Homework 5: More Limits

Assignments should be ***stapled*** *and written clearly and legibly.*

1. From any given sequence (a_n) we can form the related sequence $(b_n) = (5a_n + 2)$. Use the definition of convergence of a sequence to prove that if (a_n) converges to 20, then (b_n) converges to __________. (First fill in the blank.)

2. Suppose that a sequence (a_n) converges to 0, and that at least one term a_n of the sequence is greater than 0. Prove that the set $\{a_n\}$ has a maximum.

3. Let (a_n) and (b_n) be sequences. Suppose that (a_n) converges to 0.
   (a) Using the definition of convergence, prove that if (b_n) is bounded, then the sequence $(a_n b_n)$ converges. (Note that you may not assume that (b_n) converges.)
   (b) If the sequence (b_n) is not bounded, must the sequence $(a_n b_n)$ necessarily converge? If so, prove it. If not, give a counterexample.

4. Give an example of a sequence (a_n) such that
   (a) (a_n) converges to 0, but $a_n \neq 0$ for all n.
   (b) (a_n) is bounded but does not converge.
   (c) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ but (a_n) does not converge.
   (d) $(|a_n|)$ converges but (a_n) does not.