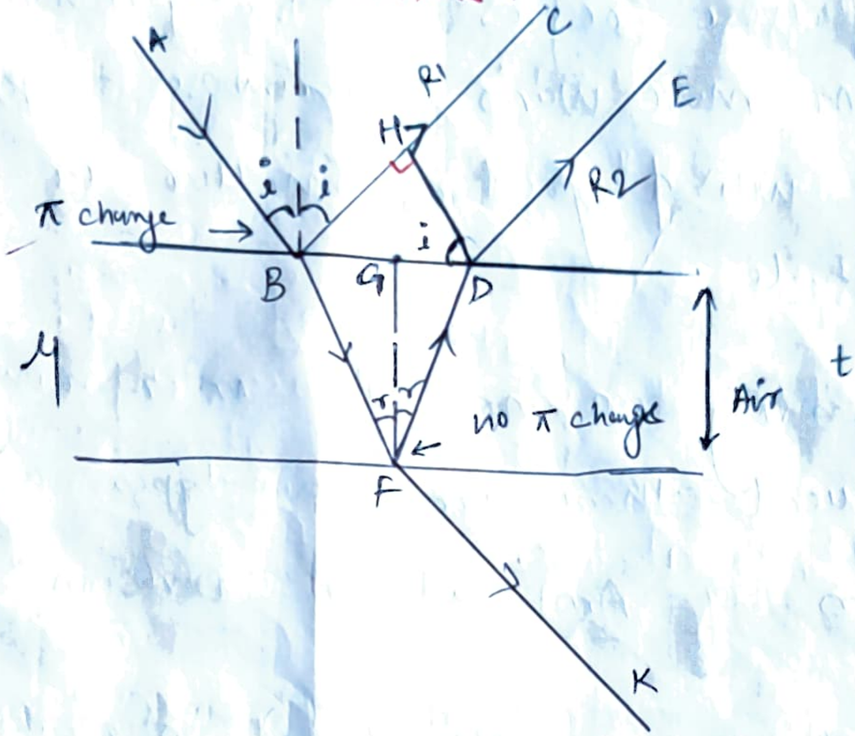
Interference

Interference refers to the phenomena where two or more waves superimpose to form a resultant wave with a greater, smaller or the same amplitude.

Depending on how the interfering waves are produced there are two types of interference

Division of Amplitude	Division of wavefront
1) Interference is achieved by splitting a single wave into two or more by changing its amplitude	1) Interference is produced by dividing the wavefront of single coherent source into two or more parts which travel different paths and recombine
2) This method typically involves the use of partially reflecting surfaces like, mirror or beam splitter, which divide the amplitude of incoming wave.	2) The wavefront is split by obstacles or slits and the resulting waves interfere after travelling along seperate trajectories
3) Ex: Newton's rings: Light is reflected from the top and bottom surface of thin air film between lens and a glassplate creating circular interference rings	3) ex young's double-slit experiment A single light source is split by two slits and the light passes through each slit interfers, creating a fringe pattern.

Q2: Expression for optical path difference in a thin film of uniform thickness:



Ray AB is divided among two rays R_1 & R_2

Let us consider a transparent film of uniform thickness 't' bounded by two parallel surface with R.I. of 'μ'

The incident ray AB is divided among two rays R_1 and R_2 i.e. BC and DE respectively.

Lets compute the optical path difference between the reflected ray BC (R_1) and refracted ray BFDE (R_2)

Draw a normal to BC starting from D to H such that rays HC and DE travels equidistance.

The reflected ray BC travels in air while the refracted ray BF + FD travels in film of R.I μ

∴ The Geometrical path difference (GPD) between R_1 and R_2

$$= (BF + FD) - BH$$

$\therefore$ Optical path difference = μ [G.P.D.]

$$\Delta_a = \mu[(BF + FD)] - 1[BH] \quad — (1)$$

In Δ^{le} BFD $\angle BFG = \angle GFD = r$

$BF = FD$ & $BG = GD$ { Isoceles Δ^{le} law

In Δ^{le} BFG and GFD $\cos r = \frac{t}{BF} = \frac{t}{FD}$

$$\therefore \boxed{BF + FD = \frac{t}{\cos r} + \frac{t}{\cos r} = \frac{2t}{\cos r}} \quad — (2)$$

Also, $\tan r = \frac{BG}{FG} \Rightarrow BG = t\tan r \quad — (3)$

In Δ^{le} BHD $\angle HBD = 90° - i$ & $\angle BHD = 90°$

$\therefore$ In Δ BDH $\sin i = \frac{BH}{BD} \Rightarrow BH = BD \sin i$

$\Rightarrow BH = 2\,BG \sin i \quad — (4)$ as $BD = BG + GD$ & $BG = GD$

from eqn 3 $BG = t\tan r$

$\therefore$ from 3 & 4 $BG = 2(t\tan r).\sin i \quad — (5)$

from snell's law $\sin i = \mu \sin r \quad — (6)$

from 5 & 6 $\quad BH = 2t \cdot \tan r \cdot \mu \sin r$

$$= 2\mu t \frac{\sin^2 r}{\cos r} \quad \text{— (7)}$$

using eqn (1) (2) and (7)

$$\Delta_a = \mu\left(\frac{2t}{\cos r}\right) - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$\Delta_a = 2\mu t \frac{(1-\sin^2 r)}{\cos r} = 2\mu t \cos r$$

∴ optical path difference $= 2\mu t \cos r$

* Imp: Stokes theorem:

when light reflected from ~~denser to rarer~~ rarer to denser medium it undergoes a phase change of π radian or in wavelength by $\frac{\lambda}{2}$ amount.

∴ for Bright band $2\mu t \cos r - \frac{\lambda}{2} = m\lambda$

$$\boxed{2\mu t \cos r = (2m+1)\frac{\lambda}{2}} \quad m = 0, 1, 2 \ldots$$

Condition for Bright Band.

For dark band.

$$2\mu t\cos r - \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}$$

$$2\mu t\cos r = (m+1)\lambda \Rightarrow m\lambda$$

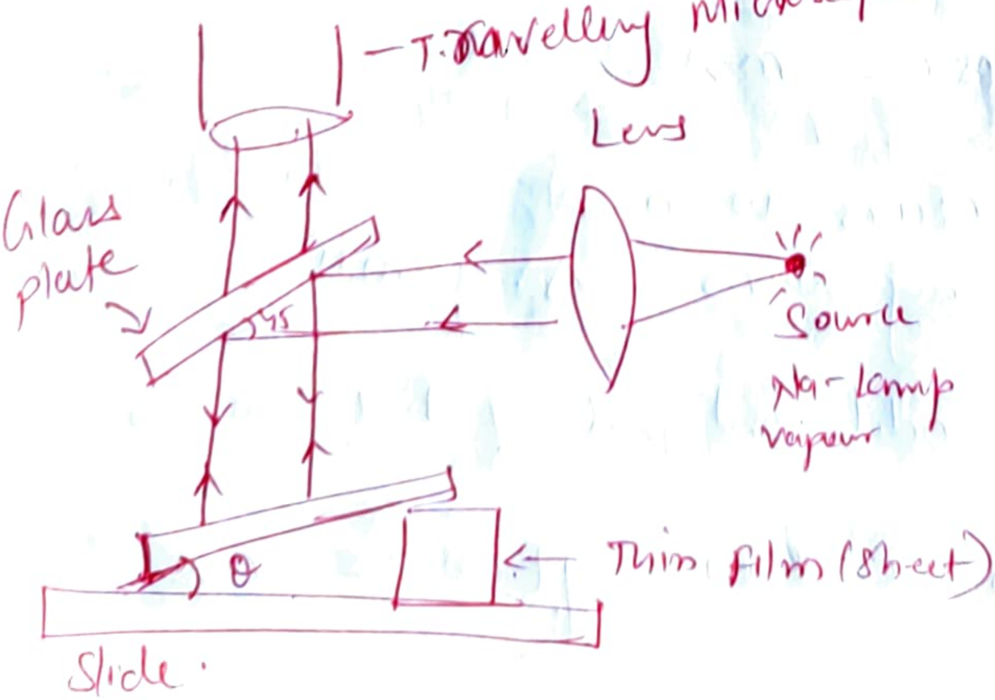
$$\boxed{2\mu t\cos r = m\lambda} \quad m=0,1,2\ldots$$

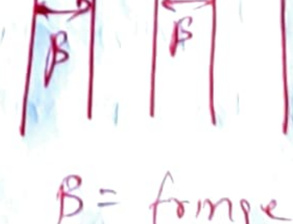
Subtraction or addition of one λ does not change phase relation of interfering waves

Condition for Dark band.

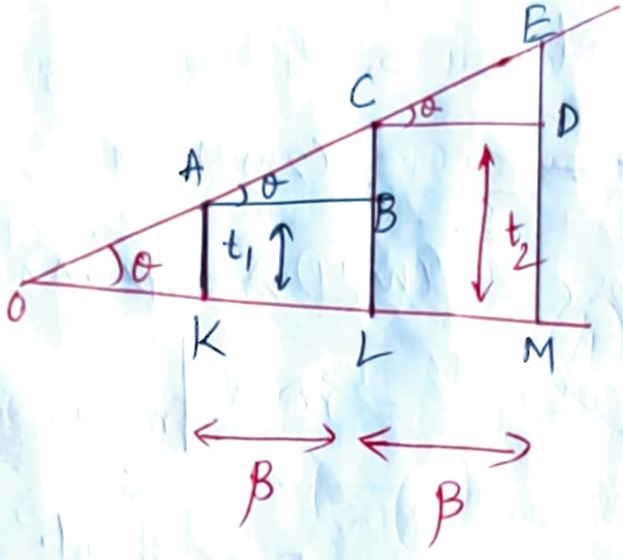
* Expression for fringe width in wedge-shaped thin film.

A wedge is a thin film of varying thickness having a zero thickness at one end and progressively increasing to a particular thickness at the another end.





β = fringe width.



Let AK = 1^{st} Dark band EM = 3^{rd} Dark band
CL = 2^{nd} Dark band

Fringe width d(KL) = d(LM)

maxima occurs when the optical path difference

$\Delta = m\lambda$ integral number of full waves

$\therefore$ from thin film derivation we know that

$$2\mu t \cos r = m\lambda$$

111^{ly} minima occurs when optical path difference between interfering waves $\Delta = (2m+1)\frac{\lambda}{2}$

~~Δ~~ ie odd integral of half waves

from thin film we know $2\mu t \cos r = m\lambda$

Let's assume the Dark fringe occurs at point A

$\therefore \quad 2\mu t \cos r = m\lambda$

at point A the angle of incidence $= 90^\circ$ & $r = 0^\circ$ and the thickness of air film is 't_1' (AK)

$\therefore$ At point A $\quad 2\mu t_1 \cos 0^\circ = m\lambda$

$\therefore 2\mu t_1 = m\lambda$ — (1)

The next dark fringe will occur at C where the thickness CL = 't_2'

At point C we can write $2\mu t_2 = (m+1)\lambda$

— (2)

fringe width β = (2) − (1)

$\beta = 2\mu(t_2 - t_1) = (m+1)\lambda - m\lambda = \lambda$

$\therefore \underbrace{t_2 - t_1}_{(BC)} = \dfrac{\lambda}{2\mu}$ — (3) (where $t_2 - t_1 = BC$

In Δ^{le} ABC, $\angle CAB = \theta$ and $\tan\theta = \dfrac{BC}{AB}$

$\therefore BC = AB \tan\theta$

But $AB = \beta$ as it is distance between two succesive dark fringes.

$\therefore BC = \beta \tan\theta$ — (4)

from (3) and (4) $\beta \tan\theta = \frac{\lambda}{2y}$

$$\therefore \boxed{\beta = \frac{\lambda}{2y \tan\theta}}$$

for small values of θ $\beta = \frac{\lambda}{2y\theta}$

$\tan\theta \simeq \theta$

$$\therefore \boxed{\theta = \frac{\lambda}{2y\beta}}$$

Wedge Angle $= \theta$

fringe width $= \beta$

$\theta \propto \frac{1}{\beta}$ $\theta \uparrow$ $\beta \downarrow$

wavelength $= \lambda$ (5893)

so $\theta \simeq 1^\circ$ the interference pattern vanishes

y = R.I of thin air film

$\therefore$ Less the θ more the seperation & you see the fringes clearly seperated.