

## Homework 19: Eigenspaces

Assignments should be **stapled** and written clearly and legibly. Problem 4 is optional.

1. §5.1, #5(a)(b)(d), 7, 21, 25(a)(c).

For Problem 21, there is no need to refer to any tables, as the instructions suggest, and calculations are not required.

2. The vector  $\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$  is an eigenvector for  $A = \begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$ .

(a) Find the eigenvalue  $\lambda$  which corresponds to  $\mathbf{x}$ . Show your work.

(b) Let  $\lambda$  be the eigenvalue you found in (a). Is it possible to find another eigenvector  $\mathbf{y}$  corresponding to eigenvalue  $\lambda$  such that  $\{\mathbf{x}, \mathbf{y}\}$  is linearly independent? Justify.

*Hint: Find the dimension of the  $\lambda$ -eigenspace of  $A$ .*

3. Let  $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ , and let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be given by  $T(\mathbf{x}) = A\mathbf{x}$ .

(a) Find all eigenvalues of  $A$ .

(b) Find a basis  $B$  such that  $[T]_B$  is diagonal.

(c) Find  $[T]_B$ .

(d) Express  $A$  in the form  $A = PDP^{-1}$  for some diagonal matrix  $D$  and invertible matrix  $P$ .

4. (Optional) §5.2, #15, 27.