

Homework 11: Expectation of Functions of Random Variables

1. Let X have pdf

$$f_X(x) = 2(1 - x), 0 \leq x \leq 1$$

Suppose that $Y = X^2$. Find $E(Y)$ in two different ways.

2. A box is to be constructed so that its height is five inches and its base is X inches by X inches, where X is a random variable with pdf $f_X(x) = 6x(1 - x)$, $0 < x < 1$. Let V be the volume of the box. Find $E(V)$.
3. Suppose that the hypotenuse of an isosceles right triangle is a random variable which is uniform over the interval $[4, 10]$. Find the expected value of the triangle's area.
4. Grades on a recent Math 120 exam were fairly low. When graphed, the distribution of grades had a shape similar to the pdf

$$f_X(x) = \frac{1}{5000}(100 - x), 0 \leq x \leq 100$$

In order to “curve” the results, the professor announces that she will replace each person's grade X with a new grade $g(X)$, where $g(X) = 10\sqrt{X}$. Will this curve raise the class average above 60?

5. Suppose that Y has pdf

$$f_Y(y) = \frac{1}{\pi(1 + y^2)},$$

for $-\infty < y < \infty$. Let $Z = 1/(1 + Y^2)$. Then Z has a pdf of the form $cz^\alpha(1 - z)^\beta$, $0 < z < 1$.

- (a) Find c , α , and β .
- (b) Find $E(Z)$.