Homework 19: Diagonalization

1. §5.2, #15, 27.

2. Let $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T(\mathbf{x}) = A\mathbf{x}$.
   (a) Find all eigenvalues of A.
   (b) Find a basis B such that $[T]_B$ is diagonal.
   (c) Find $[T]_B$.
   (d) Express A in the form $A = PDP^{-1}$ for some diagonal matrix D and invertible matrix P.

3. Let $A = \frac{1}{9}\begin{bmatrix} 7 & 4 & -4 \\ 4 & 1 & 8 \\ -4 & 8 & 1 \end{bmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T(\mathbf{x}) = A\mathbf{x}$.
   (a) Find all eigenvalues of A.
   (b) Find a basis B such that $[T]_B$ is diagonal.
   (c) Find $[T]_B$.
   (d) Express A in the form $A = PDP^{-1}$ for some diagonal matrix D and invertible matrix P.

   Note. Observe that this matrix A is the answer to Problem 3(c) of Homework 16. Given this fact, and assuming that you have already solved Problem 3(c) of Homework 16, try to solve this problem with as few calculations as possible. For example, there is no need to compute the characteristic polynomial of A.