Homework 10: Limits of Functions
(Due April 14, 2023)

*Assignments should be **stapled** and written clearly and legibly. For problems 1, 3, and 4, you must use the $\epsilon - \delta$ definition of a limit.*

1. §5.1, #7(a), #13 (the Squeeze Theorem).

2. Use the Squeeze Theorem to prove that $\lim\limits_{x \to 0} \left(x \sin \left(\dfrac{1}{x} \right) \right) = 0$. Make sure to state what D is.

3. Let $f : D \to \mathbb{R}$, where $D \subseteq \mathbb{R}$, and let a be a limit point of D. Suppose that $\lim\limits_{x \to a} f(x) > 0$. Prove that there exists a deleted neighborhood $N_\delta^*(a)$ of a such that $f(x) > 0$ for all $x \in N_\delta^*(a) \cap D$.

4. Let $a > 0$. Use the definition of limit to prove that $\lim\limits_{x \to a} \sqrt{x} = \sqrt{a}$.

   Hint: Use the inequality $|\sqrt{x} - \sqrt{a}| = \dfrac{|x - a|}{\sqrt{x} + \sqrt{a}} \leq \dfrac{|x - a|}{\sqrt{a}}$