Homework 6: Linear Independence (Part 2)

Assignments should be **stapled** and written clearly and legibly.

- 1. $\S4.3$, #5(a), 12, 15(a).
- 2. Determine whether $\{1, \ln(2x), \ln(x^2)\}$ is linearly independent in $F(0, \infty)$. Justify your answer.
- 3. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set of vectors in a vector space V, and let \mathbf{u} be any vector in V. Using only the definition of linear dependence given in class, prove that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{u}\}$ is linearly dependent.
- 4. Let $\mathbf{v}_1, \dots, \mathbf{v}_p$ be vectors in a vector space V. Prove the following:
 - (a) Let $\mathbf{v}_1, \dots, \mathbf{v}_p$ span V, and let \mathbf{u} be any vector in V. Then $\{\mathbf{u}, \mathbf{v}_1, \dots, \mathbf{v}_p\}$ is linearly dependent.
 - (b) Let $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p\}$ be linearly independent. Then $\mathbf{v}_2, \dots, \mathbf{v}_p$ cannot span V. Hint. Use Theorem 1.4 from class for both parts.