<u>Module 02: FIBRE OPTICS</u>

Short answer questions (**3 Marks each**)

1. An optical fibre refractive index 1.48 and 1.41 respectively of core ,clad Calculate i) Critical angle ii) Numerical Aperture iii) Maximum Incidence angle

   Ans.

   n1 = 1.48

   n2 = 1.41

(ii) Numerical Aperture (NA)

The Numerical Aperture is calculated using the formula:

$$NA = \sqrt{n_1^2 - n_2^2}$$

Substitute the values:

$$NA = \sqrt{1.48^2 - 1.41^2}$$

$$NA = \sqrt{2.1904 - 1.9881}$$

$$NA = \sqrt{0.2023}$$

$$NA \approx 0.45$$

(i) Critical Angle (θ_c)

The critical angle is calculated using the formula:

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Substitute the values:

$$\theta_c = \sin^{-1}\left(\frac{1.41}{1.48}\right)$$

$$\theta_c = \sin^{-1}(0.953)$$

$$\theta_c \approx 72.57^\circ$$

(iii) Maximum Incidence Angle ($\theta_{\max}$)

The maximum incidence angle is related to the Numerical Aperture and can be calculated as:

$$\sin\theta_{\max} = NA$$

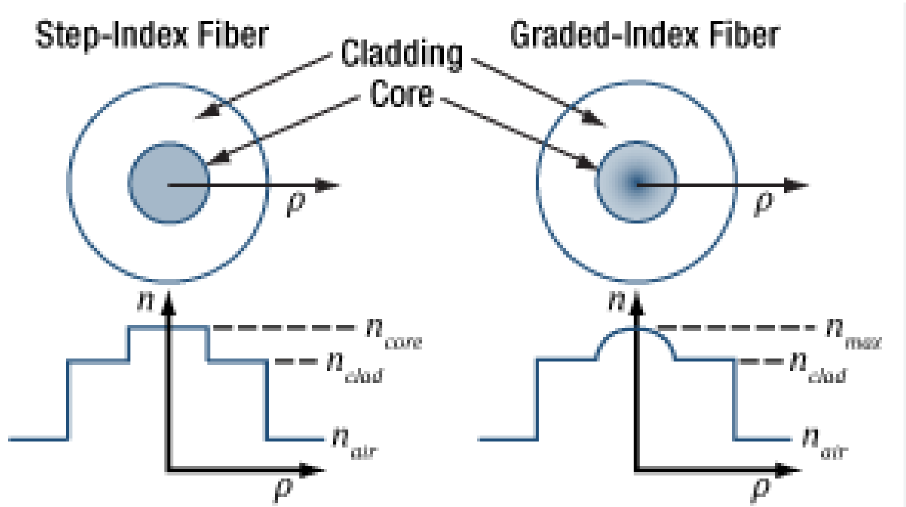
$$\theta_{\max} = \sin^{-1}(NA)$$

Substitute $NA = 0.45$:

$$\theta_{\max} = \sin^{-1}(0.45)$$

$$\theta_{\max} \approx 26.57^\circ$$

2. Distinguish between Step Index and Graded index Optical fibre. (3M)



Step index fiber	Graded index fiber
1. In step index fibers the refractive index of the core medium is uniform through and undergoes an abrupt change at the interface of core and cladding.	1. In graded index fibers, the refractive index of the core medium is varying in the parabolic manner such that the maximum refractive index is present at the center of the core.
2. The transmitted optical signal will cross the fiber axis during every reflection at the core cladding boundary.	2. The transmitted optical signal will never cross the fiber axis at any time.
3. The shape of propagation of the optical signal is in zigzag manner.	3. The shape of propagation of the optical signal appears in the helical or spiral manner or sinusoidal.
4. Attenuation is more for multimode step index fibers but Attenuation is less in single mode step index fibers	4. Attenuation is very less in graded index fibers
5. Numerical aperture is more for multimode step index fibers but it is less in single mode step index fibers	5. Numerical aperture is less in graded index fibers
6. It has both single and multimode.	6. It is only multimode.
7. Intermodal dispersion is present.	7. Intermodal dispersion is absent.
8. It is less expensive. The manufacturing is easy	8. The manufacturing is complex hence it is more expensive.

Long answer questions (**5 Marks each**)

1. Derive an expression for the angle of acceptance and then numerical aperture for a step index optical fibre.

Answer:

10.4.2 Acceptance Angle

Let us again consider a step index optical fibre into which light is launched at one end, as shown in Fig. 10.8. Let the refractive index of the core be n_1 and the refractive index of the cladding be n_2 ($n_2 < n_1$). Let n_0 be the refractive index of the medium from which light is launched into the fibre. Assume that a light ray enters the fibre at an angle θ_i to the axis of the fibre. The ray refracts at an angle θ_r and strikes the core-cladding interface at an angle ϕ. If ϕ is greater than critical angle ϕ_c, the ray undergoes total internal reflection at the interface, since $n_1 > n_2$. As long as the angle ϕ is greater than ϕ_c, the light will stay within the fibre.

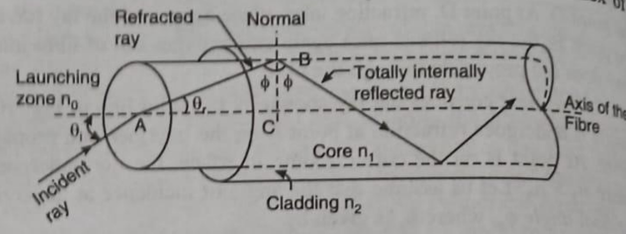


Fig. 10.8: Geometry for the calculation of acceptance angle of the fibre.

Applying Snell's law to the launching face of the fibre, we get

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_0} \tag{10.7}$$

If θ_i is increased beyond a limit, ϕ will drop below the critical value ϕ_c and the ray escapes from the sidewalls of the fibre. The largest value of θ_i occurs when $\phi = \phi_c$.

From the Δ^{le} ABC, it is seen that

$$\sin\theta_r = \sin(90° - \phi) = \cos\phi \tag{10.8}$$

Using equation (10.8) into equation (10.7), we obtain

$$\sin\theta_i = \frac{n_1}{n_0}\cos\phi$$

When $\phi = \phi_c$,

$$\sin\left[\theta_{i_{\max}}\right] = \frac{n_1}{n_0}\cos\phi_c \tag{10.9}$$

But

$$\sin\phi_c = \frac{n_2}{n_1}$$

$\therefore$

$$\cos\phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \tag{10.10}$$

Substituting the expression (10.10) into (10.9), we get

$$\sin[\theta_i(\max)] = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \tag{10.11}$$

Quite often the incident ray is launched from air medium, for which $n_0 = 1$.

Designating $\theta_i(\max) = \theta_0$, equation (10.11) may be simplified to

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$\therefore$

$$\theta_0 = \sin^{-1}\left[\sqrt{n_1^2 - n_2^2}\right] \tag{10.12}$$

The angle θ_0 is called the **acceptance angle** of the fibre. *Acceptance angle is the maximum angle that a light ray can have relative to the axis of the fibre and propagate down the fibre.*

2. What is the application of optical fibre in communication system?

Ans. 1) Fiber optics are used for short-distance image transmission in medical endoscopes, allowing doctors to view internal body parts without invasive surgery.

2) Optical fibers serve as waveguides to transmit data signals in high-speed communication systems, including telephone and internet networks.

3) In aircraft, ships, and tanks, copper wires are replaced by optical fibers to reduce weight and improve performance, enhancing fuel efficiency and communication reliability.

4) Optical fiber communication system:

An optical fiber communication system mainly consists of the following parts as shown in figure.

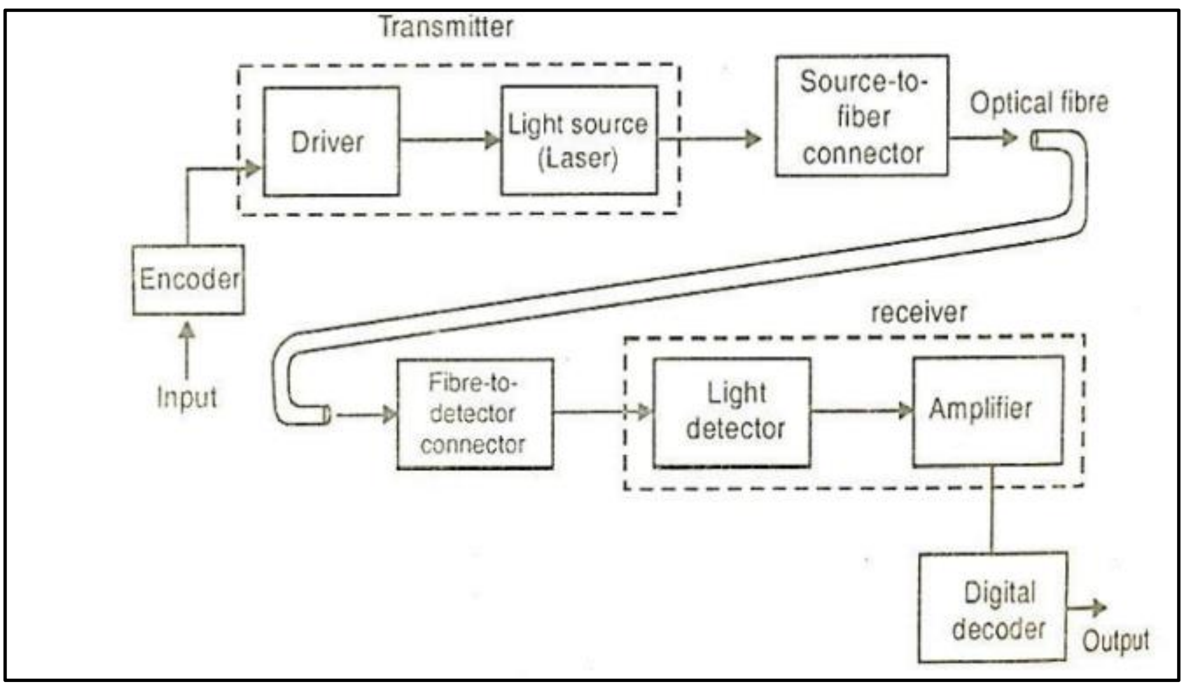
1. Encoder

2. Transmitter

3. Wave guide.

4. Receiver.

5. Decoder



1. **Encoder**: Encoder is an electronic system that converts the analog information like voice, objects etc., into binary data.

2. **Transmitter**: It contain two parts, they are drive circuit and light source. Drive circuit supplies the electric signals to the light source from the encoder in the required form. The light source converts the electrical signals into optical form. With the help of specially made connector optical signals will be injected into wave guide from the transmitter.

3. **Wave guide**: It is an optical fiber which carriers information in the form of optical signals over distances with the help of repeaters. With the help of specially made connector optical signals will be received by the receiver from the wave guide.

4. **Receiver**: It consists of three parts; they are photo detector, amplifier and signal restorer. The photo detector converts the optical signal into the equivalent electric signals and supply to amplifier. The amplifier amplifies the electric signals as they become weak during the long journey through the wave guide over longer distance. The signal restorer deeps the electric signals in a sequential form and supplies to the decoder in the suitable way.

5. **Decoder**: It converts electric signals into the analog information.

3. A step index fiber has a core diameter of 33 x10^{-6} m. the refractive indices of core and cladding are 1.56 And 1.5189 respectively. If the light of wavelength 1.3 µm is transmitted through the fiber, Determine normalized frequency of the fiber. Weather fiber supports single mode or multimode

Ans.

To determine the normalized frequency V of the fiber, we use the formula for the normalized frequency of a step-index optical fiber**:**

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

where:

a= radius of the core of the fiber (in meters),

λ= wavelength of the transmitted light (in meters),

n1 = refractive index of the core,

n2= refractive index of the cladding.

Given data

Core diameter = 33×10^{-6} m,

Refractive index of core (n_1) = 1.56,

Refractive index of cladding (n_2) = 1.5189,

Wavelength (λ) = 1.3 µm = 1.3×10^{-6} m.

The radius a is half of the core diameter:

$$a = \frac{33 \times 10^{-6}}{2} = 16.5 \times 10^{-6}\,\mathrm{m}.$$

Calculate the numerical value of $\sqrt{n_1^2 - n_2^2}$

$$\sqrt{n_1^2 - n_2^2} = \sqrt{1.56^2 - 1.5189^2}.$$

$$1.56^2 = 2.4336, \quad 1.5189^2 = 2.3035.$$

$$2.4336 - 2.3035 = 0.1301.$$

$$\sqrt{0.1301} \approx 0.360.$$

Now substitute the values into the formula:

$$V = \frac{2\pi \times 16.5 \times 10^{-6}}{1.3 \times 10^{-6}} \times 0.360.$$

$$V = \frac{2\pi \times 16.5}{1.3} \times 0.360.$$

$$V \approx \frac{103.673}{1.3} \times 0.360 \approx 79.7 \times 0.360 \approx 28.7.$$

The fiber supports **single mode** if the normalized frequency V is less than 2.405, and it supports **multimode** if V is greater than 2.405.

Since V≈28.7, which is much greater than 2.405, the fiber supports **multimode** operation.

Normalized frequency V≈28.7

The fiber supports **multimode**.

Module 03: INTEREFERENCE IN THIN FILM

1 Derive the conditions for the maxima and minima due to interference of light in a parallel thin film under reflected system.

Answer:

Let us consider a transparent film of uniform thickness 't' bounded by two parallel surfaces as shown in Fig.6.13. Let the refractive index of the material be μ. The film is surrounded by air on both the sides. Let us consider plane waves from a monochromatic source falling on the thin film at an angle of incidence 'i'. Part of a ray such as AB is reflected along BC, and part of it is transmitted into the film along BF. The transmitted ray BF makes an angle 'r' with the normal to the surface at the point B. The ray BF is in turn partly reflected back into the film along FD while a major part refracts into the surrounding medium along FK. Part of the reflected ray FD is transmitted at the upper surface and travels along DE. Since the film

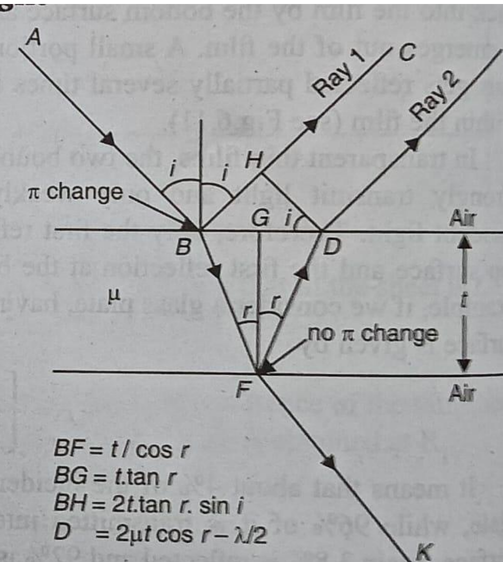


Fig. 6.13

boundaries are parallel, the reflected rays BC and DE will be parallel to each other. The waves travelling along the paths BC and BFDE are derived from a single incident wave AB. Therefore they are coherent and can produce interference if they are made to overlap by a condensing lens or the eye.

(*i*) **Geometrical Path Difference:** Let DH be normal to BC. From points H and D onwards, the rays HC and DE travel equal path. The ray BH travels in air while the ray BD travels in the film of refractive index μ along the path BF and FD. The geometric path difference between the two rays is

$$BF + FD - BH.$$

(*ii*) **Optical Path Difference:**

Optical path difference $\Delta_a = \mu L$

$$\therefore \quad \Delta_a = \mu(BF + FD) - 1(BH) \tag{6.15}$$

In the ΔBFD, $\angle BFG = \angle GFD = \angle r$

$$BF = FD$$

$$BF = \frac{FG}{\cos r} = \frac{t}{\cos r}$$

$$\therefore \quad BF + FD = \frac{2t}{\cos r} \tag{6.16}$$

Also,

$$BG = GD$$

$$\therefore \quad BD = 2BG$$

$$BG = FG \tan r = t \tan r$$

$$\therefore \quad BD = 2t \tan r$$

In the Δ^{le} BHD

$$\angle HBD = (90 - i)$$

$$\angle BHD = 90^\circ$$

$$\therefore \quad \angle BDH = i$$

$$\therefore \quad BH = BD \sin i = 2\,t \tan r \sin i \tag{6.17}$$

From Snell's law,

$$\sin i = \mu \sin r$$

$$\therefore \quad BH = 2t \tan r\,(\mu \sin r) = \frac{2\mu t \sin^2 r}{\cos r} \tag{6.18}$$

Using the equations (6.17) and (6.16) into equ.(6.15), we get

$$\Delta_a = \mu\left[\frac{2t}{\cos r}\right] - \left[\frac{2\mu t \sin^2 r}{\cos r}\right]$$

$$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

$$= \frac{2\mu t}{\cos r}\cos^2 r$$

$$\therefore \quad \Delta_a = 2\mu t \cos r \tag{6.19}$$

(*iii*) **Correction on account of phase change at reflection:** When a ray is reflected at he boundary of a rarer to denser medium, a path-change of $\lambda/2$ occurs for the ray BC (see ig.6.13). There is no path difference due to transmission at D. Including the change in path ifference due to reflection in eqn. (6.19), the true path difference is given by

$$\Delta_t = 2\mu t \cos r - \lambda/2 \tag{6.20}$$

Condition for Maxima:

$$2\mu\, t \cos r - \frac{\lambda}{2} = m\lambda$$

$$2\mu\, t \cos r = m\lambda + \lambda/2$$

$$2\mu\, t \cos r = (2m + 1)\lambda/2 \qquad \textbf{Condition for Brightness}$$

'm' is an integer and it can take the values from 0, 1, 2, 3….

Condition for Minima:

$$2\mu\, t \cos r - \lambda/2 = (2m + 1)\lambda/2$$

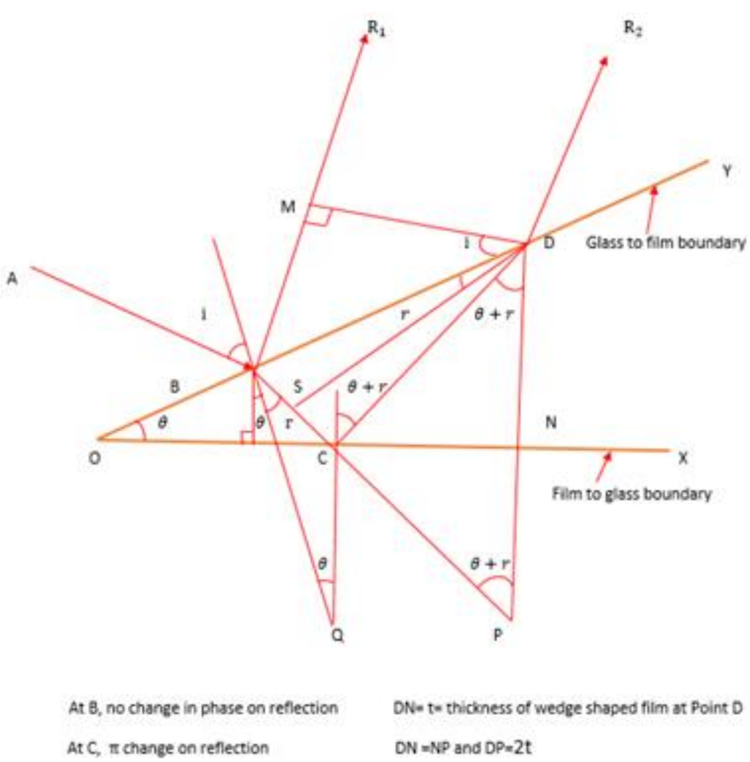
$$2\mu\, t \cos r = (m + 1)\lambda$$

OR

$$2\mu\, t \cos r = m\lambda \qquad \textbf{Condition for Darkness}$$

2. Derive the conditions for the maxima and minima due to interference of light in a wedge shaped film.

Ans.



When a parallel beam of a monochromatic light of wavelength λ incident the wedge shaped film of refractive index μ, the reflected rays interfere with each other and produce Alternate bright and dark fringes. Fringes are formed on its top surface.

Path difference between the reflected rays from the lower and upper surfaces of the air film varies along its length due to variation in film thickness.

The rays BR_1 and DR_2 reflected from the top surface and bottom surface of the air film. The rays BR_1 and $DR_2$are coherent, derived from the same ray AB (division of amplitude). The rays are very close if the thickness of the film is of the order of wavelength of light. Thickness of the glass plate is large as compared to the wavelength of incident light so that entire pattern is due to air film only.

Optical path difference between BR_1 and DR_2

$$\Delta = 2\mu t\cos(r+\theta) + \frac{\lambda}{2}$$

Where $\frac{\lambda}{2}$ is the abrupt phase change of on reflection from the boundary of air to glass interface at point c.

Condition for maxima or bright fringe (constructive interference):

$$\Delta = n\lambda$$

Or

$$2\mu t\cos(r+\theta)+\frac{\lambda}{2}=n\lambda$$

$$2\mu t\cos(r+\theta)=(n-\frac{1}{2})\lambda$$

Where $n = 1,2,3,4 \ldots\ldots$ etc.

$n = 1$ = first bright fringe or band

$n = 2$ = second bright fringe or band

Condition for minima or dark fringe (destructive interference):

$$\Delta = (n+\frac{1}{2})\lambda$$

Or

$$2\mu t\cos(r+\theta)+\frac{\lambda}{2}=(n+\frac{1}{2})\lambda$$

$$2\mu t\cos(r+\theta)=n\lambda$$

Where $n = 0,1,2,3,4 \ldots\ldots$ etc.

$n = 0$ = Zero order

$n = 1$ = first dark fringe or band

$n = 2$ = second dark fringe or band

3. How Newton's rings are formed? Explain the experimental arrangement of Newton's rings

Ans:

Explanation of the formation of Newton's rings

Division of amplitude takes place at the curved surface of the plano convex lens. The incident light is partially reflected and partially transmitted at the curved surface. The transmitted ray is reflected from the glass plate as shown in Fig. 1.4.2. These two rays

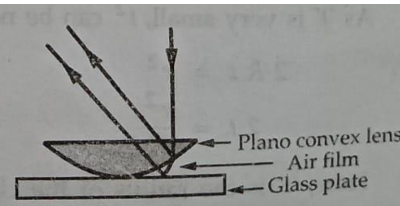


Fig. 1.4.2

interfere in reflected light. The path difference between these rays depends on thickness of the air film enclosed between the curved surfaces of lens and glass plate which increases radially outwards from the centre. The thickness of the air film is *zero* at the centre.

Shape of fringes : Thickness of air film is zero at the centre and increases radially outwards. The locus of points of constant thickness of air film and hence constant path difference is a circle as seen from the top. Hence interference fringes are circular. They are concentric alternate bright and dark rings.

Path difference : As the radius of curvature of the plano convex lens is large, the air

1.4 Newton's Rings Experiment

May-08,09,10,11,12, Dec-08,09,10,11

Experimental arrangement : A plano convex lens 'L' of large radius of curvature is placed on a plane glass plate 'G_1', with the curved surface touching the glass plate as shown in Fig. 1.4.1. An air film is enclosed between the curved surface of the lens and the glass plate.

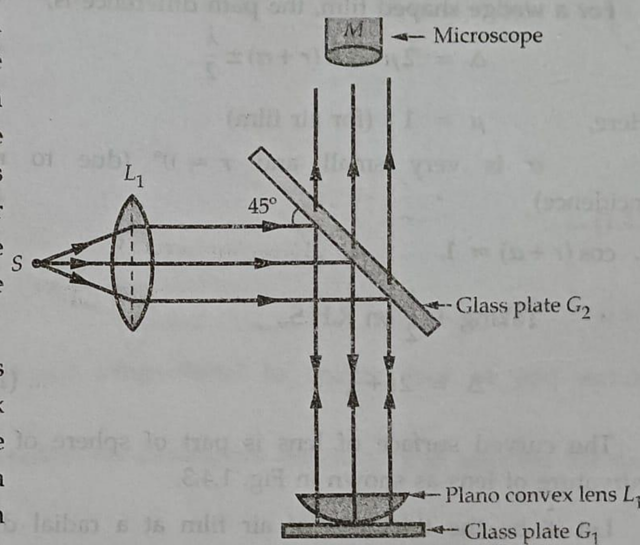


Fig. 1.4.1

A sodium vapour lamp 'S' is kept at the focus of a biconvex lens 'L_1' which converts the diverging beam of light into a parallel beam. This parallel beam of light is made to fall on a glass plate 'G_2' kept at an angle of 45° with the incident beam. A part of incident light is reflected towards the plano convex lens. This light is again reflected back, partially from the top and partially from the bottom of the air film, and transmitted by the glass plate 'G_2'. The interference of these rays is observed through a microscope 'M'.

3.Describe in detail the concept of anti-reflecting film with a proper ray diagram

Ans.

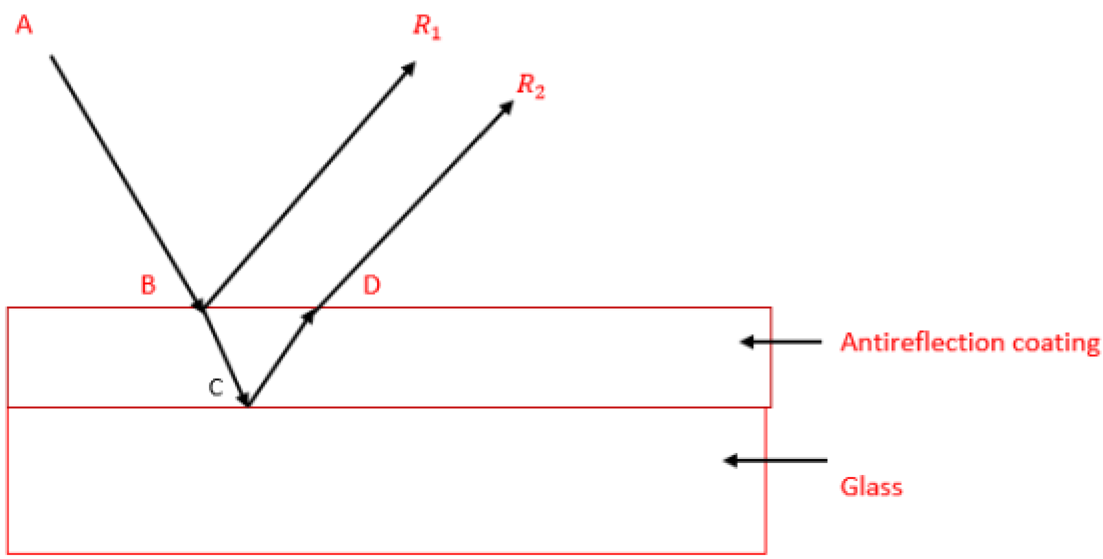
Antireflection coating

Optical instruments such as cameras and telescopes use multi-component glass lenses. When light incident on glass plate both reflection and refraction occur. When the number of reflections are large, the quality of the image produced by optical device will be poor. The reflection from a surface can be reduced by coating the surface with a thin transparent dielectric film. A transparent thin film coated on a surface to suppress the surface reflections is called an antireflection coating (AR) or nonreflecting film.

Conditions:

Phase condition: waves reflected from the top and bottom surfaces of the thin films are in opposite phase and interfere destructively.

Amplitude Condition: Reflected waves have equal amplitude. For this refractive index of the thin film should be less than substrate and $\mu_f = \sqrt{\mu_g}$



μ_f = refractive index of antireflection thin film

μ_g = refractive index of glass

Let t be the thickness of antireflection coating.

Reflected BR_1 undergoes a phase change of π radian from the top surface (air to film boundary) so path change of $\frac{\lambda}{2}$ with incident ray AB. Reflected DR_2 undergoes a phase change of π radian from the bottom surface (film to glass boundary) so path change of $\frac{\lambda}{2}$ with incident ray AB.

Optical path difference between reflected rays R_1 and R_2 is given by

$$\Delta = 2\mu_f t + \frac{\lambda}{2} + \frac{\lambda}{2}$$

$$\Delta = 2\mu_f t + \lambda$$

Addition or subtraction of a full wave (λ) does not affect the phase.

$$\text{Optical path difference} = \Delta = 2\mu_f t$$

Condition for destructive interference

$$\Delta = (n + \frac{1}{2})\lambda$$

So

$$2\mu_f t = (n + \frac{1}{2})\lambda$$

For the film to be transparent, the thickness of the film should be minimum which is possible for $n = 0$

$$2\mu_f t = \frac{\lambda}{2}$$

Or $\mu_f t = \frac{\lambda}{4}$

$$\text{Optical thickness} = \mu_f t = \frac{\lambda}{4}$$

$$\text{thickness of antireflction coating} = t = \frac{\lambda}{4\mu_f}$$

1. Find the minimum thickness of soap film, which appear yellow (Wavelength 5896Å) in reflection when it is illuminated by white light at an angle of 45^0.Given, refractive index of the film is 1.33. **(3 to 5 Marks)**

Solution : For constructive interference,

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

$$\mu = 1.33 \; ; \; i = 45^\circ$$

By Snell's law, $$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \quad 1.33 = \frac{\sin 45}{\sin r}$$

$$\therefore \quad r = 32.12^\circ$$

For minimum thickness, $n = 1$.

$$\lambda = 5896 \text{ A}^\circ = 5896 \times 10^{-8} \text{ cm}$$

$$\therefore 2 \times 1.33 \times t \times \cos 32.12 = (2\times1-1)\frac{5896\times10^{-8}}{2}$$

$$\therefore \quad \boxed{t = 1.31\times10^{-5} \text{ cm}}$$

2. A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between two consecutive bright fringes is 0.12 cm. Calculate the wavelength of light used. **(3 Marks)**

Solution : The fringe width in air is given by

$$\beta_{\text{air}} = \frac{\lambda}{2\alpha}$$

$$\beta_{\text{air}} = \frac{1.2}{10} = 0.12 \text{ cm}$$

$$\alpha = 40 \text{ sec} = \frac{40}{3600} \text{ deg} = \frac{40}{3600}\times\frac{\pi}{180} \text{ radian}$$

$$\therefore \quad \lambda = 2\alpha\beta_{air} = 2\times\frac{40}{3600}\times\frac{\pi}{180}\times 0.12$$

$$= 4.6542 \times 10^{-5} \text{ cm}$$

$$\therefore \quad \boxed{\lambda = 4654.2 \text{ A}^\circ}$$