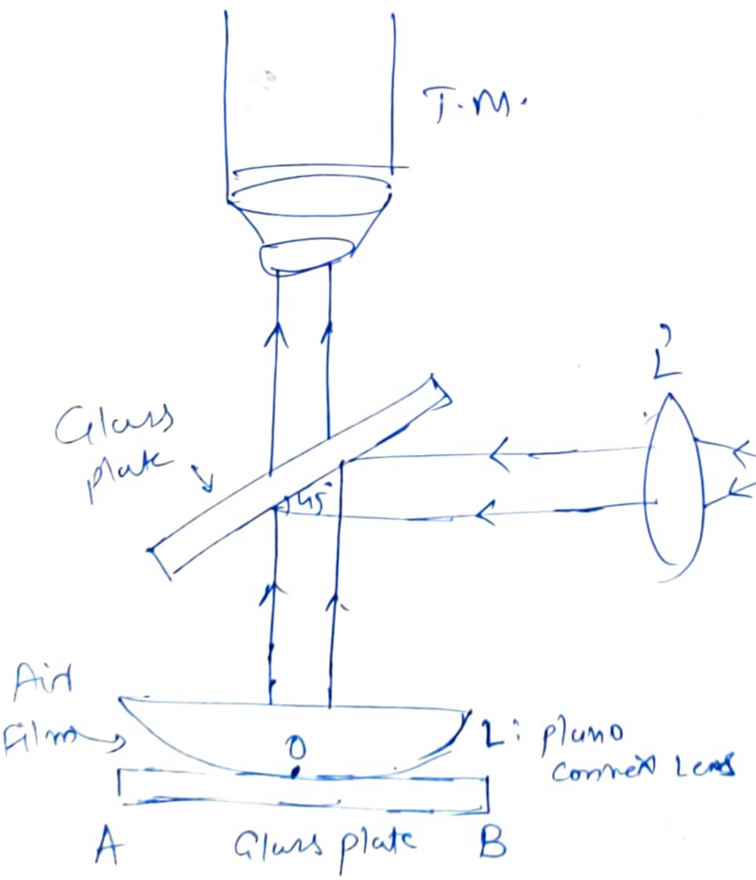
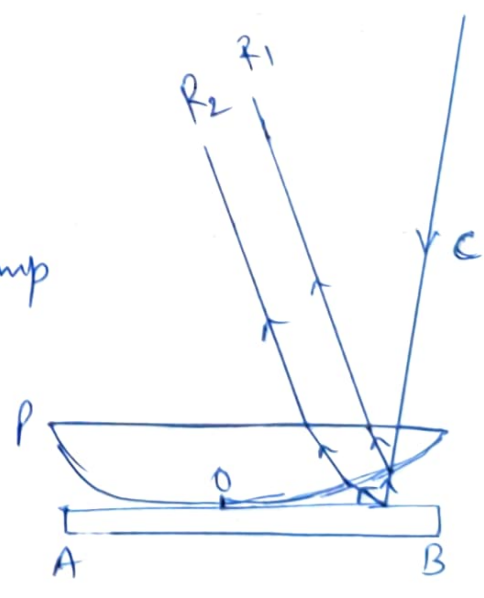
# Applications of Interference : ①

①
Newton's Ring —
- (i) Determination of wavelength
- (ii) Refractive Index of transparent Liquid
- ~~(iii) Anti~~

② Anti Reflecting Coating

## Newton's Ring Experiment



The optical path difference between the Ray $R_1$ and $R_2$ is given by $\Delta$

where $\Delta = 2\mu t \cos r - \frac{\lambda}{2}$

For the Normal Incidence of light from air

$\cos r = 1$ and $\mu = 1$

$\therefore \quad \Delta = 2t - \frac{\lambda}{2}$

Condition for maxima : $\Delta = m\lambda$

$\therefore \quad 2t - \frac{\lambda}{2} = m\lambda$ $\quad m = 1, 2, 3, \ldots$ Integral multiple

$$\therefore \quad \boxed{2t = (2m+1)\frac{\lambda}{2}}$$

Condition for minima : $\Delta = (2m+1)\frac{\lambda}{2}$

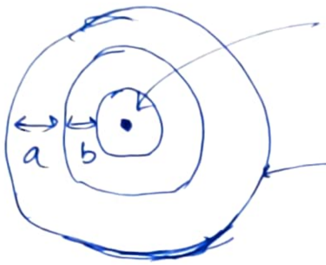
$\therefore \quad 2t - \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}$

$\therefore \quad 2t = (m+1)\lambda$ $\quad m+1 \simeq m$

$$\boxed{2t = m\lambda}$$

## Circular fringes :


$t=0$ at center point of contact




At center it is dark.

Locus of points where the air film has same thickness

Thickness of air film is constant for a given ~~circle~~ point on a circle.

Also called fringes of equal thickness.

Question

The fringes are equally spaced

$\therefore$ $a = b$ ?

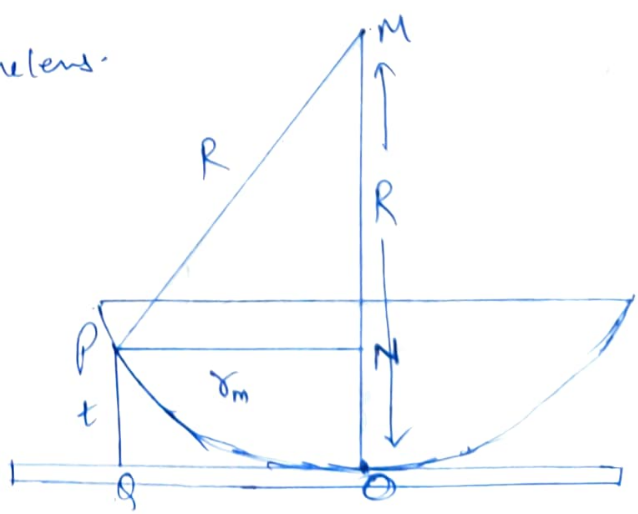
will check.

## Radii of dark fringes :

Let:

R : Radius of curvature of the lens.

Consider a dark fringe located at point Q



Therefore at point $Q$

Let the thickness of air film $PQ = t$

Let the Radius of $m^{th}$ Circular fringe at $Q$ be $r_m$

$$\therefore \boxed{r_m = OQ}$$

$\therefore$ By pythagoras theorems

$$PM^2 = PN^2 + MN^2$$

$$R^2 = r_m^2 + (R-t)^2$$

$$\therefore r_m^2 = 2Rt - t^2$$

$PM = R$

$OQ = PN = r_m$

$OM = ON + MN$

$\therefore MN = R - t$

As $R >> t$, $2Rt >> t^2$ $\therefore t^2 \to 0$

$$\therefore r_m^2 \simeq 2Rt \quad \text{—①} \Rightarrow \boxed{t = \frac{r_m^2}{2\lambda}}$$

The Condition for dark band at point $Q$

$$\therefore 2t = m\lambda \quad \text{—②}$$

$$\therefore r_m^2 = (2t)R = (m\lambda)R \qquad \text{from - ①}$$

$$\boxed{r_m = \sqrt{m\lambda R}} \qquad m = 1, 2, 3, \dots$$

The

∴ The radii of $1^{st}$ Dark fringe is for $m = 1$

$\therefore \; r_1 = \sqrt{1 \lambda R}$

for $2^{nd}$ $\; r_2 = \sqrt{2 \lambda R}$

for $3^{rd}$ $\; r_3 = \sqrt{3 \lambda R}$

$r_m \propto \sqrt{\lambda}$

$\Rightarrow r_m \propto \sqrt{\text{Natural numbers}}$

The Diameter of the ring will be

$$\boxed{D_m = 2r_m = 2\sqrt{m \lambda R}}$$

* Spacing between the fringes

The diameter is given as $D_m = 2\sqrt{m \lambda R}$ $\quad m = 1, 2, 3 \ldots$

$D_1 = 2\sqrt{1 \cdot \lambda \cdot R} = 1(2\sqrt{\lambda R}) = 1 \cdot x = x$

$D_2 = 2\sqrt{2 \cdot \lambda \cdot R} = \sqrt{2}(2\sqrt{\lambda R}) = \sqrt{2} \cdot x = 1.41x$

$D_3 = 2\sqrt{3 \cdot \lambda \cdot R} = \sqrt{3}(2\sqrt{\lambda R}) = \sqrt{3}x = 1.73x$

$D_4 = 2\sqrt{4 \lambda R} = \sqrt{4}(2\sqrt{\lambda R}) = \sqrt{4}x = 2x$

$\therefore \; D_2 - D_1 = 0.41x$

$D_3 - D_2 = 0.32x$

$D_4 - D_3 = 0.27x$

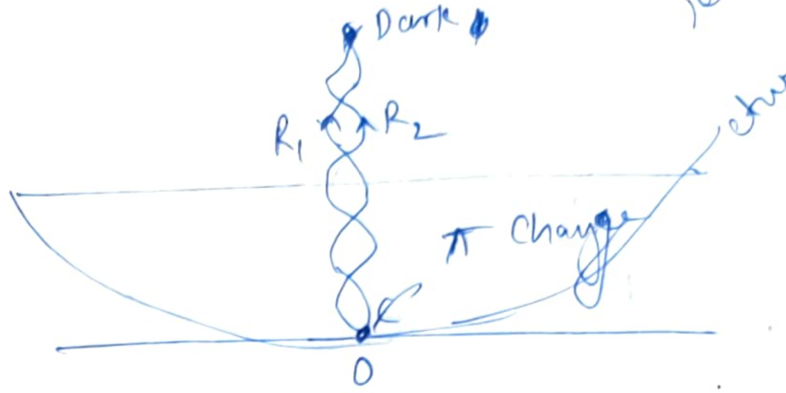
Not equally spaced.

as $m \uparrow \quad \beta \downarrow$

$\Rightarrow$ The frings's get closer and closer with increasing $m$.

* Dark Central spot

At Center point
O we know that
$t = 0$




$\therefore$ The Condition for

Darkness $\Delta = 2t - \frac{\lambda}{2}$

at O

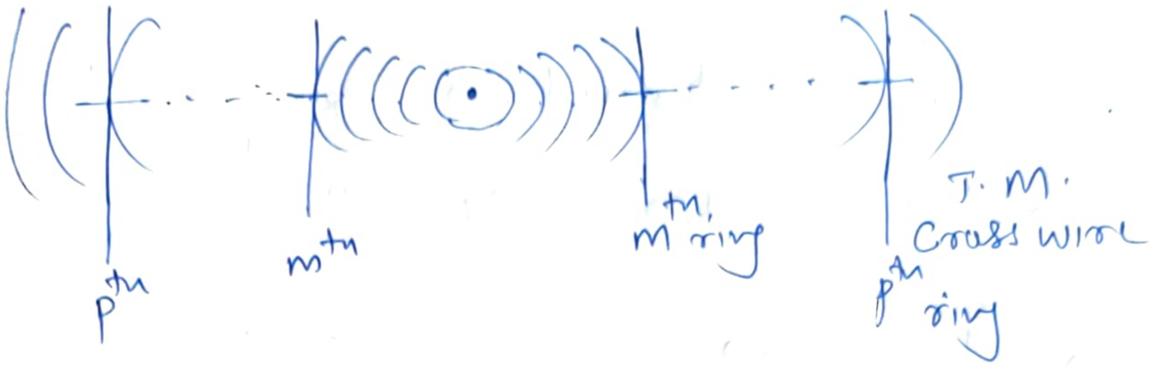
$$\Delta_0 \cong \frac{\lambda}{2} \qquad \text{Path difference is } \frac{\lambda}{2}$$

The reflected wave from surface 1 i.e. $R_1$ does not undergo change of phase.
But the reflected wave from bottom surface undergoes a phase change of $\pi$

$\Rightarrow$ The interfering waves are in opposite phase and we see a dark spot.

Determination of wavelength of light source (4)

Setup the Newton's ring interference experiment




We know that $D_m = 2\sqrt{m\lambda R}$

for $m^{th}$ ring $D_m^2 = 4m\lambda R$ — (1)

$\boxed{y = mx}$

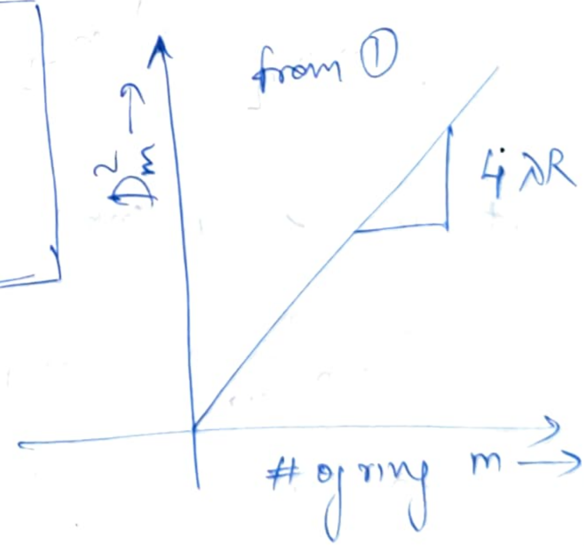
$\Downarrow$

$m = 4\lambda R$

for $(m+p)^{th}$ ring $D_{m+p}^2 = 4(m+p)\lambda R$ — (2)

Subtracting (1) from (2) $D_{m+p}^2 - D_m^2 = 4p\lambda R$

$$\Rightarrow \boxed{\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}}$$




from Graph slope $= 4\lambda R$

$$\therefore \boxed{\lambda = \frac{\text{slope}}{4R}}$$

(2) Refractive index of transparent liquid.

* In the setup the liquid whose R.I is to be determined is filled in the gap between the lens and glass plate.

Let the R.I of liquid be $\mu$ then the condition for Dark ring can be written as

$$2\mu t = m\lambda$$

The diameter of $m^{th}$ dark ring $[D_m^2]_L = \frac{4m\lambda R}{\mu}$ — (1)

III^ly for $(m+p)^{th}$ ring $[D_{m+p}^2]_L = \frac{4(m+p)\lambda R}{\mu}$ — (2)

Subtracting (1) from (2)

$$[D_{m+p}^2]_L - [D_m^2]_L = \frac{4p\lambda R}{\mu} \quad — (3)$$

But we already know that in air

$$[D_{m+p}^2]_{air} - [D_m^2]_{air} = 4p\lambda R \quad — (4)$$

from (3) and (4) we can write

R.I of liquid $\mu = \frac{[D_{m+p}^2]_{air} - [D_m^2]_{air}}{[D_{m+p}^2]_L - [D_m^2]_L}$

# Anti Reflection Coatings:

A.R. Coatings are thin transparent coating of optical thickness of one quarter wavelength given on surface in order to suppress reflection from the surface.

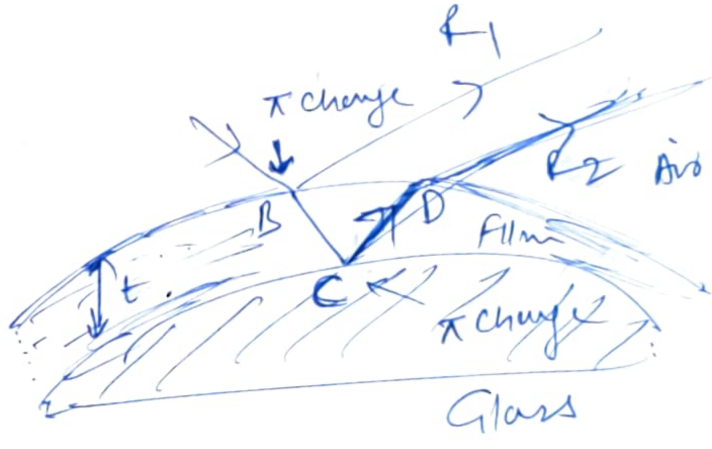
A Thin film can act as A.R. coating if

a) phase condition: The reflected waves from top and bottom surface of thin film are in opposite phase leading to destructive interference.

b) Amplitude condition: The waves have equal amplitudes.

1) Phase Condition and minimum thickness of the film.

Let the thickness of the A.R coating film be 't' and R.I of '$\mu_f$' such that $\mu_1 < \mu_f < \mu_g$




∴ The optical path difference between $R_1$ and $R_2$ must be equal to $\frac{\lambda}{2}$

$$\Delta = 2\mu_f t \cos r - \frac{\lambda}{2} - \frac{\lambda}{2}$$

air to film boundary    film to glass boundary

If we assume the normal incidence $\cos r =$ Ampl

$\therefore \Delta = 2\mu_f t - \lambda \leftarrow$ phase shift of $2\pi$. Th

$\therefore \Delta = 2\mu_f t$

The $R_1$ and $R_2$ interfere destructively

$$\therefore \quad 2\mu_f t = (2m+1)\frac{\lambda}{2} \qquad m = 0, 1, 2, \ldots$$

For the film to be transparent, its thickness should be minimum, which happens when $m = 0$

i.e. $$2\mu_f t_{min} = \frac{\lambda}{2}$$

$$\therefore \quad \boxed{t_{min} = \frac{\lambda}{4\mu_f}} \qquad \text{provided } (\mu_f < \mu_{glass})$$

$\Rightarrow$ The optical thickness of the A.R. Coating should be of one quarter wavelength.

## Amplitude Condition: ⑥

This condition requires the amplitudes of reflected rays $R_1$ and $R_2$ are equal $E_1 = E_2$

i.e. It requires $\left[\frac{\mu_f - \mu_a}{\mu_f + \mu_a}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2$ — (1)

$\mu_a$: R.I of air $\mu_f$: R.I of film $\mu_g$: R.I of glass

Eqn (1) can be re-written as

$$\left[\frac{\mu_f - 1}{\mu_f + 1}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2$$ as $\mu_a = 1$ for air

$$\frac{\mu_f^2 - 2\mu_f + 1}{\mu_f^2 + 2\mu_f + 1} = \frac{\mu_g^2 - 2\mu_g\mu_f + \mu_f^2}{\mu_g^2 + 2\mu_g\mu_f + \mu_f^2}$$

$$\Rightarrow 4\mu_f^3\mu_g + 4\mu_f\mu_g = 4\mu_f^3 + 4\mu_f\mu_g^2$$

Divide by $4\mu_f$ and rearranging we get

$$\mu_f^2 - \mu_g \mu_f + \mu_g^2 - \mu_g = 0$$

$$\therefore \mu_f^2 = \mu_g (1 + \mu_f - \mu_g)$$

If say $\mu_f \simeq \mu_g$ then

$$\mu_f^2 = \mu_g \implies \boxed{\mu_f = \sqrt{\mu_g}}$$

$\Rightarrow$ The R.I of thin film A.R. coating should be less than that of substrate and possibly nearer to its square root.

for glass $\mu_g = 1.5$ $\quad \mu_f = \sqrt{1.5} \simeq 1.22$

The materials have R.I nearer to this is

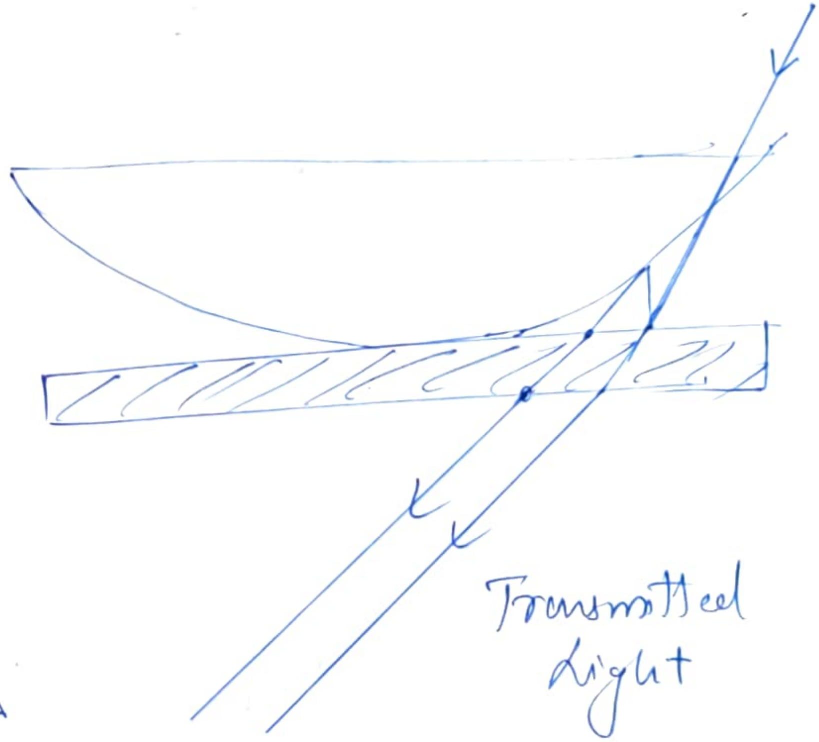
$MgF_2$ ($\mu = 1.38$) magnesium floride (cheaper)

$3NaF \cdot AlF_3$ ($\mu = 1.36$) cryolite

Film should
1) adhere well & durable
2) Scratch proof and insoluble in ordinary solvents.

mostly $MgF_2$ is used in AR coating.

# Newton's Ring by transmitted light



Transmitted light

$\Delta = 2\mu t \cos r$

$\cos r = 1$ for $\perp$ incidence

$\Delta = 2t$

For maxima $\Delta = m\lambda$

$$\boxed{2t = m\lambda}$$

For minima $\Delta = (2m+1)\frac{\lambda}{2}$

$$2t = (2m+1)\frac{\lambda}{2}$$

Radius of $m^{th}$ bright fringe $r_m = \sqrt{m\lambda R}$

for dark fringe $r_m = \sqrt{\frac{(2m+1)\lambda R}{2}}$