Homework 7: Linear Independence, Basis

*Assignments should be **stapled** and written clearly and legibly. All answers must be justified. For proofs, you should use complete sentences and grammatically correct English.*

1. §4.4, #13(b), 14(b), 27(a)(b).

2. Determine whether $\{1, \sin x, \cos x\}$ is linearly independent in $F(-\infty, \infty)$. Justify your answer.

3. Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be vectors which span a vector space V, and let $\mathbf{u}$ be a vector in V. Prove that $\text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n, \mathbf{u}\} = V$.

4. Let $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a linearly dependent set of vectors in a vector space V, and let $\mathbf{v}$ be any vector in V. Using the definition of linear independence (and no theorems on linear dependence/independence), prove that if $\mathbf{v}$ is a vector in V which is not in $\text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$, then $\{\mathbf{v}_1, \ldots, \mathbf{v}_n, \mathbf{v}\}$ is still linearly independent.