Homework 5: Limit Points

*Assignments should be **stapled** and written clearly and legibly.*

1. §2.2, #2.13(a)(c)(e)(f), 2.15.

2. Let X be a topological space and $A \subseteq X$.
   (a) Prove that if a sequence $(x_1, x_2, \ldots)$ of points in A converges to x, then $x \in \overline{A}$.
   (b) Prove that if a sequence $(x_1, x_2, \ldots)$ of points in $A \setminus \{x\}$ converges to x, then $x \in A'$.

3. A topological space X is said to be a T_1-space if finite subsets of X are closed. Prove that a Hausdorff space is a T_1-space.

   Hint. Use a theorem we proved in class about Hausdorff spaces.

4. Suppose that a topological space X is a T_1-space. Let $A \subseteq X$.
   (a) Prove that x is a limit point of A if and only if every neighborhood of x intersects A in infinitely many points.
   (b) Prove that $(A')' = A'$.
   (c) (Bonus – Real Analysis) Consider a sequence (x_n) in X. Prove that if the set $\{x_n\}$ has a limit point, then the sequence (x_n) has a convergent subsequence.