WIIT 7751

Lesson H Guided Notes

Random Variable

If we roll a fair die, the possible outcomes are the numbers 1, 2, 3, 4, 5, and 6, and each of these numbers has probability 1/6. Rolling a die is a probability experiment whose outcomes are numbers. The outcome of such an experiment is called a **random variable**.

Discrete random variables –

Continuous random variables –

Probability Distribution

A **probability distribution** for a discrete random variable specifies the probability for each possible value of the random variable. For discrete random variables, the probability distribution is often referred to as a **probability mass function**, or **pmf**.

Properties:

Example 1 Decide if each of the following is a probability distribution

x	1	2	3	4
P(x)	0.25	0.65	-0.30	0.11

x	3	4	5	6	7
P(x)	0.17	0.25	0.31	0.22	0.05

x	1	2	3	4
P(x)	1.02	0.31	0.90	0.43

Example 2 Four patients have made appointments to have their blood pressure checked at a clinic. Let X be the number of them that have high blood pressure. Based on data from the National Health and Examination Survey, the probability distribution of X is

x	0	1	2	3	4
P(x)	0.23	0.41	0.27	0.08	0.01

(a) Find P(2 or 3)

(b) Find P(More than 1)

(c) Find P(At least one)

Probability Distributions and Populations

Statisticians are interested in studying samples drawn from populations. Random variables are important because when an item is drawn from a population, the value observed is the value of a random variable.

The probability distribution of the random variable tells how frequently we can expect each of the possible values of the random variable to turn up in the sample.

Example 3 In a town with a population of 1000 households, 142 of the households have no car, 378 have one car, 423 have two cars, and 57 have three cars. A household is sampled at random. Let *X* represent the number of cars in the randomly sampled household. Find the probability distribution of *X*.

Cumulative Distribution Function

An alternative way of specifying the probabilistic properties of a random variable X is through the function

$$F(x) = P(X \text{ is at most } x)$$

Which is known as the cumulative distribution function, for which the abbreviation CDF is used.

Example 4 Airlines sometimes overbook flights. Suppose that for a plane with 50 seats, 55 passengers have tickets. Define the random variable *Y* as the number of ticketed passengers who actually show up for the flight. The probability mass function of *Y* appears in the accompanying table.

y	45	46	47	48	49	50	51	52	53	54	55
$p(y)$	0.04	0.10	0.13	0.14	0.25	0.17	0.06	0.05	0.03	0.02	0.01

(a) What is the probability that the flight will accommodate all ticketed passengers who show up?

(b) What is the probability that not all ticketed passengers who show up can be accommodated?

(c) If you are the first person on the standby list (which means you will be the first one to get on the plane if there are any seats available after all ticketed passengers have been accommodated), what is the probability that you will be able to take the flight?

(d) What is this probability if you are the third person on the standby list?