Vidyavardhini's College of Engineering and Technology, Vasai (West)

First Year Engineering

Academic Year: 2024-2025

Solution to the prevous year questions papers [2017 − 2024] [1]

Subject: BSC102/AP **Date: 20/11/2024**

Module-5: Quantum Physics

De Broglie's Hypothesis of Matter Wave

Q.1. **Explain De Broglie's hypothesis of matter waves and deduce the expression for wavelength. [3 Marks] [May-2019, May-2023]**
OR
What is e Broglie's hypothesis? Derive expression for De Broglie's wavelength. [4 Marks] [May-2022, May-2024 (3M)]
OR
State de' Broglie hypothesis and derive an expression for de' Broglie wavelength. Mention three properties of matter waves. [5 Marks] [Dec-2023]
OR
State properties of matter waves. [3 Marks] [Nov-2018, May-2023]
OR
What are matter Waves? State three properties of matter waves. [3 Marks] [Dec-2022]

Ans: Interference, diffraction requires wave nature for their explanation. In photo-electric effect Einstein visualized the incident light as a sort of particles which he called photons and accounted for the emission of electrons as due to the collision between these photons and electrons bound to the metal. During the collision, the photon transfers all its energy to the electrons which results in the emission of photo electrons. Here the behaviour of light is same as that of a particle.

Louis De Broglie put forward the dual behaviour in terms of hypothesis which states If the radiation behaves as particle under certain circumstances, then one can even expect that, entities which ordinarily behave as particles to exhibit properties attributed to only waves under appropriate circumstances. The concept that matter behaves like a wave was proposed by Louis de Broglie in 1924. It is also referred to as the de Broglie hypothesis of matter waves. On the other hand de Broglie hypothesis is the combination of wave nature and particle nature.

If E is the energy of a photon of radiation havinf frequency ν and the same energy can be written for a wave as follows

$$\text{Then its energy is } \mathrm{E} = \mathrm{h}\nu. \text{ (wave nature)}$$
$$\text{It can also be represented as } \mathrm{E} = \mathrm{mc}^2. \text{ (particle nature)}$$
$$\therefore \quad \mathrm{h}\nu = \mathrm{mc}^2$$

$$\therefore mc^2 = h\nu \implies \frac{mc^2}{c} = \frac{h\nu}{c} = \frac{h\nu c}{\lambda} \implies \therefore \lambda = \frac{h}{p}$$

where λ = De Broglie wavelength and p = momentum associated with photon which travels in free space.

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Properties of matter waves

1. The wavelength of a matter wave is inversely related to its particles momentum. if the particle moves faster, then the wavekegth will be smaller and vice versa
2. If the particle is at rest , then the de Broglie wavelength is infinite. Such waves cannot be visualized.
3. Matter wave can be reflected, refracted, diffracted and undergo interference
4. The amplitude of the matter waves at a particular region and time depends on the probability of finding the particle at the same region and time.
5. The position and momentum of the material particles cannot be determined accurately and simultaneously.
6. Matter wave is independent of the charge of the particle

Q.2. **For an electron passing through potential difference 'V', show that its wavelength is; $\lambda = \frac{12.26}{\sqrt{(V)}} A^{\circ}$. [5 Marks] [Nov-2018]**

Ans: De Broglie wavelength for a matter wave is given by

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{mv}}; \quad \text{where} \lambda = \text{DeBrogliewavelength} \qquad (1)$$

From eqn. (1) we find that, if the particles like electrons are accelerated to various velocities, we can produce waves of various wavelengths. Thus higher the electron velocity, smaller will be the de-Broglie wavelength. If velocity v is given to an electron by accelerating it through a potential difference V, then the work done is converted to kinetic energy of electron. Hence, we can write

$$\mathrm{mv} = \sqrt{2\mathrm{meV}} \qquad (2)$$

Substituting eqn.(2) in eqn.(1) we get

$$\lambda = \frac{\mathrm{h}}{\sqrt{2\mathrm{meV}}}$$

Substituting the the values of h = 6.625 x 10^{-34} J-sec, m = 1.6 x 10^{-19} C, e = = 9.1 x 10^{-31} Kg for electron we get $\lambda = \frac{12.26}{\sqrt{V}} A^{\circ}$.

Q.3. **Calculate the de Broglie wavelength of alpha particles accelerating through a potential difference of 150 volts. Given mass of Alpha particle is 6.68 x 10^{-17} Kg. [3 Marks] [May-2019]**

Ans:

Given : $V = 150$ V, $m_{\alpha} = 6.68 \times 10^{-27}$ kg

We know that charge of an α particle $q_{\alpha} = 3.2 \times 10^{-19}$ C

Required : λ_{α}

To find the wavelength λ_{α} of α particle

$$\lambda_{\alpha} = \frac{h}{\sqrt{m_{\alpha} \cdot q_{\alpha} \cdot V}} \qquad \text{...Std. formula}$$

$$\therefore \lambda_{\alpha} = \frac{6.63 \times 10^{-34}}{\sqrt{6.68 \times 10^{-27} \times 3.2 \times 10^{-19} \times 150}}$$

$$= \mathbf{1.17 \times 10^{-12}\ m} \qquad \text{...Ans.}$$

Q.4. **What is the wavelength of a beam of neutron having: (i) an energy of 0.025 eV? (ii) an electron and photon each have wavelength of 2 A°. What are their momentum and enegy?** $m_n = 1.67x10^{-27}kg, h = 6.625x10^{-34}$ **J-sec.** **[5 Marks] [May-2017, Dec2021]**

Ans: Given Data: energy of neutron = 0.025 eV.
To find : wavelength of a beam.
Calculation :

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.626x10^{-34}}{\sqrt{2x1.676x10^{-27}x0.025x10^{-19}x1.6}} = 1.8095A^\circ$$

Hence wavelength is equal to 1.8095 A°

2. Given Data $\lambda = 2A^\circ, m_n = 1.676x10^{-27}kg, h = 6.625x10^{-34}J - sec.$

To find :- momentum and energy?
Calculations : $\lambda = h/p \implies p = 3.3125x10^{-24}$ kg-m/sec.

$$\lambda = h/\sqrt{2mE} \quad \implies \quad 2x10^{-10} = \frac{6.625x10^{-34}}{\sqrt{2x1.676x10^{-27}xE}} \quad \implies \quad E = 5.721x10^{-11}\text{joules.}$$

Hence momentum = 3.3125 x 10^{-24} kg-m/sec. and energy is = 5.721 x 10^{-11} joules.

Q.5. **Calculate the frequency and wavelength of photon whose energy is 75 eV.**
[5 Marks] [May 2018]

Q.6. **Find the de Broglie wavelength of (i) an electron accelerated through a potential difference of 182 Volts and (ii) 1 Kg object moving with a speed of 1 m/s. Comparing the results, explain why is the wave nature of matter not apparent in daily observations?**
[5 Marks] [Dec-2022]

Ans:

Given :
(i) Electron accelerated through P.D. of V = 182 V.
(ii) 1 kg Object moving with a speed v_o = 1 m/s.

Required :
Explanation about the wave nature of matter not being more apparent in daily observations.

Step 1 : To find the wavelength (λ_e) of the electron.

$$\lambda_e = \frac{12.26}{\sqrt{V}} \quad \text{...Std. formula}$$

$$\therefore \lambda_e = \frac{12.26}{\sqrt{182}} = 0.9087 \text{ Å}$$

$$= \mathbf{0.9087 \times 10^{-10}\ m/s} \quad \text{...Ans.}$$

Step 2 : To find the wavelength (λ_o) of 1kg object.

$$\lambda_o = \frac{h}{m_o v_o} \quad \text{...Std. formula}$$

$$\therefore \lambda_o = \frac{6.63 \times 10^{-34}}{1 \times 1} = \mathbf{6.63 \times 10^{-34}\ m/s} \quad \text{...Ans.}$$

Step 3 : The explanation about the wave nature of matter in daily observations

The wavelength (λ_e) of electron is measurable while the wavelength (λ_o) of the object is too **small** and not measurable hence wave nature of matter is not apparent in daily observations.

Heisenberg's Uncertainty Principle

Q.7. **With Heisenberg's uncertainty principle prove that electron cannot survive in nucleus. An electron has a speed of 300 m/sec. with uncertainty of 0.01%. Find the accuracy in its position. [4 Marks] [Dec-2017].**

OR

State Heisenberg's Uncertainty Principle. Show that electron doesn't exist in the nucleus. Find the accuracy in the position of an electron moving with speed 350 m/sec with uncertainty of 0.01%. [8 Marks] [Nov-2018]

OR

Discuss Heisenberg's Uncertainty principle and prove that electrons cannot reside inside the nucleus of an atom using the same principle. [8 Marks] [May-2024]

Ans: Physical quantities like position, momentum, time, energy etc. can be measured accurately in macroscopic systems (i.e. classical mechanics). However, in the case of microscopic systems, the measurement of physical quantities for particles like electrons, protons, neutrons, photons etc are not accurate. If the measurement of one is certain and that of other will be uncertain.

Thus according to uncertainty principle states that *the position and the momentum of a particle in an atomic system cannot be determined simultaneously and accurately. If Δx is the uncertainty associated with the position of a particle and Δp_x the uncertainty associated with its momentum, then the product of these uncertainties will always be equal or greater than $h/4\pi$. That is*

$$\Delta x \Delta p_x \geq h/4\pi$$

Nonexistence of electron in the nucleus

The radius 'r' of the nucleus of any atom is of the order of 10^-14m so that if an electron is confined in the nucleus, the uncertainty in its position will be of the order of 2r = Δx (say) i.e diameter of the nucleus. According to Heisenberg's Uncertainty principle

$$\Delta x \Delta p_x \geq h/4\pi \qquad \text{where} \quad \Delta\mathrm{x} \sim 10^{-14}\mathrm{m}$$

Therefore,

$$\Delta p = h/(4\pi\Delta x) = 6.625\mathrm{x}10^{-34}/(4\pi\mathrm{x}2\mathrm{x}10^{-14}) = 2.63\mathrm{x}10^{-21}\mathrm{kg-m/s}$$

Taking Δp $\sim$ p we can calculate energy using the formula

$$\mathrm{E}^2 = \mathrm{c}^2[\mathrm{p}^2 + \mathrm{m}_0^2\mathrm{c}^2] = (3\mathrm{x}10^8)^2\mathrm{x}[(2.63\mathrm{x}10-21)^2 + (9.1\mathrm{x}10-31)^2\mathrm{x}(3\mathrm{x}10^8)^2]$$
$$= 7.932\mathrm{x}10^{-13}\mathrm{J} = 4957745\mathrm{eV} \sim 5\mathrm{MeV}$$

However, the experimental investigations on beta decay reveal that the kinetic energies of electrons must be equal to 4MeV. Since there is a disagreement between theoretical and experimental energy values we can conclude that electrons cannot be found inside the nucleus.

Q.8. **An electron has a speed of 400 m/sec with uncertainty of 0.01%. Find the accuracy in its position. [5 Marks] [May-2023]**

Ans:

Given : $v_{eb} = 400$ m/s, % accuracy of speed = 0.01

Required :

(accuracy in posion of electron) i.e. Δx_e

▸ **Step 1 :** To find the momentum (P_e)

$$P_e = m_e \cdot v_e \qquad \text{...Std. formula}$$

$$\therefore \; P_e = 9.11 \times 10^{-31} \times 400$$

$$= 3.644 \times 10^{-23} \text{ kg–m/s}$$

▸ **Step 2 :** To find $\frac{\Delta v_e}{v_e}$ (% accuracy of speed)

$$= \frac{\Delta v_e}{v_e} \times 100 = 0.01$$

$$\therefore \; \frac{\Delta v_e}{400} \times 100 = 0.01$$

or $\Delta v = 0.04$

▸ **Step 3 :** To find ΔP_e

$$P_e = m_e \cdot v_e \qquad \text{...Std formula}$$

$$\therefore \; \Delta P_e = m_e \times \Delta v_e$$

Or $\Delta P_e = 9.11 \times 10^{-31} \times 0.04 = 0.3644 \times 10^{-31}$

▸ **Step 4 :** To find Δx_e

As per Heisenberg's uncertainity principle,

$$\Delta x \cdot \Delta P_x \geq \frac{h}{2\pi} \qquad \text{...Std. theory}$$

$$\therefore \; \Delta x_e \times \Delta P_e \geq \frac{h}{2\pi}$$

$$\therefore \; \Delta x_e \times 0.3644 \times 10^{-31} \geq \frac{6.63 \times 10^{-34}}{2\pi}$$

or $\Delta x_e = 2.895 \times 10^{-3}$ m ...Ans.

Q.9. **Find the lowest energy of a neutron within a nucleus of dimension 10-14 m. given mass of a neutron $1.67X10^{-27}$ kg.** [4 Marks] [May-2023]

Ans:

Given	**Required**
mass of a neutron $m = 1.67 \times 10^{-27}$ kg,	Lowest energy of neutron E_{min} in the nucleus.

Nucleous diameter

$= 10^{-14}$ m $= (\Delta x)_{max}$

(1) According to Helsenberg's Uncertainity Principle we have,

$$(\Delta p)_{min} (\Delta x)_{max} = h$$

$$\therefore \; (\Delta p)_{min} = \frac{h}{(\Delta x)_{max}} = \frac{6.626 \times 10^{-34}}{5 \times 10^{-14}}$$

$$= 1.33 \times 10^{-20} \text{ kg-m/sec.}$$

(II) $\because$ **p cannot be less than $(\Delta p)_{min}$, we have**

$$p_{min} = (\Delta p)_{min}$$

(III) $$E_{min} = p_{min}^2 = \frac{(1.33 \times 10^{-20})^2}{2 \times 1.67 \times 10^{-27}}$$

$$= \mathbf{0.529 \times 10^{-13}} \qquad \text{...Ans.}$$

Q.10. **Arrive at Heisenberg's uncertainty principle with single slit electron diffraction. An electron has a speed of 300m/sec. with uncertainty of 0.01%. Find the accuracy in its position.** [7 Marks] [May 2018]

Ans: Similar to above problem(s)

Q.11. **Show that Non- Existence of electron in the Nucleus. Find the uncertainty in the position of electron. The speed of an to an accuracy of of 0.002%.** [8 Marks] [Dec-2019]

Ans: Similar to above problem(s)

Schrödinger's Wave Equation

Q.12. **Derive Schrödinger's time dependent wave equation for matter waves. OR Derive one dimensional Schrödinger's time dependent equation for matter waves. OR Obtain one dimensional time dependent Schrödinger equation [May-2017, 2018, 2023, Dec-2022] [5 Marks]**

Ans:

(I) The equation of the de **Broglie wave** for the motion of a free particle is given by,

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\cdot\frac{d^2\psi}{dt^2}$$

Where ψwave function and v is the phase velocity.

(II) Solution of the above second order differential equation is of the form,

$$\psi(x, t) = A\, e^{-i(Et - px)/h}$$

Where, p = mv is the momentum of the particle of the mass m and

h = Planck's constant = 6.634×10^{-34} J-s

(III) Differentiating Equation (5.3.2) partially we get,

$$\frac{d\psi}{dt} = -\frac{iE}{h} \times Ae^{-i(Et - px)/h} = -\frac{iE}{h}\cdot\psi \text{ or}$$

$$E\psi = -\frac{h}{i}\frac{d\psi}{\partial t} \qquad \text{...(5.3.3)}$$

(IV) Taking double partial differentiation of Equation (5.3.2) we get,

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{p^2}{h^2}\psi \text{ or } p^2\psi = -h^2\frac{\partial^2\psi}{\partial x^2} \qquad \text{...(5.3.4)}$$

(V) Let the particle be placed in a field of voltage V, then it will acquire a potential energy of P.E. = V.

(VI) Also it has K.E. = $\frac{p^2}{2m}$

(VII) Therefore its total energy,

$$E = \text{K.E.} + \text{P.E.} = \frac{p^2}{2m} + V \quad \text{or} \quad \frac{p^2}{2m} = E - V,$$ multiplying both the sides by ψ

We get, $$\frac{p^2}{2m}\cdot\psi = E\psi - V\psi \qquad \text{...(5.3.5)}$$

(VIII) Putting the values of Eψ from Equation (5.3.3) and $p^2\psi$ from Equation (5.3.4) in Equation (5.3.5) we get,

$$-\frac{h^2}{2m}\cdot\frac{\partial^2\psi}{\partial x^2} = -\frac{h}{i}\frac{\partial\psi}{\partial t} - V\psi$$

or $$-\frac{h^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = -\frac{h}{i}\frac{\partial\psi}{\partial t}$$

This is the required Schrodinger's time dependent wave equation.

Q.13. **Write the expression for Schrödinger's time dependent equation of matter waves and derive Schrodinger's time independent equation. [5 Marks] [May-2024]**

Ans: State the Schrödinger's time dependent equation derived in Q.12 and then derive the following

(I) The general differential equation of a matter wave travelling in x-direction is given by,

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2} \qquad ...(5.3.6)$$

where ψ is a wave function, **u** = phase velocity

(II) The general solution of the above equation is of the form

$$\psi = \psi_o \cdot e^{i(kx-\omega t)} \qquad ...(5.3.7)$$

where, ψ_o = a constant

(III) Differentiating Equation (5.3.7) partially with respect to 't' we get,

$$\frac{\partial\psi}{\partial t} = (-i\omega)\,\psi_o\, e^{i(kx-\omega t)} \qquad ...(5.3.8)$$

(IV) Differentiating Equation (5.3.8) partially with respect to 't',

$$\frac{\partial^2\psi}{\partial t^2} = (-i\omega)^2\psi_o\, e^{i(kx-\omega t)} = -\omega^2\psi_o\, e^{i(kx-\omega t)} = -\omega^2\psi \qquad ...(5.3.9)$$

(V) Substituting in Equation (5.3.6) we get,

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{\omega^2}{u^2}\cdot\psi$$

(VI) But $\omega = 2\pi\cdot\gamma = \frac{2\pi u}{\lambda}$ where, v = frequency $= \frac{u}{\lambda}$

$$\therefore \frac{\omega}{u} = \frac{2\pi}{\lambda} \qquad \therefore \frac{\omega^2}{u^2} = \frac{4\pi^2}{\lambda^2} \qquad ...(5.3.10)$$

(VII) By de-Broglie hypothesis,

$$\lambda = \frac{h}{p}$$

$\therefore$ Substituting this value of λ in Equation (5.3.10)

$$\text{we get} \therefore \frac{\omega^2}{u^2} = \frac{4\pi^2 p^2}{h^2} \qquad ...(5.3.11)$$

(VIII) We know that,

Total energy = Kinetic energy + Potential energy

i.e. T.E. = K.E. + P.E. But K.E. $= \frac{1}{2}$ mv^2

$$\therefore \text{T.E.} = \frac{1}{2}mv^2 + \text{P.E.} = \frac{1}{2}\frac{m^2v^2}{m} + \text{P.E.} = \frac{p^2}{2m} + \text{P.E.} \quad (\because p = mv)$$

$$\therefore p^2 = 2m(\text{T.E.} - \text{P.E.})$$

(IX) Substituting in Equation (5.3.11), we get,

$$\frac{\omega^2}{u^2} = \frac{8\pi^2 m}{h^2}(\text{T.E.} - \text{P.E.})$$

(X) Substituting in Equation (5.3.9), we get

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{8\pi^2 m}{h^2}(\text{T.E.} - \text{P.E.})\cdot\psi$$

$$\therefore \frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(\text{T.E.} - \text{P.E.})\,\psi = 0$$

(XI) The above equation is used when the P.E. is constant in time but varies in space.

$\therefore$ Replacing the above equation by total derivative, we get,

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(\text{T.E.} - \text{P.E.})\Psi = 0$$

This is the required one dimensional time independent Schrodinger equation for the matter wave.

Particle Enclosed in a Rigid Box

Q.14. **Derive the expression for energy eigen values for free particle in one dimensional potential well.** **[4 Marks] [May-2022]**

OR

Show that the energy of an electron in a one-dimensional deep potential well of infinite height varies as the square of the natural numbers. **[7Marks] [May-2022]**

Ans:

MU - Dec. 15, Q. 3(b), Dec. 22, 5 Marks

(I) (a) Free particle means that particle which is not acted upon by any force. Therefore its P.E. = 0 and it moves in positive x-direction.

(b) The time independent Schrodinger wave equation for such a free particle is given by,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 mE}{h^2}\psi = 0$$

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + k^2\psi = 0, \text{ where, } k^2 = \frac{8\pi^2 mE}{h^2}$$

(c) $\therefore E = \frac{h^2}{8\pi^2 m} k^2$ $\therefore \boxed{E \propto k^2}$ $\because$ h and m are constants.

Hence, the energy is continuous. ($\because$ Square is always continuous).

(II) (a) Suppose **the particle of mass m is free to move** in the x-direction only in the region from x = 0to x = a.

(b) Outside this region, its P.E.: $(V) \to \infty$ and within this region, it is 0. Further, the particle does not lose energy when it collides. Therefore its energy remains constant.

(c) According to Schrodinger equation,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

$\because$ V = 0 inside the box

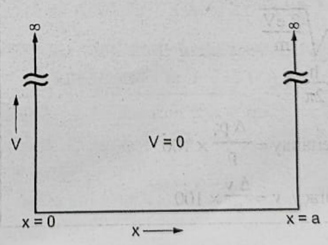
Therefore we get,

$$\therefore \frac{d^2\psi}{dx^2} + \frac{8\pi^2 m E}{h^2}\psi = 0$$

(d) This equation can be written as,

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad ...(5.3.12)$$

$$\text{where, } k^2 = \frac{8\pi^2 mE}{h^2}$$



(1B6)**Fig. 5.3.1**

(e) Solution of Equation (5.3.12) is given by,

$$\psi = A \cdot (\cos kx) + B (\sin kx)$$

When, $x = 0$ at $\psi = 0$

$\therefore 0 = A \cos 0 + B \sin 0$

$\therefore A = 0$

$\therefore B \cdot \sin (ka) = 0, \; B \neq 0$

$\therefore \sin (ka) = 0$

(f) This is possible only when, $ka = n\pi$ (where n = 0, 1, 2, ...)

where, n = Quantum number.

$$\therefore \psi_n = B \sin\left(\frac{n\pi}{a} \times x\right)$$

(g) Now, $k = \frac{n\pi}{a}$ $\therefore k^2 = \frac{n^2\pi^2}{h^2} = \frac{8\pi^2 mE}{h^2}$

$$\therefore E = \frac{n^2 h^2}{8 ma^2}$$ where n = 1, 2, 3, ... are the natural numbers

Thus, the energy of an electron in the box varies as the square of the natural numbers.

Energy levels in one dimensional box are quantized. Since they can have only certain values known as the **'Eigen Values'**.

Q.15. **The ground state energy of an electron in an infinite well is 5.6 x10^{-3} eV. What will be the ground state energy if the width of the well is doubled? [4 Marks] [May-2022]**

Ans:

Note : 1eV = 1.602×10^{-19} Joules (J) or Volts (V)

$\therefore E_0 = 5.6 \times 10^{-3} \times 1.602 \times 10^{-9} = 8.971 \times 10^{-22}$ V

(I) Let 'a' be the original width then given that new width $E_n = 2a$.

(I) $$E_0 = \frac{(1)^2 h^2}{8m_e a^2} \quad ...(1)$$

and $$E_{0n} = \frac{(1)^2 h^2}{8m_e (2a^2)} = \frac{(1)^2 h^2}{32 m_e a^2} \quad ...(2)$$

(II) Dividing Equation (2) by Equation (1)

We get, $$\frac{E_{on}}{E_o} = \frac{1}{4}$$

$$\therefore \quad E_{on} = \frac{E_o}{4} = \frac{8.971 \times 10^{-22}}{4}$$

$$= \mathbf{2.242 \times 10^{-7}\ V} \quad \textbf{...Ans.}$$

Q.16. **An electron is bound in a one-dimensional potential well of width 2 A° but of infinite height. Find its energy values in the ground state and in first excited state. [3 Marks] [Dec-2022]**

Ans:

(I) $$E_0 = \frac{(1)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9 \times 10^{-31} \times (10 \times 10^{-10})^2}$$

$$= \mathbf{0.61 \times 10^{-17}\ V} \quad \textbf{...Ans.}$$

(II) $$E_1 = \frac{(2)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9 \times 10^{-31} \times (10 \times 10^{-10})^2}$$

$$= \mathbf{2.442 \times 10^{-17}\ V} \quad \textbf{...Ans.}$$

Q.17. **The minimum energy possible for a particle trapped in a 1-d box is 3.2 x 10^{-18} J. What are the next 5 three energies in eV the particle can have? [5 Marks] [May-2023]**

Ans:

Given	Required
$E_o = 3.2 \times 10^{-18}$ J	E_1, E_2, E_3 in eV

(I) $E_0 = \dfrac{(1)^2 \times h^2}{8\ ma^2} = 3.2 \times 10^{-18}$ J or V (given)

$\therefore \dfrac{h^2 \times 8}{8\ ma^2} = 3.2 \times 10^{-18}$

(II) $E_1 = \dfrac{(2)^2 \times h^2}{8\ ma^2} = 4 \times \dfrac{h^2}{8\ ma} = 4 \times 3.2 \times - 10^{-18}$

$= \dfrac{12.8 \times 10^{-8}}{1.602 \times 10^{-19}} = \mathbf{79.9\ eV}$ **...Ans.**

Similarly,

(III) $E2 = (3)^2 \times \dfrac{(2)^2 \times h^2}{8\ ma} = 9 \times 3.2 \times 10^{-18}$

$= 28.8 \times 10^{-18}$ V

$\therefore$ E2 in eV $= \dfrac{28.8 \times 10^{-18}}{1.602 \times 10^{-19}}$

$= \mathbf{179.775\ eV}$ **...Ans.**

(IV) $E^3 = (4)^2 \times \left(\dfrac{h^2}{8\ ma}\right) = 16 \times 3.2 \times 10^{-18}$

Concert E3 in eV from Joules.

Q.18. **An electron is bound in a one-dimensional potential well of width 5 A° but of infinite height. Find its energy values in the ground state and in first two excited states. [5 Marks] [Dec-2023]**

Ans: Similar to above problem(s)