Homework 7: Subsequences, Series, Topology

*Assignments should be **stapled** and written clearly and legibly.*

1. Suppose that the sequence (a_n) converges to 7. Prove that there exists a subsequence (a_{n_k}) such that $|a_{n_k} - 7| < \dfrac{1}{10^k}$ for all k. (In other words, the subsequence converges to 7 rapidly.)
2. Let (a_n) be a sequence which does not converge to 5. Prove that there exists some $\epsilon > 0$ and a subsequence (a_{n_k}) such that $|a_{n_k} - 5| > \epsilon$ for all k.
3. Let m be a positive integer. By using the definition of convergence of a series, prove that $\displaystyle\sum_{k=1}^{\infty} a_k$ converges if and only if $\displaystyle\sum_{k=m}^{\infty} a_k$ converges.
4. Define the following: ϵ-neighborhood, open set, closed set.
5. Define the following: limit point of A.
6. §3.4, #14.