Homework 16: The Mean Value Theorem (Due May 4, 2020)

1. §6.2, #16.

2. Find a twice differentiable function $f(x)$ such that $f'(1) = -1$, $f'(4) = 7$, and $f''(x) > 3$ for all x, or prove that such a function cannot exist.

3. Suppose that $f : [a, b] \to \mathbb{R}$ has continuous derivatives f' and f'', and assume there exists $c \in (a, b)$ such that $f(a) = f(c) = f(b)$. Prove that there exists d in (a, b) such that $f''(d) = 0$.

   Hint. First prove that there exist x_1 and x_2 such that $a < x_1 < x_2 < b$ and $f'(x_1) = f'(x_2) = 0$.

4. Recall that a number c is called a **fixed point** of a function f if $f(c) = c$. Prove that if f is a differentiable function and $f'(x) \neq 1$ for all real numbers x, then f has at most one fixed point. (Hint: Consider the function $g(x) = f(x) - x$.)