Homework 9: The Quotient Topology

*Assignments should be **stapled** and written clearly and legibly.*

1. §3.3, #3.23, 3.24, 3.27, 3.33(a)(b)(c)(h)(j)(k)

2. Let X and S be sets, and let $g : X \to S$ be a surjective map. Define a relation $\sim$ on X by $x \sim y$ if $g(x) = g(y)$.

   (a) Verify that $\sim$ is an equivalence relation.

   (b) What are the equivalence classes of $\sim$?

   (c) Prove that there exists a bijection $\overline{g} : (X/\sim) \to S$.

   (d) The map $\overline{g}$ appears as a dashed line in the following diagram:

$$\begin{tikzcd} X \arrow[r, "g"] \arrow[d, "\pi"'] & S \\ X/\sim \arrow[ur, dashed, "\overline{g}"'] & \end{tikzcd}$$

   (Here π is the standard projection.) Prove that this diagram commutes, i.e., prove that $g = \overline{g} \circ \pi$. We sometimes say that *the map g factors through $X/\sim$*.

3. Let X be a topological space, S a set, and $g : X \to S$ a surjective map. Give S the quotient topology. Let C be a subset of S. Prove that C is closed in S if and only if $g^{-1}(C)$ is closed in X.