Homework 8: Limits of Functions

Directions. Do not use the Sequential Criterion for Functional Limits (Theorem 5.1.8) for any of these problems.

1. §5.1, #7(a), #13 (the Squeeze Theorem).

2. For $\lim\limits_{x \to 2}(x^2 + 2x + 3) = 11$, illustrate the $\epsilon - \delta$ definition of a limit by finding values of δ that correspond to $\epsilon = 1$ and $\epsilon = 0.1$.

3. Determine the following limits, and then use the $\epsilon - \delta$ definition of limit to prove your answers.

   (a) $\lim\limits_{x \to 3} \dfrac{x^2 + 2x - 15}{x - 3}$

   (b) $\lim\limits_{x \to 2} x^4$

4. Use the Squeeze Theorem to prove that $\lim\limits_{x \to 0} \left(x \sin \left(\dfrac{1}{x} \right) \right) = 0$. Make sure to state what D is.

5. Let $f : D \to \mathbb{R}$ and let a be a limit point of D. Suppose that $\lim\limits_{x \to a} f(x) > 0$. Prove that there exists a deleted neighborhood $N_\delta^*(a)$ of a such that $f(x) > 0$ for all $x \in N_\delta^*(a) \cap D$.