Math 221: Homework Exercises

1 The Definition of a Derivative

1. For each of the following functions, use limits to find (i) the slope of the tangent line at $x = 3$, and (ii) the equation of the tangent line at $x = 3$.

   (a) $f(x) = 3x^2 + 2x$.

   (b) $g(x) = \dfrac{5}{x}$.

2. For each of the following functions, use limits to find (i) the slope of the tangent line at $x = 1$, and (ii) the equation of the tangent line at $x = 1$.

   (a) $g(x) = \dfrac{2}{x}$.

   (b) $f(x) = 5x^2$.

   (c) $h(x) = \sqrt{x}$.

3. For each of the following functions, use limits to find (i) the slope of the tangent line at $x = 4$, and (ii) the equation of the tangent line at $x = 4$.

   (a) $f(x) = 5x^2 - 2x + 7$.

   (b) $g(x) = \dfrac{3}{x}$.

   (c) $h(x) = 3\sqrt{x}$.

4. Use limits to find $f'(x)$ for (a) $f(x) = 1/x$, (b) $f(x) = 3x^2 - 2x - 5$, (c) $f(x) = \sqrt{x}$.

5. An object is dropped from a tall building on planet Krypton. The distance the object falls in t seconds is $s(t) = 12t^2$ m.

   (a) Use limits to find $s'(t)$.

   (b) Find the object's velocity after 2 seconds.

   (c) How much time is required for the object's velocity to reach 36 m/s?

   (d) When the object's velocity is 36 m/s, how far has the object fallen?

   (e) If the height of the building is 300 m, what will be the object's velocity when it hits the ground?

6. Let $f(x) = 3x^2 + 2x$.

   (a) Use limits to find $f'(x)$.

   (b) Find the equation of the tangent line to the graph of $f(x)$ at $x = 1$.

   (c) Find the point on the graph of $f(x)$ where the tangent line has slope 20.

   (d) Find an equation of the tangent line to the graph of $f(x)$ which has slope 20.

7. An object travels along a straight line. Its position after t seconds is $s(t) = 12t - t^2$ meters.

   (a) Find the average velocity of the object over the time interval $[1, 3]$.

   (b) Find the average velocity of the object over the time interval $[1, 1 + h]$. Simplify your answer as much as possible. (Assume h is not equal to 0).

   (c) Use part (b) to find the instantaneous velocity of the object at time $t = 1$ second.