Homework 12: Indicator Random Variables

1. You randomly throw 6 balls into 10 different baskets. Let X be the number of balls which land in the first basket, and let Y be the number of baskets which are empty.
   (a) Find $f_X(x)$.
   (b) Find $E(X)$.
   (c) Find $E(Y)$. (Hint. Let I_j be the indicator of the event "basket j is empty". Note that Y is the sum of the I_j's.)

2. From a group of 8 math majors and 7 physics majors (no double majors), 4 are randomly selected for the Putnam Competition team. Let M be the number of math majors on the team. Find $E(M)$ in two ways:
   (a) Use the definition of $E(M)$. (You will need to first find $f_M(m)$.)
   (b) Use indicator random variables. Specifically, let I_j be the indicator of the event "the j-th person selected is a math major". Use the fact that $M = I_1 + I_2 + I_3 + I_4$.

3. A population of n people vote in an election. d vote democratic and $n-d$ vote republican. In the next election, the probability of a democratic voter switching to republican is p_1, and the probability of a republican voter switching to democratic is p_2. Let X be the number of democratic votes in the second election. Find $E(X)$.

4. Ten husband and wife couples are randomly seated in a circle. Find the expected number of husbands who are seated next to their wives.

5. Suppose a bent coin has a probability $p = 0.4$ of landing heads. If one flips the coin n times, what is the expected number of heads which are immediately followed by a tail. (For example, if $n = 8$, then for outcome 'THHTTHTH', two heads, namely the third and sixth flips, are immediately followed by a tail.)

6. A standard deck of cards is randomly dealt to 25 people. Each person will receive somewhere between 0 and 52 cards. Find the expected number of people who receive no cards.