Homework 9: Metric Spaces
(Due April 10, 2023)

*Assignments should be **stapled** and written clearly and legibly. Problems 4 and 5 are optional.*

1. Let X be a nonempty set and let d be the discrete metric on X. Prove that every subset of X is both open and closed.

2. Consider $\mathbb{R}$ with the discrete metric. Prove that $E = [0, 1]$ is closed and bounded in $\mathbb{R}$, but not compact. (Note that closed, bounded, and compact are in reference to the discrete metric.)

3. In Homework 7, you proved that if A is a set of real numbers, then $(\overline{A})' = A'$ (using the Euclidean metric on $\mathbb{R}$). Use this fact and a theorem from class to prove that $\overline{A}$ is closed.

4. (GRE Mathematics Subject Test. This question was answered correctly by 19% of examinees.) Let d be a metric on a set X. Which of the following is also a metric on X?

   (a) $4 + d$

   (b) $e^d - 1$

   (c) $d - |d|$

   (d) d^2

   (e) $\sqrt{d}$

5. Consider $\mathbb{Q}$, viewed as a metric subspace of $\mathbb{R}$ with Euclidean metric. Let $E = \{p \in \mathbb{Q} : 2 < p^2 < 3\}$. Prove that in this metric subspace, E is closed and bounded, but not compact.