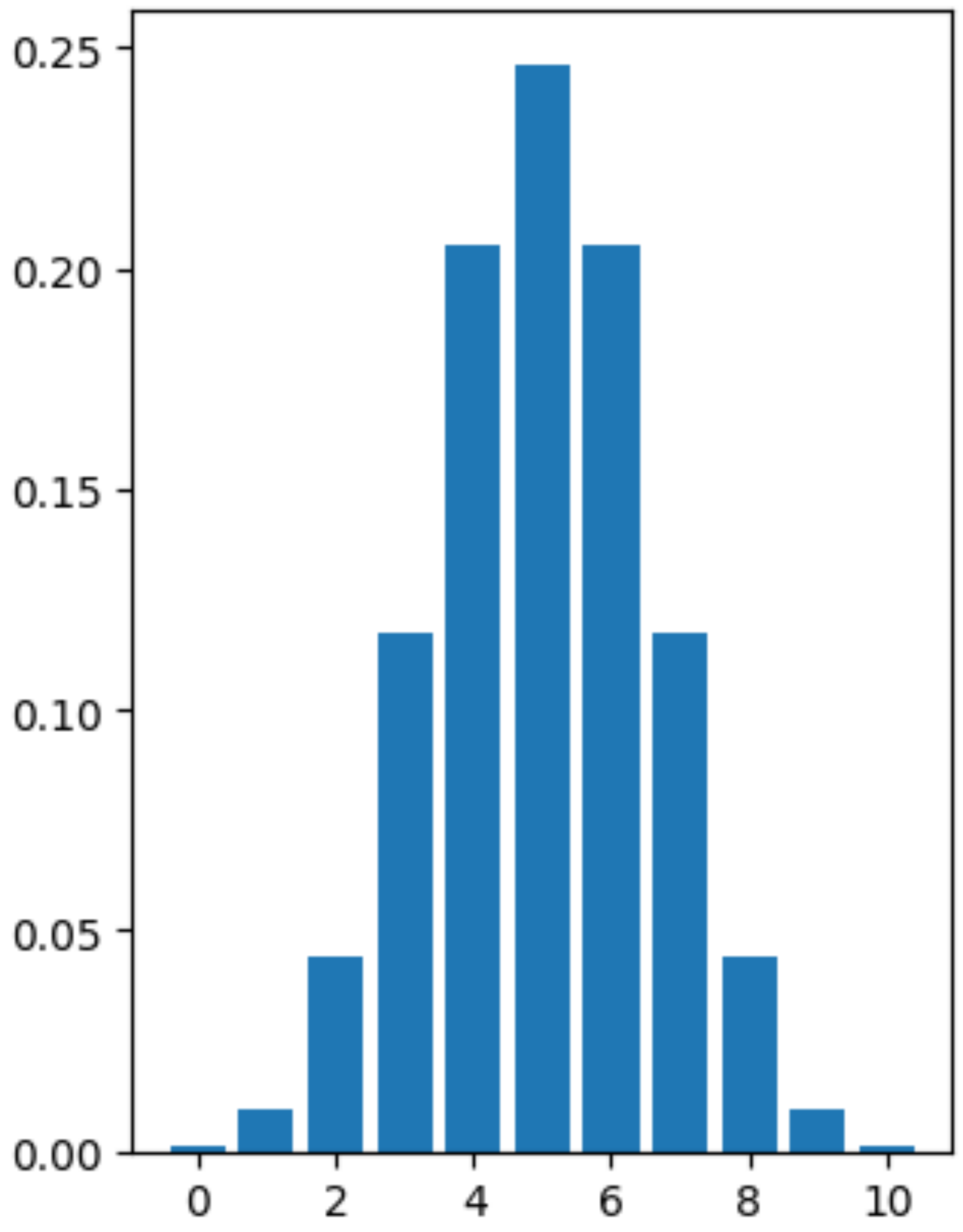
E.g. Number of heads in a sequence of coin flips

Described by n and p

- n: total number of trials
- p: probability of success

p **n**

```
binom.rvs(3, 0.5, size = 10)
```



What's the probability of 7 heads?

$P(\text{heads} = 7)$

```
# binom.pmf(num heads, num trials, prob of heads)
binom.pmf(7, 10, 0.5)
```

```
0.1171875
```

What's the probability of 7 or fewer heads?

$P(\text{heads} \leq 7)$

```
binom.cdf(7, 10, 0.5)
```

```
0.9453125
```

What's the probability of more than 7 heads?

$P(\text{heads} > 7)$

```
1 - binom.cdf(7, 10, 0.5)
```

```
0.0546875
```

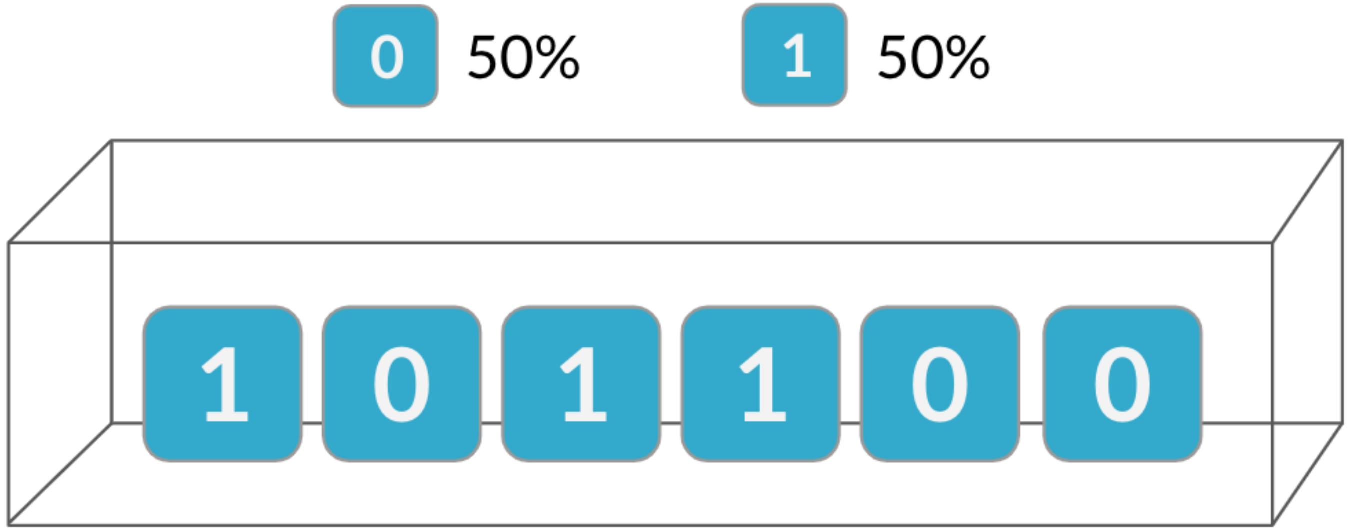
Expected value

$$\text{Expected value} = n \times p$$

$$\textit{Expected number of heads out of 10 flips} = 10 \times 0.5 = 5$$

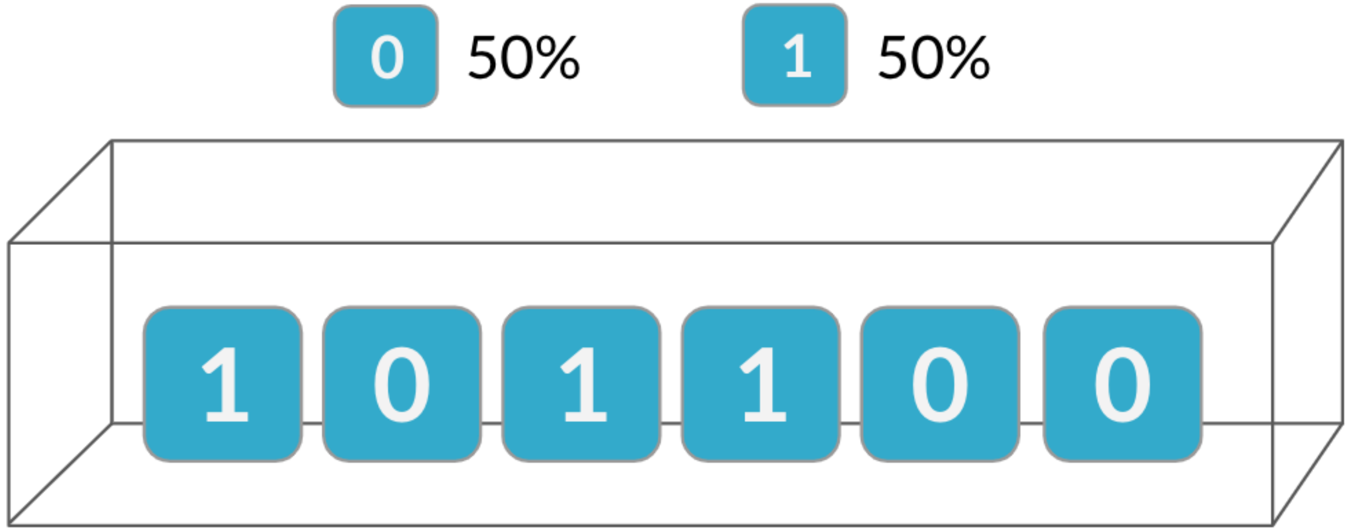
Independence

*The binomial distribution is a probability distribution of the number of successes in a sequence of **independent** trials*

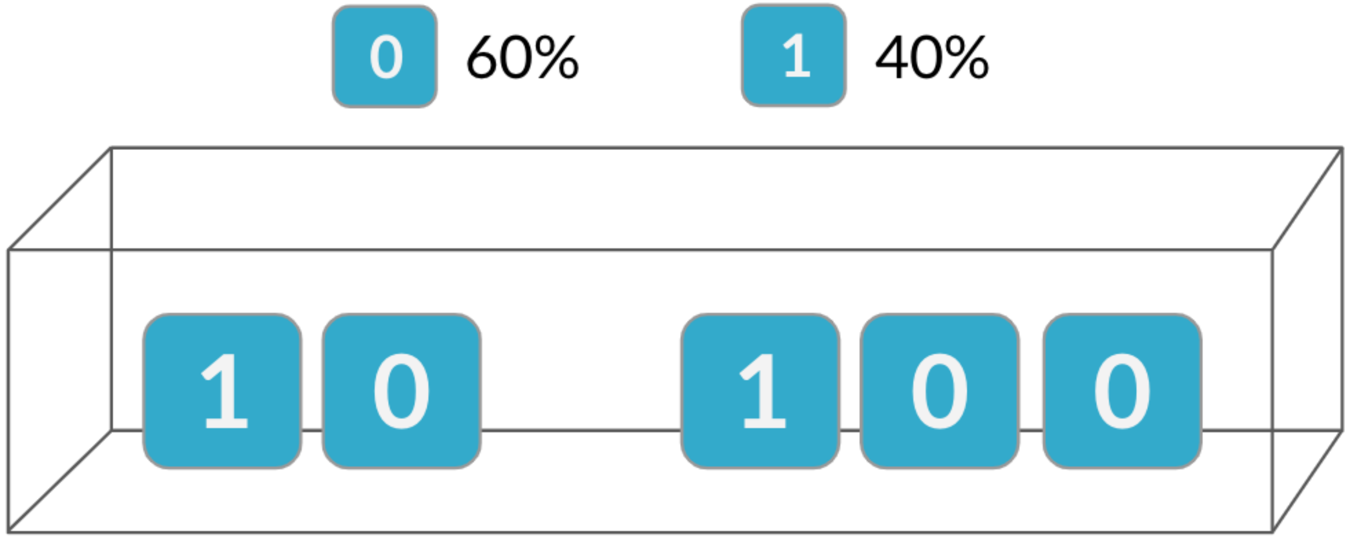


Independence

The binomial distribution is a probability distribution of the number of successes in a sequence of ***independent*** *trials*



Probabilities of second trial are altered due to outcome of the first



If trials are not independent, the binomial distribution does not apply!