Homework 7: Linear Independence, Basis

*Assignments should be **stapled** and written clearly and legibly. All answers must be justified. Problem 6 is optional.*

1. §4.4, #13(b), 14(b), 27(a)(b).

2. Determine whether $\{1, \sin x, \cos x\}$ is linearly independent in $F(-\infty, \infty)$. Justify your answer.

3. Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be vectors which span a vector space V, and let $\mathbf{u}$ be a vector in V. Prove that $\operatorname{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n, \mathbf{u}\} = V$.

4. Suppose that $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a linearly idependent set of vectors in a vector space V. Prove that
   (a) $\operatorname{Span} \mathcal{B}$ is a subspace of V.
   (b) $\mathcal{B}$ is a basis for $\operatorname{Span} \mathcal{B}$.

5. Use Problem 4 to show that in the vector space $F(-\infty, \infty)$, $\{e^x, e^{2x}\}$ is a basis for $\operatorname{Span}\{e^x, e^{2x}\}$. (Note that you must first show that $\{e^x, e^{2x}\}$ is linearly independent.)

6. Let $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a linearly independent set of vectors in a vector space V. Using the definition of linear independence (and no theorems on linear dependence/independence), prove that if $\mathbf{v}$ is a vector in V which is not in $\operatorname{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$, then $\{\mathbf{v}_1, \ldots, \mathbf{v}_n, \mathbf{v}\}$ is still linearly independent.