Homework 1: Ordered Sets, Infimums, Supremums (Due 2/6/2023)

Assignments should be ***stapled*** *and written clearly and legibly. Problems 4 and 5 are optional.*

1. §3.2, #10, 12(b).

2. §3.3, #3(a), (d), (f), (h), 5, 8.

3. Suppose that A and B are two nonempty sets of real numbers such that $x \leq y$ for all x in A and y in B. In this problem you will prove that $\sup A \leq \inf B$.

   (a) Explain how we know that A is bounded above and B is bounded below.

   (b) Explain how we know that both $\sup A$ and $\inf B$ must exist.

   (c) **Prove** that $\sup A \leq y$ for all $y \in B$.

   (d) Use part (c) and the definition of $\inf B$ to **prove** that $\sup A \leq \inf B$.

   (e) Can one say that $\max A \leq \min B$? Justify your answer.

4. (Bonus) Using the axioms of an ordered field, prove the arithmetic-geometric mean inequality: For any $a, b \in \mathbb{R}$ with $a > 0$ and $b > 0$,

$$\sqrt{ab} \leq \frac{a+b}{2}$$

   You may assume the existence of square roots.

5. (Open Question[1]) Is $e + \pi$ rational?

[1] An open question is a question which has not been answered. No one knows its answer.