Homework 12: Homeomorphisms

*Assignments should be **stapled** and written clearly and legibly.*

1. §4.2, #4.27. 4.30, 4.32(a).

2. Let $f : A \to B$ be a map of sets. Prove that f is bijective if and only if there exists a map $g : B \to A$ such that $g \circ f = \mathrm{id}_A$ and $f \circ g = \mathrm{id}_B$.

3. Give an explicit homeomorphism $f : \mathring{B}^2 \to \mathbb{R}^2$, and give its inverse function explicitly (see top of page 17 for the definition of $\mathring{B}^2$). You need not prove that f or its inverse function are continuous. However, you must explain why both f and its inverse function are well defined, and verify that they are inverse to eachother. This problem shows that "boundedness" is not a topological property.

4. Let X and Y be topological spaces. Let $y_0 \in Y$, and let $f : X \to Y$ be continuous.
   (a) Prove that the map $g : X \to X \times Y$ given by $g(x) = (x, y_0)$ is an embedding.
   (b) Prove that the map $h : X \to X \times Y$ given by $h(x) = (x, f(x))$ is an embedding. (Note that f need not be injective. Note also that the image of h is the graph of f, as defined in Exercise 4.10 of Section 4.1.)

   Hint. You may find it helpful to use the theorem from class giving rules for constructing continuous functions.