Homework 13: Standard Matrices, Compositions

Assignments should be ***stapled*** *and written clearly and legibly. Problem 8 is optional.*

1. Find the standard matrix for each of the following linear transformations.
   (a) $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by rotating 2θ **clockwise** about the origin.
   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by reflecting across the line $y = -x$.
   (c) $R : \mathbb{R}^3 \to \mathbb{R}^3$ given by rotating $180°$ about the line passing through points $(-1, 0, -1)$ and $(1, 0, 1)$.

2. Find the result of rotating the vector $\begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$ by $180°$ about the line through the points $(-1, 0, -1)$ and $(1, 0, 1)$.

3. §1.3, #5(a), (b)

4. §4.10, #9, 11.

5. In $\mathbb{R}^3$, let T be counterclockwise rotation by θ about the z-axis, and let S be counterclockwise rotation by ψ about the y-axis. (Here counterclockwise means as viewed from the positive axis.) Let R be counterclockwise rotation by θ about the z-axis followed by counterclockwise rotation by ψ in $\mathbb{R}^3$ about the y-axis. Find standard matrices for T, S, and R.

   Hint: The standard matrix for R is obtained by multiplying the other two matrices (in the correct order).

6. Give an isomorphism $T : M_{2,3} \to \mathbb{R}^6$. No justification required.

7. Suppose that there exists a linear transformation from V <u>onto</u> W, where both V and W are finite-dimensional vector spaces. Is it possible for the dimension of W to be greater than the dimension of V? Justify your answer.

8. (Optional) Give an example of a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which is bijective but is not an isomorphism.