Homework 12: Continuity in Metric Spaces (Due April 13, 2022)

*Assignments should be **stapled** and written clearly and legibly. Problem 5 is optional.*

1. §5.2, #18. Give a proof using Homework 11, Problem 4(a). (Note that another proof can be given using Problem 3 below.)

2. §5.5, #15.

3. Let (X, d) be a metric space and suppose that $f : X \to \mathbb{R}$ is continuous on X. If $D \subseteq X$ and $f(x) = 0$ for all $x \in D$, prove that $f(x) = 0$ for all $x \in \overline{D}$.

4. Let (X, d) be a metric space and suppose that $f : X \to \mathbb{R}$ is continuous on X. Let $Z(f)$, the **zero set** of f, be the set of all $x \in X$ for which $f(x) = 0$. Prove that $Z(f)$ is closed.

   Hint: The zero set of f can be identified as a pre-image. Use a theorem concerning pre-images of continuous functions.

5. §5.2, #17.