Factorization via Extraction

- Review of multi-level operations:
  - Elimination/Collapsing:
  - Decomposition
  - Simplification (via espresso)
  - Extraction of common sub-functions
    - Identify common subfunctions
    - Identify a good divisor (Kernel?)
    - Perform Division:
      - <u>Dividend = Divisor * Quotient + Remainder</u>
      - Weak Division! (already studied)
      - Kernel Extraction – know the basics

Theory of Kernels & Co-Kernels

- Cube free expressions:
  - F = ab + ac + ae ('a' can be extracted)
  - F = ace + bce + de + g
  - = (ce) * (a + b) + de + g
  - (a+b): Kernel, cube free
  - (ce): co-kernel of (a+b)
  - (de + g): remainder
- Remember Level-0 kernel?
  - F = (e) (ac + bc + d) + g
  - F = (e)((c)(a + b) + d) + g

How to identify Kernels?

- Remember procedure compute Kernel?
- Went through in class.
- Compute_Kernel(f) {
  - Initialize K = NULL;
  - For each x in sup(f)
    - if(|cubes(f,x)| >= 2) /* no single cubes */
    - C = largest cube containing x, such that cubes(f,C) = cubes(f, x)
    - K = K U Compute_Kernel(f/f_c)
- }

How to identify Kernels?

- F = ace + bce + de + g
- X = a, cubes(f, a) = ace <= 2; Abort!
- X = b, ...
- X = c, cubes(f, c) = {ace, bce}
  - Largest cube containing c & contained in both ace and bce = {ce}
  - {ce} = ?
  - f/f_c = ace + bce + de + g/ce =
  - (a + b) = kernel? Cube-free?
- And so on...
- Limitation: may re-compute same kernel....
- Remedy? Rectangular covering......

Kernel Extraction: Rectangle Covering

- Given in DeMicheli – notes given in class
- F = ace + bce + de + g

	Var:	a	b	c	d	e	g
Cube	R/C	1	2	3	4	5	6
ace	1	1	0	1	0	1	0
bce	2	0	1	1	0	1	0
de	3	0	0	0	1	1	0
g	4	0	0	0	0	0	1

- Rect: (R, C); co-Rect: (R, C'); C' = ?
- (R_1, C_1) contains (R_2, C_2), iff.....
- Prime rectangle: not contained in any other

Kernel Extraction: Rectangle Covering

- F = ace + bce + de + g
- Var: a b c d e g
- Cube R/C 1 2 3 4 5 6
- ace 1 1 0 1 0 1 0
- bce 2 0 1 1 0 1 0
- de 3 0 0 0 1 1 0
- g 4 0 0 0 0 0 1
- Prime Rect: ({1, 2}, {3, 5}), coRect:
- Prime rect = ?
- Corresponding co-Rect =?

Kernel Extraction: Rectangle Covering

- F = ace + bce + de + g

	Var:	a	b	c	d	e	g
Cube	R/C	1	2	3	4	5	6
ace	1	1	0	**1**	0	**1**	0
bce	2	0	1	**1**	0	**1**	0
de	3	0	0	0	1	**1**	0
g	4	0	0	0	0	0	1

- Co-kernel: {ce} corresponds to columns
- Kernel corresponds to rows: {ace, bce}
- But kernel = (a + b)
- Similarly, co-kernel = e; rows: {ace, bce, de}

Extract Kernels from Matrix representation

- F1 = ace + bce + de + g
- F2 = cde + b
- Construct matrix for: F = F1 + F2
- Identify all prime rectangles
- Co-kernel: prime rectangle (R, C) w/ |R| >= 2
- How to get kernel?
- Get co-rectangle (R, C')
- Sum the terms (cubes) corresponding to R.
- Drop the variables of the co-kernel. Or, in other words, restrict the sum terms to variables in C'.

Kernel Extraction: Rectangle Covering

Cube	R/C	1	2	3	4	5	6
	Var:	a	b	c	d	e	g
ace	1	1	0	1	0	1	0
bce	2	0	1	1	0	1	0
de	3	0	0	0	1	1	0
g	4	0	0	0	0	0	1
cde	5	0	0	1	1	1	0
b	6	0	1	0	0	0	0

- Identify prime rectangles:

In the textbook....

- Chapter 10, Intro 10.1 to 10.4
- 10.4 - Division: F = G H + R
  - Studied Boolean Division (not in text)
  - Remainder = 0, Div = factorization
  - Weak_Div:
  - Kernels & co-Kernels: study definitions from the textbook, also done in class
  - Two procedures for Kernel extraction: compute_kernel + rect_cov.
  - Text: gen_factor/quick_factor + cube-intersection matrix (level-0 kernels only)
- Next topic: Don't cares – Ch. 11, good discussion in the text-book, will follow it.