# Homework 15: Orthogonal Bases

1. §6.2, #42.
2. §6.3, #10, 14, 38, 42(b).

   *Note. For problem 42(b), you should use the fact that the Legendre polynomials are orthogonal with respect to the inner product given in the first sentence of Example 9.*

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be rotation by $\theta$ about the line through $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (counterclockwise, as viewed from the tip of $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$). Let $\mathcal{B}$ be the the following basis of $\mathbb{R}^3$:

   $$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \right\}.$$

   Let $\mathcal{B}' = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, the standard basis for $\mathbb{R}^3$.

   (a) Confirm that $\mathcal{B}$ is an orthonormal basis for $\mathbb{R}^3$.

   (b) Find $[T]_{\mathcal{B}}$.

   (c) Use the change of basis formula to find $[T]_{\mathcal{B}'}$, the standard matrix for $T$.

   (d) Using (c), find the rotation of $\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ by $\pi/3$ about the line through $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

   *Note. For this problem, you should use the Euclidean inner product (i.e., the dot product) on $\mathbb{R}^3$.*