Homework 4: Conditional Probability, Bayes' Theorem

1. §2.1, #13, §2.3, #5

2. A desk has three drawers. The first contains two gold coins, the second contains two silver coins, and the third contains one gold and one silver coin. A coin is drawn from a drawer selected at random. Suppose the selected coin is silver. What is the probability that the other coin in the drawer is gold?

3. English and American spellings are *rigour* and *rigor*, respectively. A man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel. If 40% of the English-speaking men at the hotel are English and 60% are American, what is the probability that the writer is an Englishman?

4. A multiple choice exam has 5 choices per question. On 75% of the questions, you think you know the answer; on the other 25% of the questions, you just guess at random. Unfortunately, when you think you know the answer, you are right only 80% of the time.

   (a) Find the probability of getting an arbitrary question right.

   (b) If you do get a question right, what is the probability that it was a lucky guess (i.e., that it was one of the questions whose answers you just guessed at random)?

5. A deck is missing one card. You draw two cards, and both are diamonds. What is the probability that the missing card is diamonds?

6. Scotty has n keys of which one will open the door to the engine room.

   (a) If he tries the keys at random, discarding those that do not work, what is the probability that he will open the door on the kth try?

   (b) What if he does not discard the previously tried keys?

7. A laboratory blood test is 95% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a false positive result for 1% of the healthy persons tested. (That is, if a healthy person is tested, then there is a 1% probability that the test will imply that he or she has the disease.) Suppose that only 0.5% of the population actually has the disease. If a person tests positive for the disease, what is the probability that the person has the disease?

8. Draw a card. If it's diamonds, place it back in the deck and draw another card. If the first card is not diamonds, then draw a second card without replacing the first card.

   (a) Find the probability that the second card is the queen or king of diamonds.

   (b) If the second card is the two of clubs, find the probability that the first card was diamonds.