Homework 6: Linear Independence, Basis

1. §4.4, #13(b), 14(b), 27(a)(b).

2. Determine whether $\{1, \ln(2x), \ln(x^2)\}$ is linearly independent in $F(0, \infty)$. Justify your answer.

3. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set of vectors in a vector space V, and let $\mathbf{u}$ be any vector in V. Using only the definition of linearly dependendence given in class, prove that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{u}\}$ is linearly dependent.

4. Let $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a set of vectors in a vector space V such that every vector $\mathbf{u}$ in V can be written in exactly one way as a linear combination of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$. Prove that $\mathcal{B}$ is a basis for V.

   Note that in this problem you are being asked to prove the converse of Theorem 1.6. You must prove two things: $\mathcal{B}$ spans V, and $\mathcal{B}$ is linearly independent.