Homework 16: The Gram-Schmidt Orthogonalization Process

1. §6.3, #37, 43.

2. §2.1, #15, 22, 30.

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by reflecting across the plane $x_1 - 2x_2 + 2x_3 = 0$. The goal of this problem is to find the standard matrix for T.

   (a) Find an orthogonal basis $B = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$ such that $\mathbf{v}_1, \mathbf{v}_2$ span the plane. There are several ways to do this. Here is one:

      1. Find a vector $\mathbf{v}_3$ which is orthogonal to all vectors in the plane. (This was covered in Math 223. No calculations are required.)
      2. Find two vectors $\mathbf{u}_1, \mathbf{u}_2$ which span the plane.
      3. Use the Gram-Schmidt process to replace $\mathbf{u}_1, \mathbf{u}_2$ by two orthogonal vectors $\mathbf{v}_1, \mathbf{v}_2$ which span the plane.

   (b) Find $[T]_B$.

   (c) Use the change of basis formula to find $[T]_{B'}$, where B' is the standard basis for $\mathbb{R}^3$.

   (d) Find the reflection of $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ across the plane.

Answer to 3(c): $[T]_{B'} = \dfrac{1}{9}\begin{bmatrix} 7 & 4 & -4 \\ 4 & 1 & 8 \\ -4 & 8 & 1 \end{bmatrix}$