Assume circuit is rectifiable, i.e. U exists and $U = U(\dots, x_k, \dots)$, with $x_k < x_i$

Observations (1) h_i is a polyn in variables $< x_i$ -

(2) h_i' is a polyn in variables $< x_i$

(3) Combining (8), (10), (13) from page 10

$$\Rightarrow h_i(U - h_i') = \sum_{j=i+1}^{s} (h_j - h_j') f_j + \sum_{x_\ell \in X_{PI}} (H_\ell - H_\ell')(x_\ell^2 -$$

(4) For all $p \in \{0,1\}^{X_{PI}}$, $\exists!\, p'$ s.th $p'|_{X_{PI}} = p$

and $f_{i+1}(p') = \dots = f_s(p') =$

Thm if U exists and if $p \in \{0,1\}^{X_{PI}}$, then for p' s.th $p'|_{X_{PI}} = p$ and $f_{i+1}(p') = \dots = f_s(p') =$ we have $h_i(p') \neq 0$, then

$$h_i'(p') = U(p').$$

Pf Use (3):

$$h_i(p')\cdot(U(p')-h_i'(p'))=0 \quad \text{since}$$

$$f_j(p')=0,\ \forall j=i+1,\dots,s \quad \text{and} \quad x_\ell^2=x_\ell$$

$$\forall x_\ell \in X_{PI}$$
$$\shortparallel$$
$$p$$

Since $h_i(p')\neq 0$

$$\Rightarrow \quad U(p')=h_i'(p').$$

Cor if U exists and $\forall p\in\{0,1\}^{X_{PI}}$

then $h_i(p')\neq 0$, $\forall p'$ s.th $p'|_{X_{PI}}=p$,

then U is unique and can be obtained

by procedure on pages 9,10.

Pf Any other h_i' satisfies (1)—(4)

So $U(p')=h_i'(p')$, $\forall p'$.

<u>Note</u> observations (1), (2) hold by construction.

(1) is by GB division

(2) is it correct that extended GB gives this as well?

Example in Sect 5 confirms it.