

## Homework 7: Linear Independence, Basis

Assignments should be **stapled** and written clearly and legibly. All answers must be justified. For proofs, you must use complete sentences and grammatically correct English.

1. §4.4, #13(b), 14(b), 27(a)(b).
2. Determine whether  $\{1, \sin x, \cos x\}$  is linearly independent in  $F(-\infty, \infty)$ . Justify your answer.
3. Let  $\mathbf{v}_1, \dots, \mathbf{v}_n$  be vectors which span a vector space  $V$ , and let  $\mathbf{u}$  be a vector in  $V$ . Prove that  $\text{Span}\{\mathbf{v}_1, \dots, \mathbf{v}_n, \mathbf{u}\} = V$ .
4. Let  $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$  be a linearly independent set of vectors in a vector space  $V$ . Using the definition of linear independence (and no theorems on linear dependence/independence), prove that if  $\mathbf{v}$  is a vector in  $V$  which is not in  $\text{Span}\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ , then  $\{\mathbf{v}_1, \dots, \mathbf{v}_n, \mathbf{v}\}$  is still linearly independent.