Homework 13: The Riemann Integral

*Directions. Assignments should be **stapled** and written clearly and legibly. Problems 5 and 6 are optional.*

1. Let $f(x) = x^2 - x$ and $P = \left\{0, \frac{1}{2}, 1, \frac{3}{2}, 2\right\}$. Find $U(f, P)$ and $L(f, P)$.

2. If $a < b < c < d$ and f is integrable on $[a, d]$, prove that f is integrable on $[b, c]$.

3. Suppose that $f : [a, b] \to \mathbb{R}$ is continuous, $f(x) \geq 0$ for all $x \in [a, b]$, and $f(x) > 0$ for at least one value $c \in [a, b]$. Using definitions, prove that $\int_a^b f > 0$. (You may assume that f is integrable.)