Homework 8: Independent Random Variables

1. Suppose that X and Y are independent continuous random variables with

$$f_X(x) = \begin{cases} \frac{3x^2}{8}, & 0 < x < 2 \\ 0, & x < 0 \end{cases}$$

$$f_Y(y) = \begin{cases} y^{-2}, & 1 < y \\ 0, & y < 1 \end{cases}$$

   (a) Find $f_{X,Y}(x,y)$, the joint density function of X and Y.

   (b) Find $P(XY > 1)$.

2. Let X and Y be jointly continuous with joint pdf $f_{X,Y}(x,y) = (24/5)(x+y)$, for $0 \le 2y \le x \le 1$.

   (a) Find $f_X(x)$ and $f_Y(y)$.

   (b) Are X and Y independent?

3. A point is chosen at random from the interior of a circle whose equation is $x^2 + y^2 \le 4$. Let the random variables X and Y denote the x- and y-coordinates of a sampled point.

   (a) Find $f_{X,Y}(x,y)$.

   (b) Find $f_X(x)$ and $f_Y(y)$.

   (c) Are X and Y independent?

4. A hat contains 3 red and 6 white slips of paper. You draw 2 slips at random, without replacement. Let X equal 1 if the first slip is red, and X equal 0 otherwise. Let Y equal 1 if the second slip is red, and Y equal 0 otherwise. Use the definition of independence of random variables to determine whether X and Y are independent.