

Lesson I Guided Notes

Mean of a Random Variable

The expected value of a discrete random variable is the weighted sum of the outcomes using the probabilities as weights. If X is a discrete random variable, then

$$E(X) = \mu_X = \sum_{\text{all } x} x \cdot p(x)$$

$E(X)$ provides a summary measure of the average value taken by the random variable and is also known as the mean of the random variable.

This value can be interpreted as a weighted average of the values.

Example 1 Suppose a fair coin is flipped four times and the number of heads is observed. The probability distribution table is given below.

x	0	1	2	3	4
$p(x)$	0.0625	0.25	0.375	0.25	0.0625

Find the expected value of this distribution.

Example 2 Consider another discrete random variable with the given probability distribution for the number of landline phones in a house.

x	0	1	2	3
$p(x)$	0.55	0.30	0.10	0.05

Find the expected value

Interpretation of Expected Value

Sometimes, the expected value will be one of the possible values for the random variable, but will also sometimes not be one of the values the random variable can take on.

We can think of the expected value as follows: suppose we could repeat the experiment over and over, many times. If we were to take the mean of these values, it would tend towards the expected value as the number of trials of the experiment increased.

There are many occasions on which people want to predict how much they are likely to gain or lose if they make a certain decision or take a certain action. Often, this is done by computing the mean of a random variable.

In such situations, the mean is sometimes called the “expected value” and is sometimes denoted by $E(X)$. If the expected value is positive, it is an expected gain, and if it is negative, it is an expected loss.

Example 3 A mineral economist estimated that a particular mining venture had probability 0.4 of a \$30 million loss, probability 0.5 of a \$20 million profit, and probability 0.1 of a \$40 million profit. Let X represent the profit, in millions of dollars. Find the probability distribution of the profit and the expected value of the profit. Does this venture represent an expected gain or an expected loss?

Variance

Let X have expected value μ . Then the variance of X , denoted by $V(X)$ or σ_X^2 (or just σ^2) is

$$V(X) = \sum_{\text{all } x} (x - \mu)^2 \cdot p(x) = E[(X - \mu)^2]$$

The standard deviation of X is $\sigma_X = \sqrt{\sigma_X^2}$

A shortcut formula for the variance is as follows:

$$V(X) = \left[\sum_{\text{all } x} x^2 \cdot p(x) \right] - \mu^2$$

Example 4 Suppose a fair coin is flipped four times and the number of heads is observed. The probability distribution table is given below. Find the variance and standard deviation of this distribution.

x	0	1	2	3	4
$p(x)$	0.0625	0.25	0.375	0.25	0.0625

Example 5 Consider another discrete random variable with the given probability distribution for the number of landline phones in a house. Find the variance and standard deviation of this distribution.

x	0	1	2	3
$p(x)$	0.55	0.30	0.10	0.05