Homework 3: Combinatorics and Probability (Part 2)

1. §1.8, #7, 11

2. §1.9, #9

3. Alice buys 5 soup cans and Ben buys 4 soup cans from a shelf with 10 dented and 15 nondented cans. Find the probabilities that (a) Alice gets 2 dented and Ben gets 3 dented cans; (b) Alice gets 3 dented cans.

4. You are dealt 5 cards one by one from a standard deck. After picking up the first two cards, you see that one is the ace of hearts and the other is the seven of hearts. Knowing this, find the probability that after you pick up the remaining three cards you will have (a) a flush in hearts; (b) 3 aces and 2 sevens; (c) a full house; (d) a pair of aces and a pair of sevens.

5. Find the number of rearrangements of the word MISSISSIPPI such that (a) the four S's are next to eachother; (b) no two S's are next to eachother; (c) the leftmost letter is an S.

6. Five people, named A, B, C, D, and H. Wolfeschlegelsteinhausenbergerdorff, line up for a photograph. Find the number of lineups with (a) A second from the right; (b) B at either end; (c) A next to B but to the right of B; (d) A next to B; (e) H. Wolfeschlegel-steinhausenbergerdorff in the middle.

7. In how many ways can 8 people be seated in a row if

    (a) there are no restrictions on the seating arrangement?

    (b) Han and Luke must be seated next to eachother?

    (c) there are 4 men and 4 women and no 2 men or 2 women can sit next to each other?

    (d) there are 5 men and they must sit next to eachother?

    (e) there are 4 married couples and each couple must sit together?

8. Consider a grid of points in space at locations (i, j, k), where i, j, k are integers such that $1 \leq i \leq 6$, $1 \leq j \leq 8$, $1 \leq k \leq 9$. A *crossing* in this grid is defined to be a path made of line segments moving from $(1, 1, 1)$ to $(6, 8, 9)$, such that from each grid point the path can move only to another grid point one unit away from it in the positive x, y, or z directions. How many crossings are there? What is the probability that a crossing misses the point $(3, 4, 5)$?