Vidyavardhini's College of Engineering and Technology, Vasai (West)

First Year Engineering

Academic Year: 2024-2025

Assignment No.: 1 (Measurements and Errors)

Subject: BSC2023/EP **Date: 24/01/2025**
Max Marks: 30 **Duration: 1 Hr**

CO1: To provide students with a basic understanding of measurements in the field of basic engineering.

Sample Mean

Q1 (a): A quality control team collects the following data of defective items in 5 production batches: $3, 7, 2, 8, 5$.
1. Calculate the sample mean.
2. If a 6^{th} batch is added with 10 defective items, calculate the new sample mean.

Solution: 1. Sample mean for 5 batches:

$$\bar{x} = \frac{\sum x}{n} = \frac{3+7+2+8+5}{5} = \frac{25}{5} = 5$$

2. New mean with 6 batches:

$$\bar{x}_{\text{new}} = \frac{\sum x + x_6}{n+1} = \frac{25+10}{6} = \frac{35}{6} \approx 5.83$$

Final Answer: Original Mean: 5 New Mean: 5.83

Sample Standard Deviation

Q1 (b): A researcher measures the heights of 6 plants (in cm): $48, 52, 49, 51, 50, 53$.
1. Calculate the sample mean.
2. Determine the sample standard deviation.

Solution: 1. Sample mean:

$$\bar{x} = \frac{48+52+49+51+50+53}{6} = \frac{303}{6} = 50.5\,\text{cm}$$

2. Variance:

$$\text{s}^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$\text{s}^2 = \frac{(48-50.5)^2 + (52-50.5)^2 + (49-50.5)^2 + (51-50.5)^2 + (50-50.5)^2 + (53-50.5)^2}{6-1}$$

$$\mathrm{s}^2 = \frac{6.25 + 2.25 + 2.25 + 0.25 + 0.25 + 6.25}{5} = \frac{17.5}{5} = 3.5$$

Standard Deviation:

$$\mathrm{s} = \sqrt{3.5} \approx 1.87\,\mathrm{cm}$$

Final Answer: Mean: 50.5 cm Sample Standard Deviation: 1.87 cm

Population Mean

Q2 (a): A small population of $N = 8$ students scored the following marks in a physics test: $75, 80, 82, 70, 85, 90, 88, 76$.
1. Calculate the population mean.
2. If a new student scores 92, recalculate the population mean.

Solution: 1. Population mean:

$$\mu = \frac{\sum x}{N} = \frac{75 + 80 + 82 + 70 + 85 + 90 + 88 + 76}{8} = \frac{646}{8} = 80.75$$

2. New population mean:

$$\mu_{\text{new}} = \frac{\sum_{i=1}^{8} x + x_9}{N + 1} = \frac{646 + 92}{9} = \frac{738}{9} = 82$$

Final Answer: Original Mean: 80.75 New Mean: 82

Population Standard Deviation

Q2 (b): The weights (in kg) of a population of 6 individuals are $58, 60, 62, 64, 66, 68$.
1. Calculate the population mean.
2. Determine the population standard deviation.

Solution: 1. Population mean:

$$\mu = \frac{\sum x}{N} = \frac{58 + 60 + 62 + 64 + 66 + 68}{6} = \frac{378}{6} = 63\,\mathrm{kg}$$

Variance:

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

$$\sigma^2 = \frac{(58-63)^2 + (60-63)^2 + (62-63)^2 + (64-63)^2 + (66-63)^2 + (68-63)^2}{6}$$

$$\sigma^2 = \frac{25 + 9 + 1 + 1 + 9 + 25}{6} = \frac{70}{6} \approx 11.67$$

2. Standard Deviation:

$$\sigma = \sqrt{11.67} \approx 3.42\,\mathrm{kg}$$

Final Answer: Population Mean: 63 kg Population Standard Deviation: 3.42 kg

Principles of Least Squares

Q3: A researcher measures the following data:

$$(x_1, y_1) = (1, 3),\ (x_2, y_2) = (2, 5),\ (x_3, y_3) = (3, 7),\ (x_4, y_4) = (4, 10)$$

1. Find the equation of the best-fit line $y = mx + c$ using the least squares method.

Solution: Calculate required sums:

$$\sum x = 1 + 2 + 3 + 4 = 10, \quad \sum y = 3 + 5 + 7 + 10 = 25$$

$$\sum xy = (1)(3) + (2)(5) + (3)(7) + (4)(10) = 74$$

$$\sum x^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30, \quad n = 4$$

Use the formulas for slope m and intercept c:

$$m = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = \frac{4(74) - (10)(25)}{4(30) - (10)^2} = \frac{296 - 250}{120 - 100} = \frac{46}{20} = 2.3$$

$$c = \frac{\sum y - m \sum x}{n} = \frac{25 - 2.3(10)}{4} = \frac{25 - 23}{4} = \frac{2}{4} = 0.5$$

Final equation: $y = 2.3x + 0.5$

Final Answer: Best-Fit Line: $y = 2.3x + 0.5$