

## Homework 7: Basis

*Assignments should be **stapled** and written clearly and legibly. All answers must be justified.*

1. §4.4, #13(b), 24(a)(b), 27(a)(b).
2. Suppose that  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are vectors in a vector space  $V$ .
  - (a) Suppose that  $\mathbf{u}$  is not a scalar multiple of  $\mathbf{v}$ , and  $\mathbf{v}$  is not a scalar multiple of  $\mathbf{u}$ . Is  $\{\mathbf{u}, \mathbf{v}\}$  necessarily linearly independent? If so, prove it. If not, give a concrete counterexample.
  - (b) If it is only known that  $\mathbf{u}$  is not a scalar multiple of  $\mathbf{v}$ , is  $\{\mathbf{u}, \mathbf{v}\}$  necessarily linearly independent? If so, prove it. If not, give a concrete counterexample.
  - (c) Suppose that none of the three vectors  $\mathbf{u}_1$ ,  $\mathbf{u}_2$ , and  $\mathbf{u}_3$  is a scalar multiple of either of the other two vectors. In other words,  $\mathbf{u}_1$  is not a scalar multiple of  $\mathbf{u}_2$  or of  $\mathbf{u}_3$ ;  $\mathbf{u}_2$  is not a scalar multiple of  $\mathbf{u}_1$  or of  $\mathbf{u}_3$ ; and  $\mathbf{u}_3$  is not a scalar multiple of  $\mathbf{u}_1$  or of  $\mathbf{u}_2$ . Is  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  necessarily linearly independent? If so, prove it. If not, give a concrete counterexample.
3. Suppose that  $\mathcal{B} = \{\mathbf{v}_1, \dots, \mathbf{v}_n\}$  is a linearly independent set of vectors in a vector space  $V$ . Prove that
  - (a)  $\text{Span } \mathcal{B}$  is a subspace of  $V$ .
  - (b)  $\mathcal{B}$  is a basis for  $\text{Span } \mathcal{B}$ .
4. In this problem you will prove that  $\{e^x, e^{2x}\}$  is a basis for  $\text{Span}\{e^x, e^{2x}\}$ .
  - (a) Prove that  $\{e^x, e^{2x}\}$  is linearly independent in  $F(-\infty, \infty)$ .
  - (b) Prove that  $\text{Span}\{e^x, e^{2x}\}$  is a subspace of  $F(-\infty, \infty)$ .
  - (c) Use Problem 3 to prove that  $\{e^x, e^{2x}\}$  is a basis for  $\text{Span}\{e^x, e^{2x}\}$ .