# Homework 13: The Intermediate Value Theorem (Due April 28, 2023)

*Assignments should be* ***stapled*** *and written clearly and legibly. Problems 4 and 5 are optional.*

1. §5.3, #5, 7, 10.

2. Let $f$ be continuous on $[0,1]$ with $f(0) = f(1)$. Prove that there exists $c \in [0, \frac{1}{2}]$ such that $f(c) = f(c + \frac{1}{2})$.

3. Prove that there exists a real number $x$ such that
$$x^{177} + \frac{165}{1 + x^8 + \sin^2 x} = 125.$$

4. Prove that if $f : [a,b] \to \mathbb{R}$ is injective and continuous, then the inverse function $f^{-1}$ is also continuous.

5. (Putnam Exam) Suppose that the real numbers $a_0, a_1, \ldots, a_n$ and $x$, with $0 < x < 1$, satisfy
$$\frac{a_0}{1-x} + \frac{a_1}{1-x^2} + \cdots + \frac{a_n}{1-x^{n+1}} = 0.$$
Prove that there exists a real number $y$ with $0 < y < 1$ such that
$$a_0 + a_1 y + \cdots + a_n y^n = 0.$$