WIIT 7751

Lesson G Guided Notes

For an experiment with a given sample space, the objective of probability is to assign each event A a number P(A), called the probability of the event A.

The probability of an event gives a precise measurement of the chance that A will occur.

Let A be an event consisting of a fixed set of outcomes in an experiment. The probability of event A is the long-run proportion of the time that the experiment outcome is in A, when the experiment is repeated many times under the same condition.

That is, $P(A) = \lim_{n \to \infty} \frac{n(A)}{n}$

where n(A) is the number of times A occurs in n repetitions.

<u>Probabilities with Equally Likely Events</u>

If a sample space has *n* equally likely outcomes, and an event *A* has *k* outcomes, then

<u>Example 3</u> A fair die is rolled. Find the probability that an odd number comes up.

Example 4 A family wants to have three pets, but only wants to have cats and dogs. Denoting a cat by C and a dog by D, we can denote the type of pet in the order of acquisition. For example, CDC means the first pet is a cat, the second pet is a dog, and the third is a cat. There are eight possible outcomes: CCC, CCD, CDC, CDD, DCC, DCD, DDC, and DDD. Assume the outcomes are equally likely. What is the probability that all three pets are of the same type?

Probability Rules

Sampling from a Population

Sampling an individual from a population is a probability experiment. The population is the sample space and members of the population are equally likely outcomes.
There are 10,000 families in a certain town categorized as follows:

Own a house	Own a condo	Rent a house	Rent an apartment
4753	1478	912	2857

A pollster samples a single family from this population. What is the probability that the sampled family rents?

A **compound event** is an event that is formed by combining two or more events.

One type of compound event is of the form ***A* or *B***.

The event *A* or *B* occurs whenever *A* occurs, *B* occurs, or *A* and *B* both occur.

<u>The General Addition Rule</u>

To compute probabilities of the form $P(A \text{ or } B)$, we use the **General Addition Rule**.

For any two events *A* and *B*,

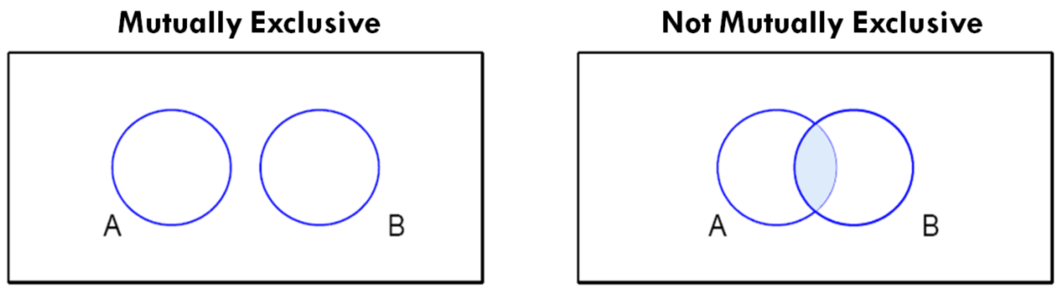
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

<u>Example 5</u> Consider the following table which presents the result of a survey in which 1000 adults were asked whether they favored a law that would provide government support for higher education. Each person was also asked whether they voted in the last election. Find the probability that a randomly selected person is likely to vote or favors the law.

	Favor	Oppose	Undecided
Likely to Vote	372	262	87
Not likely to vote	151	103	25

Mutually Exclusive Events

Two events are said to be **mutually exclusive** if it is impossible for both events to occur.



Example 6

a) A die is rolled. Event A is that the die comes up 3, and event B is that the die comes up an even number.

b) A fair coin is tossed twice. Event A is that one of the tosses is heads, and Event B is that one of the tosses is tails.

The Addition Rule for Mutually Exclusive Events

If events A and B are mutually exclusive, then $P(A \text{ and } B) = 0$. This leads to a simplification of the General Addition Rule.

If *A* and *B* are mutually exclusive events, then

$$P(A \text{ or } B) = P(A) + P(B)$$

Example 7 In the 2008 Olympic Games, a total of 11,028 athletes participated. Of these, 596 represented the United States, 332 represented Canada, and 85 represented Mexico. What is the probability that an Olympic athlete chosen at random represents the U.S. or Canada?

Complements

If there is a 60% chance of rain today, then there is a 40% chance that it will not rain. The events "Rain" and "No rain" are complements. The **complement** of an event says that the event does not occur.

If A is any event, the complement of A is the event that A does not occur. The complement of A is denoted A^c.

The Rule of Complements states that $P(A^c) = 1 - P(A)$.

Example 8 According to the *Wall Street Journal*, 42% of cars sold in May 2008 were small cars. What is the probability that a randomly chosen car sold in May 2008 is not a small car?

Independence

Two events are **independent** if the occurrence of one does not affect the probability that the other event occurs. If two events are not independent, we say they are **dependent**.

Example 9 Determine whether the following pairs of events are independent:

a) A college student is chosen at random. The events are “being a freshman” and “being less than 20 years old.”

b) A college student is chosen at random. The events are “born on a Sunday” and “taking a statistics class.”

The Multiplication Rule for Independent Events

When two events, A and B, are independent, then knowing that A occurred does not affect the probability that B occurs. This leads to the multiplication rule for independent events.

For any two independent events A and B,

$$P(A \text{ and } B) = P(A)P(B)$$

Example 10 According to recent figures from the U.S. Census Bureau, the percentage of people under the age of 18 was 23.5% in New York City, 25.8% in Chicago, and 26.0% in Los Angeles. If one person is selected from each city, what is the probability that all of them are under 18?

Sampling With and Without Replacement

When we sample two items from a population, we can proceed in either of two ways. We can replace the first item drawn before sampling the second. This is known as **sampling with replacement**. The other option is to leave the first item out when sampling the second one. This is known as **sampling without replacement**.

When sampling with replacement, each draw is made from the entire population, so the probability of drawing a particular item on the second draw does not depend on the first draw.

When sampling **with replacement**, the draws are **independent**.
When sampling **without replacement**, the draws are **not independent**.

"At Least One" Type of Probabilities

Sometimes we need to find the probability that an event occurs at least once in several independent trials.

The easiest way to calculate these probabilities is by finding the probability of the complement.

Remember that the complement of "At least one event occurs" is "No events occur".

Example 11 A fair coin is tossed five times. What is the probability that it comes up heads at least once?