Vidyavardhini's College of Engineering & Technology, Vasai

First Year Engineering
Academic Year 2024-25
Solution with Marking Scheme
Internal Assessment – II

Sub: BSC102/AP | **Year/Sem: - FE/ I** | **Date: 04/11/24**
Max. Marks: 15 | | **Duration: -1 Hr**

Course Outcomes:

Learners will be able to:

CO4: Illustrate the significance of Maxwell's equations in the field of modern technology.
CO5: Apply the foundations of quantum mechanics for the development of modern technology.
CO6: Explain the types of semiconductors based on variations in fermi level with temperature and doping concentration

BL: Blooms Taxonomy level | **CO: Course Outcome**

Q. No.	Solutions	Marking Scheme	CO	BL
	Attempt the following questions.	Total Marks 05		
1. a)	What is the divergence of a vector field? Find the divergence of a field for $\overline{A} = x^2y\,\boldsymbol{i} - 3xyz^2\,\boldsymbol{j} + 2xy\,\boldsymbol{k}$ at (1,1,1)		4	2
Ans	The divergence of a vector field is a scalar measurement that measures how much a vector field is expanding (diverging) or contracting (converging) at a given point. The divergence of vector field $\overline{A}$ is $\vec{\nabla}\cdot\vec{A}$ $\vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(x^2y\hat{i} - 3xyz^2\hat{j} + 2xy\hat{k}\right)$ $\vec{\nabla}\cdot\vec{A} = 2xy - 3xz^2$ $= 2(1)(1) - 3(1)(1)^2 = -1$ $\therefore \vec{\nabla}\cdot\vec{A} = -1$ at $(1,1,1)$			

What is the curl of a vector field? Find the curl of a Vector field for $\vec{E} = 4x\,\boldsymbol{i} + 2y\,\boldsymbol{j} + 3z\,\boldsymbol{k}$ Ans) The curl of a vector field measures the tendency of the field to rotate around a point. It quantifies the rotation or circulation of the vector field. Given $\vec{E} = 4x\hat{i} + 2y\hat{j} + 3z\hat{k}$ $\therefore$ the curl of $\vec{E}$ is defined as $\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4x & 2y & 3z \end{vmatrix} = 0 - 0 + 0$ $\therefore \vec{\nabla}\times\vec{E} = \bar{0}$		4	2
Derive Maxwell's 3rd equation in differential form, which describes how the electric field circulates around the time-varying magnetic field. Ans) According to the Faraday's law of electromagnetic Induction we know that $e = -\frac{d\phi}{dt}$ — ① where ϕ is the magnetic flux associated with the coil. Therefore $\phi = \int_s \vec{B}\cdot\vec{ds}$ — ② where B is the magnetic field intensity and the lines of force passing through the infinitesimal surface area ds.		4	3

from eqn ① and ② we can write

$$e = - \int_s \frac{\partial}{\partial t} (\vec{B} \cdot ds) \quad — ③$$

Also we know that the e.m.f 'e' induced inside the coil can be written as

$$e = \oint_l \vec{E} \cdot \vec{dl} \quad — ④$$

from eqn ③ and ④ we can write

$$\oint_l \vec{E} \cdot \vec{dl} = - \int_s \left(\frac{\partial \vec{B}}{\partial t} \right) \cdot \vec{ds} \qquad \left\{ \text{as } B(x, t) \right.$$

By using stoke's curl theorem we can write

$$\int_s (\vec{\nabla} \times \vec{E}) \cdot \vec{ds} = - \int_s \left(\frac{\partial \vec{B}}{\partial t} \right) \cdot \vec{ds}$$

$$\therefore \int_s \left(\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) \cdot \vec{ds} = 0$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}}$$

The above integral is for any arbitary surface which implies $\vec{\nabla} \times \vec{E}$ and $\frac{\partial B}{\partial t}$ must be zero as the element $ds \neq 0$

	Derive Maxwell's first equation in differential form for static electric field produced by charge enclosed within a closed surface. According to Guass Law $\oint_s \vec{E}\cdot d\vec{s} = \frac{q}{\epsilon_0}$ where q: charge enclosed by a closed surface E: electric field due to charge 'q' and ϵ_0 is Permitivity in free space. we know that $q = \int_v \rho \, dV$ where — ① ρ is the volume charge density. ∴ By using Guass divergence theorem we can write $\oint_s \vec{E}\cdot d\vec{s} = \int_v (\vec{\nabla}\cdot\vec{E}) dV$ ② from ① & ② we can write $\int_v (\vec{\nabla}\cdot\vec{E}) dv = \int_v \frac{\rho dv}{\epsilon_0} \Rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0}$		4	3

Q. No. 2)	Solve any one	Total marks 04	CO	BL
a)	What is Heisenberg's Uncertainty Principle? Prove that electron cannot exist in the nucleus using H.U.P. Hisenberg's uncertanity principle states that it is impossible to determine the exact position and momentum of particle accurately. i.e. mathematically $\Delta x \, \Delta P_x > \frac{\hbar}{2\pi}$ $\hbar = h/2\pi$ h: plank's constant Δx: uncertanity in the position. ΔP_x: uncertanity in the momentum along the x direction Now, let us assume if an electron		5	3

Can exist inside a nucleus.

The radius 'r' of the nucleus of any atom is the order of 10^{-15} m so that if an electron is confined in the nucleus, the uncertanity in its posistion will be of the order '$2r$' $= \Delta x$ (say) i.e. diameter of the nucleus.

But according to Hisenbergs uncertanity principle

$$\Delta x \cdot \Delta P_x > \frac{\hbar}{2} \Rightarrow \Delta P_x \geq \frac{\hbar}{2\Delta x}$$

$\therefore$ Substituting the values we get $\Delta P_x \geq \frac{1.054 \times 10^{-34}}{2 \times 10^{-15}}$

$$\Rightarrow \Delta P_x \geq 5.27 \times 10^{-20} \text{ kgm/s}$$

We know that the momentum transfoom's as $P = mv$

$\therefore$ ΔP_e will tranfoom as $\Delta P_e = m_e \Delta v_e$

where m_e: mass of an electron

Δv_e: Velocity of electron

$$\therefore \Delta v_e = \frac{\Delta P_e}{m_e} = \frac{5.27 \times 10^{-20}}{9.11 \times 10^{-31}} = 5.78 \times 10^{10} \text{ m/s}$$

The uncertanity in the Velocity of electrons Comes out to be 5.78×10^{8} m/s which exceeds the speed of light

Hence a Contradiction to our assumption as to remain confined inside an nucleus an electron needs to have such a enormous Velocity which is not Physical.

b) Derive Schrodinger Time dependent Wave Equation.

Consider a particle of mass 'm' moving with velocity 'v' having total energy 'E' associated is given as

$$E = K.E + P.E.$$

$$\therefore E = \frac{p^2}{2m} + V \quad -(1)$$

where p : momentum & V : Potential energy.

As we know that the particle has wavefunction $\psi(x,t)$ associated with it along its trajectory when in motion.

The wavefunction can be written as

$$\psi(x,t) = A e^{i/\hbar (px - Et)} \quad \text{— (2)}$$

differentiating eqn (2) w.r.t 'x'

$$\frac{\partial \psi(x,t)}{\partial x} = A e^{i/\hbar (px-Et)} \; i/\hbar \,(p)$$

differentiating once again w.r.t 'x' we get

$$\frac{\partial \psi(x,t)}{\partial x} = A e^{i/\hbar (px-Et)} \left(i/\hbar\right)^2 (p)^2$$

$$\therefore \frac{\partial^2 \psi}{\partial x^2} = -p^2/\hbar^2 \, \psi(x,t) \implies p^2 = \frac{-\hbar^2}{\psi(x,t)} \frac{\partial^2 \psi(x,t)}{\partial x^2} \quad \text{— (3)}$$

Now differentiating eqn (2) w.r.t 't'

$$\frac{\partial \psi(x,t)}{\partial t} = -\frac{i}{\hbar} E \left(A e^{i/\hbar (px-Et)}\right) = -i/\hbar \, E \, \psi(x,t)$$

$$\therefore E = \frac{i\hbar}{\psi(x,t)} \frac{\partial \psi(x,t)}{\partial t} \quad \text{— (4)}$$

Subsituting eqn (4) & (3) in eqn (1) we get

$$\frac{i\hbar}{\psi(x,t)} \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m\psi(x,t)} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V$$

After simplyfing we get

$$\boxed{i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V \psi(x,t)}$$

This is the required S.T.D.W.E

Q.No. 3)	Solve any one	Total marks 05	CO	BL
a)	Explain conductivity and mobility. Calculate the conductivity of a Ge specimen if the donor impurity added to Ge is 1.5 x 10^{25} atoms / m^3. Given mobility of electron is 3900 cm^2/V-sec			

Conductivity: It is the physical property that characterizes the conducting ability of a material. It depends on the number of charge carriers (electrons or holes) and the mobility of the material. It is denoted as σ

$\sigma = q(\mu_e + \mu_p) n$ where

q = charge of the carrier 1.6×10^{-19} C

n: Carrier concentration

μ_e: mobility of electron

μ_p: mobility of holes.

mobility: The mobility is defined as the magnitude of drift velocity acquired by the charge carriers in a unit electric field.

The mobility is denoted as μ

$$\therefore \mu = \frac{v_d}{E}$$

where v_d : drift velocity of the charge carrier

E : Applied electric field.

<u>Given data:</u> $n_e = 1.5 \times 10^{25}$ atoms / m^3

$\mu_e = 3900$ cm^2 / V-sec $= 0.39$ m^2 / V-sec

To find: Conductivity σ

Formula: $\sigma = e \mu_e n_e$

$$= 1.6 \times 10^{-19} \times 0.39 \times 1.5 \times 10^{25}$$

$$\therefore \sigma = 936000 \ \mho / m$$

a) Explain Fermi-Dirac distribution function. If the fermi level in K is 2.2eV, Calculate the energy for which the probability of occupancy at 300°K is 0.98?

The probablity of finding an electron in the energy state E at the temprecture T is mathematically expressed as Fermi-Dirac distribution.

$$F(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

where K : Boltzman Constant

E_F : Fermi Energy and

T : absolute Tempreature.

1) AT $T = 0K$ & $E < E_F$: $E - E_F$ is negative

$$\therefore f(E) = \frac{1}{1+e^{-\infty}} = \frac{1}{1+0} = 1$$

$f(E) = 1$ implies that all energy levels lying below E_F are occupied by the electrons.

2) AT $T = 0K$ & $E > E_F$ $E - E_F$ is positive

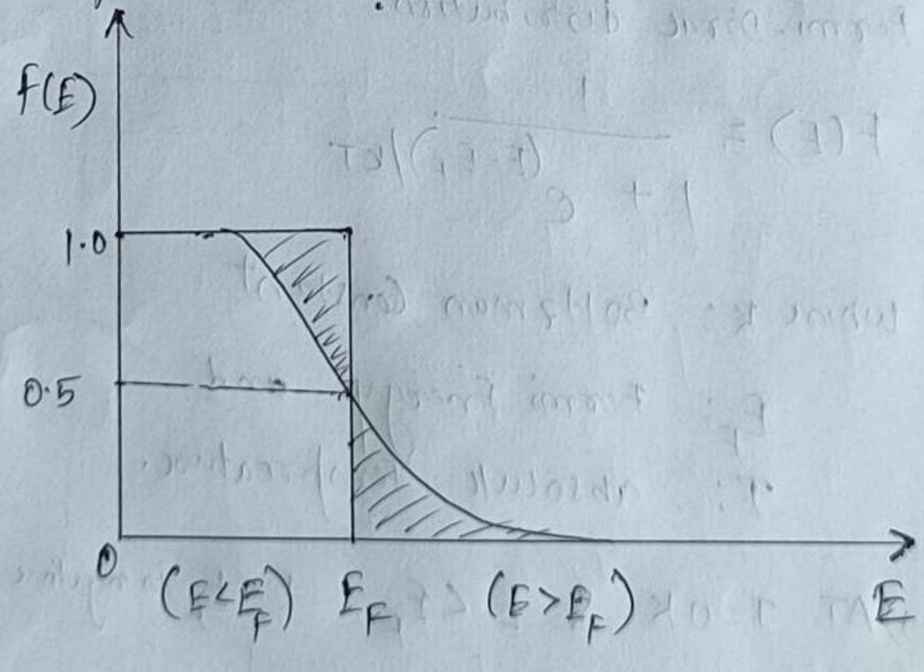
$$\therefore f(E) = \frac{1}{1+e^{\infty}} = \frac{1}{\infty} = 0$$

$f(E) = 0$ implies that all levels above E_F are vacant

3) At $T > 0K$ & $E = E_F$ $E - E_F = 0$

$$\therefore f(E) = \frac{1}{1+e^{0}} = \frac{1}{1+1} = \frac{1}{2}$$

$f(E) = 0.5$ implies the occupancy of fermi level at any temperature above 0K is 50%



Given data: $E_F = 2.2\,eV$

$T = 300K \quad f(E) = 0.98$

To find: $E = ?$

Formula: $f(E) = \dfrac{1}{1 + e^{(E-E_F)/KT}}$

Solving for E

$\therefore \quad e^{(E-E_F)/KT} = \dfrac{0.02}{0.98}$

Taking ln on both the side

$(E-E_F)/KT = \ln(0.02/0.98)$

$\therefore \quad E = E_F + KT \ln(0.02/0.98)$

Substituting the values & KT in eV we get

$E = 2.0993\,eV$

$\therefore$ The energy 'E' for which the probablity of occupancy is 0.98 is 2.0993 eV.