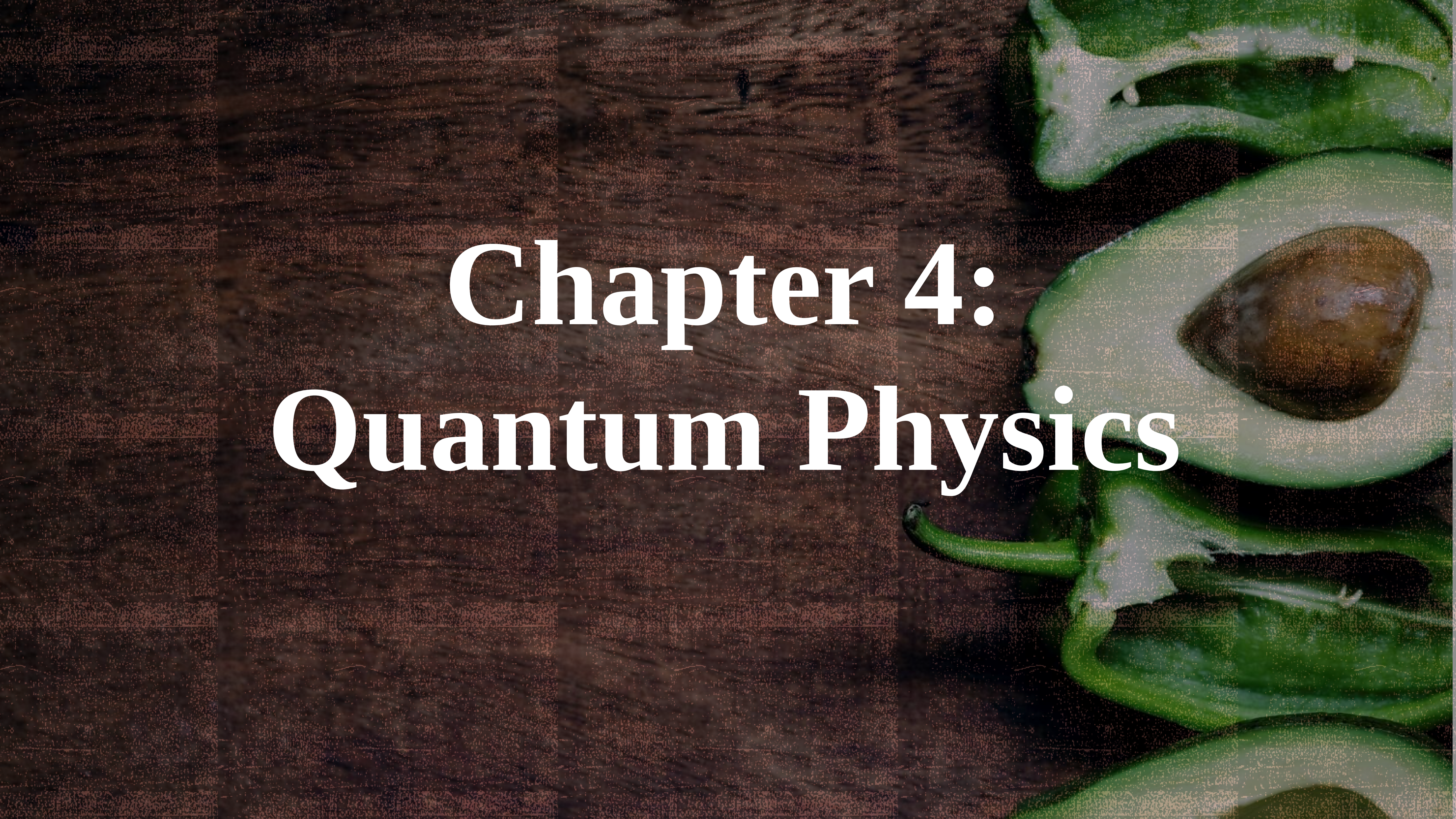

Chapter 4:
Quantum Physics

Photons

Eistein proposed that:

- Light is quantized
- For light of frequency f, quanta have energy **ε = hf**

Where h is Planck's constant, $\mathbf{h = 6.63 \times 10^{-34} Js}$

- Light is both emitted and absorbed as quanta

$$\boxed{\varepsilon = hf = \hbar\omega = \frac{hc}{\lambda}}$$

$$\boldsymbol{\hbar = h/2\pi = 1.05\times10^{-34}\,Js}$$

The Photoelectric Effect

$$\boxed{\mathrm{hf} = \mathrm{K}_{\mathrm{max}} + \Phi = \mathrm{eV}_{\mathrm{stop}} + \Phi}$$

Cutoff wavelength λ_c

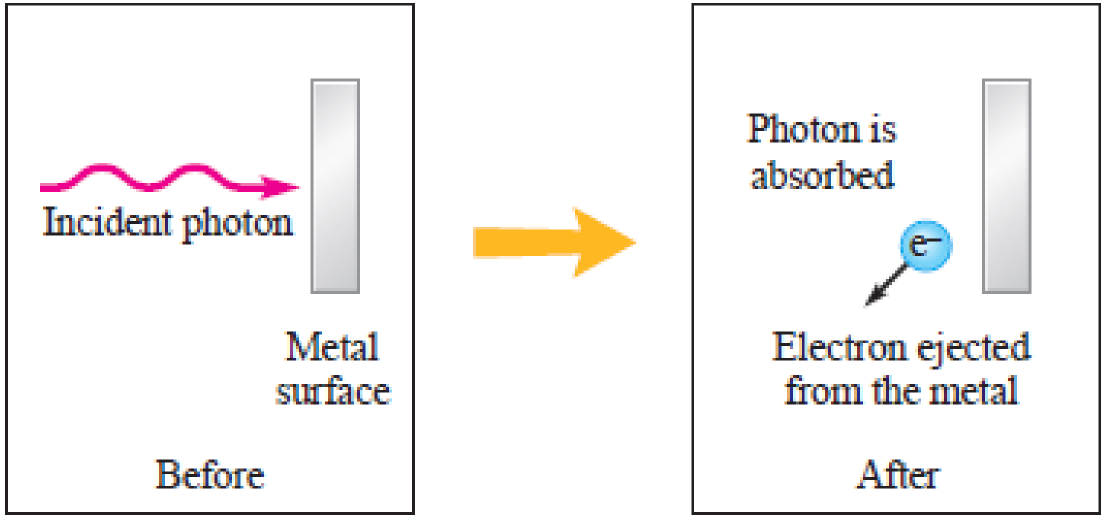
$$\boxed{\lambda_{\mathrm{c}} = \frac{\mathrm{c}}{\mathrm{f}_{\mathrm{c}}} = \frac{\mathrm{c}}{\Phi/\mathrm{h}} = \frac{\mathrm{hc}}{\phi}}$$

Photon Momentum

Light of frequency f and wavelength λ has photons of energy and momentum

$$\boxed{\varepsilon = hf}$$

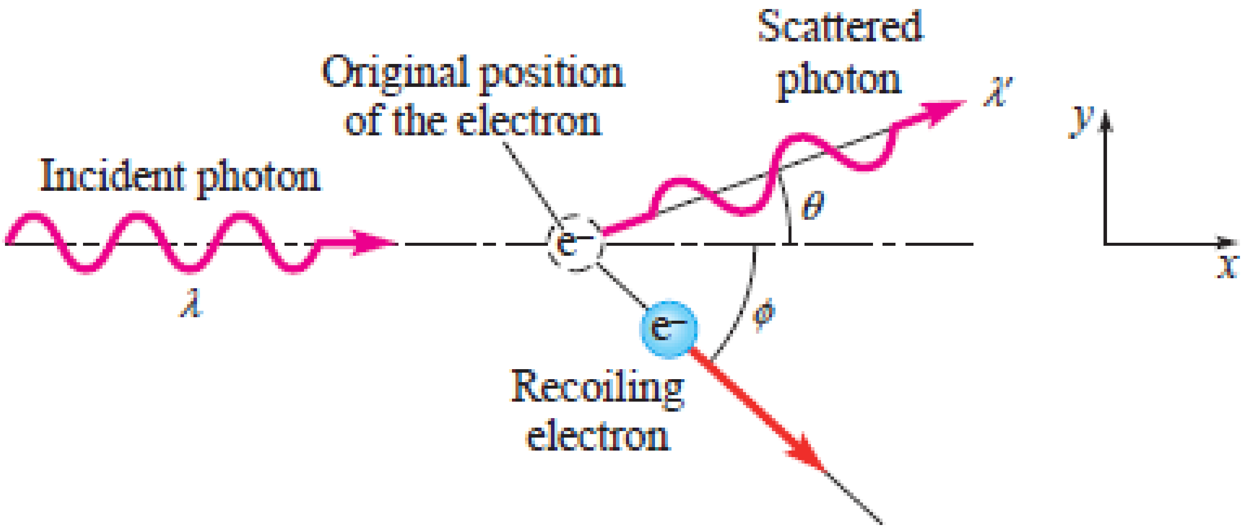
$$\boxed{p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{\varepsilon}{c}}$$



Compton Scattering

Compton shift:

$$\boxed{\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)}$$



The momentum of a photon can be expressed as

$$p = \frac{h}{\lambda}$$

the magnitude of the momentum of a particle of mass m and speed u is $p = mu$, the **de Broglie wavelength** of that particle is

$$\lambda = h/p = h/mu$$

The Uncertainty Principle

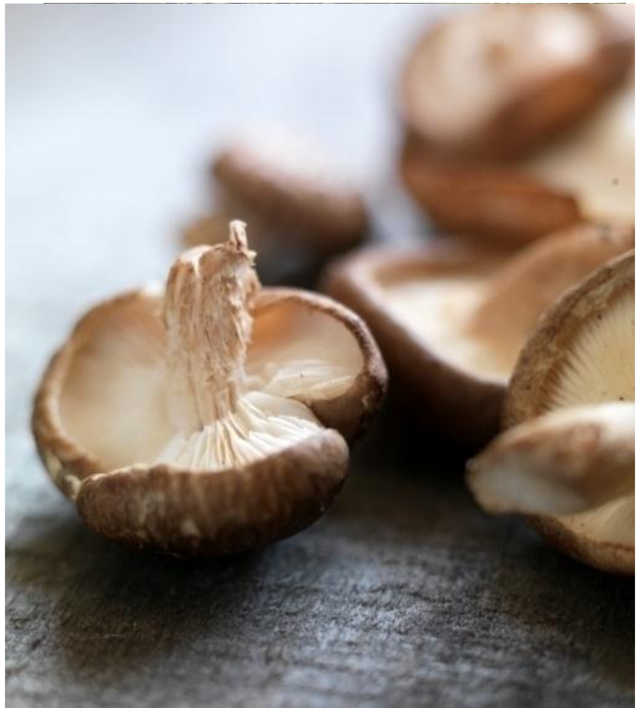
If a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its x component of momentum is made with uncertainty $\Delta\, p_x$, the product of the two uncertainties can never be smaller than $\hbar/2$:

$$\boxed{\Delta x\, \Delta p_x \geq \hbar/2}$$

Δx: uncertainty in the position
Δp_x: uncertainty in the x – momentum of the partical

$$\boxed{\Delta E\, \Delta \mathrm{t} \geq \hbar/2}$$

Δt: uncertainty in the time
ΔE: uncertainty in the energy



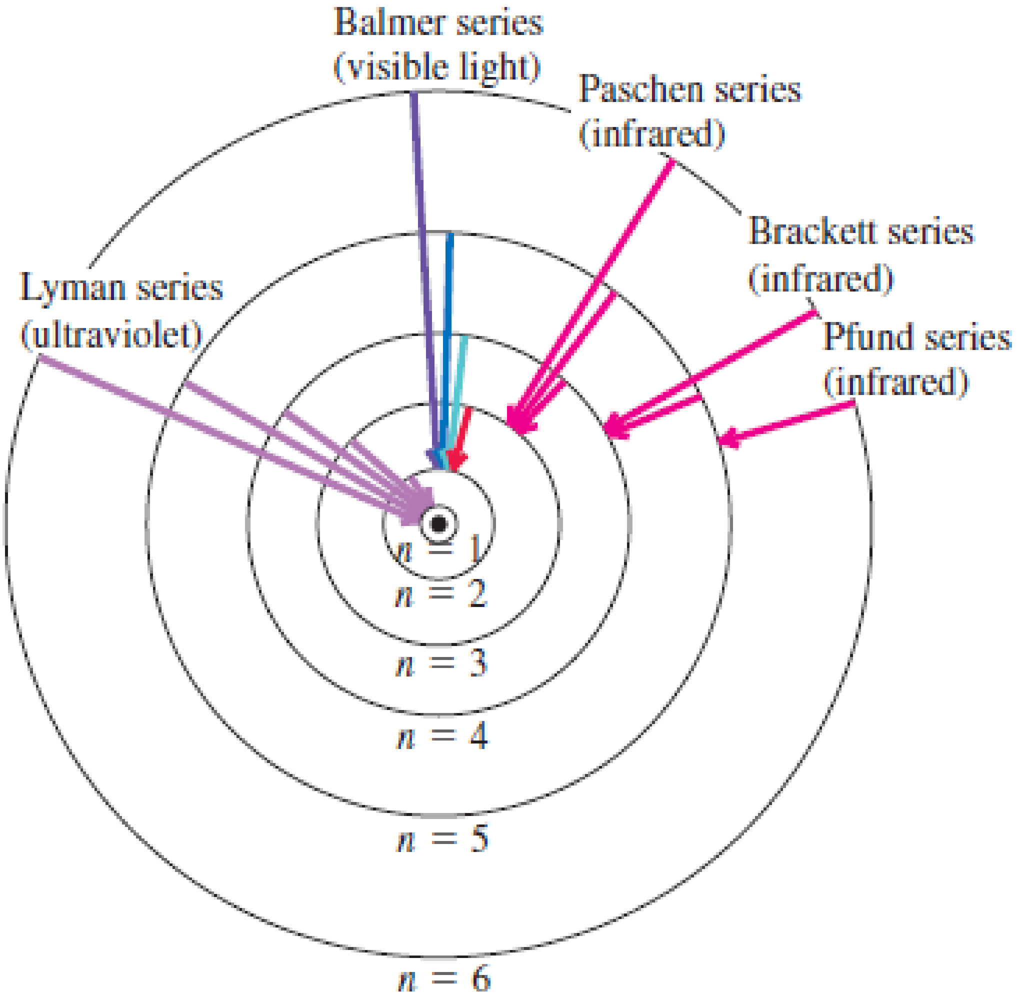
The Bohr Model

Bohr's hypothesis

If is the initial energy of an atom before a transition from one energy level to another, is the atom's (smaller) final energy after the transition, and the energy of the emitted photon is then

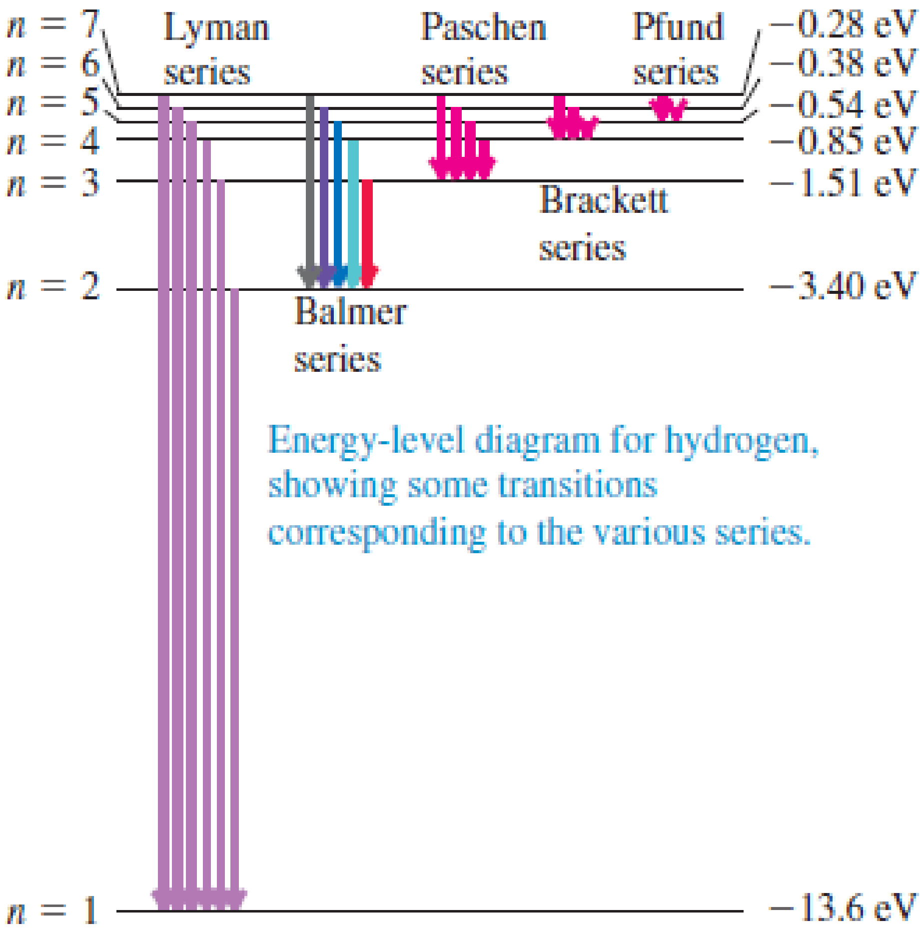
$$\mathbf{hf} = E_i - E_f$$

The Bohr Model



"Permitted" orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.

(a)



Energy-level diagram for hydrogen, showing some transitions corresponding to the various series.

(b)

The Bohr Model

The **energy levels** in a hydrogen atom:

$$\boxed{E_n = -\frac{me^4}{8\varepsilon_0{}^2h^2}\frac{1}{n^2} = -\frac{hcR}{n^2} = -\frac{13.60\ \text{eV}}{n^2}}$$

where $R = \dfrac{me^4}{8\varepsilon_0{}^2h^3c} = 1.1\times10^7\ \text{m}^{-1}$ is the Rydberg constant.

Transitions involve photons of energy

$$\Delta E = E_{high} - E_{low} = hcR\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) = 13.60\ \text{eV}\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right)$$

and wavelength $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_{low}^2} - \dfrac{1}{n_{high}^2}\right)$

The Bohr Model

Balmer's formula for the hydrogen spectrum

Balmer's formula is

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right),$$

where λ is the wavelength, R is a constant called the **Rydberg constant,** and n may have the integer values 3, 4, 5,· · ·. If λ is in meters, the numerical value of R (determined from measurements of wavelengths) is

$$R = 1.097 \times 10^7\ \mathrm{m}^{-1}.$$

1. The wavelength of the yellow spectral emission line of sodium is 590 nm. Find the photon energy (in electron volts) and the photon momentum.

1. The wavelength of the yellow spectral emission line of sodium is 590 nm. Find the photon energy (in electronvolts) and the photon momentum.

$\lambda = 590\text{nm} \Rightarrow \varepsilon = ?\ p = ?$

Photon energy is

$$\varepsilon = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{590\times10^{-9}} = 0.0337\times10^{-17}(\text{J}) = 2.11(\text{eV})$$

Photon momentum is

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{590\times10^{-9}} = 1.12\times10^{-27}(\text{kgm/s})$$

2. A helium-neon laser emits red light at wavelength 633 nm in a beam of diameter 3.5 mm and at an energy emission rate of 5.0 mW. A detector in the beam's path totally absorbs the beam. At what rate per unit area does the detector absorb photons?

2. A helium-neon laser emits red light at wavelength 633 nm in a beam of diameter 3.5 mm and at an energy emission rate of 5.0 mW. A detector in the beam's path totally absorbs the beam. At what rate per unit area does the detector absorb photons?

$$\begin{cases} \lambda = 633\text{nm} \\ d = 3.5\text{mm} \Rightarrow R = 1.75\text{mm} \\ E = 5\text{mW} = 5\text{mJ}/\text{s} = 5\times10^{-3}(\text{J}/\text{s}) \end{cases}$$

The number of photons which reach to the detector in 1 s is

$$E = n\varepsilon = n\frac{hc}{\lambda} \Rightarrow n = \frac{E\lambda}{hc} = \frac{5\times10^{-3}\times6.33\times10^{-9}}{6.63\times10^{-34}\times3\times10^{8}} = 159.125\times10^{14}(\text{photons})$$

The detector absorb photons with rate is

$$\frac{n}{A} = \frac{n}{\pi R^2} = \frac{159.125\times10^{14}}{\pi\times(1.75\times10^{-3})^2} = 1.65\times10^{21}(\text{photons}/\text{m}^2\text{s})$$

3. Under ideal conditions, a visual sensation can occur in the human visual system if light of wavelength 550 nm is absorbed by the eye's retina at a rate as low as 100 photons per second. What is the corresponding rate at which energy is absorbed by the retina?

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λ = 550 nm
Rate = 100 photons

Energy of one photon is $\varepsilon = \dfrac{hc}{\lambda} = \dfrac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}} = 0.0361\times10^{-17}\,(\mathrm{J})$

Energy is absorbed by the retina

$$E = 100\varepsilon = 3.61\times10^{-17}\,(\mathrm{J/s})$$

4. You wish to pick an element for a photocell that will operate via the photoelectric effect with visible light. Which of the following are suitable? (Work functions are in brackets.) Crome (4.2 eV), Coban (5.0 eV), aluminium (4.2 eV), barium (2.5 eV), lithium (2.3 eV).

4. You wish to pick an element for a photocell that will operate via the photoelectric effect with visible light. Which of the following are suitable? (Work functions are in brackets.) Tantalum (4.2 eV), tungsten (4.5 eV), aluminium (4.2 eV), barium (2.5 eV), lithium (2.3 eV).

Photo electric effect formular is $hf = K_{max} + \Phi = eV_{stop} + \Phi$

Visible light $\lambda = 400 - 700\text{nm}$

Limiting wave $$\varepsilon = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{400\times10^{-9}} = 3.1(\text{eV})$$

➡ Barium and lithium

5. Light of wavelength 200 nm shines on an aluminium surface; 4.20 eV is required to eject an electron. What is the kinetic energy of (a) the fastest and (b) the slowest ejected electrons? (c) What is the stopping potential for this situation? (d) What is the cut-off wavelength for aluminium?

5. Light of wavelength 200 nm shines on an aluminium surface; 4.20 eV is required to eject an electron. What is the kinetic energy of (a) the fastest and (b) the slowest ejected electrons? (c) What is the stopping potential for this situation? (d) What is the cut-off wavelength for aluminium?

$$\begin{cases} \lambda = 200\text{nm} \\ \Phi = 4.2\text{eV} \end{cases}$$

a) The kinetic energy of the fastest ejected electrons

$$K_{max} = \frac{hc}{\lambda} - \Phi = \frac{6.63\times10^{-34}\times3\times10^{8}}{200\times10^{-9}} - 4.2\times1.6\times10^{-19} = 6.22(\text{eV}) - 4.2(\text{eV}) = 2.02(\text{eV})$$

b) $K_{min} = 0$

c) The stopping potential for this situation is

$$eV = K_{max}$$

$$\Rightarrow V = \frac{K_{max}}{e} = \frac{2.02\times1.6\times10^{-19}}{1.6\times10^{-19}} = 2.02(\text{Volt})$$

d) The cut-off wavelength of aluminium is

$$\Phi = \frac{hc}{\lambda} \qquad (K_{max} = 0)$$

$$\Rightarrow \lambda = \frac{hc}{\Phi} = \frac{6.63\times10^{-34}\times3\times10^{8}}{4.2\times1.6\times10^{-19}} = 2.95\times10^{-7}(m) = 295(nm)$$

6. Molybdenum has a work function of 4.20 eV. (a) Find the cutoff wavelength and cutoff frequency for the photoelectric effect. (b) What is the stopping potential if the incident light has a wavelength of 180 nm?

6. Molybdenum has a work function of 4.20 eV. (a) Find the cutoff wavelength and cutoff frequency for the photoelectric effect. (b) What is the stopping potential if the incident light has a wavelength of 180 nm?

a) Cutoff wavelength is $\lambda_c = \frac{hc}{\phi} = \frac{6.63\times10^{-34}\times3\times10^{8}}{4.2\times1.6\times10^{-19}} = 295\ \text{nm}$

Cutoff frequency is $f_c = \frac{c}{\lambda_c} = \frac{3\times10^{8}}{295\times10^{-9}} = 1.02 \times 10^{15}\ \text{Hz}$

b) The stopping potential is $hf = \frac{hc}{\lambda} = eV_{stop} + \Phi$

$eV_{stop} = \frac{hc}{\lambda} - \Phi = \frac{6.63\times10^{-34}\times3\times10^{8}}{180\times10^{-9}} - 4.2 \times 1.6 \times 10^{-19} = 4.33 \times 10^{-19}$

$V_{stop} = 4.33 \times 10^{-19}/1.6 \times 10^{-19} = 2.7$ (volt)

7. The work function for zinc is 4.31 eV. (a) Find the cutoff wavelength for zinc. (b) What is the lowest frequency of light incident on zinc that releases photoelectrons from its surface? (c) If photons of energy 5.50 eV are incident on zinc, what is the maximum kinetic energy of the ejected photoelectrons?

7. The work function for zinc is 4.31 eV. (a) Find the cutoff wavelength for zinc. (b) What is the lowest frequency of light incident on zinc that releases photoelectrons from its surface? (c) If photons of energy 5.50 eV are incident on zinc, what is the maximum kinetic energy of the ejected photoelectrons?

a) Cutoff wavelength is $\lambda_c = \frac{hc}{\phi} = \frac{6.63\times10^{-34}\times3\times10^{8}}{4.31\times1.6\times10^{-19}} = 288 \text{ nm}$

b) Cutoff frequency is $f_c = \frac{c}{\lambda_c} = \frac{3\times10^{8}}{288\times10^{-9}} = 1.04 \times 10^{15} \text{ Hz}$

c) The stopping potential is $\varepsilon = hf = K_{max} + \Phi$

$K_{max} = \varepsilon - \Phi = 5.5 - 4.31 = 1.19 \text{ eV}$

8. Two light sources are used in a photoelectric experiment to determine the work function for a particular metal surface. When green light from a mercury lamp (λ= 546.1 nm) is used, a stopping potential of 0.376 V reduces the photocurrent to zero. (a) Based on this measurement, what is the work function for this metal? (b) What stopping potential would be observed when using the yellow light from a helium discharge tube (λ = 587.5 nm)?

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a) The work function for this metal $\mathrm{hf} = \frac{hc}{\lambda} = eV_{stop} + \Phi$

$$\Phi = \frac{hc}{\lambda} - eV_{stop} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{546.1 \times 10^{-9}} - 0.376 \times 1.6 \times 10^{-19}$$

$\Phi = 3.04 \times 10^{-19}$ (J) = 1.9 eV

b) The stopping potential is

$$eV_{stop} = \frac{hc}{\lambda} - \Phi = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{587.5 \times 10^{-9}} - 3.04 \times 10^{-19}$$

V_{stop} = 0.21 (volt)

9. Lithium, beryllium, and mercury have work functions of 2.30 eV, 3.90 eV, and 4.50 eV, respectively. Light with a wavelength of 400 nm is incident on each of these metals. (a) Determine which of these metals exhibit the photoelectric effect for this incident light. Explain your reasoning. (b) Find the maximum kinetic energy for the photoelectrons in each case.

9. Lithium, beryllium, and mercury have work functions of 2.30 eV, 3.90 eV, and 4.50 eV, respectively. Light with a wavelength of 400 nm is incident on each of these metals. (a) Determine which of these metals exhibit the photoelectric effect for this incident light. Explain your reasoning. (b) Find the maximum kinetic energy for the photoelectrons in each case.

a) The photon energy is $\varepsilon = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{400\times10^{-9}} = 4.9725\times10^{-19}$ (J)

➡ $\varepsilon = 3.1$ eV

➡ Lithium

b) The maximum kinetic energy is $\varepsilon = hf = K_{max} + \Phi$

➡ $K_{max} = \varepsilon - \Phi = 3.1 - 2.3 = 0.8$ eV

10. Electrons are ejected from a metallic surface with speeds of up to 4.60 x 10^5 m/s when light with a wavelength of 625 nm is used. (a) What is the work function of the surface? (b) What is the cutoff frequency for this surface?

1 eV = 1.6 x $10^{-19} J$

4.9725 × 10^{-19} (J) / (1.6 x 10^{-19}) = …. (eV)

10. Electrons are ejected from a metallic surface with speeds of up to 4.60 x 10^5 m/s when light with a wavelength of 625 nm is used. (a) What is the work function of the surface? (b) What is the cutoff frequency for this surface?

a) The maximum kinetic energy of electron

$$K_{max} = \frac{1}{2}mu^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (4.6 \times 10^5)^2 = 9.64\times 10^{-20} (J)$$

The work function is

$$hf = \frac{hc}{\lambda} = K_{max} + \Phi$$

$$\Phi = \frac{hc}{\lambda} - K_{max} = \frac{6.63\times10^{-34}\times3\times10^8}{625\times10^{-9}} - 9.64 \times 10^{-20} = 2.2\times 10^{-19}\ (J)$$

b) The cutoff wavelength is

$$\lambda_c = \frac{hc}{\phi} = \frac{6.63\times10^{-34}\times3\times10^8}{2.2\times10^{-19}} = 9.04\times 10^{-7}\ \ (m)$$

The cutoff frequency is

$$f_c = \frac{c}{\lambda_c} = \frac{3\times10^8}{9.04\times10^{-7}} = 3.32 \times 10^{14}\ Hz$$

11. Light of wavelength 2.40 pm is directed onto a target containing free electrons. Find the wavelength of light scattered at (a) 30^0, and (b) 120^0 from the incident direction.

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$$\lambda = 2.4\text{pm} = 2.4\times10^{-12}(\text{m})$$

a) $\phi = 30^0$

b) $\phi = 120^0$

a) The wavelength of light scattered is

Φ

ϕ

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = \frac{6.63\times10^{-34}(1-\cos 30^0)}{9.11\times10^{-31}\times3\times10^8} = 0.325\times10^{-12}(\text{m})$$

$$\lambda' = 2.4\times10^{-12} + 0.325\times10^{-12} = 2.73(\text{pm})$$

b) The wavelength of light scattered is

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = \frac{6.63\times10^{-34}(1-\cos 120^0)}{9.11\times10^{-31}\times3\times10^8} = 0.36\times10^{-11}(\text{m})$$

$$\lambda' = 2.4\times10^{-12} + 3.6\times10^{-12} = 6(\text{pm})$$

12. What is the maximum wavelength shift for a Compton collision between a photon and a free *proton*

12. What is the maximum wavelength shift for a Compton collision between a photon and a free *proton*

$\theta = 180^0$

The maximum wavelength shift for a Compton collision is

$$\Delta\lambda = \frac{6.63\times10^{-34}\times2}{m_p c} = \frac{6.63\times10^{-34}\times2}{1.67\times10^{-27}\times3\times10^{8}} = 2.64\times10^{-15}(\mathrm{m})$$

13. X-rays are scattered from a target at an angle of 55.0° with the direction of the incident beam. Find the wavelength shift of the scattered x-rays.

13. X-rays are scattered from a target at an angle of 55.0° with the direction of the incident beam. Find the wavelength shift of the scattered x-rays.

Compton shift: $$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

Wavelength shift of the x rays

$$\lambda' - \lambda_0 = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}(1 - \cos 55°)$$

$$\Delta\lambda = 1.03 \times 10^{-3}\ \text{nm}$$

14. Find the (a) wavelength and (b) frequency of a 3.1-eV photon.

Ans: (a) 400 nm (b) 7.5×10^{14}Hz

14. Find the (a) wavelength and (b) frequency of a 3.1-eV photon.

$$\varepsilon = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\varepsilon} = \frac{6.63\times10^{-34}\times3\times10^{8}}{3.1\times1.6\times10^{-19}} = 4.01 \times 10^{-7}\ (\mathrm{m})$$

$\varepsilon = hf$ $f = \varepsilon / h = 3.1 \times 1.6 \times 10^{-19}/6.63 \times 10^{-34} = 7.48 \times 10^{14}\ (\mathrm{Hz})$

15. A rubidium surface has a work function of 2.16 eV. (a) What is the maximum kinetic energy of ejected electrons if the incident radiation is of wavelength 413 nm? (b) What is the threshold wavelength for this surface?

Ans: (a) 0.84 eV (b) 574 nm

15. A rubidium surface has a work function of 2.16 eV. (a) What is the maximum kinetic energy of ejected electrons if the incident radiation is of wavelength 413 nm? (b) What is the threshold wavelength for this surface?

We have $$\mathrm{hf} = \frac{\mathrm{hc}}{\lambda} = K_{\max} + \Phi$$

Maximum kinetic energy

$$K_{\max} = \frac{\mathrm{hc}}{\lambda} - \Phi = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{413 \times 10^{-9}} - 2.16 \times 1.6 \times 10^{-19}$$
$$= 1.35 \times 10^{-19}\ \text{(J)} = 0.85\ \text{eV}$$

The threshold wavelength

$$\lambda_c = \frac{\mathrm{hc}}{\phi} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2.16 \times 1.6 \times 10^{-19}} = 5.76 \times 10^{-7}\ \text{(m)} = 576\ \text{(nm)}$$

16. The minimum energy required to remove an electron from a metal is 2.60 eV. What is the longest wavelength photon that can eject an electron from this metal?

16. The minimum energy required to remove an electron from a metal is 2.60 eV. What is the longest wavelength photon that can eject an electron from this metal?

$$\varepsilon = hf = \frac{hc}{\lambda} = K_{max} + \Phi \quad \Rightarrow \quad \varepsilon = hf = \frac{hc}{\lambda} = 0 + \Phi$$

the longest wavelength photon

$$\lambda_c = \frac{hc}{\phi} = \frac{hc}{\varepsilon} = \frac{6.63\times10^{-34}\times3\times10^{8}}{2.6\times1.6\times10^{-19}} = 4.78\times 10^{-7} \ \text{(m)}$$

17. X-rays of wavelength 10.0 pm are incident on a target. Find the wavelengths of the x-rays scattered at (a) 45.0 ° and (b) 90.0 ° . (tutorial: Compton scattering)

Ans: (a) 10.7 pm (b) 12.4 pm

17. X-rays of wavelength 10.0 pm are incident on a target. Find the wavelengths of the x-rays scattered at (a) 45.0° and (b) 90.0°.(tutorial: Compton scattering)

Compton scattering $\lambda' - \lambda_0 = \frac{h}{m_e c}(1\text{- Cos }\theta)$

a) $\lambda' - \lambda_0 = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1\text{- Cos }45.0°)$

$\Rightarrow$ $\lambda' - \lambda_0 = 7.11\times 10^{-13}$

$\Rightarrow$ $\lambda' = 7.11\times 10^{-13} + \lambda_0$ $\Rightarrow$ $\lambda' = 7.11\times 10^{-13} + 10\times 10^{-12}$

$\Rightarrow$ $\lambda' = 7.11\times 10^{-13} + 10\times 10^{-12} = 1.07\times 10^{-11}\ (m)$

17. X-rays of wavelength 10.0 pm are incident on a target. Find the wavelengths of the x-rays scattered at (a) 45.0° and (b) 90.0°.(tutorial: Compton scattering)

Compton scattering $\lambda' - \lambda_0 = \frac{h}{m_e c}(1\text{- Cos }\theta)$

b) $\lambda' - \lambda_0 = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1\text{- Cos }90.0°)$

$\Rightarrow \lambda' - \lambda_0 = 2.43\times 10^{-12}$

$\Rightarrow \lambda' = 2.43\times 10^{-12} + \lambda_0$ $\Rightarrow \lambda' = 2.43\times 10^{-12} + 10 \times 10^{-12}$

$\Rightarrow \lambda' = 12.43 \times 10^{-12}\ (m)$

18. An x-ray photon of wavelength 0.150 nm collides with an electron initially at rest. The scattered photon moves off at an angle of 80.0 ° from the direction of the incident photon. Find (a) the Compton shift in wavelength and (b) the wavelength of the scattered photon.

18. An x-ray photon of wavelength 0.150 nm collides with an electron initially at rest. The scattered photon moves off at an angle of 80.0 ° from the direction of the incident photon. Find (a) the Compton shift in wavelength and (b) the wavelength of the scattered photon.

Compton scattering $\lambda' - \lambda_0 = \frac{h}{m_e c}(1\text{- Cos }\theta)$

$$\lambda' - \lambda_0 = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1\text{- Cos }80.0°)$$

$$\Rightarrow \lambda' - \lambda_0 = 2.01\times 10^{-12}\ (\text{m})$$

$$\Rightarrow \lambda' = 2.01\times 10^{-12} + \lambda_0 \quad \Rightarrow \lambda' = 2.01\times 10^{-12} + 0.15 \times 10^{-9}$$

$$\Rightarrow \lambda' = 1.52\times 10^{-10}\ (m)$$

19. An incident beam of photons is scattered through 100.0°; the wavelength of the scattered photons is 124.65 pm. What is the wavelength of the incident photons?

19. An incident beam of photons is scattered through 100.0°; the wavelength of the scattered photons is 124.65 pm. What is the wavelength of the incident photons?

Compton scattering $\lambda' - \lambda_0 = \frac{h}{m_e c}(1\text{- Cos } \theta)$

$$\lambda' - \lambda_0 = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1\text{- Cos } 100°)$$

$$\lambda' - \lambda_0 = 2.85\times 10^{-12}$$

$$\lambda_0 = \lambda'\text{- } 2.85\times 10^{-12}$$

$$\lambda_0 = 124.65 \times 10^{-12} - 2.85\times 10^{-12} = 1.218 \times 10^{-10}\ (m)$$

20. An incident photon of wavelength 0.0100 nm is Compton scattered; the scattered photon has a wavelength of 0.0124 nm. What is the change in kinetic energy of the electron that scattered the photon?

Ans: 2.4×10^4 eV

20. An incident photon of wavelength 0.0100 nm is Compton scattered; the scattered photon has a wavelength of 0.0124 nm. What is the change in kinetic energy of the electron that scattered the photon?

$$p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{\varepsilon}{c}$$

We have:

$$K = \frac{1}{2}\frac{p^2}{m} = \frac{1}{2}\frac{h^2}{\lambda^2 m} = \frac{1}{2}\frac{(6.63\times10^{-34})^2}{(0.01\times10^{-9})^2 9.1\times10^{-31}} = 2.415\times10^{-15}\ (\mathrm{J})$$

$$K' = \frac{1}{2}\frac{p'^2}{m} = \frac{1}{2}\frac{h^2}{\lambda'^2 m} = \frac{1}{2}\frac{(6.63\times10^{-34})^2}{(0.0124\times10^{-9})^2 9.1\times10^{-31}} = 1.57\times10^{-15}\ (\mathrm{J})$$

The change in kinetic energy of the electron

$$\Delta K = K - K' = 2.415\times10^{-15} - 1.57\times10^{-15} = 8.44\times10^{-16}\ (\mathrm{J})$$

21. Find the de Broglie wavelength of a neutron of kinetic energy of 0.50 keV.

21. Find the de Broglie wavelength of a neutron of kinetic energy of 0.50 keV.

$$K = 0.5\text{keV} = 0.5\times10^{3}\times1.6\times10^{-19} = 0.8\times10^{-16}(\text{J})$$

We have formula $$K = \frac{1}{2}\frac{p^2}{m} \Rightarrow p = \sqrt{2mK}$$

Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times0.8\times10^{-16}}} = 1.28\times10^{-12}(\text{m})$$

22. In Q1 we considered photons of light of wavelength 590 nm.

(a) Suppose an *electron* has kinetic energy equal to the energy of a 590 nm photon. Find the speed, the momentum and the de Broglie wavelength of the electron.

(b) At what kinetic energy and what speed would an electron have de Broglie wavelength 590 nm?

22. In Q1 we considered photons of light of wavelength 590 nm.
(a) Suppose an *electron* has kinetic energy equal to the energy of a 590 nm photon. Find the speed, the momentum and the de Broglie wavelength of the electron.
(b) At what kinetic energy and what speed would an electron have de Broglie wavelength 590 nm?

a) $$\varepsilon = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{590\times10^{-9}} = 33.71\times10^{-20}(J)$$

Because the kinetic energy of an electron equals to energy of a photon

$$K = \varepsilon = 33.71\times10^{-20}(J)$$

$$K = \frac{1}{2}m_e v^2 \Rightarrow v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\times33.71\times10^{-20}}{9.11\times10^{-31}}} = 8.6\times10^{5}(m/s)$$

The momentum of electron is

$$p = m_e v = 9.11\times10^{-31}\times8.6\times10^{5} = 78.34\times10^{-26}(Ns)$$

Broglie wavelength of the electron is

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{78.34\times10^{-26}} = 0.0846\times10^{-8}(m)$$

b) $p = \dfrac{h}{\lambda}$

The kinetic energy of electron is

$$K = \frac{1}{2}\frac{p^2}{m_e} = \frac{1}{2}\frac{h^2}{\lambda^2 m_e} = 6.9\times10^{-25}(J) = 4.3\times10^{-6}(eV)$$

The speed of electron is $v = \sqrt{\dfrac{2K}{m_e}} = \sqrt{\dfrac{2\times6.9\times10^{-25}}{9.11\times10^{-31}}} = 1229(m/s)$

23. Suppose you want to perform Young's double slit experiment using electrons. If the slit separation is 0.5 mm and you want to obtain interference maxima separated by 1.0 mm on a screen 1.0 m from the slits, what energy (in eV) must the electrons have?

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$$\begin{cases} D = 1\text{m} \\ \Delta x = 1\text{mm} \\ K = ? \end{cases}$$

$$\Delta x = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{\Delta x d}{D} = \frac{10^{-3} \times 0.5 \times 10^{-3}}{1} = 0.5 \times 10^{-6}(\text{m})$$

Kinetic energy of electron must have

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = 9.65 \times 10^{-25}(\text{J}) = 6.03(\text{MeV})$$

24. An electron and a 0.020 0-kg bullet each have a velocity of magnitude 500 m/s, accurate to within 0.0100%. Within what lower limit could we determine the position of each object along the direction of the velocity?

24. An electron and a 0.020 0-kg bullet each have a velocity of magnitude 500 m/s, accurate to within 0.0100%. Within what lower limit could we determine the position of each object along the direction of the velocity?

$m = 0.02$ kg

$v = 500$ m/s $\pm$ 0.01%

Take $\Delta v = 500 \times \frac{0.01}{100} = 0.05$

$p = mv$ ➡ $\Delta p = m\Delta v = 0.02 \times 0.05 = 0.001 \; (\frac{kgm}{s})$

➡ $\Delta x \Delta p \geq \hbar/2$ ➡ $\Delta x \geq \hbar/(2\Delta p)$ ➡ $\Delta x \geq \frac{h}{4\pi\Delta p}$

➡ $\Delta x \geq \frac{6.63 \times 10^{-34}}{4\pi 0.001}$ ➡ $\Delta x \geq 5.28 \times 10^{-32} \; (m)$

24. An electron and a 0.020 0-kg bullet each have a velocity of magnitude 500 m/s, accurate to within 0.0100%. Within what lower limit could we determine the position of each object along the direction of the velocity?

$m = 9.1\times 10^{-31}$ kg

$v = 500$ m/s $\pm$ 0.01%

Take $\Delta v = 500 \times \frac{0.01}{100} = 0.05$

$p = mv$ ➡ $\Delta p = m\Delta v = 9.1\times 10^{-31} \times 0.05 = 4.55 \times 10^{-32} \left(\frac{kgm}{s}\right)$

➡ $\Delta x \Delta p \geq \hbar/2$ ➡ $\Delta x \geq \hbar/(2\Delta p)$ ➡ $\Delta x \geq \frac{h}{4\pi\Delta p}$

➡ $\Delta x \geq \frac{6.63 \times 10^{-34}}{4\pi 4.55 \times 10^{-32}}$ ➡ $\Delta x \geq 1.159\times 10^{-3}$ (m)

25. A 0.500-kg block rests on the frictionless, icy surface of a frozen pond. If the location of the block is measured to a precision of 0.150 cm and its mass is known exactly, what is the minimum uncertainty in the block's speed?

25. A 0.500-kg block rests on the frictionless, icy surface of a frozen pond. If the location of the block is measured to a precision of 0.150 cm and its mass is known exactly, what is the minimum uncertainty in the block's speed?

$m = 0.5$ kg

$\Delta x = 0.15$ cm

$\Delta x \Delta p \geq \hbar/2 \Rightarrow \Delta p \geq \hbar/(2\Delta x) \Rightarrow \Delta p \geq \dfrac{h}{4\pi\Delta x}$

$\Rightarrow \Delta p \geq \dfrac{6.63 \times 10^{-34}}{4\pi 0.15 \times 10^{-2}} \Rightarrow \Delta p \geq 3.5 \times 10^{-32}\ (kgm/s)$

$p = mv \Rightarrow \Delta p = m\Delta v \Rightarrow \Delta v = \Delta p/m = 3.5 \times 10^{-32}/0.5 = 7 \times 10^{-32}$ (m/s)

26. The average lifetime of a muon is about 2 μs. Estimate the minimum uncertainty in the rest energy of a muon.

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$\Delta E\,\Delta t \geq \hbar/2$ ⇒ $\Delta E \geq \hbar/(2\Delta t)$

⇒ $\Delta E \geq \dfrac{h}{2\pi 2\Delta t}$ ⇒ $\Delta E \geq \dfrac{6.63\times10^{-34}}{2\pi\times2\times2\times10^{-6}}$

⇒ $\Delta E \geq 2.63 \times 10^{-29}$ (J)

27. A bullet with mass 10.00 g has a speed of 300.00 m/s; the speed is accurate to within 0.04%. (a) Estimate the minimum uncertainty in the position of the bullet, according to the uncertainty principle. (b) An electron has a speed of 300.00 m/s, accurate to 0.04%. Estimate the minimum uncertainty in the position of the electron. (c) What can you conclude from these results?

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a) m = 10 g

v = 300 m/s ± 0.04%

Take $\Delta v = 300 \times \frac{0.04}{100} = 0.12$ (m/s)

p = mv ⇒ $\Delta p = m\Delta v = 0.01 \times 0.12 = 1.2 \times 10^{-3}$ (kgm/s)

⇒ $\Delta x \Delta p \geq \hbar/2$ ⇒ $\Delta x \geq \hbar/(2\Delta p)$ ⇒ $\Delta x \geq \frac{h}{4\pi\Delta p}$

⇒ $\Delta x \geq \frac{6.63 \times 10^{-34}}{4\pi 1.2 \times 10^{-3}}$ ⇒ $\Delta x \geq 4.39 \times 10^{-32}\ (m)$

27. A bullet with mass 10.00 g has a speed of 300.00 m/s; the speed is accurate to within 0.04%. (a) Estimate the minimum uncertainty in the position of the bullet, according to the uncertainty principle. (b) An electron has a speed of 300.00 m/s, accurate to 0.04%. Estimate the minimum uncertainty in the position of the electron. (c) What can you conclude from these results?

b) $m = 9.1 \times 10^{-31}$ kg Take $\Delta v = 300 \times \frac{0.04}{100} = 0.12\ (\frac{m}{s})$

$v = 300$ m/s $\pm\ 0.04\%$

$p = mv \Rightarrow \Delta p = m\Delta v = 9.1 \times 10^{-31} \times 0.12 = 1.092 \times 10^{-31}$ (kgm/s)

$\Rightarrow \Delta x \Delta p \geq \hbar/2 \Rightarrow \Delta x \geq \hbar/(2\Delta p) \Rightarrow \Delta x \geq \frac{h}{4\pi\Delta p}$

$\Rightarrow \Delta x \geq \frac{6.63 \times 10^{-34}}{4\pi 1.092 \times 10^{-31}} \Rightarrow \Delta x \geq 4.83 \times 10^{-4}\ (m)$

28. The omega particle (Ω) decays on average about 0.1 ns after it is created. Its rest energy is 1672 MeV. Estimate the fractional uncertainty in the Ω 's rest energy ($\Delta E_0 / E_0$).

28. The omega particle (Ω) decays on average about 0.1 ns after it is created. Its rest energy is 1672 MeV. Estimate the fractional uncertainty in the Ω 's rest energy ($\Delta E_0 / E_0$).

$$\Delta E\,\Delta t \geq \hbar/2 \;\Rightarrow\; \Delta E \geq \hbar/(2\Delta t) \;\Rightarrow\; \Delta E \geq \frac{h}{2\pi 2\Delta t}$$

$$\Rightarrow\; \Delta E \geq \frac{6.63\times10^{-34}}{2\pi\times2\times0.1\times10^{-9}} \;\Rightarrow\; \Delta E \geq 5.28\times10^{-25}\ (\mathrm{J})$$

$$\Rightarrow\; \frac{\Delta E}{E} = \frac{5.28\times10^{-25}}{1672\times10^{6}\times1.6\times10^{-19}} = 1.97\times10^{-15}$$

29. Suppose Fuzzy, a quantum mechanical duck, lives in a world in which $h = 2\pi$ Js. Fuzzy has a mass of 2.00 kg and is initially known to be within a pond 1.00 m wide. What is the minimum uncertainty in his speed?

29. Suppose Fuzzy, a quantum mechanical duck, lives in a world in which $h = 2\pi$ Js. Fuzzy has a mass of 2.00 kg and is initially known to be within a pond 1.00 m wide. What is the minimum uncertainty in his speed?

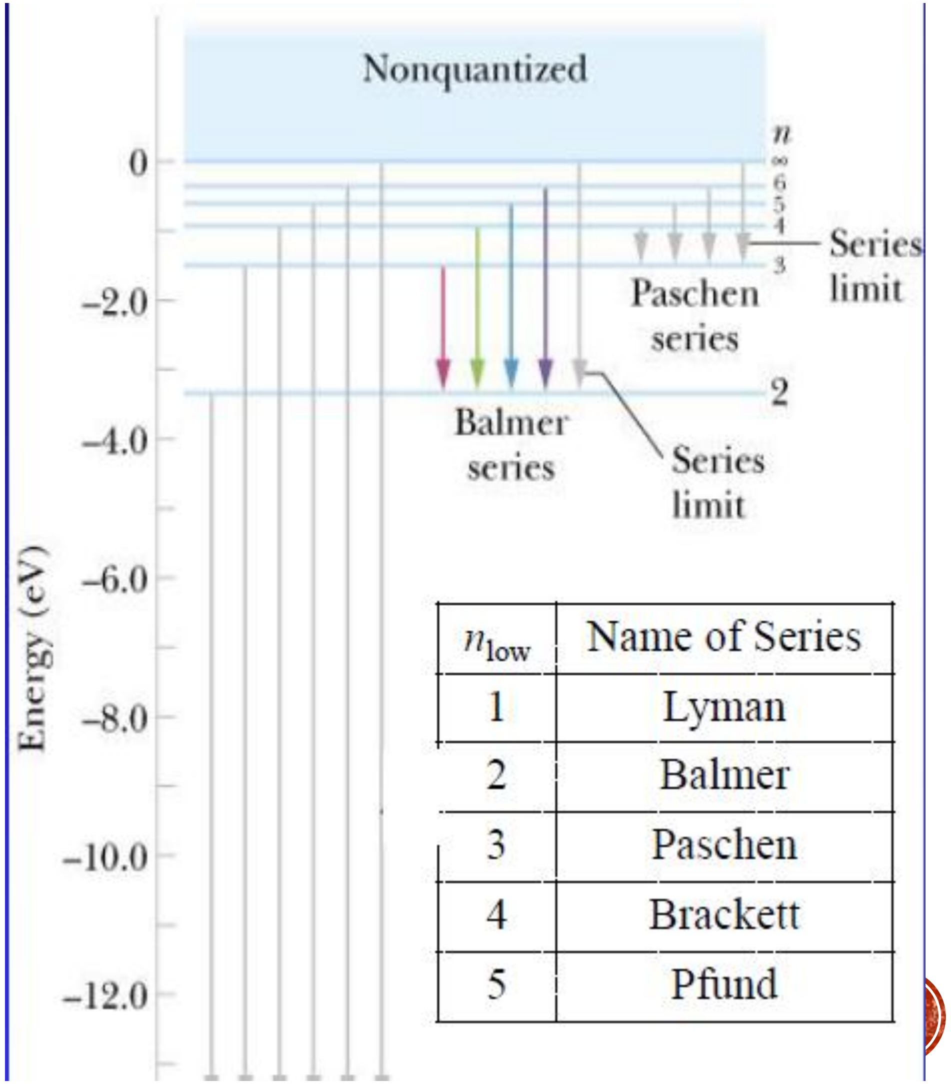
m = 2 kg

Δx = 1 m

$$\Delta x \Delta p \geq \hbar/2 \Rightarrow \Delta p \geq \hbar/(2\Delta x) \Rightarrow \Delta p \geq \frac{h}{4\pi\Delta x} \Rightarrow \Delta p \geq \frac{2\pi}{4\pi 1} \Rightarrow \Delta p \geq \frac{1}{2}$$

$$p = mv \Rightarrow \Delta p = m\Delta v \Rightarrow \Delta v = \Delta p/m = 0.5/2 = 0.25 \text{ m/s}$$

30. Determine the wavelength range and frequency range of the Balmer series of the hydrogen spectrum.



n_{low}	Name of Series
1	Lyman
2	Balmer
3	Paschen
4	Brackett
5	Pfund

30. Determine the wavelength range and frequency range of the Balmer series of the hydrogen spectrum.

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) = 1.1\times10^7\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

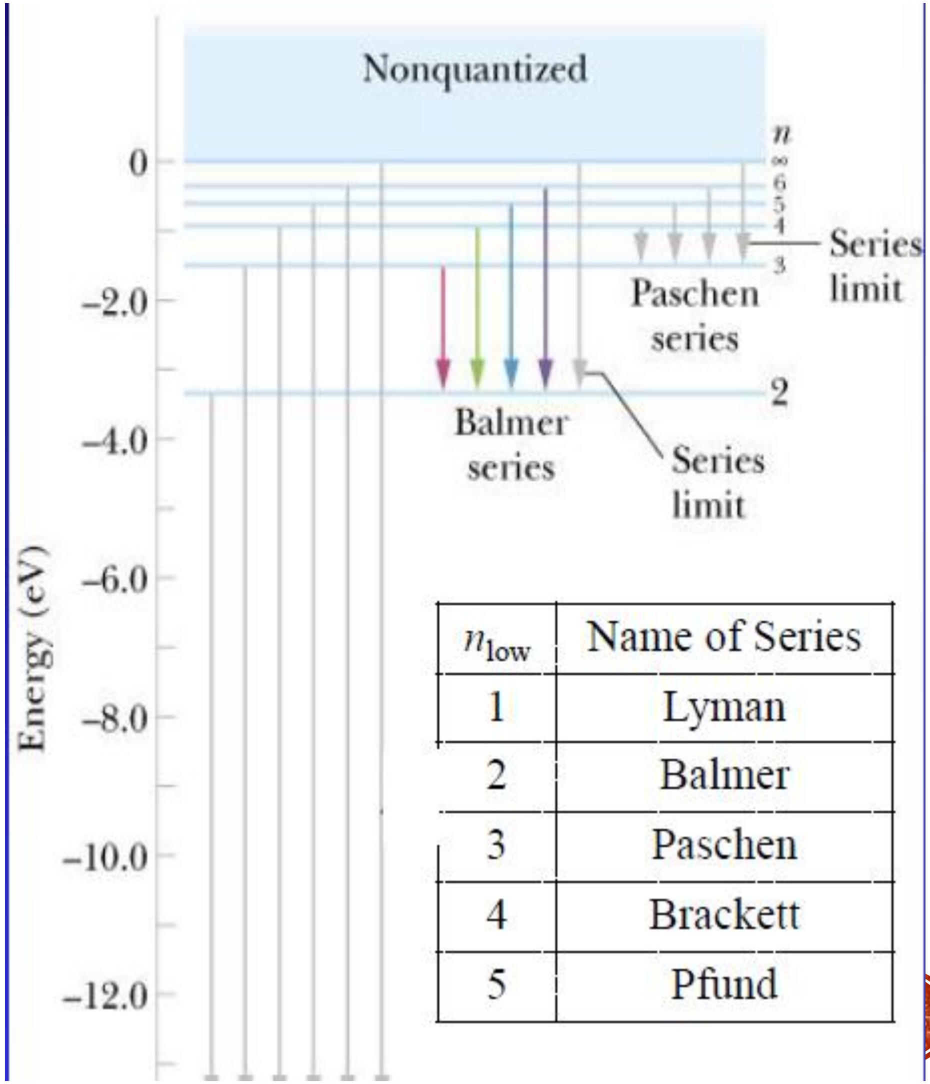
$$\Rightarrow \lambda_{max} = 6.54\times10^{-7}(m)$$

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) = 1.1\times10^7\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$$

$$\Rightarrow \lambda_{min} = 3.64\times10^{-7}(m)$$

$$f = \frac{c}{\lambda} \Rightarrow f_{max} = \frac{c}{\lambda_{min}} = 8.24\times10^{14}(Hz)$$

$$f = \frac{c}{\lambda} \Rightarrow f_{min} = \frac{c}{\lambda_{max}} = 4.6\times10^{14}(Hz)$$



n_{low}	Name of Series
1	Lyman
2	Balmer
3	Paschen
4	Brackett
5	Pfund

31. A hydrogen atom, initially at rest in the $n = 4$ quantum state, undergoes a transition to the ground state, emitting a photon in the process. Find (a) the energy of the photon, (b) the wavelength of the photon

31. A hydrogen atom, initially at rest in the n = 4 quantum state, undergoes a transition to the ground state, emitting a photon in the process. Find (a) the energy of the photon, (b) the wavelength of the photon

a) $$\Delta E = 13.6\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 12.75(\text{eV})$$

b) $$\frac{1}{\lambda} = R\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) = 1.1\times10^7\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$$

$$\Rightarrow \lambda = 97\times10^{-9}(\text{m})$$

32. A hydrogen atom emits light of wavelength 121.6 nm. (a) What are the quantum numbers of the higher and lower energy levels involved in the transition? (b) To which series does this line belong?

32. A hydrogen atom emits light of wavelength 121.6 nm. (a) What are the quantum numbers of the higher and lower energy levels involved in the transition? (b) To which series does this line belong?

$\lambda = 121.6$ nm

$$\frac{1}{\lambda} = R\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) \Rightarrow \frac{1}{n_{low}^2} - \frac{1}{n_{high}^2} = \frac{1}{\lambda R} = \frac{1}{1.1\times10^7\times121.6\times10^{-9}} = 0.75$$

$$\Rightarrow \quad n_{low} = 1$$

$$n_{high} = 2$$

33. A hydrogen atom in its ground state absorbs a photon of energy 12.1 eV. To what energy level is the atom excited?

Ans: third level

33. A hydrogen atom in its ground state absorbs a photon of energy 12.1 eV. To what energy level is the atom excited?
Ans: second excited state

$$\Delta E = E_{high} - E_{low} = hcR\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) = 13.6\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right)$$

$$\Delta E = E_{high} - E_{low} = 13.6\left(\frac{1}{1^2} - \frac{1}{n_{high}^2}\right) = 12.1$$

$$n_{high}^2 = 9$$

$$n_{high} = 3$$

Second excited state

34. The *Paschen series* in the hydrogen emission spectrum is formed by electron transitions from $n_i > 3$ to $n_f = 3$. (a) What is the longest wavelength in the Paschen series? (b) What is the wavelength of the series limit (the lower bound of the wavelengths in the series)?

Ans: 81 μm – 1.87 μm

34. The *Paschen series* in the hydrogen emission spectrum is formed by electron transitions from $n_i > 3$ to $n_f = 3$. (a) What is the longest wavelength in the Paschen series? (b) What is the wavelength of the series limit (the lower bound of the wavelengths in the series)?

$$\frac{1}{\lambda} = R\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) \Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_{high}^2}\right)$$

$$\Rightarrow \frac{1}{\lambda_{max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 1.1\times 10^7 \left(\frac{1}{3^2} - \frac{1}{4^2}\right) \Rightarrow \lambda max = 1.87\times 10^{-6}\ (\mathrm{m})$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) \Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_{high}^2}\right)$$

$$\Rightarrow \frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = 1.1\times 10^7 \left(\frac{1}{3^2} - 0\right) \Rightarrow \lambda min = 8.1\times 10^{-7}\ (\mathrm{m})$$

$\Rightarrow$ 81 µm – 1.87 µm

35. Find the energy for a hydrogen atom in the stationary state $n = 4$.
Ans: -0.85 eV

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Ans: -0.85 eV

The energy for a hydrogen atom in the stationary state

$$E_n = -\frac{13.6}{n^2} = -\frac{13.6}{4^2} = -0.85 \text{ eV}$$

36. Find the wavelength of the radiation emitted when a hydrogen atom makes a transition from the $n = 6$ to the $n = 3$ state.

36. Find the wavelength of the radiation emitted when a hydrogen atom makes a transition from the n = 6 to the n = 3 state.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) = 1.1 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{6^2}\right)$$

$$=> \lambda = 1.09 \times 10^{-6}\ (m)$$

Exercises in class 1

1. In a photoelectric effect experiment it is found that no current flows unless the incident light has a wavelength shorter than 289 nm. (a) What is the work function of the metal surface? (b) What stopping potential will be needed to halt the current if light of 225 nm falls on the surface

2. Two different monochromatic light sources, one yellow (580 nm) and one violet (425 nm), are used in a photoelectric effect experiment. The metal surface has a photoelectric threshold frequency of 6.20×10^{14} Hz. (a) Are both sources able to eject photoelectrons from the metal? Explain. (b) How much energy is required to eject an electron from the metal?

3. A photon is incident on an electron at rest. The scattered photon has a wavelength of 2.81 pm and moves at an angle of 29.5° with respect to the direction of the incident photon. What is the wavelength of the incident photon?

1. A bullet with mass 20 g has a speed of 200 m/s; the speed is accurate to within 0.02%. (a) Estimate the minimum uncertainty in the position of the bullet, according to the uncertainty principle. (b) An electron has a speed of 200 m/s, accurate to 0.02%. Estimate the minimum uncertainty in the position of the electron. (c) What can you conclude from these results?

2. A fluorescent solid absorbs a photon of ultraviolet light of wavelength 320 nm. If the solid dissipates 0.500 eV of the energy and emits the rest in a single photon, what is the wavelength of the emitted light?

3. An ultraviolet lamp emits light of wavelength 400 nm at the rate of 400 W. An infrared lamp emits light of wavelength 700 nm, also at the rate of 400 W. (a) Which lamp emits photons at the greater rate and (b) what is that greater rate?

- If a photon of wavelength 0.04250 nm strikes a free electron and is scattered at an angle of 35.0° from its original direction, find (a) the change in the wavelength of this photon, (b) the wavelength of the scattered light, (c) the change in energy of the photon (is it a loss or a gain?), and (d) the energy gained by the electron.

At a baseball game, a radar gun measures the speed of a 144-g baseball to be 137.32 ± 0.10 km/h. (a) What is the minimum uncertainty of the position of the baseball? (b) If the speed of a proton is measured to the same precision, what is the minimum uncertainty in its position?

19. (a) 1.3 × 10^-32 m (b) 1.1 × 10^-6 m