# Homework 11: One-to-One, Onto

*Assignments should be **stapled** and written clearly and legibly. Problem 4 is optional.*

1. §8.2, #1(c), 6, 7, 19(a)(c), 26. Make sure to justify your answer to the question asked in Exercise 7.

2. Let $T : V \to W$ be linear transformation, and let $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ be linearly independent in $V$.

   (a) Prove that if $T$ is one-to-one, then $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ is linearly independent in $W$.

   *Hint. Begin the proof as follows:*

   *"Suppose $c_1 T(\mathbf{v}_1) + \cdots + c_p T(\mathbf{v}_p) = \mathbf{0}$. I must show that $c_1 = \cdots = c_p = 0$."*

   (b) Give an example showing that if $T$ is not one-to-one, then $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ need not be linearly independent in $W$.

3. Let $T : V \to W$ be a linear transformation, with $\dim V = n$, $\dim W = m$. Prove the following:

   (a) $\dim(R(T)) \leq n$.

   (b) $\dim(R(T)) = n$ if and only if $T$ is one-to-one.

   (c) $\dim(R(T)) = m$ if and only if $T$ is onto.

4. (Challenge) Let $T : V \to V$ be a linear transformation such that $T \circ T = T$, and let $\mathbf{u}$ be a vector in $V$. Prove that $\{\mathbf{u}, T(\mathbf{u})\}$ is linearly dependent if and only if $T(\mathbf{u}) = \mathbf{u}$ or $T(\mathbf{u}) = \mathbf{0}$.