Mod: 05 : Quantum Physics

Derive Schrodinger's Time dependent Wave equation.

Ans:— Consider a particle of mass 'm' moving with velocity 'v' & has the total energy associated given as —

$$E = K.E + P.E$$

$$E = \frac{p^2}{2m} + V \quad \text{—— (1)} \quad (V \text{ is the Potential Energy})$$

As we know that the particle has wave nature, so it should have some wave function which gives its trajectory at any instant. The wave fn is given as —

$$\psi(x,t) = A e^{\frac{i}{\hbar}(px - Et)} \quad \text{—— (2)}$$

differentiating eqn (2) w.r.t 'x' —

$$\frac{\partial \psi(x,t)}{\partial x} = A e^{\frac{i}{\hbar}(px - Et)} \times \frac{i}{\hbar} \times p$$

differentiating once again w.r.t 'x'

We get, $\frac{\partial \psi(x,t)}{\partial x} = Ae^{i/\hbar(px-Et)} \times \frac{i}{\hbar}p$

Differentiating w.r.t 'x' again, we get,

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = Ae^{i/\hbar(px-Et)} \times \left(\frac{i}{\hbar}\right)^2 p^2$$

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = -\frac{p^2}{\hbar^2} \times \psi(x,t) \text{ - (from equ (2))}$$

$$\therefore p^2 = \frac{-\hbar^2}{\psi(x,t)} \times \frac{\partial^2 \psi(x,t)}{\partial x^2} \quad \text{— (3)}$$

Now, differentiating equ (2) w.r.t time 't'

$$\therefore \frac{\partial \psi(x,t)}{\partial t} = Ae^{i/\hbar(px-Et)} \times -\frac{i}{\hbar}E$$

$$\therefore \frac{\partial \psi(x,t)}{\partial t} = -\frac{iE}{\hbar}\psi(x,t) \text{ --- (from equ (2))}$$

$$\therefore E = \frac{\hbar}{-i\,\psi(x,t)} \times \frac{\partial \psi(x,t)}{\partial t}$$

$$\therefore E = \frac{i\hbar}{\psi(x,t)} \times \frac{\partial \psi(x,t)}{\partial t} \quad \text{— (4)}$$

Substituting the values of equ (3) & (4) in equ (1) —

$$\frac{i\hbar}{\psi(x,t)} \times \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m\,\psi(x,t)} \frac{\partial^2 \psi}{\partial x^2} + V$$

After simplifying, we get —

$$\boxed{i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V\psi(x,t)}$$

S.T.D.W.E

Derive schrodinger's time independent wave equation.

Ans: — We know that, Schrodinger's time dependent wave equation is given as —

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V\psi(x,t) = i\hbar \frac{\partial}{\partial t}\psi(x,t) \quad \text{— (1)}$$

Using variable separable method —

$$\psi(x,t) = \psi(x).\phi(t).$$

Substituting in the above equ —

$$\frac{-\hbar^2}{2m} \frac{\partial^2 [\psi(x).\phi(t)]}{\partial x^2} + V[\psi(x).\phi(t)] = i\hbar \frac{\partial [\psi(x)\phi(t)]}{\partial t}$$

$$\therefore \frac{\hbar^2}{2m} \phi(t) \frac{d^2 \psi(x)}{dx^2} + V[\psi(x).\phi(t)] = i\hbar\,\psi(x) \frac{d\phi(t)}{dt}$$

As we know that, potential Energy is always position dependent

We get,

$$\therefore V \equiv V(x) \text{ —}$$

$$\therefore \frac{\hbar^2}{2m} \phi(t) \frac{d^2 \psi(x)}{dx^2} + V(x)[\psi(x).\phi(t)] = i\hbar\,\psi(x) \frac{d\phi(t)}{dt}$$

Now, dividing throughout by $[\psi(x)\phi(t)]$ —

We get,

$$\frac{\frac{-\hbar^2}{2m}\cancel{\phi(t)}\frac{d^2\psi(x)}{dx^2}}{[\psi(x)\cdot\cancel{\phi(t)}]} + \frac{V(x)[\psi(x)\cdot\phi(t)]}{[\psi(x)\cdot\phi(t)]} = \frac{i\hbar\,\cancel{\psi(x)}\frac{d}{dt}\phi(t)}{[\psi(x)\cdot\phi(t)]}$$

$$\therefore \frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2} + V(x) = \frac{i\hbar}{\phi(t)}\frac{d\phi(t)}{dt}$$

We can equate the above equation with some constant, which can be the total energy (E) of the particle.

$$\therefore \frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\cdot\frac{d^2\psi(x)}{dx^2} + V(x) = \frac{i\hbar}{\phi(t)}\cdot\frac{d\phi(t)}{dt} = E$$

$$\therefore \frac{-\hbar^2}{2m}\times\frac{1}{\psi(x)}\cdot\frac{d^2\psi(x)}{dx^2} + V(x) = E$$

$$\therefore \frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\cdot\psi(x) = E\psi(x)$$

It is the schrodinger's time independent wave equation.