Homework 4: Limits of Sequences
(Due 2/21/22)

Assignments should be ***stapled*** *and written clearly and legibly.*

1. §4.1, #6(c).

2. Using only the definition of a limit, prove

   (a) $\lim_{n\to\infty} \frac{2n-2}{5n+3} = \frac{2}{5}$

   (b) $\lim_{n\to\infty} \left(5 - \frac{1}{\sqrt{n+\sqrt{n}+12}}\right) = 5$

3. Consider the following definition:

   Definition: A sequence (a_n) is said to ***reverge*** *to L if there exists $\epsilon > 0$ such that for every $N \in \mathbb{N}$, whenever $n \geq N$, we have $|a_n - L| < \epsilon$.*

   (a) Give an example of a sequence that reverges.

   (b) If a sequence reverges, must it also converge? If not, give a counterexample.

   (c) Is it possible for a sequence to reverge to two different values?

   (d) (optional) If possible, give a simpler definition of revergence.

4. Does the sequence (a_n) defined by

$$a_n = \begin{cases} 2, & \text{if } n = 2^m \text{ for some } m \in \mathbb{N} \\ 1 + \frac{1}{n}, & \text{otherwise} \end{cases}$$

   converge? Justify your answer. (A formal proof is not required.)

5. Suppose that a sequence (a_n) converges to 0, and that at least one term a_n of the sequence is greater than 0. Prove that the set $\{a_n\}$ has a maximum.