

## Homework 2: Ordered Fields, More Supremums (Due 2/12/2021)

*Assignments should be **stapled** and written clearly and legibly.*

1. §3.2, # 3(j).
2. Find a rational number between  $\pi$  and  $\sqrt{10}$ . Express your answer in the form  $a/b$ , where  $a, b \in \mathbb{Z}$ .
3. Suppose that  $A$  and  $B$  are bounded sets and that  $\sup A < \sup B$ . Prove that there exists  $b \in B$  that is an upper bound for  $A$ . Then show by example that this is not always the case if we only assume  $\sup A \leq \sup B$ .
4. Let  $a < b$  be real numbers and consider the set  $T = \mathbb{Q} \cap [a, b]$ .
  - (a) Prove that  $\sup T = b$ . You may use any of the theorems we've proved in class.
  - (b) Show by example that it is not always the case that  $\max T = b$  (proof not needed for this part).
5. §2.3, #5, 7(b), (c), (e).