

Homework 9: Functions of a Random Variable

1. §3.8, #1, 8
2. Suppose the velocity of a gas molecule of mass m is a random variable V with pdf $f_V(v) = av^2e^{-bv^2}$, $v \geq 0$, where a and b are positive constants depending on the gas. Find the pdf of the kinetic energy $E = \frac{1}{2}mV^2$ of such a molecule.
3. Suppose that X is uniform on $[-1, 2]$. Find the pdf of X^2 .
4. (Bonus) Suppose that a particle is fired from the origin in the (x, y) plane in a random angle θ to the x -axis. Let Y be the y -coordinate of the place where the particle hits the line $x = 1$. Show that if θ is uniform on $(-\pi/2, \pi/2)$, then

$$f_Y(y) = \frac{1}{\pi(1 + y^2)}$$

This is called the *Cauchy distribution*.

5. (Putnam Exam) A dart, thrown at random, hits a square target. Assuming that any two parts of the target of equal area are equally likely to be hit, find the probability that the point hit is nearer to the center than to any edge. Express your answer in the form $(a\sqrt{b} + c)/d$, where a , b , c , and d are positive integers.