Homework 1: Ordered Sets, Infimums, Supremums
(Due 2/7/2022)

*Assignments should be **stapled** and written clearly and legibly. Problems 4 and 5 are optional.*

1. §3.2, #10, 12(b).

2. §3.3, #3(a), (d), (f), (h), 5, 8.

3. Suppose that A and B are two nonempty sets of real numbers such that $x \leq y$ for all x in A and y in B.

    (a) Explain how we know that A is bounded above and B is bounded below.

    (b) Explain how we know that both $\sup A$ and $\inf B$ must exist.

    (c) **Prove** that $\sup A \leq y$ for all $y \in B$.

    (d) Use part (c) and the definition of $\inf B$ to **prove** that $\sup A \leq \inf B$.

    (e) Can one say that $\max A \leq \min B$? Justify your answer.

4. (Bonus) Using the axioms of an ordered field, prove the arithmetic-geometric mean inequality: For any $a, b \in \mathbb{R}$ with $a > 0$ and $b > 0$,

$$\sqrt{ab} \leq \frac{a+b}{2}$$

    You may assume the existence of square roots.

5. (Open Question[1]) Is $e + \pi$ rational?

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[1]An open question is a question which has not been answered. No one knows its answer.