

Homework 13: The Matrix of a Linear Transformation

1. §8.4, #8.
2. Let $T : V \rightarrow W$ be linear, and let $\mathcal{B} = \{\mathbf{u}_1, \mathbf{u}_2\}$ be a basis for V and $\mathcal{B}' = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ a basis for W . Suppose that $T(\mathbf{u}_1) = 3\mathbf{w}_1 + 5\mathbf{w}_2 - 7\mathbf{w}_3$ and $T(\mathbf{u}_2) = 2\mathbf{w}_1 + 4\mathbf{w}_3$.
 - (a) Find $[T]_{\mathcal{B}', \mathcal{B}}$.
 - (b) If $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$, then what is $[T(\mathbf{x})]_{\mathcal{B}'}$?
3. Let \mathcal{B} be the standard basis of \mathbb{R}^2 , let \mathcal{C} be the basis of \mathbb{R}^3 consisting of the three vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, and let \mathcal{D} be the standard basis of \mathbb{R}^3 . Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation defined by $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = a\mathbf{u}_1 + b\mathbf{u}_2 + (a+b)\mathbf{u}_3$. Find $[T]_{\mathcal{C}, \mathcal{B}}$ and $[T]_{\mathcal{D}, \mathcal{B}}$.
4. §8.5, #10, 14.
5. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be reflection across the line $y = \frac{1}{3}x$. Let $\mathcal{B} = \left\{\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}\right\}$ and $\mathcal{C} = \{\mathbf{e}_1, \mathbf{e}_2\}$, two bases of \mathbb{R}^2 .
 - (a) Find $[T]_{\mathcal{B}}$.
 - (b) Use (a) and the change of basis formula to find $[T]_{\mathcal{C}}$, the standard matrix for T .
 - (c) Find the reflection of $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$ across the line $y = \frac{1}{3}x$, i.e., find $T\left(\begin{bmatrix} 3 \\ 7 \end{bmatrix}\right)$.