Homework 11: One-to-One, Onto

*Assignments should be **stapled** and written clearly and legibly.*

1. §8.2, #1(c), 6, 7, 19(a)(c), 26. Make sure to justify your answer to the question asked in Exercise 7.

2. Let $T : V \to W$ be linear transformation, and let $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ be linearly independent in V.
   (a) Prove that if T is one-to-one, then $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ is linearly independent in W.
   Hint. Begin the proof as follows:
   "Suppose $c_1T(\mathbf{v}_1) + \cdots + c_pT(\mathbf{v}_p) = \mathbf{0}$. I must show that $c_1 = \cdots = c_p = 0$."
   (b) Give an example showing that if T is not one-to-one, then $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ need not be linearly independent in W.

3. Let $T : V \to W$ be a linear transformation, with $\dim V = n$, $\dim W = m$. Prove the following:
   (a) $\dim(R(T)) \leq n$.
   (b) $\dim(R(T)) = n$ if and only if T is one-to-one.
   (c) $\dim(R(T)) = m$ if and only if T is onto.