Homework 6: Linear Independence (Part 2)

*Assignments should be **stapled** and written clearly and legibly.*

1. §4.3, #5(a), 12, 15(a).

2. Determine whether $\{1, \ln(2x), \ln(x^2)\}$ is linearly independent in $F(0, \infty)$. Justify your answer.

3. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set of vectors in a vector space V, and let $\mathbf{u}$ be any vector in V. Using only the definition of linear dependendence given in class, prove that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{u}\}$ is linearly dependent.

4. Let $\mathbf{v}_1, \ldots, \mathbf{v}_p$ be vectors in a vector space V. Prove the following:

   (a) Let $\mathbf{v}_1, \ldots, \mathbf{v}_p$ span V, and let $\mathbf{u}$ be any vector in V. Then $\{\mathbf{u}, \mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly dependent.

   (b) Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ be linearly independent. Then $\mathbf{v}_2, \ldots, \mathbf{v}_p$ cannot span V.

   Hint. Use Theorem 1.4 from class for both parts.