Homework 22: Sample Means and the Central Limit Theorem

1. On average, Supercell Premium AAA batteries last 60 days with a standard deviation of 3 days. Suppose that 25 of these batteries are bought. Find the probability that the average lifespan of the 25 batteries is greater than 61 days.

2. A random sample $X_1, \ldots, X_{15}$ is drawn from a random variable X with pdf $f_X(x) = 3(1-x)^2$, $0 \le x \le 1$. Let $\widetilde{X} = \left(\frac{1}{15}\right) \sum_{i=1}^{15} X_i$.

   (a) Find $P(\frac{1}{8} \le X \le \frac{7}{8})$

   (b) Use the Central Limit Theorem to approximate $P(\frac{1}{8} \le \widetilde{X} \le \frac{7}{8})$.

3. On any given day 1000 cars are expected to cross a certain intersection in Kenosha. Find the probability that over a 25 day period, an average of at least 1010 cars per day cross the intersection.

   You should assume that the number of cars crossing the intersection is Poisson distributed. For the variance of a Poisson distribution, see page 290 of the textbook.