Homework 7: Cauchy Sequences, Series
(Due March 7, 2020)

Assignments should be ***stapled*** *and written clearly and legibly. Problems 5 and 6 are optional. In this and future assignments, you may use any result proved or stated in class, unless a problem states otherwise.*

1. §3.1, #7.
2. §4.3, #15.
3. §8.2, #9.
4. Let (a_n) be a bounded sequence.
   (a) Prove that the sequence (b_n), defined by $b_n = \sup\{a_k : k \geq n\}$, converges.
   (b) The **limit superior** of (a_n), denoted by $\limsup a_n$, is defined to be the limit of the sequence (b_n) of part (a). Give a reasonable definition for $\liminf a_n$, and briefly explain why it must exist.
   (c) Prove that if (c_n) and (d_n) are convergent sequences such that $c_n \leq d_n$ for all n, then $\lim\limits_{n\to\infty} c_n \leq \lim\limits_{n\to\infty} d_n$.
   (d) Prove that $\liminf a_n \leq \limsup a_n$. Give an example of a bounded sequence (a_n) for which this inequality is strict.
5. Let (a_n) be a bounded sequence. Prove that $\liminf a_n = \limsup a_n$ if and only if $\lim\limits_{n\to\infty} a_n$ exists. In this case, all three share the same value.
6. (Challenge) Let (a_n) have the property that every subsequence has a subsequence converging to L. Prove that (a_n) converges to L.