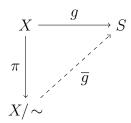
## Homework 9: The Quotient Topology

Assignments should be **stapled** and written clearly and legibly.

- 1.  $\S 3.3, \# 3.23, 3.24, 3.27, 3.33(a)(b)(c)(h)(j)(k)$
- 2. Let X and S be sets, and let  $g: X \to S$  be a surjective map. Define a relation  $\sim$  on X by  $x \sim y$  if g(x) = g(y).
  - (a) Verify that  $\sim$  is an equivalence relation.
  - (b) What are the equivalence classes of  $\sim$ ?
  - (c) Prove that there exists a bijection  $\overline{g}:(X/\sim)\to S$ .
  - (d) The map  $\overline{g}$  appears as a dashed line in the following diagram:



(Here  $\pi$  is the standard projection.) Prove that this diagram commutes, i.e., prove that  $g = \overline{g} \circ \pi$ . We sometimes say that the map g factors through  $X/\sim$ .

3. Let X be a topological space, S a set, and  $g: X \to S$  a surjective map. Give S the quotient topology. Let C be a subset of S. Prove that C is closed in S if and only if  $g^{-1}(C)$  is closed in X.