Vidyavardhini's College of Engineering and Technology, Vasai (West)

First Year Engineering

Academic Year: 2024–2025 (NEP Syllabus)

Applied Physics (PYQ)

Course Outcomes

CO1: Illustrate the use of laser in LiDAR and Barcode reading

CO2: Apply the foundation of fiber optics in the development of modern communication technology

CO3: Determine the wavelength of light and refractive index of liquid using the interference phenomenon

CO4: Illustrate the significance of Maxwell's equations in the field of modern technology

CO5: Apply the foundations of quantum mechanics for the development of modern technology

CO6: Explain the types of semiconductors based on variations in Fermi level with temperature and doping concentration

In physics, you don't have to go around making trouble for yourself, nature does it for you. Nothing in life is to be feared, it is only to be understood. Now is the time to understand more, so that we may fear less. – Frank Wilczek & Allan Sandage

Module: 1 LASER

Q.1 Explain the quantum processes? (3m)

OR

Q.2 Differentiate between spontaneous and stimulated emission. (3m)

1.2.1 Absorption

- An atom in lower energy state E_1 may absorb the incident photon and may be excited to E_2 as shown in Fig. 1.2.1. This transition is known as stimulated absorption corresponding to each transition made by an atom one photon disappears from the incident beam.

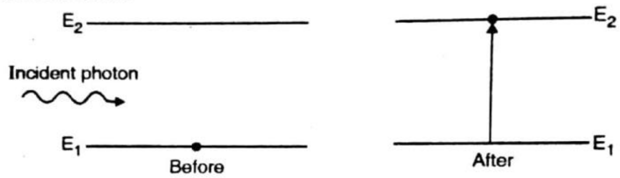


Fig. 1.2.1 : Induced absorption

The transition may be written as

$$A + h\nu = A^* \qquad ...(1.2.1)$$

Where A = Atom in lower energy state

A^* = Atom in excited state

The number of atoms N_{ab} excited during the time Δt is given by

$$N_{ab} = B_{12} N_1 Q \Delta t \qquad ...(1.2.2)$$

Where N_1 = Number of atoms in state E_1

Q = Energy density of the incident beam

B_{12} = Probability of an absorption transition.

[redacted] Spontaneous Emission

- Excited state with higher energy is inherently unstable because of a natural tendency of atoms to seek out lowest energy configuration. Therefore excited atoms do not stay in the excited state for a relatively longer time but tend to return to the lower state by giving up the excess energy $h\nu = E_2 - E_1$ in the form of spontaneous emission or stimulated emission.
- The excited atom in the state E_2 may return to the lower state E_1 on its own out of natural tendency to attain the minimum potential energy condition.
- During the transition the excess energy is released as a photon of energy $h\nu = E_2 - E_1$. This type of process in which photon emission occurs without any external agency is called **spontaneous** or **naturalemission.** Fig. 1.2.2 represents natural emission and shows the transition.

$$A^* \rightarrow A + h\nu \qquad ...(1.2.3)$$

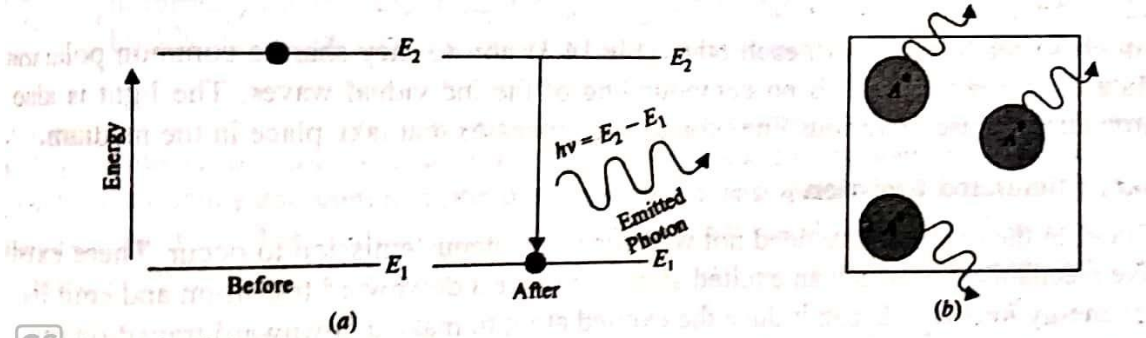
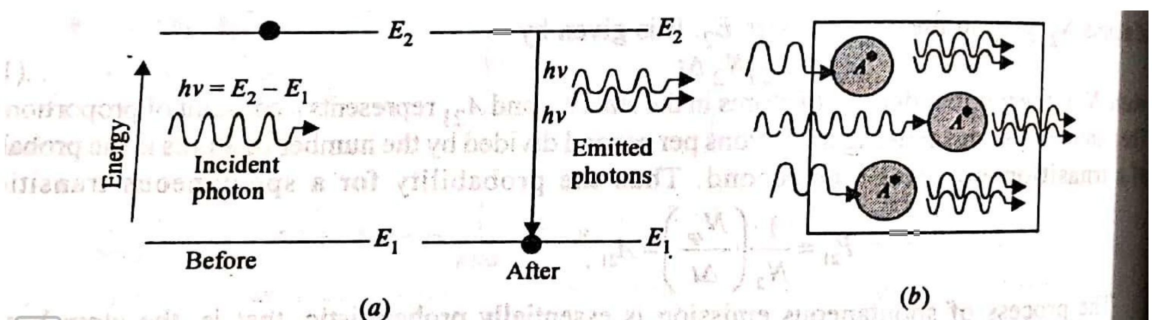


FIG. 14.2. Spontaneous emission (*a*) Emission process (*b*) Material emits photons haphazardly.

Stimulated Emission:

- An atom in excited state need not wait for spontaneous emission to occur. There exists an additional possibility according to which an excited atom can make a downward transition and emit a radiation.
- A photon of energy $h\nu = E_2 - E_1$ can induce the excited atom to make a downward transition releasing the energy in the form of a photon.
- Thus the interaction of a photon with an excited atom triggers the excited atom to drop to the lower energy state giving up a photon.
- This phenomenon is called **forcedemission** or **stimulatedemission** as shown in Fig. 1.2.3. The process may be represented as

$$A^* + h\nu = A + 2h\nu \quad \text{...(1.2.5)}$$



Stimulated emission. (*a*) Emission process. (*b*) Material emits photons in a coordinated manner.

Q.3 What is a population inversion state? Explain its significance in the operation of LASER.(3m)

OR

Explain the population inversion.(3m)

Ans:

1) A **population inversion state** refers to a condition in a system where more particles (atoms or molecules) occupy an excited energy state than the lower energy state.
2) This is the opposite of thermal equilibrium, where the majority of particles would normally reside in the lower-energy state at equilibrium conditions.
3) In a typical system without external intervention, particles tend to occupy lower energy states (ground states or nearby low-energy states) due to Boltzmann distribution.
4) A population inversion is necessary for achieving certain quantum processes such as **stimulated emission**, which is the key mechanism in the operation of a LASER.

Significance in LASER operation:

1) **Stimulated emission** occurs when an incident photon causes an excited atom or molecule to drop to a lower energy state, releasing another photon of the same frequency, phase, polarization, and direction as the incident photon. For stimulated emission to dominate over absorption, a system must have more particles in the excited state than in the lower energy state. This is achieved by creating a **population inversion**.
2) In a laser, the process of creating a population inversion ensures that stimulated emission can outcompete absorption, allowing light to build up and form a coherent beam of photons in a controlled manner.
1) Without a population inversion, the rate of absorption would prevent a chain reaction of stimulated emissions, and the system would not produce coherent, amplified light.

Q.4 What is an optical resonator? What is it's role in lasing? (3m)

OR

What is resonant cavity? Explain its use in the generation of laser beam.(3m)

Ans:

1) It is a structure that confines light through multiple reflections between two or more mirrors to enable the amplification of light through **stimulated emission**.

2) The optical resonator allows the photons emitted by stimulated emission to build up in number by repeatedly passing through the gain medium, thereby producing the coherent, monochromatic light characteristic of a laser.

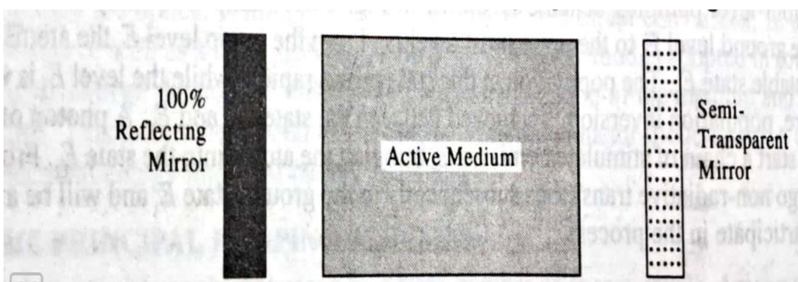


Fig. 14.10. Fabry-Perot optical resonator.

a) A photon emitted by stimulated emission travels through the gain medium and reflects between the mirrors.

b) Each time the photon passes through the gain medium, it stimulates additional emissions from excited atoms or molecules, amplifying its intensity.

c) The repeated reflections create a standing wave condition in the optical resonator. Standing waves are patterns formed by the superposition of traveling waves reflecting back and forth between the mirrors.

d) When the light reaches a sufficient intensity, a fraction of it exits through the partially reflective mirror as the **laser output beam**.

Types of optical resonators:

a) Fabry-Pérot Cavity

b) Spherical or Confocal Resonators

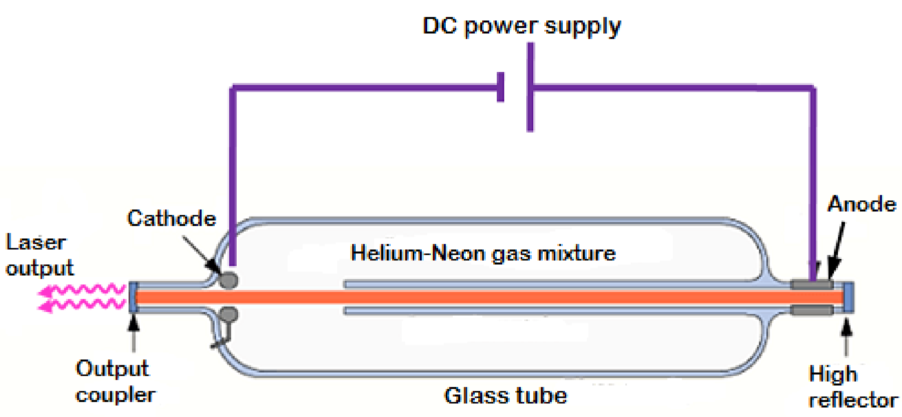
c) Ring Resonators

Q.5 With a neat energy level diagram describe the construction and working of He-Ne laser.(5m)

Ans:

The **Helium-Neon (He-Ne) laser** is one of the most well-known types of gas lasers. It operates on the principle of **stimulated emission of radiation** and produces a coherent, monochromatic light, typically at a wavelength of **632.8 nm**, which is in the red part of the visible spectrum.

Construction:



The main components of a typical He-Ne laser are:

a) **Gas Discharge Tube**:
   - A sealed glass tube containing a mixture of **helium (He)** and **neon (Ne)** gases.
   - The usual mixture is around **80% He and 20% Ne** by pressure.

b) **Electrodes**:
   - Electrodes are attached at either end of the gas discharge tube to apply an electric field.
   - The electric field ionizes the gas and excites helium atoms to a higher energy state.

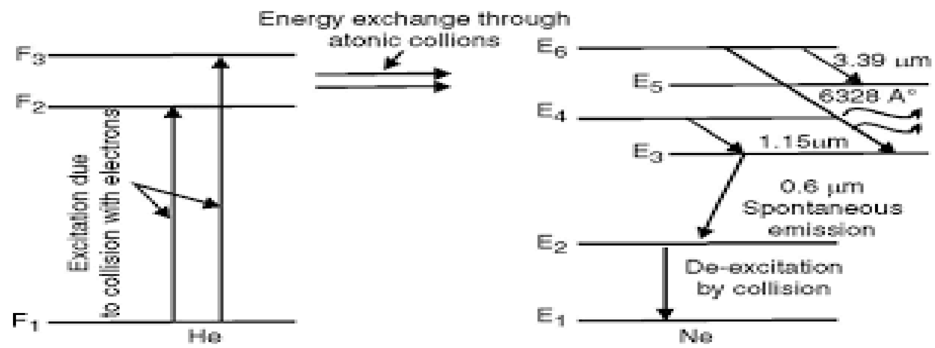
c) **Optical Resonator (Cavity)**:
   - Two mirrors are placed at the ends of the gas discharge tube.
      - One is a **highly reflective mirror**, and the other is a **partially reflective mirror**.

- The optical cavity allows photons to bounce back and forth through the neon gas multiple times, passing through the gain medium and undergoing stimulated emission.

d) **Power Supply**:

- A power supply is connected to create an electrical discharge through the gas tube, which excites the helium atoms.

Energy Level Diagram and Working:



a) The power supply generates an electric discharge that excites helium atoms to a higher energy state.

b) Excited helium atoms collide with neon atoms in the gas mixture, transferring their excitation energy to the neon atoms.

c) This process creates a population inversion in the neon gas because more neon atoms are excited to a higher energy state (E2E_2E2) than are in the ground state (E1E_1E1).

d) A photon passing through the neon gas can stimulate excited neon atoms to drop to the lower energy state (E1E_1E1), releasing photons of wavelength **632.8 nm**.

e) The optical resonator (formed by two mirrors) allows photons to reflect back and forth through the gain medium, amplifying the number of photons by repeated stimulated emissions.

f) A portion of the amplified photons exits through the partially reflective mirror as a coherent and monochromatic laser beam.

The **He-Ne laser** works by exciting helium atoms using electrical discharge, which transfer energy to neon atoms. The excited neon atoms emit light at **632.8 nm** through stimulated emission facilitated by a resonator (optical cavity). The **population inversion**, optical feedback, and repeated stimulated emission in the optical cavity lead to the production of a stable, coherent laser beam.

Q.6 Explain application of LASER in industry and medical field. Discuss any one of them in detail. (5m)

Ans:

Lasers have found a wide range of applications across various industries and the medical field due to their unique properties such as coherence, monochromaticity, and directionality.

(1) For welding and melting

- In very short time metal can be melted and then it will be evaporated. Thus, accurate welding and melting of the hard material can be done very easily. Using laser, perfect and non-porous joints of metals are possible. The typical laser welding machine is as shown in the Fig. 1.14.3.
- Due to increased power output it is possible to use this as a welding tool. Generally, CO_2 gas laser is used as cutting tool. Thus due to laser welding very high welding rate is possible.
- It is possible to weld dissimilar metals. Complex counters can be welded using easy turning of laser beam.
- Working material is not stressed because of non-contact method.

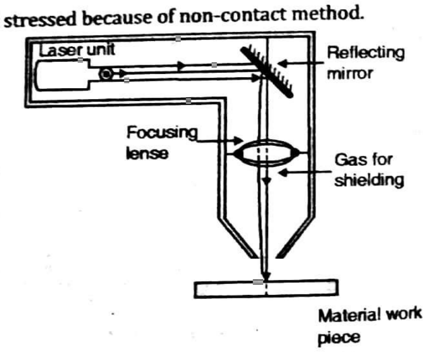


Fig. 1.14.3 : Laser welding machine

Laser application in medical field:

Among the various applications, **Laser Vision Correction**, specifically **LASIK (Laser-Assisted in Situ Keratomileusis)**, stands out as one of the most impactful uses of laser technology in the medical field.

a) LASIK is a surgical procedure that corrects refractive vision problems like **myopia (nearsightedness)**, **hyperopia (farsightedness)**, and **astigmatism**. It uses a specialized excimer laser to reshape the cornea of the eye, allowing light entering the eye to focus correctly on the retina.
b) In refractive vision problems, the shape of the cornea (the transparent front part of the eye) does not properly bend light to focus on the retina.
c) LASIK aims to reshape the cornea so that light focuses correctly.
d) A surgical instrument (microkeratome or femtosecond laser) is used to create a thin, hinged flap in the cornea.

e) The excimer laser is used to ablate (remove) a small amount of corneal tissue in a precise, controlled manner to reshape the cornea.

f) The corneal flap is placed back into its original position to heal naturally.

g) The **excimer laser** is a type of ultraviolet laser that provides high precision and can be programmed to remove very exact amounts of tissue from the cornea.

Module 02: FIBRE OPTICS

Short answer questions (**3 Marks each**)

1. An optical fibre refractive index 1.48 and 1.41 respectively of core ,clad Calculate i) Critical angle ii) Numerical Aperture iii) Maximum Incidence angle

Ans.

n1 = 1.48

n2 = 1.41

(ii) Numerical Aperture (NA)

The Numerical Aperture is calculated using the formula:

$$NA = \sqrt{n_1^2 - n_2^2}$$

Substitute the values:

$$NA = \sqrt{1.48^2 - 1.41^2}$$

$$NA = \sqrt{2.1904 - 1.9881}$$

$$NA = \sqrt{0.2023}$$

$$NA \approx 0.45$$

(i) Critical Angle (θ_c)

The critical angle is calculated using the formula:

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Substitute the values:

$$\theta_c = \sin^{-1}\left(\frac{1.41}{1.48}\right)$$

$$\theta_c = \sin^{-1}(0.953)$$

$$\theta_c \approx 72.57^\circ$$

(iii) Maximum Incidence Angle (θ_{max})

The maximum incidence angle is related to the Numerical Aperture and can be calculated as:

$$\sin\theta_{\text{max}} = NA$$

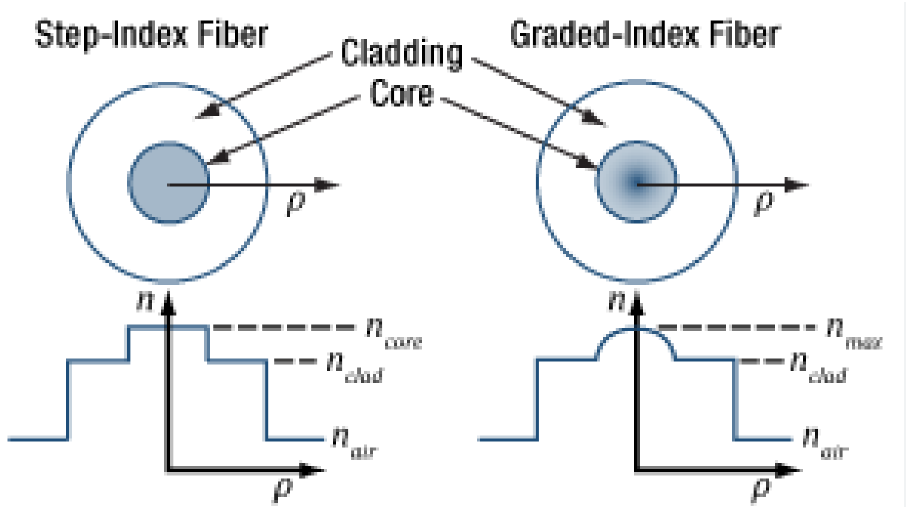
$$\theta_{\text{max}} = \sin^{-1}(NA)$$

Substitute $NA = 0.45$:

$$\theta_{\text{max}} = \sin^{-1}(0.45)$$

$$\theta_{\text{max}} \approx 26.57^{\circ}$$

2. Distinguish between Step Index and Graded index Optical fibre. (3M)



Step index fiber	Graded index fiber
1. In step index fibers the refractive index of the core medium is uniform through and undergoes an abrupt change at the interface of core and cladding.	1. In graded index fibers, the refractive index of the core medium is varying in the parabolic manner such that the maximum refractive index is present at the center of the core.
2. The transmitted optical signal will cross the fiber axis during every reflection at the core cladding boundary.	2. The transmitted optical signal will never cross the fiber axis at any time.
3. The shape of propagation of the optical signal is in zigzag manner.	3. The shape of propagation of the optical signal appears in the helical or spiral manner or sinusoidal.
4. Attenuation is more for multimode step index fibers but Attenuation is less in single mode step index fibers	4. Attenuation is very less in graded index fibers
5. Numerical aperture is more for multimode step index fibers but it is less in single mode step index fibers	5. Numerical aperture is less in graded index fibers
6. It has both single and multimode.	6. It is only multimode.
7. Intermodal dispersion is present.	7. Intermodal dispersion is absent.
8. It is less expensive. The manufacturing is easy	8. The manufacturing is complex hence it is more expensive.

Long answer questions (**5 Marks each**)

1. Derive an expression for the angle of acceptance and then numerical aperture for a step index optical fibre.

Answer:

10.4.2 Acceptance Angle

Let us again consider a step index optical fibre into which light is launched at one end, as shown in Fig. 10.8. Let the refractive index of the core be n_1 and the refractive index of the cladding be n_2 ($n_2 < n_1$). Let n_0 be the refractive index of the medium from which light is launched into the fibre. Assume that a light ray enters the fibre at an angle θ_i to the axis of the fibre. The ray refracts at an angle θ_r and strikes the core-cladding interface at an angle ϕ. If ϕ is greater than critical angle ϕ_c, the ray undergoes total internal reflection at the interface, since $n_1 > n_2$. As long as the angle ϕ is greater than ϕ_c, the light will stay within the fibre.

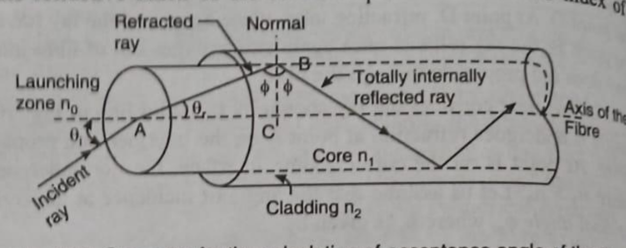


Fig. 10.8: Geometry for the calculation of acceptance angle of the fibre.

Applying Snell's law to the launching face of the fibre, we get

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_0} \tag{10.7}$$

If θ_i is increased beyond a limit, ϕ will drop below the critical value ϕ_c and the ray escapes from the sidewalls of the fibre. The largest value of θ_i occurs when $\phi = \phi_c$.

From the Δ^{le} ABC, it is seen that

$$\sin\theta_r = \sin(90° - \phi) = \cos\phi \tag{10.8}$$

Using equation (10.8) into equation (10.7), we obtain

$$\sin\theta_i = \frac{n_1}{n_0}\cos\phi$$

When $\phi = \phi_c$,

$$\sin\left[\theta_{i_{\max}}\right] = \frac{n_1}{n_0}\cos\phi_c \tag{10.9}$$

But

$$\sin\phi_c = \frac{n_2}{n_1}$$

$$\therefore \quad \cos\phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \tag{10.10}$$

Substituting the expression (10.10) into (10.9), we get

$$\sin[\theta_i(\max)] = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \tag{10.11}$$

Quite often the incident ray is launched from air medium, for which $n_0 = 1$.

Designating $\theta_i(\max) = \theta_0$, equation (10.11) may be simplified to

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad \theta_0 = \sin^{-1}\left[\sqrt{n_1^2 - n_2^2}\right] \tag{10.12}$$

The angle θ_0 is called the **acceptance angle** of the fibre. *Acceptance angle is the maximum angle that a light ray can have relative to the axis of the fibre and propagate down the fibre.*

2. What is the application of optical fibre in communication system?

Ans. 1) Fiber optics are used for short-distance image transmission in medical endoscopes, allowing doctors to view internal body parts without invasive surgery.

2) Optical fibers serve as waveguides to transmit data signals in high-speed communication systems, including telephone and internet networks.

3) In aircraft, ships, and tanks, copper wires are replaced by optical fibers to reduce weight and improve performance, enhancing fuel efficiency and communication reliability.

4) Optical fiber communication system:

An optical fiber communication system mainly consists of the following parts as shown in figure.

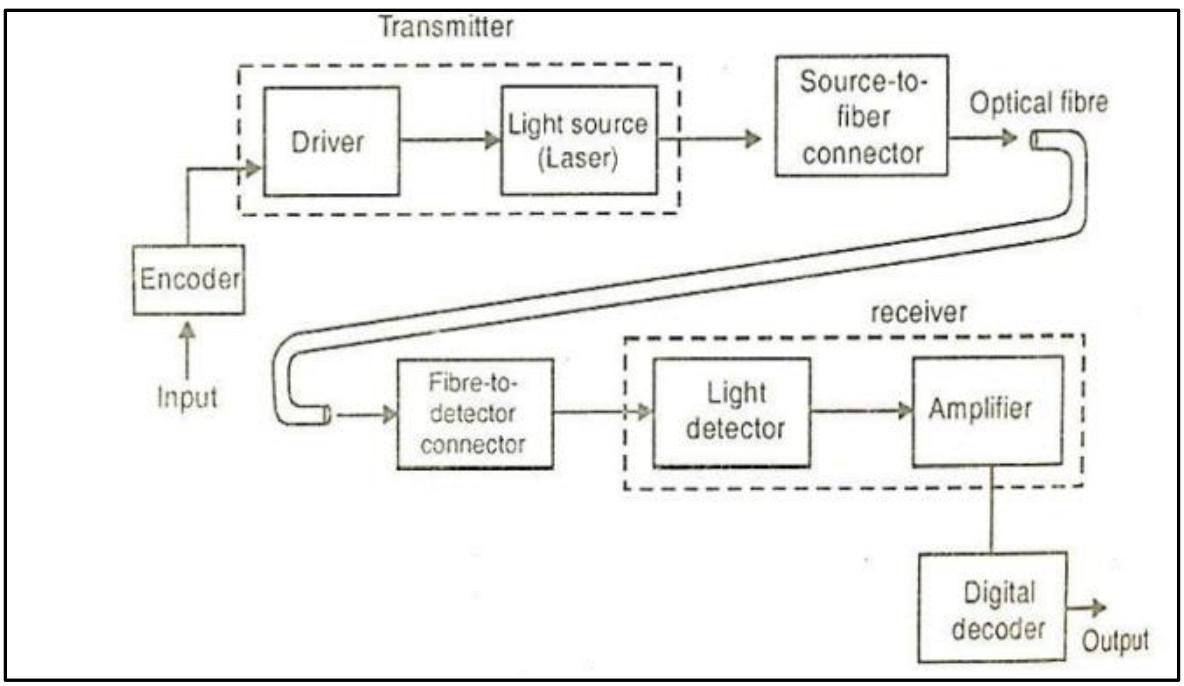
1. Encoder

2. Transmitter

3. Wave guide.

4. Receiver.

5. Decoder



1. **Encoder**: Encoder is an electronic system that converts the analog information like voice, objects etc., into binary data.

2. **Transmitter**: It contain two parts, they are drive circuit and light source. Drive circuit supplies the electric signals to the light source from the encoder in the required form. The light source converts the electrical signals into optical form. With the help of specially made connector optical signals will be injected into wave guide from the transmitter.

3. **Wave guide**: It is an optical fiber which carriers information in the form of optical signals over distances with the help of repeaters. With the help of specially made connector optical signals will be received by the receiver from the wave guide.

4. **Receiver**: It consists of three parts; they are photo detector, amplifier and signal restorer. The photo detector converts the optical signal into the equivalent electric signals and supply to amplifier. The amplifier amplifies the electric signals as they become weak during the long journey through the wave guide over longer distance. The signal restorer deeps the electric signals in a sequential form and supplies to the decoder in the suitable way.

5. **Decoder**: It converts electric signals into the analog information.

3. A step index fiber has a core diameter of 33 x10^{-6} m. the refractive indices of core and cladding are 1.56 And 1.5189 respectively. If the light of wavelength 1.3 µm is transmitted through the fiber, Determine normalized frequency of the fiber. Weather fiber supports single mode or multimode

Ans.

To determine the normalized frequency V of the fiber, we use the formula for the normalized frequency of a step-index optical fiber**:**

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

where:

a= radius of the core of the fiber (in meters),

λ= wavelength of the transmitted light (in meters),

n1 = refractive index of the core,

n2= refractive index of the cladding.

Given data

Core diameter = $33 \times 10^{-6}\,\mathrm{m}$,

Refractive index of core (n_1) = 1.56,

Refractive index of cladding (n_2) = 1.5189,

Wavelength (λ) = 1.3 µm = $1.3 \times 10^{-6}\,\mathrm{m}$.

The radius a is half of the core diameter:

$$a = \frac{33 \times 10^{-6}}{2} = 16.5 \times 10^{-6}\,\mathrm{m}.$$

Calculate the numerical value of $\sqrt{n_1^2 - n_2^2}$

$$\sqrt{n_1^2 - n_2^2} = \sqrt{1.56^2 - 1.5189^2}.$$

$$1.56^2 = 2.4336, \quad 1.5189^2 = 2.3035.$$

$$2.4336 - 2.3035 = 0.1301.$$

$$\sqrt{0.1301} \approx 0.360.$$

Now substitute the values into the formula:

$$V = \frac{2\pi \times 16.5 \times 10^{-6}}{1.3 \times 10^{-6}} \times 0.360.$$

$$V = \frac{2\pi \times 16.5}{1.3} \times 0.360.$$

$$V \approx \frac{103.673}{1.3} \times 0.360 \approx 79.7 \times 0.360 \approx 28.7.$$

The fiber supports **single mode** if the normalized frequency V is less than 2.405, and it supports **multimode** if V is greater than 2.405.

Since V≈28.7, which is much greater than 2.405, the fiber supports **multimode** operation.

Normalized frequency V≈28.7

The fiber supports **multimode**.

Module 03: INTEREFERENCE IN THIN FILM

1 Derive the conditions for the maxima and minima due to interference of light in a parallel thin film under reflected system.

Answer:

Let us consider a transparent film of uniform thickness 't' bounded by two parallel surfaces as shown in Fig.6.13. Let the refractive index of the material be μ. The film is surrounded by air on both the sides. Let us consider plane waves from a monochromatic source falling on the thin film at an angle of incidence 'i'. Part of a ray such as AB is reflected along BC, and part of it is transmitted into the film along BF. The transmitted ray BF makes an angle 'r' with the normal to the surface at the point B. The ray BF is in turn partly reflected back into the film along FD while a major part refracts into the surrounding medium along FK. Part of the reflected ray FD is transmitted at the upper surface and travels along DE. Since the film

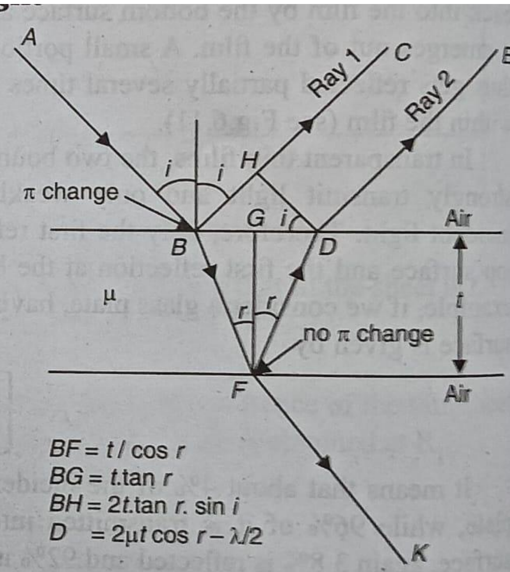


Fig. 6.13

boundaries are parallel, the reflected rays BC and DE will be parallel to each other. The waves travelling along the paths BC and BFDE are derived from a single incident wave AB. Therefore they are coherent and can produce interference if they are made to overlap by a condensing lens or the eye.

(*i*) **Geometrical Path Difference:** Let DH be normal to BC. From points H and D onwards, the rays HC and DE travel equal path. The ray BH travels in air while the ray BD travels in the film of refractive index μ along the path BF and FD. The geometric path difference between the two rays is

$$BF + FD - BH.$$

(*ii*) **Optical Path Difference:**

Optical path difference $\Delta_a = \mu L$

$$\therefore \quad \Delta_a = \mu (BF + FD) - 1(BH) \tag{6.15}$$

In the ΔBFD, $\angle BFG = \angle GFD = \angle r$

$$BF = FD$$

$$BF = \frac{FG}{\cos r} = \frac{t}{\cos r}$$

$$\therefore \quad BF + FD = \frac{2t}{\cos r} \tag{6.16}$$

Also, $BG = GD$

$\therefore$ $BD = 2BG$

$BG = FG \tan r = t \tan r$

$\therefore$ $BD = 2t \tan r$

In the Δ^{le} BHD $\angle HBD = (90 - i)$

$\angle BHD = 90°$

$\therefore$ $\angle BDH = i$

$$\therefore \quad BH = BD \sin i = 2\, t \tan r \sin i \tag{6.17}$$

From Snell's law,

$$\sin i = \mu \sin r$$

$$\therefore \quad BH = 2t \tan r\, (\mu \sin r) = \frac{2\mu t \sin^2 r}{\cos r} \tag{6.18}$$

Using the equations (6.17) and (6.16) into equ.(6.15), we get

$$\Delta_a = \mu\left[\frac{2t}{\cos r}\right] - \left[\frac{2\mu t \sin^2 r}{\cos r}\right]$$

$$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

$$= \frac{2\mu t}{\cos r}\cos^2 r$$

$$\Delta_a = 2\mu\, t \cos r \tag{6.19}$$

(*iii*) **Correction on account of phase change at reflection:** When a ray is reflected at the boundary of a rarer to denser medium, a path-change of $\lambda/2$ occurs for the ray BC (see Fig.6.13). There is no path difference due to transmission at D. Including the change in path difference due to reflection in eqn. (6.19), the true path difference is given by

$$\Delta_t = 2\mu\, t \cos r - \lambda/2 \tag{6.20}$$

Condition for Maxima:

$$2\mu\, t \cos r - \frac{\lambda}{2} = m\lambda$$

$$2\mu\, t \cos r = m\lambda + \lambda/2$$
$$2\mu\, t \cos r = (2m+1)\lambda/2 \qquad \textbf{Condition for Brightness}$$

'm' is an integer and it can take the values from 0, 1, 2, 3….

Condition for Minima:

$$2\mu\, t \cos r - \lambda/2 = (2m+1)\lambda/2$$

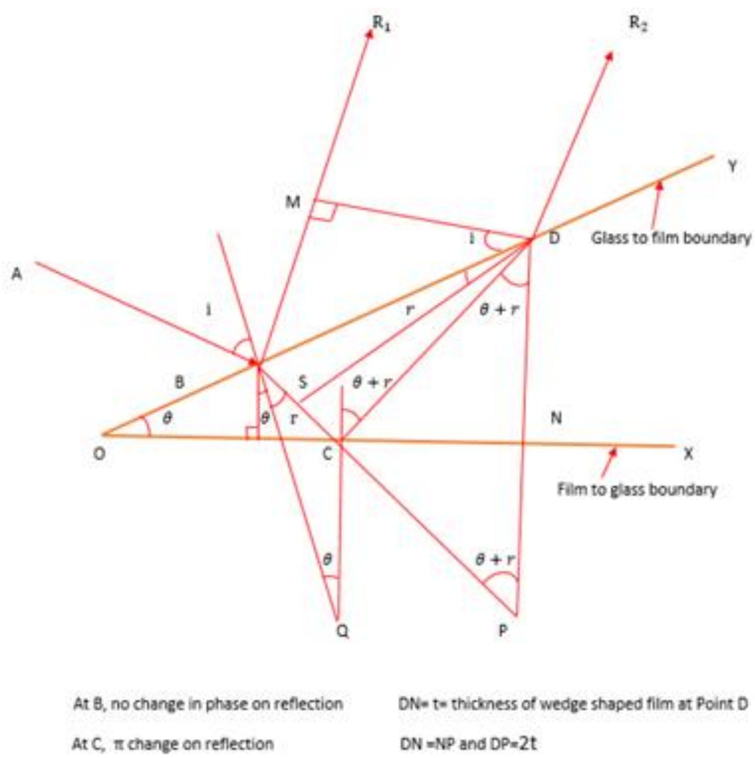
$$2\mu\, t \cos r = (m+1)\lambda$$

OR

$$2\mu\, t \cos r = m\lambda \qquad \textbf{Condition for Darkness}$$

2. Derive the conditions for the maxima and minima due to interference of light in a wedge shaped film**.**

Ans.



When a parallel beam of a monochromatic light of wavelength λ incident the wedge shaped film of refractive index μ, the reflected rays interfere with each other and produce Alternate bright and dark fringes. Fringes are formed on its top surface.

Path difference between the reflected rays from the lower and upper surfaces of the air film varies along its length due to variation in film thickness.

The rays BR_1 and DR_2 reflected from the top surface and bottom surface of the air film. The rays BR_1 and $DR_2$are coherent, derived from the same ray AB (division of amplitude). The rays are very close if the thickness of the film is of the order of wavelength of light. Thickness of the glass plate is large as compared to the wavelength of incident light so that entire pattern is due to air film only.

Optical path difference between BR_1 and DR_2

$$\Delta = 2\mu t \cos(r + \theta) + \frac{\lambda}{2}$$

Where $\frac{\lambda}{2}$ is the abrupt phase change of on reflection from the boundary of air to glass interface at point c.

Condition for maxima or bright fringe (constructive interference):

$$\Delta = n\lambda$$

Or

$$2\mu t\cos(r+\theta)+\frac{\lambda}{2}=n\lambda$$

$$2\mu t\cos(r+\theta)=(n-\frac{1}{2})\lambda$$

Where $n = 1,2,3,4 \ldots\ldots$ etc.

$n = 1 =$ first bright fringe or band

$n = 2 =$ second bright fringe or band

Condition for minima or dark fringe (destructive interference):

$$\Delta=(n+\frac{1}{2})\lambda$$

Or

$$2\mu t\cos(r+\theta)+\frac{\lambda}{2}=(n+\frac{1}{2})\lambda$$

$$2\mu t\cos(r+\theta)=n\lambda$$

Where $n = 0,1,2,3,4 \ldots\ldots$ etc.

$n = 0 =$ Zero order

$n = 1 =$ first dark fringe or band

$n = 2 =$ second dark fringe or band

3. How Newton's rings are formed? Explain the experimental arrangement of Newton's rings

Ans:

Explanation of the formation of Newton's rings

Division of amplitude takes place at the curved surface of the plano convex lens. The incident light is partially reflected and partially transmitted at the curved surface. The transmitted ray is reflected from the glass plate as shown in Fig. 1.4.2. These two rays

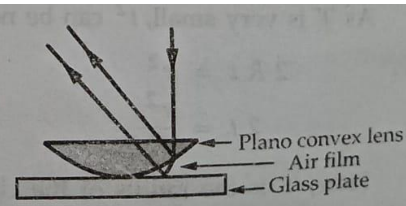


Fig. 1.4.2

interfere in reflected light. The path difference between these rays depends on thickness of the air film enclosed between the curved surfaces of lens and glass plate which increases radially outwards from the centre. The thickness of the air film is zero at the centre.

Shape of fringes : Thickness of air film is zero at the centre and increases radially outwards. The locus of points of constant thickness of air film and hence constant path difference is a circle as seen from the top. Hence interference fringes are circular. They are concentric alternate bright and dark rings.

Path difference : As the radius of curvature of the plano convex lens is large, the air

1.4 Newton's Rings Experiment

May-08,09,10,11,12, Dec.-08,09,10,11

Experimental arrangement : A plano convex lens 'L' of large radius of curvature is placed on a plane glass plate 'G_1', with the curved surface touching the glass plate as shown in Fig. 1.4.1. An air film is enclosed between the curved surface of the lens and the glass plate.

A sodium vapour lamp 'S' is kept at the focus of a biconvex lens 'L_1' which converts the diverging beam of light into a parallel beam. This parallel beam of light is made to fall on a glass plate 'G_2' kept at an angle of 45° with the incident beam. A part of incident light is reflected towards the plano convex lens. This light is again reflected back, partially from the top and partially from the bottom of the air film, and transmitted by the glass plate 'G_2'. The interference of these rays is observed through a microscope 'M'.

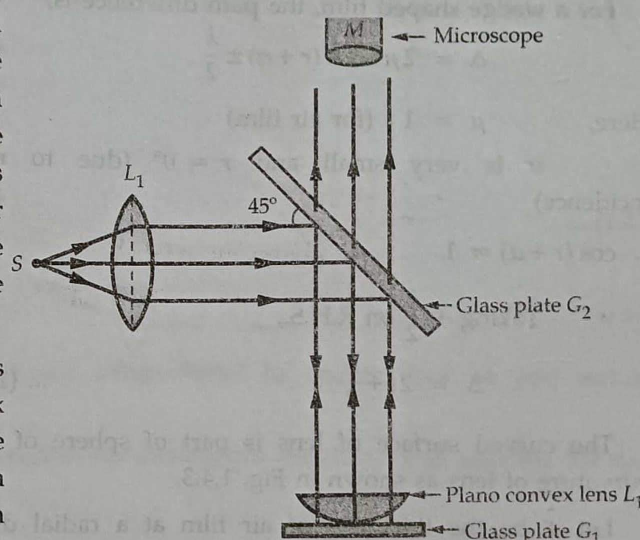


Fig. 1.4.1

3.Describe in detail the concept of anti-reflecting film with a proper ray diagram

Ans.

<u>Antireflection coating</u>

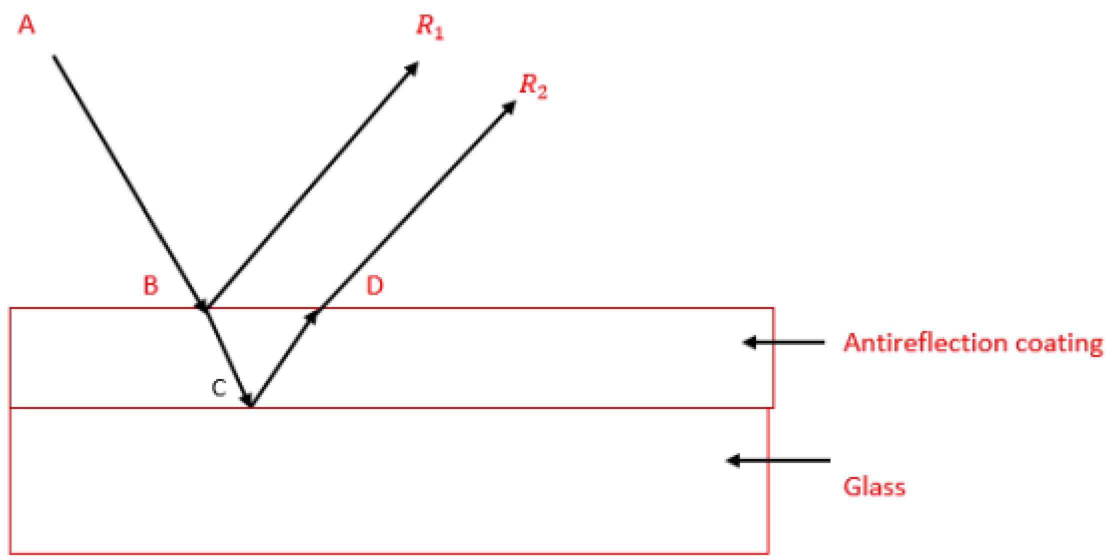
Optical instruments such as cameras and telescopes use multi-component glass lenses. When light incident on glass plate both reflection and refraction occur. When the number of reflections are large, the quality of the image produced by optical device will be poor. The reflection from a surface can be reduced by coating the surface with a thin transparent dielectric film. A transparent thin film coated on a surface to suppress the surface reflections is called an antireflection coating (AR) or nonreflecting film.

<u>Conditions</u>:

Phase condition: waves reflected from the top and bottom surfaces of the thin films are in opposite phase and interfere destructively.

Amplitude Condition: Reflected waves have equal amplitude. For this refractive index of

the thin film should be less than substrate and $\mu_f = \sqrt{\mu_g}$



μ_f = refractive index of antireflection thin film

μ_g = refractive index of glass

Let t be the thickness of antireflection coating.

Reflected BR_1 undergoes a phase change of π radian from the top surface (air to film boundary) so path change of $\frac{\lambda}{2}$ with incident ray AB. Reflected DR_2 undergoes a phase change of π radian from the bottom surface (film to glass boundary) so path change of $\frac{\lambda}{2}$ with incident ray AB.

Optical path difference between reflected rays R_1 and R_2 is given by

$$\Delta = 2\mu_f t + \frac{\lambda}{2} + \frac{\lambda}{2}$$

$$\Delta = 2\mu_f t + \lambda$$

Addition or subtraction of a full wave (λ) does not affect the phase.

$$\text{Optical path difference} = \Delta = 2\mu_f t$$

Condition for destructive interference

$$\Delta = (n + \frac{1}{2})\lambda$$

So

$$2\mu_f t = (n + \frac{1}{2})\lambda$$

For the film to be transparent, the thickness of the film should be minimum which is possible for $n = 0$

$$2\mu_f t = \frac{\lambda}{2}$$

Or $\mu_f t = \frac{\lambda}{4}$

$$\text{Optical thickness} = \mu_f t = \frac{\lambda}{4}$$

$$\text{thickness of antireflction coating} = t = \frac{\lambda}{4\mu_f}$$

1. Find the minimum thickness of soap film, which appear yellow (Wavelength 5896Å) in reflection when it is illuminated by white light at an angle of 45^0.Given, refractive index of the film is 1.33. **(3 to 5 Marks)**

Solution : For constructive interference,

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

$$\mu = 1.33 \; ; \; i = 45^\circ$$

By Snell's law, $\mu = \dfrac{\sin i}{\sin r}$

$$\therefore \quad 1.33 = \frac{\sin 45}{\sin r}$$

$$\therefore \quad r = 32.12^\circ$$

For minimum thickness, $n = 1$.

$$\lambda = 5896 \text{ A}^\circ = 5896 \times 10^{-8} \text{ cm}$$

$$\therefore 2 \times 1.33 \times t \times \cos 32.12 = (2\times1-1)\frac{5896\times10^{-8}}{2}$$

$$\therefore \quad \boxed{t = 1.31 \times 10^{-5} \text{ cm}}$$

2. A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between two consecutive bright fringes is 0.12 cm. Calculate the wavelength of light used. **(3 Marks)**

Solution : The fringe width in air is given by

$$\beta_{\text{air}} = \frac{\lambda}{2\alpha}$$

$$\beta_{\text{air}} = \frac{1.2}{10} = 0.12 \text{ cm}$$

$$\alpha = 40 \text{ sec} = \frac{40}{3600}\text{deg} = \frac{40}{3600}\times\frac{\pi}{180} \text{ radian}$$

$$\therefore \quad \lambda = 2\alpha\beta_{air} = 2\times\frac{40}{3600}\times\frac{\pi}{180}\times 0.12$$

$$= 4.6542 \times 10^{-5} \text{ cm}$$

$$\therefore \quad \boxed{\lambda = 4654.2 \text{ A}^\circ}$$

Module-4: Electrodynamics

Maxwell's Equations

Q.1. **Explain Gauss's laws for static electric and static magnetic fields in differential and integral forms.** **[5 Marks] [May-2022, May-2023]**

Ans:

UQ. Derive Maxwell's First equation and state its significance. **MU - Q. 4(C), Dec. 22, 4 Marks**

(I) In the differential or point form

UQ. State and derive maxwell's equation which describes how the electric field circulates around the time varying magnetic field (Differential form).
MU - Q. 2(B), May 22, 4 Marks, Q. 4(B), May 23, 5 Marks, Q. 5(A), May 23, 5 Marks

(i) Consider an arbitrary surface S bounding an arbitrary volume V in a dielectric medium. We know that for any dielectric medium, the total charge density is the sum of free charge density ρ_v and polarized charge density ρ_p.

(ii) According to the Gauss's law of electrostatics, the total electric flux ψ crossing the closed surface = the total charge enclosed by that surface i.e.

$$\psi = \iint \vec{E}\, dS = \oint_S \vec{E}\, dS = \frac{1}{\epsilon_0}\oint_v (\rho_v + \rho_p)\, dV$$

Where, $q = \rho_v$ and $\rho_p = -\nabla \bar{P}$

$$\therefore \oint_S \epsilon_0 \vec{E}\, dS = \int_v (\rho_v - \nabla \vec{P})\, dV$$

(iii) Using the divergence theorem, convert surface integral to volume integral,

(Electrodynamics) ....Page no. (4-12)

$$\oint_S \epsilon_0 E\, dS = \oint_v \nabla(\epsilon_0 \vec{E})\, dV = \oint (\rho_v - \nabla \vec{P})\, dV$$

$$\therefore \oint_v \nabla(\epsilon_0 \vec{E} + P)\, dV = \oint \rho_v\, dV$$

(iv) Since, electrical displacement vector $\vec{D} = \epsilon_0 \vec{E} + P$

$$\therefore \oint_v \nabla \cdot \vec{D}\, dV = \oint \rho_v\, dV$$

$$\text{Or} \quad \oint (\nabla \cdot \vec{D} - \rho_v)\, dV = 0$$

$$\therefore \nabla \cdot \vec{D} - \rho_v = 0 \quad \text{or} \quad \nabla \cdot \vec{D} = \rho_v$$

This is the Maxwell's first equation in the differential or point form.

(II) In the integral form

(i) the Maxwell's first equation in the differential or point form i.e.

$$\nabla \cdot \vec{D} = \rho_v$$

$\therefore$ integrating both the sides w.r.t. V, we get,

$$\int_V \nabla \cdot \vec{D}\, dV = \int_V \rho_v\, dV \qquad \text{...(4.6.1)}$$

(ii) from the Gauss's theorem we know that,

$$\int_V \nabla \cdot \vec{D}\, dV = \oint_S \bar{D} \cdot dS \qquad \text{...(4.6.2)}$$

$\therefore$ from equations (4.6.1) and (4.6.2), we get

$$\oint_S \vec{D} \cdot dS = \int_v \rho_v\, dV \qquad \text{...(4.6.3)}$$

(iii) But $\int_V \rho_V\, dV = q$

$$\therefore \oint_S \vec{D} \cdot dS = \int_v \rho_V\, dV$$

This is the Maxwell's first equation in the integral form as required.

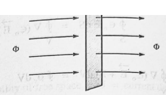
Significance of Maxwell's first equations :

It signifies that 'the total electric displacement through the surface enclosing a volume, is equal to the total charge within the volume.'

(2) Maxwell's second equation

(I) In the differential or point form

(i) We know that the number of magnetic lines flux entering any surface normally is exactly the same as the number of magnetic lines of flux leaving that surface as shown in the Fig. 4.6.1.



(2C24)**Fig. 4.6.1 : Magnetic lines of flux entering and leaving a surface**

$$\therefore \oint_V \bar{B}\, dS = 0$$

(ii) Using Gauss's divergence theorem, we get,

$$\oint_S \bar{B}\, dS = \oint_V \bar{\nabla} \cdot \bar{B}\, dV = 0$$

$$\therefore \nabla \cdot \bar{B} = 0$$

This is the Maxwell's second equation in the differential or point form as required.

(II) In the integral form

(i) From the Maxwell's second equation in the differential or point form i.e.

$$\nabla \cdot \bar{B} = 0 \quad \therefore \int_V \nabla \cdot \bar{B}\, dV = 0$$

(ii) Using Gauss's divergence theorem, we get

$$\oint_S \bar{B} \cdot d\bar{S} = 0$$

This the Maxwell's second equation in the integral form as required.

Q.2. **State and derive Maxwell's equation in differential form which describes how the electric field circulates around the time-varying magnetic field. [5 Marks] [Dec-2018, May-2022, Dec-2022]**

Ans:

(3) Maxwell's Third Equation

UQ. Derive Maxwell's third equation. **MU - Q. 5(b), Dec. 18, 5 Marks**

(I) In the differential form

UQ. How will you state Faraday's law in differential (in point) form explain with appropriate derivation. **MU - Q. 6(c), Dec. 22, Q. 1(4), May 22, 5 Marks**

(i) According to Faraday's second law of electromagnetic induction we know with the usual notation that e.m.f. induced in a closed loop is given by $e = -\frac{d\phi}{dt}$

(ii) Total magnetic flux over any arbitrary surface is given by $\phi = \oint_S \bar{B}\, dS$

$$\therefore \quad e = -\frac{d}{dt}\left(\oint_S \bar{B} \cdot dS\right) = -\oint_S \left(\frac{\partial \bar{B}}{\partial t}\right) dS$$

(iii) The emf is the work done in carrying a unit charge around the closed path.

$$\therefore \quad e = \oint_l \bar{E}\, dl \qquad \therefore \quad \oint_l \bar{E}\, dl = -\oint_S \left(\frac{\partial \bar{B}}{\partial t}\right) dS$$

(iv) Now using Stoke's theorem to convert the line integral into the surface integral, we get

$$\oint_l \bar{E} \cdot dl = \oint_S (\bar{\nabla} \times \bar{E})\, dS \qquad \therefore \quad \oint_S (\bar{\nabla} \times \bar{E})\, dS = -\oint_S \frac{\partial \bar{B}}{\partial t}\, dS$$

$$\therefore \quad \oint_S \left(\bar{\nabla} \times \bar{E} + \frac{\partial \bar{B}}{\partial t}\right) dS = 0 \quad \text{or} \quad \bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

This is the Maxwell's third equation in the differential form. It can also be called as the statement of Faraday;s law in the differential form.

(II) In the integral form

(i) Form the Maxwell's third equation in the differential form i.e.

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

(ii) $\therefore \oint_S (\bar{\nabla} \times \bar{E})\, dS = -\int_S \frac{\partial \bar{B}}{\partial t}\, dS$

(iii) Using Stoke's theorem, we get $\oint_l \bar{E} \cdot \bar{dl} = -\int_S \frac{\partial \bar{B}}{\partial t}\, dS$

This is the Maxwell's third equation in the integral form.

Q.3. **Obtain Ampere's circuital law for a static magnetic field in differential and integral forms. [5 Marks] Dec-2018, May-2022, Dec-2022]**

Ans:

(4) Maxwell's Fourth Equation

Q. Derive Maxwell's fourth equation.

(I) In the differential form

(i) According to Ampere's circuital law, $\oint_l \bar{B} \cdot d\bar{l} = \mu_0 I$ ...(4.6.4)

(ii) According to stoke's theorem, $\oint_l \bar{B} \cdot d\bar{l} = \oint_S (\bar{\nabla} \times \bar{B})\, d\bar{S}$ ...(4.6.5)

(iii) $\therefore$ from Equations (4.6.4) and (4.6.5) above we get, $\oint_S (\bar{\nabla} \times \bar{B})\, d\bar{S} = \mu_0 I$

(iv) But we know that,

$$I = \oint_S \bar{J} \cdot d\bar{S}$$

$$\therefore \oint_S (\bar{\nabla} \times \bar{B})\, dS = \mu_0 \oint_S \bar{J} \cdot d\bar{S}$$

Or $\oint_S \left(\bar{\nabla} \times \bar{B} - \mu_0 \bar{J} \right) \cdot d\bar{S} = 0 \quad \therefore \bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$ but $\bar{B} = \mu_0 \bar{H}$

$$\therefore \bar{\nabla} \times \mu_0 \bar{H} = \mu_0 \bar{J} \quad \therefore \bar{\nabla} \times \bar{H} = \bar{J} \qquad ...(4.6.6)$$

(v) But the above Equation (4.6.5) is true for static fields only. Therefore, for removing this discrepancy and to make the Equation (4.6.6) generalized, Maxwell introduced the concept of the displacement current by replacing $\bar{J}$ by $\left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right)$ where $\frac{\partial \bar{D}}{\partial t}$ represents the displacement current density.

(vi) The term $\frac{\partial \bar{D}}{\partial t}$ is known as Maxwell's correction.

$$\therefore \bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

This is the Maxwell's fourth equation in the differential form.

(II) In the integral form

(i) From the Maxwell's fourth equation in the differential form i.e.

$$\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

(ii) $\therefore \oint_S (\nabla \times \bar{H})\, dS = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right) dS$

(iii) Using Stoke's theorem, we get $\oint_l \bar{H} \cdot d\bar{l} = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right) dS = \bar{J} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\bar{S}$

This is the Maxwell's fourth equation in the integral form.

Q1. Gradient, Divergence and Curl[1]

(a) **What are scalar and vector fields? How is a del operator expressed? [3 Marks] [May-2019, May-2023]**

(b) **If $\phi(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\phi$ at the point $(-1, -2, 1)$. [3 Marks] [May-2019, May-2023]**

(c) **What is the divergence of a vector field? Find the divergence of a field $\mathbf{F} = xz\hat{i} + y^2z^3\hat{j} - xyz\hat{k}$ at a point $(3, -1, 2)$. Interpret the result you obtain. [3 Marks] [May-2017, May-2022 Dec-2022, Dec-2023]**

(d) **Explain the term 'curl of a vector' and state its significance. Show that the divergence of the curl of a vector is zero. [3 Marks] [Dec-17,May-2023, Dec-2023]**

[1]Similar numericals based on the same concept were asked; however, only one example is presented here. As it is a numerical problem, students are encouraged to practice similar problems for better understanding.

Ans:

Q.1. a) # SCALAR FIELDS:

- A scalar field is mathematical function that assigns a scalar value (a single number) to each point in space.

Ex: Temperature distribution in a room, density of material, electric potential etc.

VECTOR FIELDS:

- A vector field is a mathematical function that assigns a vector to each point in space

Ex: Fluid velocity in pipe, electric field, etc.

Vector fields are represented by function $F(x,y,z) = (F_x, F_y, F_z)$ that takes three spatial coordinates as input and returns a vector with three components

- Whereas, scalar is represented by function $f(x,y,z)$ that takes three spatial coordinates as input and returns a scalar value.

DELL OPERATOR ($\vec{\nabla}$):

Dell operator is expressed as: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

- It acts like a vector and also like a differential eq^n thus it will obey rules relating to vectors as well as differential operator. It is used for finding:

i) Gradient $(\vec{\nabla} \cdot f)$

ii) Divergence $(\vec{\nabla} \cdot F)$

iii) curl $(\vec{\nabla} \times F)$

b) $\Phi(x,y,z) = 3x^2y - y^3z^2$

To find: $\vec{\nabla} \cdot \phi$

$$\Phi_{(x,y,z)} = 3x^2y - y^3z^2$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla}\cdot\Phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(3x^2y - y^3z^2)$$

$$= \frac{\partial \Phi}{\partial x}\hat{i} + \frac{\partial \Phi}{\partial y}\hat{j} + \frac{\partial \Phi}{\partial z}\hat{k}$$

$$= \frac{\partial}{\partial x}(3x^2y - y^3z^2)\hat{i} + \frac{\partial}{\partial y}(3x^2y - y^3z^2)\hat{j} + \frac{\partial}{\partial z}(3x^2y - y^3z^2)\hat{k}$$

$$= 6xy\,\hat{i} + (3x^2 - 3y^2z^2)\hat{j} + (-y^3 2z)\hat{k}$$

$$(\vec{\nabla}\cdot\Phi)_{(-1,-2,1)} = 6(-1)(-2)\hat{i} + 3(-1)^2 - 3(-2)^2(1)^2\hat{j} - 2(-2)^3(1)\hat{k}$$

$$= 12\hat{i} - 9\hat{j} + 16\hat{k}$$

c) The divergence of vector field $F(x,y,z) = (F_x, F_y, F_z)$ is a scalar value that represents flux of vector field at a given point. It measures how much vector field diverges or converges at that point.

$$\text{div}\,F = \vec{\nabla}\cdot F = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$F = xz\hat{i} + y^2z^3\hat{j} - xyz\hat{k}$ (Given)

$\vec{\nabla}\cdot F$ – To find

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\text{div}F = \vec{\nabla}\cdot F = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(xz\hat{i} + y^2z^3\hat{j} - xyz\hat{k}\right)$$

$$= \frac{\partial xz}{\partial x} + \frac{\partial y^2z^3}{\partial y} + \frac{\partial(-xyz)}{\partial z}$$

$$= z + 2yz^3 - xy$$

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$$\vec{\nabla}\cdot F_{(3,-1,2)} = 2 + 2(-1)(2)^3 - (3(-1))$$

$$= 2 - 16 + 3 = 2 - 13$$

$$\boxed{\vec{\nabla}\cdot F_{(3,-1,2)} = -11}$$

- The negative divergence indicates that vector field is converging
- It means more vectors are entering the region around this point than leaving it

d) - The curl of vector field $F(x,y,z) = (F_x, F_y, F_z)$ is a vector that measures rotation or circulation of field around a point.
- Denoted by $\vec{\nabla}\times F$ and it signifies rotation, circulation, magnetic field, etc

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\therefore \vec{\nabla}\cdot(\vec{\nabla}\times\vec{F}) = \vec{\nabla}\cdot\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \vec{\nabla}\cdot\left[\hat{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \hat{j}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\right]$$

$$= \vec{\nabla}\cdot 0\hat{i} - 0\hat{j} + 0\hat{k}$$

$$= 0$$

Hence proved, divergence of curl is zero.

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Module-5: Quantum Physics

De Broglie's Hypothesis of Matter Wave

Q.1. **Explain De Broglie's hypothesis of matter waves and deduce the expression for wavelength. [3 Marks] [May-2019, May-2023]**
OR
What is e Broglie's hypothesis? Derive expression for De Broglie's wavelength. [4 Marks] [May-2022, May-2024 (3M)]
OR
State de' Broglie hypothesis and derive an expression for de' Broglie wavelength. Mention three properties of matter waves. [5 Marks] [Dec-2023]
OR
State properties of matter waves. [3 Marks] [Nov-2018, May-2023]
OR
What are matter Waves? State three properties of matter waves. [3 Marks] [Dec-2022]

Ans: Interference, diffraction requires wave nature for their explanation. In photo-electric effect Einstein visualized the incident light as a sort of particles which he called photons and accounted for the emission of electrons as due to the collision between these photons and electrons bound to the metal. During the collision, the photon transfers all its energy to the electrons which results in the emission of photo electrons. Here the behaviour of light is same as that of a particle.

Louis De Broglie put forward the dual behaviour in terms of hypothesis which states If the radiation behaves as particle under certain circumstances, then one can even expect that, entities which ordinarily behave as particles to exhibit properties attributed to only waves under appropriate circumstances. The concept that matter behaves like a wave was proposed by Louis de Broglie in 1924. It is also referred to as the de Broglie hypothesis of matter waves. On the other hand de Broglie hypothesis is the combination of wave nature and particle nature.

If E is the energy of a photon of radiation havinf frequency ν and the same energy can be written for a wave as follows

$$\text{Then its energy is } \mathrm{E} = \mathrm{h}\nu. \text{ (wave nature)}$$
$$\text{It can also be represented as } \mathrm{E} = \mathrm{mc}^2. \text{ (particle nature)}$$
$$\therefore \quad \mathrm{h}\nu = \mathrm{mc}^2$$

$$\therefore mc^2 = h\nu \implies \frac{mc^2}{c} = \frac{h\nu}{c} = \frac{h\nu c}{\lambda} \implies \therefore \lambda = \frac{h}{p}$$

where λ = De Broglie wavelength and p = momentum associated with photon which travels in free space.

Properties of matter waves

1. The wavelength of a matter wave is inversely related to its particles momentum. if the particle moves faster, then the wavekegth will be smaller and vice versa
2. If the particle is at rest , then the de Broglie wavelength is infinite. Such waves cannot be visualized.
3. Matter wave can be reflected, refracted, diffracted and undergo interference
4. The amplitude of the matter waves at a particular region and time depends on the probability of finding the particle at the same region and time.
5. The position and momentum of the material particles cannot be determined accurately and simultaneously.
6. Matter wave is independent of the charge of the particle

Q.2. **For an electron passing through potential difference 'V', show that its wavelength is;** $\lambda = \frac{12.26}{\sqrt{(V)}} A^{\circ}$**.** **[5 Marks] [Nov-2018]**

Ans: De Broglie wavelength for a matter wave is given by

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{mv}}; \quad \text{where}\lambda = \text{DeBrogliewavelength} \qquad (1)$$

From eqn. (1) we find that, if the particles like electrons are accelerated to various velocities, we can produce waves of various wavelengths. Thus higher the electron velocity, smaller will be the de-Broglie wavelength. If velocity v is given to an electron by accelerating it through a potential difference V, then the work done is converted to kinetic energy of electron. Hence, we can write

$$\mathrm{mv} = \sqrt{2\mathrm{meV}} \qquad (2)$$

Substituting eqn.(2) in eqn.(1) we get

$$\lambda = \frac{\mathrm{h}}{\sqrt{2\mathrm{meV}}}$$

Substituting the the values of h = 6.625 x 10^{-34} J-sec, m = 1.6 x 10^{-19} C, e = = 9.1 x 10^{-31} Kg for electron we get $\lambda = \frac{12.26}{\sqrt{V}} A^{\circ}$.

Q.3. **Calculate the de Broglie wavelength of alpha particles accelerating through a potential difference of 150 volts. Given mass of Alpha particle is 6.68 x** 10^{-17} **Kg.** **[3 Marks] [May-2019]**

Ans:

Given : V = 150 V, $m_{\alpha} = 6.68 \times 10^{-27}$ kg

We know that charge of an α particle $q_{\alpha} = 3.2 \times 10^{-19}$ C

Required : λ_{α}

To find the wavelength λ_{α} of α particle

$$\lambda_{\alpha} = \frac{h}{\sqrt{m_{\alpha} \cdot q_{\alpha} \cdot V}} \qquad \text{...Std. formula}$$

$$\therefore \lambda_{\alpha} = \frac{6.63 \times 10^{-34}}{\sqrt{6.68 \times 10^{-27} \times 3.2 \times 10^{-19} \times 150}}$$

$$= \mathbf{1.17 \times 10^{-12}\ m} \qquad \text{...Ans.}$$

Q.4. **What is the wavelength of a beam of neutron having: (i) an energy of 0.025 eV? (ii) an electron and photon each have wavelength of 2 A°. What are their momentum and enegy?** $m_n = 1.67x10^{-27}kg, h = 6.625x10^{-34}$ **J-sec.** **[5 Marks] [May-2017, Dec2021]**

Ans: Given Data: energy of neutron = 0.025 eV.
To find : wavelength of a beam.
Calculation :

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.626x10^{-34}}{\sqrt{2x1.676x10^{-27}x0.025x10^{-19}x1.6}} = 1.8095A^\circ$$

Hence wavelength is equal to 1.8095 A°

2. Given Data $\lambda = 2A^\circ, m_n = 1.676x10^{-27}kg, h = 6.625x10^{-34}J - sec.$

To find :- momentum and energy?
Calculations : $\lambda = h/p \implies p = 3.3125x10^{-24}$ kg-m/sec.

$$\lambda = h/\sqrt{2mE} \implies 2x10^{-10} = \frac{6.625x10^{-34}}{\sqrt{2x1.676x10^{-27}xE}} \implies E = 5.721x10^{-11}\text{joules.}$$

Hence momentum = 3.3125 x 10^{-24} kg-m/sec. and energy is = 5.721 x 10^{-11} joules.

Q.5. **Calculate the frequency and wavelength of photon whose energy is 75 eV. [5 Marks] [May 2018]**

Q.6. **Find the de Broglie wavelength of (i) an electron accelerated through a potential difference of 182 Volts and (ii) 1 Kg object moving with a speed of 1 m/s. Comparing the results, explain why is the wave nature of matter not apparent in daily observations? [5 Marks] [Dec-2022]**

Ans:

Given :
(i) Electron accelerated through P.D. of V = 182 V.
(ii) 1 kg Object moving with a speed v_o = 1 m/s.

Required :
Explanation about the wave nature of matter not being more apparent in daily observations.

Step 1 : To find the wavelength (λ_e) of the electron.

$$\lambda_e = \frac{12.26}{\sqrt{V}} \quad \text{...Std. formula}$$

$$\therefore \lambda_e = \frac{12.26}{\sqrt{182}} = 0.9087 \text{ Å}$$

$$= \mathbf{0.9087 \times 10^{-10}\ m/s} \quad \text{...Ans.}$$

Step 2 : To find the wavelength (λ_o) of 1kg object.

$$\lambda_o = \frac{h}{m_o v_o} \quad \text{...Std. formula}$$

$$\therefore \lambda_o = \frac{6.63 \times 10^{-34}}{1 \times 1} = \mathbf{6.63 \times 10^{-34}\ m/s}$$

...Ans.

Step 3 : The explanation about the wave nature of matter in daily observations

The wavelength (λ_e) of electron is measurable while the wavelength (λ_o) of the object is **too small** and not measurable hence wave nature of matter is not apparent in daily observations.

Heisenberg's Uncertainty Principle

Q.7. **With Heisenberg's uncertainty principle prove that electron cannot survive in nucleus. An electron has a speed of 300 m/sec. with uncertainty of 0.01%. Find the accuracy in its position. [4 Marks] [Dec-2017].**
OR
State Heisenberg's Uncertainty Principle. Show that electron doesn't exist in the nucleus. Find the accuracy in the position of an electron moving with speed 350 m/sec with uncertainty of 0.01%. [8 Marks] [Nov-2018]
OR
Discuss Heisenberg's Uncertainty principle and prove that electrons cannot reside inside the nucleus of an atom using the same principle. [8 Marks] [May-2024]

Ans: Physical quantities like position, momentum, time, energy etc. can be measured accurately in macroscopic systems (i.e. classical mechanics). However, in the case of microscopic systems, the measurement of physical quantities for particles like electrons, protons, neutrons, photons etc are not accurate. If the measurement of one is certain and that of other will be uncertain.

Thus according to uncertainty principle states that *the position and the momentum of a particle in an atomic system cannot be determined simultaneously and accurately. If Δx is the uncertainty associated with the position of a particle and Δp_x the uncertainty associated with its momentum, then the product of these uncertainties will always be equal or greater than $h/4\pi$. That is*

$$\Delta x \Delta p_x \geq h/4\pi$$

Nonexistence of electron in the nucleus

The radius 'r' of the nucleus of any atom is of the order of 10^-14m so that if an electron is confined in the nucleus, the uncertainty in its position will be of the order of 2r = Δx (say) i.e diameter of the nucleus. According to Heisenberg's Uncertainty principle

$$\Delta x \Delta p_x \geq h/4\pi \qquad \text{where} \quad \Delta \text{x} \sim 10^{-14}\text{m}$$

Therefore,

$$\Delta p = h/(4\pi\Delta x) = 6.625\text{x}10^{-34}/(4\pi\text{x}2\text{x}10^{-14}) = 2.63\text{x}10^{-21}\text{kg} - \text{m/s}$$

Taking Δp $\sim$ p we can calculate energy using the formula

$$\text{E}^2 = \text{c}^2[\text{p}^2 + \text{m}_0^2\text{c}^2] = (3\text{x}10^8)^2\text{x}[(2.63\text{x}10 - 21)^2 + (9.1\text{x}10^{-31})^2\text{x}(3\text{x}10^8)^2]$$
$$= 7.932\text{x}10^{-13}\text{J} = 4957745\text{eV} \sim 5\text{MeV}$$

However, the experimental investigations on beta decay reveal that the kinetic energies of electrons must be equal to 4MeV. Since there is a disagreement between theoretical and experimental energy values we can conclude that electrons cannot be found inside the nucleus.

Q.8. **An electron has a speed of 400 m/sec with uncertainty of 0.01%. Find the accuracy in its position. [5 Marks] [May-2023]**

Ans:

Given : $v_{eb} = 400$ m/s, % accuracy of speed = 0.01

Required :

(accuracy in posion of electron) i.e. Δx_e

▶ **Step 1 :** To find the momentum (P_e)

$$P_e = m_e \cdot v_e \quad \text{...Std. formula}$$

$$\therefore P_e = 9.11 \times 10^{-31} \times 400$$

$$= 3.644 \times 10^{-23} \text{ kg–m/s}$$

▶ **Step 2 :** To find $\frac{\Delta v_e}{v_e}$ (% accuracy of speed)

$$\frac{\Delta v_e}{v_e} \times 100 = 0.01$$

$$\therefore \frac{\Delta v_e}{400} \times 100 = 0.01$$

$$\text{or} \quad \Delta v = 0.04$$

▶ **Step 3 :** To find ΔP_e

$$P_e = m_e \cdot v_e \quad \text{...Std formula}$$

$$\therefore \Delta P_e = m_e \times \Delta v_e$$

$$\text{Or} \quad \Delta P_e = 9.11 \times 10^{-31} \times 0.04 = 0.3644 \times 10^{-31}$$

▶ **Step 4 :** To find Δx_e

As per Heisenberg's uncertainity principle,

$$\Delta x \cdot \Delta P_x \geq \frac{h}{2\pi} \quad \text{...Std. theory}$$

$$\therefore \Delta x_e \times \Delta P_e \geq \frac{h}{2\pi}$$

$$\therefore \Delta x_e \times 0.3644 \times 10^{-31} \geq \frac{6.63 \times 10^{-34}}{2\pi}$$

$$\text{or } \Delta x_e = 2.895 \times 10^{-3} \text{ m} \quad \text{...Ans.}$$

Q.9. **Find the lowest energy of a neutron within a nucleus of dimension 10-14 m. given mass of a neutron $1.67X10^{-27}$ kg. [4 Marks] [May-2023]**

Ans:

Given	**Required**
mass of a neutron $m = 1.67 \times 10^{-27}$ kg, Nucleous diameter $= 10^{-14}$ m $= (\Delta x)_{max}$	Lowest energy of neutron E_{min} in the nucleus.

(1) According to Helsenberg's Uncertainity Principle we have,

$$(\Delta p)_{min} (\Delta x)_{max} = h$$

$$\therefore (\Delta p)_{min} = \frac{h}{(\Delta x)_{max}} = \frac{6.626 \times 10^{-34}}{5 \times 10^{-14}}$$

$$= 1.33 \times 10^{-20} \text{ kg-m/sec.}$$

(II) $\because$ **p cannot be less than $(\Delta p)_{min}$, we have**

$$p_{min} = (\Delta p)_{min}$$

(III) $$E_{min} = p^2_{min} = \frac{(1.33 \times 10^{-20})^2}{2 \times 1.67 \times 10^{-27}}$$

$$= 0.529 \times 10^{-13} \quad \text{...Ans.}$$

Q.10. **Arrive at Heisenberg's uncertainty principle with single slit electron diffraction. An electron has a speed of 300m/sec. with uncertainty of 0.01%. Find the accuracy in its position. [7 Marks] [May 2018]**

Ans: Similar to above problem(s)

Q.11. **Show that Non- Existence of electron in the Nucleus. Find the uncertainty in the position of electron. The speed of an to an accuracy of of 0.002%. [8 Marks] [Dec-2019]**

Ans: Similar to above problem(s)

Schrödinger's Wave Equation

Q.12. **Derive Schrödinger's time dependent wave equation for matter waves. OR Derive one dimensional Schrödinger's time dependent equation for matter waves. OR Obtain one dimensional time dependent Schrödinger equation [May-2017, 2018, 2023, Dec-2022] [5 Marks]**

Ans:

(I) The equation of the de **Broglie wave** for the motion of a free particle is given by,

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\cdot\frac{d^2\psi}{dt^2}$$

Where ψ wave function and v is the phase velocity.

(II) Solution of the above second order differential equation is of the form,

$$\psi(x, t) = A e^{-i(Et - px)/h}$$

Where, p = mv is the momentum of the particle of the mass m and

h = Planck's constant = 6.634×10^{-34} J-s

(III) Differentiating Equation (5.3.2) partially we get,

$$\frac{d\psi}{dt} = -\frac{iE}{h} \times Ae^{-i(Et-px)/h} = -\frac{iE}{h}\cdot\psi \text{ or}$$

$$E\psi = -\frac{h}{i}\frac{d\psi}{\partial t} \quad ...(5.3.3)$$

(IV) Taking double partial differentiation of Equation (5.3.2) we get,

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{p^2}{h^2}\psi \text{ or } p^2\psi = -h^2\frac{\partial^2\psi}{\partial x^2} \quad ...(5.3.4)$$

(V) Let the particle be placed in a field of voltage V, then it will acquire a potential energy of P.E. = V.

(VI) Also it has K.E. $= \frac{p^2}{2m}$

(VII) Therefore its total energy,

$$E = K.E. + P.E. = \frac{p^2}{2m} + V \quad \text{or} \quad \frac{p^2}{2m} = E - V,$$ multiplying both the sides by ψ

We get, $$\frac{p^2}{2m}\cdot\psi = E\psi - V\psi \quad ...(5.3.5)$$

(VIII) Putting the values of $E\psi$ from Equation (5.3.3) and $p^2\psi$ from Equation (5.3.4) in Equation (5.3.5) we get,

$$-\frac{h^2}{2m}\cdot\frac{\partial^2\psi}{\partial x^2} = -\frac{h}{i}\frac{\partial\psi}{\partial t} - V\psi$$

or $$-\frac{h^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = -\frac{h}{i}\frac{\partial\psi}{\partial t}$$

This is the required Schrodinger's time dependent wave equation.

Q.13. **Write the expression for Schrödinger's time dependent equation of matter waves and derive Schrodinger's time independent equation. [5 Marks] [May-2024]**

Ans: State the Schrödinger's time dependent equation derived in Q.12 and then derive the following

(I) The general differential equation of a matter wave travelling in x-direction is given by,

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 \psi}{\partial t^2} \quad ...(5.3.6)$$

where ψ is a wave function, **u** = phase velocity

(II) The general solution of the above equation is of the form

$$\psi = \psi_o \cdot e^{i(kx - \omega t)} \quad ...(5.3.7)$$

where, ψ_o = a constant

(III) Differentiating Equation (5.3.7) partially with respect to 't' we get,

$$\frac{\partial \psi}{\partial t} = (-i\omega)\,\psi_o\, e^{i(kx-\omega t)} \quad ...(5.3.8)$$

(IV) Differentiating Equation (5.3.8) partially with respect to 't',

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)^2 \psi_o\, e^{i(kx-\omega t)} = -\omega^2 \psi_o\, e^{i(kx-\omega t)} = -\omega^2 \psi \quad ...(5.3.9)$$

(V) Substituting in Equation (5.3.6) we get,

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{\omega^2}{u^2}\cdot\psi$$

(VI) But $\omega = 2\pi \cdot \gamma = \frac{2\pi u}{\lambda}$ where, v = frequency $= \frac{u}{\lambda}$

$$\therefore \frac{\omega}{u} = \frac{2\pi}{\lambda} \qquad \therefore \frac{\omega^2}{u^2} = \frac{4\pi^2}{\lambda^2} \quad ...(5.3.10)$$

(VII) By de-Broglie hypothesis,

$$\lambda = \frac{h}{p}$$

$\therefore$ Substituting this value of λ in Equation (5.3.10)

$$\text{we get} \therefore \frac{\omega^2}{u^2} = \frac{4\pi^2 p^2}{h^2} \quad ...(5.3.11)$$

(VIII) We know that,

Total energy = Kinetic energy + Potential energy

i.e. T.E. = K.E. + P.E. But K.E. $= \frac{1}{2} mv^2$

$$\therefore \text{T.E.} = \frac{1}{2} mv^2 + \text{P.E.} = \frac{1}{2}\frac{m^2 v^2}{m} + \text{P.E.} = \frac{p^2}{2m} + \text{P.E.} \quad (\because p = mv)$$

$$\therefore p^2 = 2m(\text{T.E.} - \text{P.E.})$$

(IX) Substituting in Equation (5.3.11), we get,

$$\frac{\omega^2}{u^2} = \frac{8\pi^2 m}{h^2}(\text{T.E.} - \text{P.E.})$$

(X) Substituting in Equation (5.3.9), we get

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{8\pi^2 m}{h^2}(\text{T.E.} - \text{P.E.})\cdot\psi$$

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(\text{T.E.} - \text{P.E.})\,\psi = 0$$

(XI) The above equation is used when the P.E. is constant in time but varies in space.

$\therefore$ Replacing the above equation by total derivative, we get,

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(\text{T.E.} - \text{P.E.})\Psi = 0$$

This is the required one dimensional time independent Schrodinger equation for the matter wave.

Particle Enclosed in a Rigid Box

Q.14. **Derive the expression for energy eigen values for free particle in one dimensional potential well.** **[4 Marks] [May-2022]**

OR

Show that the energy of an electron in a one-dimensional deep potential well of infinite height varies as the square of the natural numbers. **[7Marks] [May-2022]**

Ans:

MU - Dec. 15, Q. 3(b), Dec. 22, 5 Marks

(I) (a) Free particle means that particle which is not acted upon by any force. Therefore its P.E. = 0 and it moves in positive x-direction.

(b) The time independent Schrodinger wave equation for such a free particle is given by,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 mE}{h^2}\psi = 0$$

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + k^2\psi = 0, \text{ where, } k^2 = \frac{8\pi^2 mE}{h^2}$$

(c) $\therefore E = \frac{h^2}{8\pi^2 m} k^2 \qquad \therefore \boxed{E \propto k^2} \qquad \because$ h and m are constants.

Hence, the energy is continuous. ($\because$ Square is always continuous).

(II) (a) Suppose **the particle of mass m is free to move** in the x-direction only in the region from x = 0to x = a.

(b) Outside this region, its P.E.: (V)$\rightarrow\infty$ and within this region, it is 0. Further, the particle does not lose energy when it collides. Therefore its energy remains constant.

(c) According to Schrodinger equation,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \qquad \because V = 0 \text{ inside the box}$$

Therefore we get,

$$\therefore \frac{d^2\psi}{dx^2} + \frac{8\pi^2 mE}{h^2}\psi = 0$$

(d) This equation can be written as,

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \qquad ...(5.3.12)$$

$$\text{where, } k^2 = \frac{8\pi^2 mE}{h^2}$$

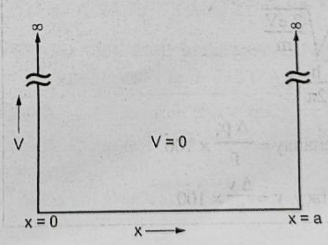
(e) Solution of Equation (5.3.12) is given by,

$$\psi = A \cdot (\cos kx) + B (\sin kx)$$

When, $x = 0$ at $\psi = 0$

$\therefore 0 = A\cos 0 + B\sin 0$

$\therefore A = 0$



(1B6)Fig. 5.3.1

$\therefore B \cdot \sin(ka) = 0, \; B \neq 0$

$\therefore \sin(ka) = 0$

(f) This is possible only when, $ka = n\pi$ (where n = 0, 1, 2, ...)

where, n = Quantum number.

$$\therefore \psi_n = B\sin\left(\frac{n\pi}{a} \times x\right)$$

(g) Now, $k = \frac{n\pi}{a} \qquad \therefore k^2 = \frac{n^2\pi^2}{h^2} = \frac{8\pi^2 mE}{h^2}$

$$\therefore E = \frac{n^2 h^2}{8 ma^2} \quad \text{where n = 1, 2, 3, ... are the natural numbers}$$

Thus, the energy of an electron in the box varies as the square of the natural numbers.

Energy levels in one dimensional box are quantized. Since they can have only certain values known as the **'Eigen Values'**.

Q.15. **The ground state energy of an electron in an infinite well is 5.6 x10^{-3} eV. What will be the ground state energy if the width of the well is doubled? [4 Marks] [May-2022]**

Ans:

Note : 1eV = 1.602×10^{-19} Joules (J) or Volts (V)

$\therefore E_0 = 5.6 \times 10^{-3} \times 1.602 \times 10^{-9} = 8.971 \times 10^{-22}$ V

(I) Let 'a' be the original width then given that new width $E_n = 2a$.

(I) $$E_0 = \frac{(1)^2 h^2}{8m_e a^2} \quad \ldots(1)$$

and $$E_{0n} = \frac{(1)^2 h^2}{8m_e (2a^2)} = \frac{(1)^2 h^2}{32 m_e a^2} \quad \ldots(2)$$

(II) Dividing Equation (2) by Equation (1)

We get, $$\frac{E_{on}}{E_o} = \frac{1}{4}$$

$$\therefore \quad E_{on} = \frac{E_o}{4} = \frac{8.971 \times 10^{-22}}{4}$$

$$= \mathbf{2.242 \times 10^{-7}\ V} \quad \ldots\textbf{Ans.}$$

Q.16. **An electron is bound in a one-dimensional potential well of width 2 A° but of infinite height. Find its energy values in the ground state and in first excited state. [3 Marks] [Dec-2022]**

Ans:

(I) $$E_0 = \frac{(1)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9 \times 10^{-31} \times (10 \times 10^{-10})^2}$$

$$= \mathbf{0.61 \times 10^{-17}\ V} \quad \ldots\textbf{Ans.}$$

(II) $$E_1 = \frac{(2)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9 \times 10^{-31} \times (10 \times 10^{-10})^2}$$

$$= \mathbf{2.442 \times 10^{-17}\ V} \quad \ldots\textbf{Ans.}$$

Q.17. **The minimum energy possible for a particle trapped in a 1-d box is 3.2 x 10^{-18} J. What are the next 5 three energies in eV the particle can have? [5 Marks] [May-2023]**

Ans:

Given	**Required**
$E_o = 3.2 \times 10^{-18}$ J	E_1, E_2, E_3 in eV

$$\text{(I)} \quad E_0 = \frac{(1)^2 \times h^2}{8\ ma^2} = 3.2 \times 10^{-18} \text{ J or V (given)}$$

$$\therefore \frac{h^2 \times 8}{8\ ma^2} = 3.2 \times 10^{-18}$$

$$\text{(II)} \quad E_1 = \frac{(2)^2 \times h^2}{8\ ma^2} = 4 \times \frac{h^2}{8\ ma} = 4 \times 3.2 \times -10^{-18}$$

$$= \frac{12.8 \times 10^{-8}}{1.602 \times 10^{-19}} = \mathbf{79.9\ eV} \quad \textbf{...Ans.}$$

Similarly,

$$\text{(III)} \quad E2 = (3)^2 \times \frac{(2)^2 \times h^2}{8\ ma} = 9 \times 3.2 \times 10^{-18}$$

$$= 28.8 \times 10^{-18}\ \text{V}$$

$$\therefore E2 \text{ in eV} = \frac{28.8 \times 10^{-18}}{1.602 \times 10^{-19}}$$

$$= \mathbf{179.775\ eV} \quad \textbf{...Ans.}$$

$$\text{(IV)} \quad E^3 = (4)^2 \times \left(\frac{h^2}{8\ ma}\right) = 16 \times 3.2 \times 10^{-18}$$

Convert E3 in eV from Joules.

Q.18. **An electron is bound in a one-dimensional potential well of width 5 A° but of infinite height. Find its energy values in the ground state and in first two excited states. [5 Marks] [Dec-2023]**

Ans: Similar to above problem(s)

Module 6: Basics of semiconductor Physics

Q.1 Define drift current, diffusion current and mobility of charge. (3m)

Ans:

Drift current is the electric current that results from the motion of charge carriers (such as electrons or holes) under the influence of an **applied electric field**.

When an electric field is applied across a conducting material (like a semiconductor or conductor), it exerts a force on the free charge carriers, causing them to move in a particular direction. This directed motion of carriers constitutes **drift current**.

The drift current density (J_d) can be expressed as:

$$J_d = n.q.\ \mu.E$$

Where,

n = number of charge carriers per unit volume,

q = charge of a single carrier (e.g., electron charge $=1.6\times10^{-19}$ C),

μ = mobility of the charge carriers,

E = applied electric field.

Diffusion current is the electric current that arises due to the **concentration gradient** of charge carriers.

If charge carriers (electrons or holes) have a higher concentration in one region than another, they will naturally move from the region of higher concentration to the region of lower concentration. This movement of carriers leads to **diffusion current**.

The diffusion current density (J_{diff}) can be expressed as:

$$J_{diff} = -q \cdot D \cdot \nabla n$$

Where,

q = charge of the carrier,

D= diffusion coefficient,

∇n = gradient of the carrier concentration (n).

The negative sign indicates that the current flows from regions of higher carrier concentration to lower concentrations.

The **mobility** of a charge carrier refers to the ability of a charge carrier to move in response to an applied electric field.

Mobility (μ) is a measure of how quickly a charged particle (electron or hole) moves through a material when exposed to an electric field.

Mobility (μ) is defined as:

$$\mu = V_d / E$$

where,

V_d = drift velocity of the carriers,

E = applied electric field.

The unit of mobility is typically expressed in **$m^2/V \cdot s$** (square meters per volt-second).

The **conductivity** of a semiconductor refers to its ability to conduct electric current. It is a measure of how easily charge carriers (electrons and holes) can move through the material under the influence of an electric field.

In semiconductors, conductivity depends on the presence of **free charge carriers** (electrons in the conduction band and holes in the valence band), which are influenced by factors such as temperature, doping, and electric fields.

The **electrical conductivity (σ)** of a semiconductor is given by:

$$\sigma = q\,(n\,\mu_n + p\,\mu_p)$$

where,

q = charge of a single electron or hole (1.6×10^{-19} coulombs),

n = number of electrons in the **conduction band** per unit volume,

p = number of holes in the **valence band** per unit volume,

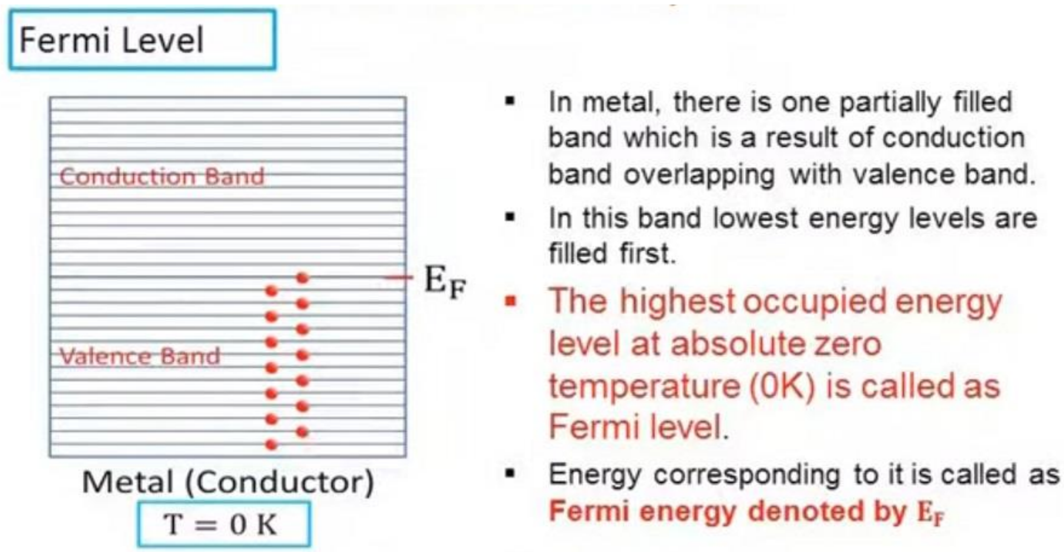
μ_n = mobility of electrons,

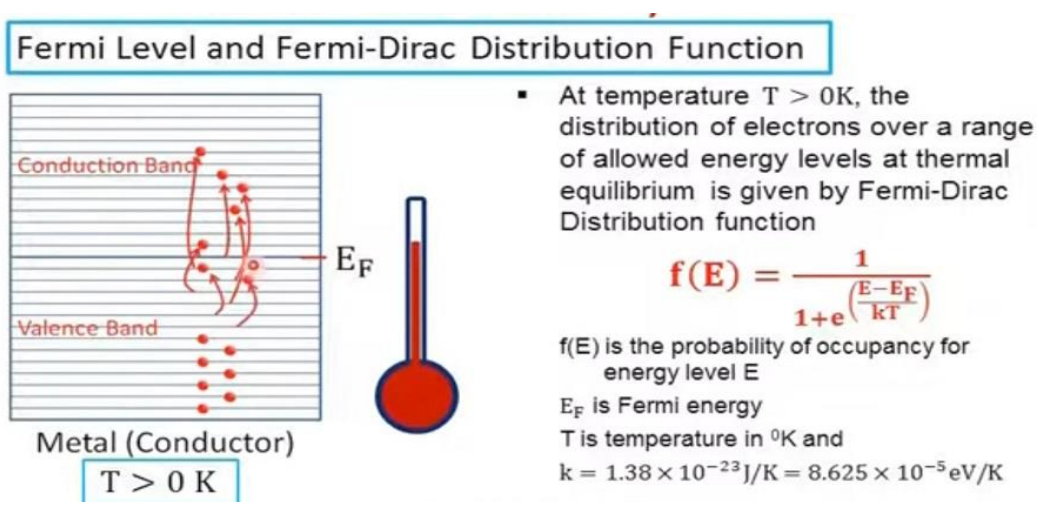
μ_p = mobility of holes.

For **intrinsic semiconductors**, n=p (equal number of electrons and holes).

For **extrinsic semiconductors**, the number of electrons or holes can vary depending on the type of doping (n-type or p-type).

Q.2 What is Fermi level? Write Fermi Dirac distribution function.(3m)





When E<E$_F$:

- The exponent $(E-E_F)/(k_BT)$ is negative because E is less than E_F,
- f(E) is close to 1, meaning that states below the Fermi level are highly likely to be occupied.

When $E>E_F$:

- The exponent $(E-E_F)/(k_BT)$ is positive because E is greater than E_F ,
- f(E) is close to 0, meaning that states above the Fermi level are unlikely to be occupied.

At T = 0 K:

- $f(E)=1$ for $E<E_f$,
- $f(E)=0$ for $E>E_F$,
- The distribution becomes a step function at T=0 K.

Q. 3 For intrinsic semiconductor show that the Fermi level lies in the centre of the forbidden energy gap.(3m)

- It can be shown for intrinsic semiconductors, Fermi energy level E_F lies midway between conduction and valence band. The proof is given below.
- At any temperature $T > 0°$ K,

n_e = Number of electrons in conduction band

n_v = Number of holes in valence band

We have $n_e = N_C e^{-(E_C - E_F)/KT}$ ...(6.7.4)

Where N_C = Effective density of states in conduction band

And $n_v = N_V e^{-(E_F - E_V)/KT}$ ...(6.7.5)

Where N_V = effective density of states in valance band

For best approximation $N_C = N_V$ ...(6.7.6)

For intrinsic semiconductor $n_C = n_v$

$$\therefore N_C \cdot e^{-(E_C - E_F)/KT} = N_V \cdot e^{-(E_F - E_V)/KT}$$

$$\therefore \frac{e^{-(E_C - E_F)/KT}}{e^{-(E_F - E_V)/KT}} = \frac{N_V}{N_C}$$

$$\therefore e^{-(E_C - E_F - E_F + E_V)/KT} = \frac{N_V}{N_C}$$

$$\therefore e^{-(E_C + E_V - 2E_F)/KT} = \frac{N_V}{N_C}$$

as $N_V = N_C = 1$

$$e^{-(E_C + E_V - 2E_F)/KT} = 1$$

$\therefore$ Taking *ln* on both sides

$$\frac{-(E_C + E_V - 2E_F)}{KT} = 0$$

$$\therefore (E_C + E_V) = 2E_F$$

$$\therefore E_F = \frac{E_C + E_V}{2} \quad ...(6.7.7)$$

Thus, the Fermi level in an intrinsic semiconductor lies at the center of forbidden energy gap.

Q.4 Discuss the effect of variation in temperature on the Fermi energy level of n-type Semiconductor with the help of labelled diagram.(3m)

(i) **At low temperature :** When the temperature in the semiconductor is low, only few donor atoms get ionized and electrons move from the donor level to the conduction band.

Hence, Fermi level for n-type semiconductor at low temperature lies midway between the bottom of the conduction band and donor level.

(ii) **At moderate temperature :** At moderate temperature all donor atoms are ionized. So, the concentration of electrons in conduction band is equal to the concentration of donor atoms.

When the temperature increases upto moderate value, Fermi level slowly shifts away from the conduction band and moves towards the center of the forbidden gap.

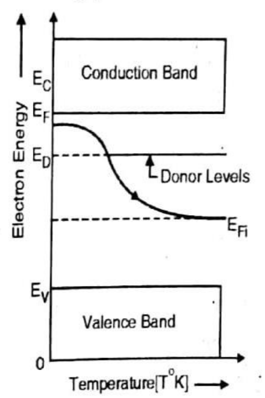


Fig. 6.8.1: Variation of E_F with temperature in n-type material

(iii) **At higher temperature :** At high temperature, the concentration of transfer of electrons from valence band to conduction band is more compared to concentration of electrons from donor atoms and Fermi level is shifted to middle of the forbidden gap.

The variation of Fermi level with temperature for n-type of material is shown in Fig. 6.8.1.

Q.5 Discuss the effect of variation in temperature on the Fermi energy level of p-type Semiconductor with the help of labelled diagram.(3m)

(i) At low temperature : At low temperature only few acceptor levels are occupied, and simultaneously holes are produced in valence band.

So, Fermi level lies in the middle of the top of the valence band and the acceptor level.

(ii) At moderate temperature : At moderate temperature, all acceptor levels are filled.

So, at moderate temperature, Fermi level gradually moves up i.e. moves towards the middle of the forbidden gap.

(iii) At higher temperature : At very high temperature, the contribution of conduction band for the formation of holes in the valence band is more compared to acceptor impurity.

Hence, at very higher temperature, Fermilevel approaches the middle of the energy gap i.e. the position of E_{Fi} for intrinsic semiconductor. The variation of E_F with temperature in p- type material is shown in Fig. 6.8.2.

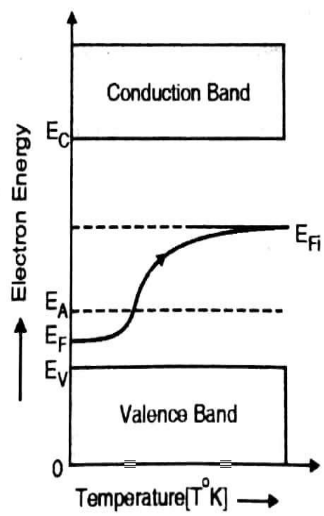


Fig. 6.8.2 : Variation of E_F with temperature in p-type material

Q.6 Draw and explain Fermi level diagram of p-n junction diode? (5m)

Ans: Refer solution of Q.4 and Q.5 for this question.

Q.7 Draw Fermi level and explain it in detail for conductors.(5m)

(a) At T = 0° K

- At 0° K electrons occupy the lower energy levels in the conduction band leaving upper energy levels vacant.
- The band is filled up to a certain energy level E_F, therefore Fermi level may be regarded as the uppermost filled energy level in conductor at 0°K. Let us see some important conclusions from Equation (6.6.1).

At T = 0° K, levels below E_F have $E < E_F$

$$\therefore \quad f(E) = \frac{1}{1 + e^{(E - E_F)/KT}}$$

$$= \frac{1}{1 + e^{-\infty}}$$

$$= \frac{1}{1 + 0} = 1$$

f(E) = 1 means all the levels below E_F are occupied by electrons.

At T = 0° K, levels above E_F have $E > E_F$

$$\therefore f(E) = \frac{1}{1 + e^{(E - E_F)/KT}} = \frac{1}{1 + e^{\infty}}$$

$$= \frac{1}{1 + \infty} \doteq 0$$

$\therefore$ f(E) = 0 means all the levels above E_F are vacant.

At T = 0° K, for $E = E_F$

$$f(E) = \frac{1}{1 + e^{(E - E_F)/KT}} = \frac{1}{1 + e^{\%}}$$

$\therefore$ f(E) is indeterminable.

This is summarized in Fig. 6.6.2.

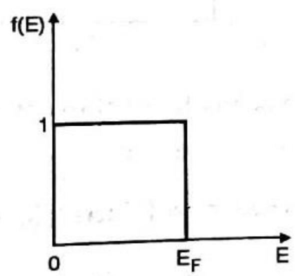


Fig. 6.6.2 : Fermi-Dirac distribution at T = 0°K

(b) At T > 0° K

- At temperature above 0° K, few electrons are excited to vacant levels above E_F. **This happens to those electrons which are close to E_F hence probability to find an electron at $E > E_F$ will become greater than unity** which was zero at T = 0° K.
- Similarly, due to excitation of electrons, few levels just below E_F will be become vacant and f(E) will be slightly reduced which was unity at T = 0° K.
- In a simple way one can understand that, what increase in f(E) at T > 0° K above $E = E_F$ we get is equal to reduction in f(E) below $E = E_F$. This is shown as below in Fig. 6.6.3.

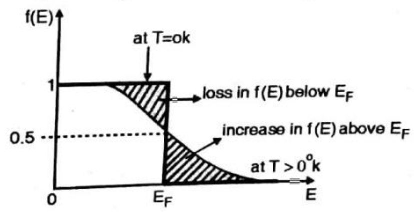


Fig. 6.6.3 : Electron occupancy at T > 0°K

At $E = E_F$ for T > 0° K

$$f(E) = \frac{1}{1+e^{0}} = \frac{1}{1+1} = \frac{1}{2} = 0.5$$