Homework 18: Eigenspaces

1. §5.1, #5(a)(b)(d), 7, 21, 25(a)(c).

   For Problem 21, there is no need to refer to any tables, as the instructions suggest, and calculations are not required.

2. The vector $\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is an eigenvector for $A = \begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$.

   (a) Find the eigenvalue λ which corresponds to $\mathbf{x}$. Show your work.

   (b) Let λ be the eigenvalue you found in (a). Is it possible to find another eigenvector $\mathbf{y}$ corresponding to eigenvalue λ such that $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent? Justify. *Hint: What is the dimension of the λ-eigenspace of A?*

3. Let $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T(\mathbf{x}) = A\mathbf{x}$.

   (a) Find all eigenvalues of A.

   (b) Find a basis B such that $[T]_B$ is diagonal.

   (c) Find $[T]_B$.

   (d) Express A in the form $A = PDP^{-1}$ for some diagonal matrix D and invertible matrix P.