Homework 5: Linear Independence

1. §4.3, #3(a), 4(a), 15(b), 16(a)(c).

   Try to solve # 15(b) and 16(a) with as few computations as possible, and try to be fairly rigorous for 16(c).

2. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ be a linearly independent set of vectors in a vector space V. Using only the definition of linear independence, prove that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent as well.

   Hint. Begin the proof as follows:

   "Suppose $k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3 = \mathbf{0}$. I must show that $k_1 = k_2 = k_3 = 0$."

3. Let $\mathbf{v}_1, \ldots, \mathbf{v}_p$ be vectors in a vector space V. Prove the following:

   (a) Let $\mathbf{v}_1, \ldots, \mathbf{v}_p$ span V, and let $\mathbf{u}$ be any vector in V. Then $\{\mathbf{u}, \mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly dependent.

   (b) Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ be linearly independent. Then $\mathbf{v}_2, \ldots, \mathbf{v}_p$ cannot span V.

   Hint. Use Theorem 4 from class for both parts.

4. Using only the definition of linear independence, prove that if $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent, then so is $\{\mathbf{u}+\mathbf{v}, \mathbf{u}+2\mathbf{w}, \mathbf{v}+3\mathbf{w}\}$.

   Hint. Begin the proof as follows:

   "Suppose $c(\mathbf{u}+\mathbf{v}) + d(\mathbf{u}+2\mathbf{w}) + e(\mathbf{v}+3\mathbf{w}) = \mathbf{0}$. I must show that $c = d = e = 0$."