# Homework 4: Limits of Sequences

*Assignments should be **stapled** and written clearly and legibly.*

1. §4.1, #6(c), (d), (f).

2. For each of the following sequences, guess a limit, and then give an $\epsilon, N$ proof that your guess is correct:

   (a) $\left\{\dfrac{5}{\sqrt{n}}\right\}$

   (b) $\left\{\dfrac{3n}{4n+5}\right\}$

   (c) $\left\{\dfrac{2}{n^2+1}\right\}$

3. Consider the following definition:

   *Definition: A sequence $(a_n)$ is said to **reverge** to $L$ if there exists $\epsilon > 0$ such that for every $N \in \mathbb{N}$, whenever $n \geq N$, we have $|a_n - L| < \epsilon$.*

   (a) Give an example of a sequence that reverges.

   (b) If a sequence reverges, must it also converge? If not, give a counterexample.

   (c) Is it possible for a sequence to reverge to two different values?

4. Does the sequence $\{a_n\}$ defined by

$$a_n = \begin{cases} 2, & \text{if } n = 2^m \text{ for some } m \in \mathbb{N} \\ 1 + \frac{1}{n}, & \text{otherwise} \end{cases}$$

   converge? Justify your answer. (A formal proof is not required.)