Math 221: Homework Exercises

1 The Definition of a Derivative

1. For each of the following functions, use limits to find (i) the slope of the tangent line at $x = 4$, and (ii) the equation of the tangent line at $x = 4$.
   (a) $f(x) = 5x^2 - 2x$.
   (b) $g(x) = \dfrac{3}{x}$.
   (c) $h(x) = 3\sqrt{x}$.
2. Use limits to find $f'(x)$ for (a) $f(x) = 1/x$, (b) $f(x) = \sqrt{x}$.
3. An object is dropped from a tall building on planet Krypton. The distance the object falls in t seconds is $s(t) = 12t^2$ m.
   (a) Use limits to find $s'(t)$.
   (b) Find the object's velocity after 2 seconds.
   (c) How much time is required for the object's velocity to reach 36 m/s?
   (d) When the object's velocity is 36 m/s, how far has the object fallen?
   (e) If the height of the building is 300 m, what will be the object's velocity when it hits the ground?
4. Let $f(x) = 3x^2 + 2x$.
   (a) Use limits to find $f'(x)$.
   (b) Find the equation of the tangent line to the graph of $f(x)$ at $x = 1$.
   (c) Find the point on the graph of $f(x)$ where the tangent line has slope 20.
   (d) Find the equation of the tangent line to the graph of $f(x)$ at the point on the graph where the tangent line has slope 20.