

**UNIVERSITY OF MASSACHUSETTS**  
**Dept. of Electrical & Computer Engineering**

**Introduction to Cryptography**  
**ECE 597XX/697XX**

**Part 4**

**The Advanced Encryption Standard (AES)**

**Israel Koren**

ECE597/697 Koren Part.4 .1

Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

**Content of this part**

- ♦ Overview of the AES algorithm
- ♦ Galois Fields
- ♦ Internal structure of AES
  - Byte Substitution layer
  - Diffusion layer
  - Key Addition layer
  - Key schedule
- ♦ Decryption
- ♦ Practical issues

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## Some Basic Facts

- AES is the most widely used symmetric cipher today
- The algorithm for AES was chosen by the US *National Institute of Standards and Technology* (NIST) in a multi-year selection process
- The requirements for all AES candidate submissions were:
  - Block cipher with **128-bit block size**
  - **Three supported key lengths**: 128, 192 and 256 bit
  - Security relative to other submitted algorithms
  - **Efficiency** in software and hardware

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

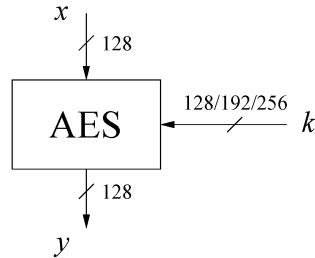
## Chronology of the AES Selection

- ♦ The need for a new block cipher announced by NIST in January, 1997
- ♦ 15 candidates algorithms accepted in August, 1998
- ♦ 5 finalists announced in August, 1999:
  - *Mars* - IBM Corporation
  - *RC6* - RSA Laboratories
  - *Rijndael* - J. Daemen & V. Rijmen
  - *Serpent* - E. Biham et al.
  - *Twofish* - B. Schneier et al.
- ♦ In October 2000, *Rijndael* was chosen as the AES
- ♦ AES was formally approved as a US federal standard in November 2001

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## AES: Overview



- ♦ The number of rounds depends on the chosen key length:

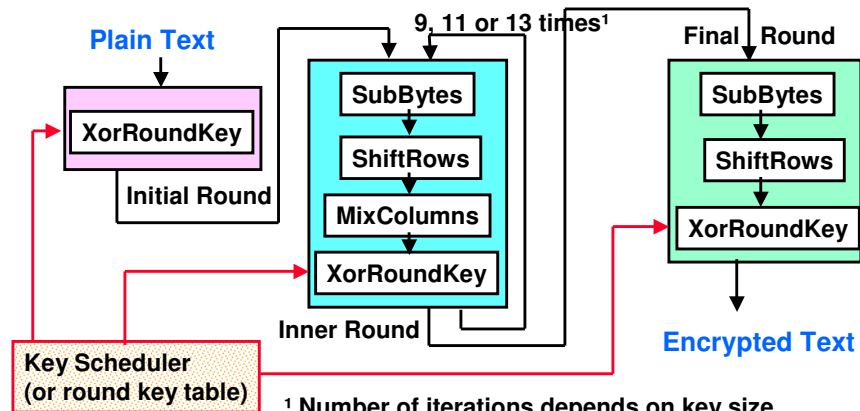
| Key length (bits) | Number of rounds |
|-------------------|------------------|
| 128               | 10               |
| 192               | 12               |
| 256               | 14               |

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## AES: Overview

- Iterated cipher with 10/12/14 rounds for key length of 128, 192, 256 bits, respectively ( $N_r+1$  round keys)
- Each round consists of "Layers"



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## Internal Structure of AES

- ♦ AES is a byte-oriented cipher
- ♦ The state  $A$  (i.e., the 128-bit data path) can be arranged in a 4x4 matrix:

|       |       |          |          |
|-------|-------|----------|----------|
| $A_0$ | $A_4$ | $A_8$    | $A_{12}$ |
| $A_1$ | $A_5$ | $A_9$    | $A_{13}$ |
| $A_2$ | $A_6$ | $A_{10}$ | $A_{14}$ |
| $A_3$ | $A_7$ | $A_{11}$ | $A_{15}$ |

|          |          |          |          |
|----------|----------|----------|----------|
| $S_{00}$ | $S_{01}$ | $S_{02}$ | $S_{03}$ |
| $S_{10}$ | $S_{11}$ | $S_{12}$ | $S_{13}$ |
| $S_{20}$ | $S_{21}$ | $S_{22}$ | $S_{23}$ |
| $S_{30}$ | $S_{31}$ | $S_{32}$ | $S_{33}$ |

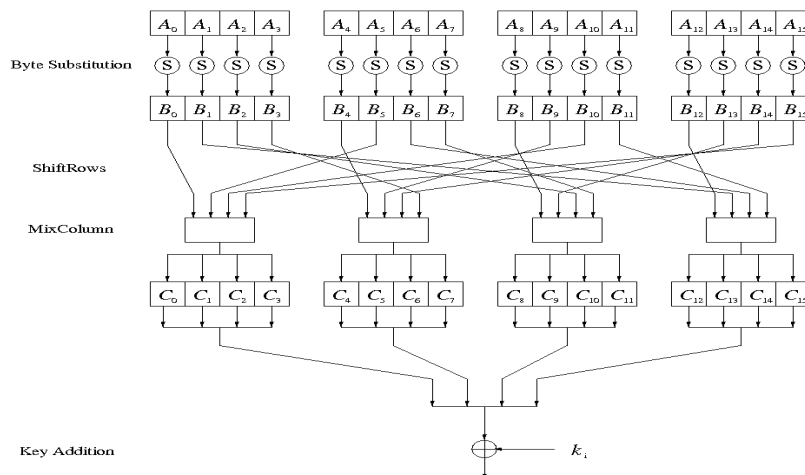
with  $A_0, \dots, A_{15}$  denoting the 16-byte input of AES

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## Internal Structure of AES

- Round function for rounds  $1, 2, \dots, N_r - 1$ :



- Note: In the last round, the MixColumn transformation is omitted

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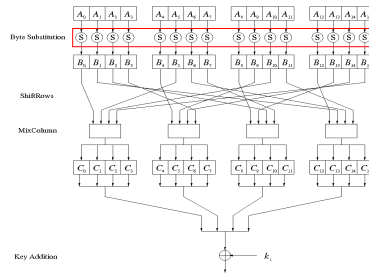
## Byte Substitution Layer

- ♦ The Byte Substitution layer consists of 16 **S-Boxes** with the following properties:

The S-Boxes are

- **identical**
- the only **nonlinear** elements of AES, i.e.,  $\text{ByteSub}(A_i) + \text{ByteSub}(A_j) \neq \text{ByteSub}(A_i + A_j)$ , for  $i, j = 0, \dots, 15$
- **bijective**, i.e., there exists a one-to-one mapping of input and output bytes  
 $\Rightarrow$  S-Box can be uniquely reversed

- ♦ In software implementations, the S-Box is usually realized as a lookup table



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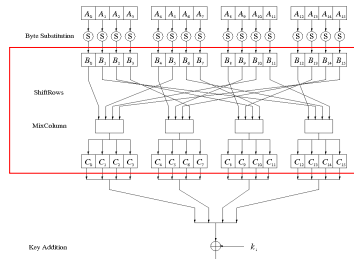
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## Diffusion Layer

The Diffusion layer

- ♦ provides diffusion over all input state bits
- ♦ consists of two sublayers:
  - **ShiftRows Sublayer**: Permutation of the data on a byte level
  - **MixColumn Sublayer**: Matrix operation which combines ("mixes") blocks of four bytes
- ♦ performs a linear operation on state matrices  $A$ ,  $B$ , i.e.,

$$\text{DIFF}(A) + \text{DIFF}(B) = \text{DIFF}(A + B)$$



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## ShiftRows Sublayer

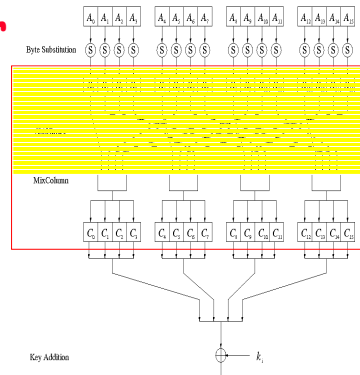
- ♦ Rows of the state matrix are shifted cyclically:

Input matrix

|       |       |          |          |
|-------|-------|----------|----------|
| $B_0$ | $B_4$ | $B_8$    | $B_{12}$ |
| $B_1$ | $B_5$ | $B_9$    | $B_{13}$ |
| $B_2$ | $B_6$ | $B_{10}$ | $B_{14}$ |
| $B_3$ | $B_7$ | $B_{11}$ | $B_{15}$ |

Output matrix

|          |          |          |          |
|----------|----------|----------|----------|
| $B_0$    | $B_4$    | $B_8$    | $B_{12}$ |
| $B_5$    | $B_9$    | $B_{13}$ | $B_1$    |
| $B_{10}$ | $B_{14}$ | $B_2$    | $B_6$    |
| $B_{15}$ | $B_3$    | $B_7$    | $B_{11}$ |



no shift

← one position left rotate

← two positions left rotate

← three positions left rotate

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Introduction to Galois Fields

- ♦ Substitution & Mix-column steps based on Galois field arithmetic
- ♦ A Galois field consists of a finite set of elements with the operation: add, subtract, multiply and invert
- ♦ A **group** is a set of elements with one operation that is **closed** and **associative**, the set has a neutral (identity) element „1" and each element  $a$  has an inverse so that  $a \circ a^{-1} = 1$  and  $a \circ 1 = a$
- ♦ A group is **commutative** if the operation is commutative.
- ♦ The set  $\{0, 1, \dots, m-1\}$  with the addition mod  $m$  is a group but this set with the operation multiply mod  $m$  is not.
- ♦ A **field** is a set of elements that form an additive group with the operation  $+$  and a multiplicative group (except 0) with the operation  $\times$ , and the **distributivity** rule holds.
- ♦ A **finite field** of order  $m$  has  $m = p^n$  elements with  $p$  a prime number.
- ♦ For  $n=1$  the field  $GF(p)$  consists of the integers  $0, 1, \dots, p-1$  and add/multiply mod  $p$  - all non-zero elements have an inverse - special case of a **ring**.

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## Galois Fields

- ♦  $GF(5)$ : multiplicative inverses for 0,1,2,3,4 are: none,1,3,2,4
- ♦  $GF(2)$  and its extension field  $GF(2^8)$  are important for AES
  - AES operates on bytes that have 256 possible values
  - But  $2^8$  is not a prime and we cannot use add/multiply mod  $2^8$  (why)
- ♦ Define the extension field  $GF(2^8)$  as consisting of 256 polynomials
 
$$A(x) = a_7x^7 + \dots + a_1x + a_0, \quad a_i \in GF(2) = \{0,1\}$$

$$A = (a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0)$$
- ♦  $GF(2^m)$ :
 
$$C(x) = A(x) + B(x) = \sum_{i=0}^{m-1} c_i x^i, \quad c_i \equiv a_i + b_i \pmod{2}$$

$$C(x) = A(x) - B(x) = \sum_{i=0}^{m-1} c_i x^i, \quad c_i \equiv a_i - b_i \equiv a_i + b_i \pmod{2}.$$
- ♦ Example:  $A=(1101\ 0011) + B=(0101\ 1010)$  in binary and polynomial notation

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## Extension Field

- ♦ Multiplication would generate a polynomial of degree  $2m$  - we divide the product by a given polynomial and use the remainder
  - The modulo reduction should use an irreducible polynomial
- ♦ Define multiplication in  $GF(2^m)$  as:  $C(x) \equiv A(x) \cdot B(x) \pmod{P(x)}$ , where  $P(x)$  is an irreducible polynomial
- ♦ For AES  $P(x) = x^8 + x^4 + x^3 + x + 1$
- ♦ Example  $A=(0010\ 0010), B=(0001\ 0101) \Rightarrow C(1011\ 1100)$

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## Inversion

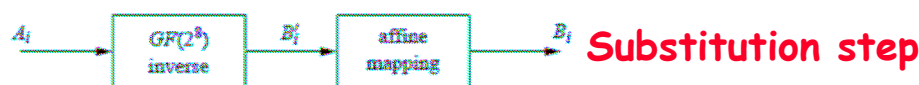
- ◆ Basic operation (but not the only one) behind the substitution step in AES
- ◆ Definition:  $A^{-1}(x) \cdot A(x) = 1 \bmod P(x)$
- ◆ For small fields commonly done using lookup tables
- ◆ Example: Inverse of  $A=(0010\ 0010)=(22)_{16}$  is  $(5A)$

$$P(x) = x^8 + x^4 + x^3 + x + 1$$

|     | Y  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|     | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | A  | B  | C  | D  | E  | F  |
| 0   | 00 | 01 | 8D | F6 | CB | 52 | 7B | D1 | E8 | 4F | 29 | C0 | B0 | E1 | E5 | C7 |
| 1   | 74 | B4 | AA | 4B | 99 | 2B | 60 | 5F | 58 | 3F | FD | CC | FF | 40 | EE | B2 |
| 2   | 3A | 6E | 5A | F1 | 55 | 4D | A8 | C9 | C1 | 0A | 98 | 15 | 30 | 44 | A2 | C2 |
| 3   | 2C | 45 | 92 | 6C | F3 | 39 | 66 | 42 | F2 | 35 | 20 | 6F | 77 | BB | 59 | 19 |
| 4   | 1D | FE | 37 | 67 | 2D | 31 | F5 | 69 | A7 | 64 | AB | 13 | 54 | 25 | E9 | 09 |
| 5   | ED | 5C | 05 | CA | 4C | 24 | 87 | BF | 18 | 3E | 22 | F0 | 51 | EC | 61 | 17 |
| 6   | 16 | 5E | AF | D3 | 49 | A6 | 36 | 43 | F4 | 47 | 91 | DF | 33 | 93 | 21 | 3B |
| 7   | 79 | B7 | 97 | 85 | 10 | B5 | BA | 3C | B6 | 70 | D0 | 06 | A1 | FA | 81 | 82 |
| X 8 | 83 | 7E | 7F | 80 | 96 | 73 | BE | 56 | 9B | 9E | 95 | D9 | F7 | 02 | B9 | A4 |
| 9   | DE | 6A | 32 | 6D | D8 | 8A | 84 | 72 | 2A | 14 | 9F | 88 | F9 | DC | 89 | 9A |
| A   | FB | 7C | 2E | C3 | 8F | B8 | 65 | 48 | 26 | C8 | 12 | 4A | CE | E7 | D2 | 62 |
| B   | 0C | E0 | 1F | EF | 11 | 75 | 78 | 71 | A5 | 8E | 76 | 3D | BD | BC | 86 | 57 |
| C   | 0B | 28 | 2F | A3 | DA | D4 | E4 | 0F | A9 | 27 | 53 | 04 | 1B | FC | AC | E6 |
| D   | 7A | 07 | AE | 63 | C5 | DB | E2 | EA | 94 | 8B | C4 | D5 | 9D | F8 | 90 | 6B |
| E   | B1 | 0D | D6 | EB | C6 | 0E | CF | AD | 08 | 4E | D7 | E3 | 5D | 50 | 1E | B3 |
| F   | 5B | 23 | 38 | 34 | 68 | 46 | 03 | 8C | DD | 9C | 7D | A0 | CD | 1A | 41 | 1C |

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources



- ◆ Inverse:  $B'_i = A_i^{-1}$

$$B' = (b'_7, \dots, b'_0)$$

- ◆ Multiply by a constant matrix and add a constant

$$\begin{pmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \bmod 2$$

Typically replaced by a table lookup

Example:

$$A_i = (11000010)_2 = (C2)_{hex}$$

From previous slide:

$$A_i^{-1} = B'_i = (2F)_{hex} = (00101111)_2$$

$$B_i = (00100101) = (25)_{hex}$$

|     | Y  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|     | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | A  | B  | C  | D  | E  | F  |
| 0   | 63 | 7C | 77 | 7B | F2 | 6B | 6F | C5 | 30 | 01 | 67 | 2B | FE | D7 | AB | 76 |
| 1   | CA | 82 | C9 | 7D | FA | 59 | 47 | F0 | AD | D4 | A2 | AF | 9C | A4 | 72 | C0 |
| 2   | B7 | FD | 93 | 26 | 36 | 3F | F7 | CC | 34 | A5 | E5 | F1 | 71 | D8 | 31 | 15 |
| 3   | 04 | C7 | 23 | C3 | 18 | 96 | 05 | 9A | 07 | 12 | 80 | E2 | EB | 27 | B2 | 75 |
| 4   | 09 | 83 | 2C | 1A | 1B | 6E | 5A | A0 | 52 | 3B | D6 | B3 | 29 | E3 | 2F | 84 |
| 5   | 53 | D1 | 00 | ED | 20 | FC | B1 | 5B | 6A | CB | BE | 39 | 4A | 4C | 58 | CF |
| 6   | D0 | EF | AA | FB | 43 | 4D | 33 | 85 | 45 | F9 | 02 | 7F | 50 | 3C | 9F | A8 |
| 7   | 51 | A3 | 40 | 8F | 92 | 9D | 38 | F5 | BC | B6 | DA | 21 | 10 | FF | F3 | D2 |
| X 8 | CD | 0C | 13 | EC | 5F | 97 | 44 | 17 | C4 | A7 | 7E | 3D | 64 | 5D | 19 | 73 |
| 9   | 60 | 81 | 4F | DC | 22 | 2A | 90 | 88 | 46 | EE | B8 | 14 | DE | 5E | 0B | DB |
| A   | E0 | 32 | 3A | 0A | 49 | 06 | 24 | 5C | C2 | D3 | AC | 62 | 91 | 95 | E4 | 79 |
| B   | E7 | C8 | 37 | 6D | 8D | D5 | 4E | A9 | 6C | 56 | F4 | EA | 65 | 7A | AE | 08 |
| C   | BA | 78 | 25 | 2E | 1C | A6 | B4 | C6 | E8 | DD | 74 | 1F | 4B | BD | 8B | 8A |
| D   | 70 | 3E | B5 | 66 | 48 | 03 | F6 | 0E | 61 | 35 | 57 | B9 | 86 | CI | 1D | 9E |
| E   | E1 | F8 | 98 | 11 | 69 | D9 | 8E | 94 | 9B | 1E | 87 | E9 | CE | 55 | 28 | DF |
| F   | 8C | A1 | 89 | 0D | BF | E6 | 42 | 68 | 41 | 99 | 2D | 0F | B0 | 54 | BB | 16 |

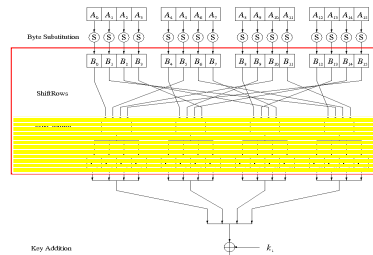
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Adapted fr



## MixColumn Sub-layer

- ◆ Linear transformation which mixes each column of the state matrix



- ◆ Each 4-byte column is considered as a vector and multiplied by a fixed 4x4 matrix, e.g.,

$$\begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \cdot \begin{pmatrix} B_0 \\ B_5 \\ B_{10} \\ B_{15} \end{pmatrix}$$

where 01, 02 and 03 are given in hexadecimal notation

- ◆ All arithmetic is done in the Galois field  $GF(2^8)$

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## MixColumn Sublayer

- ◆ **MixColumns:**  $\alpha = x = 02_{16}$ ;  $\beta = x+1 = 03_{16}$ ,  $\otimes$  and  $\oplus$  are mod 2 multiply and add; both modulo AES generator polynomial  
 $P(x) = x^8 + x^4 + x^3 + x + 1$

$$\text{Example: } 03 \otimes 5d = e7 \equiv (x+1) \otimes (x^6 + x^4 + x^3 + x^2 + 1)$$

$$\text{Example: } 02 \otimes bf = 17e \text{ mod } P(x) = 65 \equiv$$

$$f(x) = (x) \otimes (x^7 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

If  $f(x)$  is of degree 8:

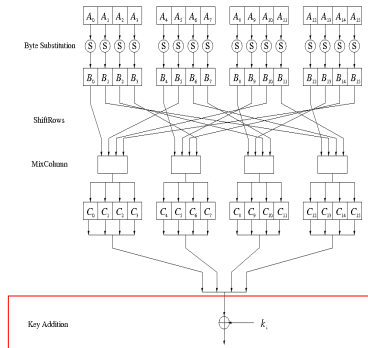
$$f(x) \text{ mod } (x^8 + x^4 + x^3 + x + 1) = f(x) \oplus P(x)$$

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Key Addition Layer

- ◆ **Inputs:**
  - 16-byte state matrix  $S$
  - 16-byte subkey  $k_i$
- ◆ **Output:**  $S \oplus k_i$
- ◆ The subkeys are generated in the key schedule



|          |          |          |          |
|----------|----------|----------|----------|
| $S_{00}$ | $S_{01}$ | $S_{02}$ | $S_{03}$ |
| $S_{10}$ | $S_{11}$ | $S_{12}$ | $S_{13}$ |
| $S_{20}$ | $S_{21}$ | $S_{22}$ | $S_{23}$ |
| $S_{30}$ | $S_{31}$ | $S_{32}$ | $S_{33}$ |

 $\oplus$ 

|          |          |          |          |
|----------|----------|----------|----------|
| $k_{00}$ | $k_{01}$ | $k_{02}$ | $k_{03}$ |
| $k_{10}$ | $k_{11}$ | $k_{12}$ | $k_{13}$ |
| $k_{20}$ | $k_{21}$ | $k_{22}$ | $k_{23}$ |
| $k_{30}$ | $k_{31}$ | $k_{32}$ | $k_{33}$ |

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Key Schedule

- ◆ Subkeys are derived recursively from the original 128/192/256-bit input key
- ◆ Each round has 1 subkey, plus 1 subkey at the beginning of AES

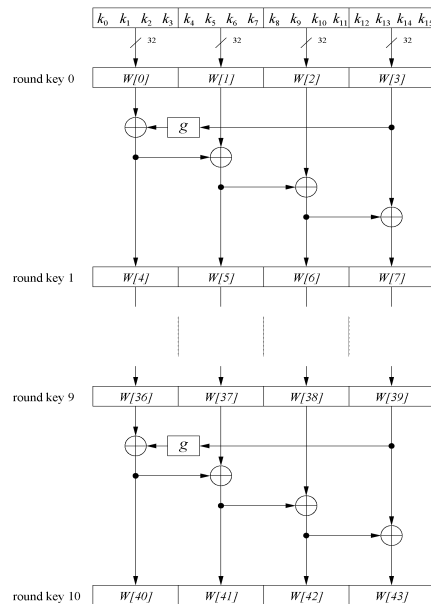
| Key length (bits) | Number of subkeys |
|-------------------|-------------------|
| 128               | 11                |
| 192               | 13                |
| 256               | 15                |

- ◆ **Key whitening:** Subkey is used both at the input and output of AES  
 $\Rightarrow \# \text{ subkeys} = \# \text{ rounds} + 1$
- ◆ There are different key schedules for the different key sizes

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## Key Schedule



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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

### Example: Key schedule for 128-bit key AES

- Word-oriented: 1 word = 4 bytes = 32 bits
- 11 subkeys are stored in  $W[0]... W[3]$ ,  $W[4]... W[7]$ , ...,  $W[40]... W[43]$
- First subkey  $W[0]... W[3]$  is the original AES key

$$W[4i] = W[4(i-1)] + g(W[4i-1])$$

$$W[4i+j] = W[4i+j-1] + W[4(i-1)+j]$$

## Key Schedule

- ♦ Function  $g$  rotates its four input bytes and performs a bitwise S-Box substitution  $\Rightarrow$  nonlinearity
- ♦ Round coefficient  $RC$  is only added to leftmost byte and varies from round to round:

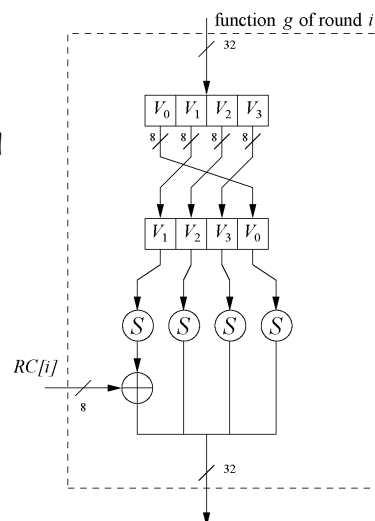
$$RC[1] = x^0 = (00000001)_2$$

$$RC[2] = x^1 = (00000010)_2$$

$$RC[3] = x^2 = (00000100)_2$$

$$\vdots$$

$$RC[10] = x^9 = (00110110)_2$$



- ♦  $x^i$  represents an element in a Galois field  $GF(2^8)$

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## AES key Schedule

♦ **Nr=10,12,14 rounds**

♦ **Nk=4,6,8 words**

```

KeyExpansion(byte key[4 * Nk], word w[4 * (Nr + 1)], Nk)
begin
  word temp
  i = 0
  while (i < Nk)
    w[i] = word(key[4 * i], key[4 * i + 1], key[4 * i + 2], key[4 * i + 3])
    i = i + 1
  end while
  i = Nk
  while (i < 4 * (Nr + 1))
    temp = w[i - 1]
    if (i mod Nk = 0)
      temp = SubWord(RotWord(temp)) xor Rcon[i/Nk]
    else if (Nk > 6 and i mod Nk = 4)
      temp = SubWord(temp)
    end if
    w[i] = w[i - Nk] xor temp
    i = i + 1
  end while
end

```

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Example

**Plaintext = 32 43 f6 a8 88 5a 30 8d 31 31 98 a2 e0 37 07 34**

**128-bit key = 2b 7e 15 16 28 ae d2 a6 ab f7 15 88 09 cf 4f 3c**

|    |    |    |    |
|----|----|----|----|
| 32 | 88 | 31 | e0 |
| 43 | 5a | 31 | 37 |
| f6 | 30 | 98 | 07 |
| a8 | 8d | a2 | 34 |

(a) Initial state matrix.

|    |    |    |    |
|----|----|----|----|
| 2b | 28 | ab | 09 |
| 7e | ae | f7 | cf |
| 15 | d2 | 15 | 4f |
| 16 | a6 | 88 | 3c |

(b) Key added in round 1.

|    |    |    |    |
|----|----|----|----|
| 19 | a0 | 9a | e9 |
| 3d | f4 | c6 | f8 |
| e3 | e2 | 8d | 48 |
| be | 2b | 2a | 08 |

(c) State matrix - end of round 1.

|    |    |    |    |
|----|----|----|----|
| d4 | e0 | b8 | 1e |
| 27 | bf | b4 | 41 |
| 11 | 98 | 5d | 52 |
| ae | f1 | e5 | 30 |

(d) After SubBytes.

♦ **MixColumns:**

$$\begin{aligned}
 s_{0,0} &= (\alpha \otimes s_{0,0}) \oplus (\beta \otimes s_{1,0}) \oplus s_{2,0} \oplus s_{3,0} \\
 &= (02 \otimes d4) \oplus (03 \otimes bf) \oplus 5d \oplus 30 \\
 &= 1b8 \oplus 1c1 \oplus 5d \oplus 30 = 04
 \end{aligned}$$

|    |    |    |    |
|----|----|----|----|
| d4 | e0 | b8 | 1e |
| bf | b4 | 41 | 27 |
| 5d | 52 | 11 | 98 |
| 30 | ae | f1 | e5 |

(e) After ShiftRows.

|    |    |    |    |
|----|----|----|----|
| 04 | e0 | 48 | 28 |
| 66 | cb | f8 | 06 |
| 81 | 19 | d3 | 26 |
| e5 | 9a | 7a | 4c |

(f) After MixColumns.

$$s_{1,0} = s_{0,0} \oplus (\alpha \otimes s_{1,0}) \oplus (\beta \otimes s_{2,0}) \oplus s_{3,0}$$

$$= d4 \oplus (02 \otimes bf) \oplus (03 \otimes 5d) \oplus 30 = d4 \oplus 17e \oplus e7 \oplus 30 = 17d$$

$$17d \bmod p(x) = 17d \oplus p(x) = 7d \oplus (x^4 + x^3 + x + 1) = 7d \oplus 1b = 66$$

ECE597/697 Koren Part.4 .24

Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

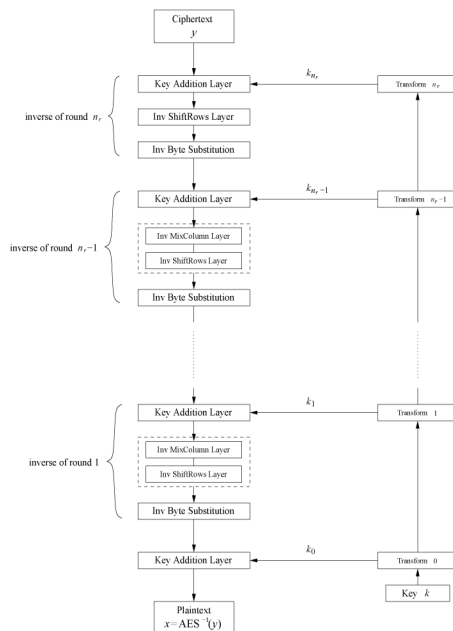
## Effect of bit flips

- ♦ Plaintext:  
32 43 f6 a8 88 5a 30 8d 31 31 98 a2 e0 37 07 34
- ♦ 128-bit key:  
2b 7e 15 16 28 ae d2 a6 ab f7 15 88 09 cf 4f 3c
- ♦ Ciphertext:  
39 25 84 1d 02 dc 09 fb dc 11 85 97 19 6a 0b 32
- ♦ A single bit flip in the plaintext:  
30 43 f6 a8 88 5a 30 8d 31 31 98 a2 e0 37 07 34
- ♦ Results in the ciphertext:  
c0 06 27 d1 8b d9 e1 19 d5 17 6d bc ba 73 37 c1
- ♦ A single bit flip in the key:  
2a 7e 15 16 28 ae d2 a6 ab f7 15 88 09 cf 4f 3c
- ♦ Results in the ciphertext:  
c4 61 97 9e e4 4d e9 7a ba 52 34 8b 39 9d 7f 84
- ♦ A single bit flip results in a totally scrambled output

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Decryption



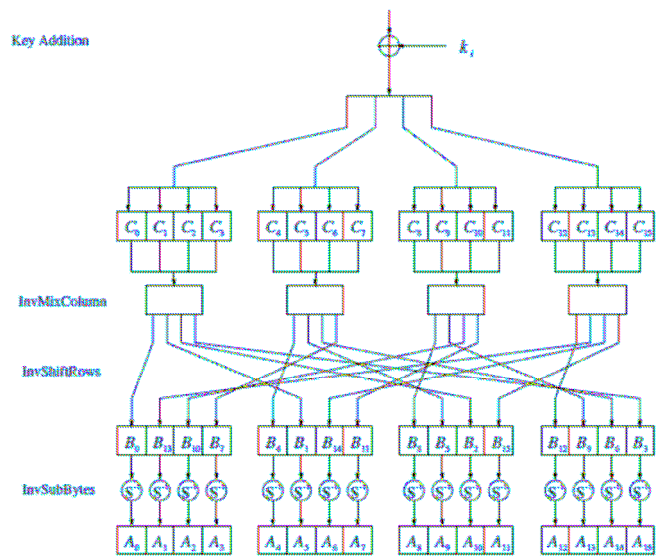
- ♦ AES is not based on a Feistel network
- ⇒ All layers must be inverted for decryption:

- MixColumn layer → Inv MixColumn layer
- ShiftRows layer → Inv ShiftRows layer
- Byte Substitution layer → Inv Byte Substitution layer
- Key Addition layer is its own inverse

ECE597/697 Koren Part.4 .26

Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Decryption- details



ECE597/697 Koren Part.4 .27

Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Decryption - Inv Mixcolumn

### ◆ Inv MixColumn layer:

- To reverse the MixColumn operation, each column of the state matrix  $C$  must be multiplied with the inverse of the 4x4 matrix, e.g.,

$$\begin{pmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{pmatrix} \cdot \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{pmatrix} \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix}$$

where 09, 0B, 0D and 0E are given in hexadecimal notation

- ◆ All arithmetic done in the Galois field  $GF(2^8)$

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Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Decryption - Inv Shift Rows

### ♦ Inv ShiftRows layer:

- All rows of the state matrix  $B$  are shifted to the opposite direction:

|               |          |          |          |          |                                |
|---------------|----------|----------|----------|----------|--------------------------------|
| Input matrix  | $B_0$    | $B_4$    | $B_8$    | $B_{12}$ |                                |
|               | $B_1$    | $B_5$    | $B_9$    | $B_{13}$ |                                |
|               | $B_2$    | $B_6$    | $B_{10}$ | $B_{14}$ |                                |
|               | $B_3$    | $B_7$    | $B_{11}$ | $B_{15}$ |                                |
| Output matrix | $B_0$    | $B_4$    | $B_8$    | $B_{12}$ | no shift                       |
|               | $B_{13}$ | $B_1$    | $B_5$    | $B_9$    | → one position right rotate    |
|               | $B_{10}$ | $B_{14}$ | $B_2$    | $B_6$    | → two positions right rotate   |
|               | $B_7$    | $B_{11}$ | $B_{15}$ | $B_3$    | → three positions right rotate |

ECE597/697 Koren Part.4 .29

Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Decryption - Inv S-Box

### ♦ Inv Byte Substitution layer:

- Since the S-Box is bijective, it is possible to construct an inverse, such that

$$A_i = S^{-1}(B_i) = S^{-1}(S(A_i))$$

⇒ The inverse S-Box is used for decryption. It is usually realized as a lookup table

### ♦ Decryption key schedule:

- Subkeys are needed in reversed order (compared to encryption)
- In practice, for encryption and decryption, the same key schedule is used. This requires that all subkeys must be computed before the encryption of the first block can begin

ECE597/697 Koren Part.4 .30

Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Implementation in Software

- ♦ One requirement of AES was the possibility of an efficient software implementation
- ♦ Straightforward implementation is well suited for 8-bit processors (e.g., smart cards), but inefficient on 32-bit or 64-bit processors
- ♦ A more sophisticated approach: Merge all round functions (except the key addition) into one table look-up
  - This results in four tables with 256 entries, where each entry is 32 bits wide
  - One round can be computed with 16 table look-ups
- ♦ Typical SW speeds are more than 1.6 Gbit/s on modern 64-bit processors

ECE597/697 Koren Part.4 .31

Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources

## Security

- ♦ **Brute-force attack:** Due to the key length of 128, 192 or 256 bits, a brute-force attack is not possible
- ♦ **Analytical attacks:** There is no efficient analytical attack known that is sufficiently better than brute-force (e.g., complexity of  $2^{126}$ )
- ♦ **Side-channel attacks:**
  - Many side-channel attacks have been published
  - Note that side-channel attacks do not attack the underlying algorithm but the implementation of it

ECE597/697 Koren Part.4 .32

Adapted from Paar & Pelzl, "Understanding Cryptography," and other sources



## Lessons Learned

- ♦ AES is a block cipher which supports three key lengths of 128, 192 and 256 bit. It provides excellent long-term security against brute-force attacks.
- ♦ AES has been studied intensively since the late 1990s and no attacks have been found that are better than brute-force.
- ♦ AES is not based on Feistel networks. Its basic operations use Galois field arithmetic and provide strong diffusion and confusion.
- ♦ AES is part of numerous open standards such as Ipsec (Internet Protocol Security) or TLS (Transport Layer Security), in addition to being the mandatory encryption algorithm for US government applications. It is likely to be the dominant encryption algorithm for many years to come.
- ♦ AES is efficient in software and hardware.