

## Homework 8: Null Space and Column Space of a Matrix

1. Without using a calculator or computer, find a nonzero vector in  $\text{Nul } A$ , where

$$A = \begin{bmatrix} 51 & 5 & 58 & 2 & 7 \\ 7 & 2021 & 9 & 1 & 2 \\ 3 & \sqrt{2} & 5 & 0 & 2 \\ 9 & 2049 & 15 & 3 & 6 \\ 3 & \pi & 8 & 37 & 5 \\ 7 & 3 & 23 & 19 & 16 \\ 11 & 1 & 14 & 0 & 3 \end{bmatrix}$$

2. Suppose that  $\mathcal{B} = \{\mathbf{v}_1, \dots, \mathbf{v}_n\}$  is a linearly independent set of vectors in a vector space  $V$ . Prove that

- (a)  $\text{Span } \mathcal{B}$  is a subspace of  $V$ .
- (b)  $\mathcal{B}$  is a basis for  $\text{Span } \mathcal{B}$ .

3. Let  $H$  be the set of all vectors of the form  $\begin{bmatrix} a \\ a \\ a \\ b \end{bmatrix}$ . Prove that  $H$  is a subspace of  $\mathbb{R}^4$ , and

then find a basis for  $H$ . Prove that your basis is indeed a basis.

*Hint. Express  $H$  as the span of vectors.*

4. A matrix  $A$  and an echelon form of  $A$  are given:

$$A = \begin{bmatrix} 1 & 2 & -4 & 3 & 3 \\ 5 & 10 & -9 & -7 & 8 \\ 4 & 8 & -9 & -2 & 7 \\ -2 & -4 & 5 & 0 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -4 & 3 & 3 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) Find a basis for  $\text{Nul } A$ . What is  $\dim(\text{Nul } A)$ ?
  - (b) Find a basis for  $\text{Col } A$ . What is  $\dim(\text{Col } A)$ ?
5. For each of the following vector spaces, find a matrix  $A$  such that the vector space is equal to  $\text{Nul } A$ . Then find a basis for the vector space.
- (a) The line  $y = 5x$  in  $\mathbb{R}^2$ .
  - (b) The plane  $x + 2y + 3z = 0$  in  $\mathbb{R}^3$ .
6. Find a basis for  $\text{Col } \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 2 & 4 \end{bmatrix}$  without doing **any** calculations.
7. Determine whether  $\{1, 1 + 2x + 3x^2\}$  is a basis for  $P_2$ . Prove your answer.