Homework 11: Isomorphisms, The Standard Matrix

1. §8.3, #4, 8. (For these two, provide brief justifications.)

2. Find the standard matrix for each of the following linear transformations.
   (a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by rotating θ **clockwise** about the origin.
   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by projecting onto the line $y = -x$.
   (c) $R : \mathbb{R}^3 \to \mathbb{R}^3$ given by rotating 180° about the line passing through points $(0, -1, -1)$ and $(0, 1, 1)$.

3. Find the result of rotating the vector $\begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$ by 180° about the line through the points $(0, -1, -1)$ and $(0, 1, 1)$.

4. Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $\ker(T) = R(T)$.

5. Let $T : U \to V$ and $S : V \to W$ be linear. Prove that if $S \circ T$ is one-to-one, then T is one-to-one.

6. Prove that the linear transformation $T : P_n \to P_n$ given by $T(p(x)) = x^n p\left(\frac{1}{x}\right)$ is an isomorphism. (You need not prove that it is linear; you can assume this.)

7. (Bonus) Consider $T : P_2 \to P_2$ given by $T(p(x)) = p(x) + p'(x)$.
   (a) Prove that T is linear.
   (b) Prove that T is an isomorphism.