Homework 6: Linear Independence, Basis

Assignments should be ***stapled*** *and written clearly and legibly.*

1. §4.3, #5(a), 12, 15(a), §4.4, #19(a)(b), 20

2. Determine whether $\{1, \ln(2x), \ln(x^2)\}$ is linearly independent in $F(0, \infty)$. Justify your answer.

3. Let $\mathbf{v}_1, \ldots, \mathbf{v}_p$ be vectors in a vector space V. Prove the following:

   (a) Let $\mathbf{v}_1, \ldots, \mathbf{v}_p$ span the vector space V, and let $\mathbf{u}$ be any vector in V. Then $\{\mathbf{u}, \mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly dependent.

   (b) Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ be linearly independent. Then $\mathbf{v}_2, \ldots, \mathbf{v}_p$ cannot span V.

   Hint. Use Theorem 1.4 from class for both parts.

4. Let $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a set of vectors in a vector space V such that every vector $\mathbf{u}$ in V can be written in exactly one way as a linear combination of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$. Prove that $\mathcal{B}$ is a basis for V.

   Note that in this problem you are being asked to prove the converse of Theorem 1.6. You must prove two things: $\mathcal{B}$ spans V, and $\mathcal{B}$ is linearly independent.