Homework 9: The Quotient Topology
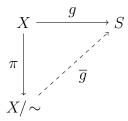
*Assignments should be **stapled** and written clearly and legibly.*

1. §3.3, #3.23, 3.24, 3.27, 3.33(a)(b)(c)(h)(j)(k)

2. Let X and S be sets, and let $g : X \to S$ be a surjective map. Define a relation $\sim$ on X by $x \sim y$ if $g(x) = g(y)$.

   (a) Verify that $\sim$ is an equivalence relation.

   (b) What are the equivalence classes of $\sim$?

   (c) Prove that there exists a bijection $\bar{g} : (X/\sim) \to S$.

   (d) The map $\bar{g}$ appears as a dashed line in the diagram below, where π is the standard projection:

$$
\begin{array}{ccc}
X & \xrightarrow{\ \ g\ \ } & S \\
{\scriptstyle \pi}\downarrow & \nearrow{\scriptstyle \bar{g}} & \\
X/\sim & &
\end{array}
$$

   Prove that this diagram commutes, i.e., prove that $\bar{g} \circ \pi = g$. We sometimes say that *the map g factors through $X/\sim$*.

3. Let X be a topological space, S a set, and $g : X \to S$ a surjective map. Give S the quotient topology. Let C be a subset of S. Prove that C is closed in S if and only if $g^{-1}(C)$ is closed in X.