Description of Coefficient Check.nb

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The Mathematica notebook Coefficient Check.nb was constructed to determine the right hand side coefficient, designated below as **?**, such that the following equation holds true:

$$\frac{i}{2}\eta_{\phi\sigma}\left(\gamma^5\gamma^\sigma\gamma^5\gamma^\mu\right)^c{}_e\left(\gamma^5\right){}_c{}^f\partial_\mu\zeta_f - \frac{i}{2}\eta_{\phi\sigma}\left(\gamma^5\gamma^\sigma\gamma^5\gamma^\mu\right)^c{}_e\left(\gamma^5\right){}_c{}^f\partial_\mu\zeta_f + \frac{i}{4}\eta_{\phi\sigma}\left(\gamma^5\gamma^\sigma\gamma^5\gamma^\mu\right)^c{}_e\left(\gamma^5\gamma^\nu\gamma_\mu\right){}_c{}^f\partial_\nu\zeta_f - \frac{i}{4}\eta_{\phi\sigma}\left(\gamma^5\gamma^\sigma\gamma^5\gamma^\mu\right)^c{}_e\left(\gamma^\nu\gamma_\mu\right){}_c{}^f\partial_\nu\zeta_f = \mathbf{?}\left(\gamma^5\right){}_c{}^f\partial_\nu\zeta_f$$

This was a necessary step in confirming the coefficients of the Complex Linear Supermultiplet zeta commutator relation. The code confirmed that $2i$ was the correct coefficient for the right hand side of the equation above.

The code begins by clearing all associations using the $ClearAll$["`*"] command and initializing the notation pallet. The Notation Palette allows users to associate an external representation, for example a set of symbols or characters, to a particular internal representation of symbols, variables, or functions. The notation pallet is used here to construct indexable symbolic super derivative operators, and indexable symbolic zetas. This is done by creating functions of variables that can be looped over, and assigning those variables as corresponding subscripts to another set of symbols. Additionally other useful objects are also defined, this includes the $2x2$ identity matrix, the gamma matrices, and the eta and spinor metric.

Next, using Mathematica's Part function, [[_]] to account for indices, the left hand side of the above equation is constructed. In Mathematica the left hand side of this equation is described as a function of many variables, each of which correspond to a tensor index. These variables are selected via the part function, and can be looped over to draw from the definitions in the first section of the code.

A $Table$ loop is then used to symbolically determine all the values for the left hand side of the equation. The result is a $\{4x4x4x4x4x4x4\}$ matrix of super derivative operators and zetas with coefficients 1/4 and -1/4, as well as many 0 elements. Then using another $Table$ loop and the Sum function, the previously determined values are summed over according to the summations originally prescribed by the indicies in the left hand side of the above equation. The result is a $\{4x4\}$ solution matrix for the left hand side of the above equation containing superderivative operators and zetas, all of which have coefficient -2 or 2.

The same process is then conducted for the right hand side of the equation. A right hand side solution matrix emerges with all the resulting combinations of superderiviative operators and zetas. However this time, all the coefficients are $-i$, or i.

Now if we compare each $\partial_\mu{}^{th}\,\zeta_f{}^{th}$ element of the left hand side solution matrix and the right hand side solution matrix, disregarding their position in either matrix, as this varies due to how the indicies are summed over in Mathematica. We see that if each $\partial_\mu{}^{th}\,\zeta_f{}^{th}$ element in the right hand side solutions matrix, if multiplied by $2i$, it is equal to it's corresponding $\partial_\mu{}^{th}\,\zeta_f{}^{th}$ element in the left hand side solution matrix.

This shows that $LHS = 2i\,RHS$. Thus the right hand side **?** coefficient is $2i$. A quick explicit check using the $Equal$ function is performed solidifying the result.