Statistics Assignment

Problem Statement

Comprehension

The pharmaceutical company Sun Pharma is manufacturing a new batch of painkiller drugs, which are due for testing. Around 80,000 new products are created and need to be tested for their time of effect (which is measured as the time taken for the drug to completely cure the pain), as well as the quality assurance (which tells you whether the drug was able to do a satisfactory job or not).

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.
b.) Calculate the required probability.

Answer:

a.) **Binomial probability distribution** will accurately portray the above scenario and the formula is given by

$$P(X = r) = {}^{n}C_{r}(p)^{r}(1-p)^{n-r}$$

The three conditions that this distribution follows:

1. Total number of trials is fixed at n
2. Each trial is binary, i.e., has only two possible outcomes - success or failure
3. Probability of success is same in all trials, denoted by p

b.) Let S be the event that drug produce satisfactory results and B be the event that the drug do not produce satisfactory result then $4P(B) = P(S)$ with condition P(S) + P(B) = 1
Now we get P(S) = 4/5 and P(B) = 1/5

Let say X be the event that out of 10 trials at most, 3 drugs are not able to do a satisfactory job, then X is a random variable with a Binomial Distribution, where n=10, r=3 and p=1/5.
Now the probability is given by

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$
$$= 0.107+0.268+0.3019+0.2013$$
$$= 0.878$$

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.
b.) Find the required range

Answer:

a.) The main methodology is central limit theorem which states that no matter how the original population is distributed, the sampling distribution will follow these three properties:
    1. Sampling distribution's mean ($\mu_{\bar{X}}$) = Population mean (μ)
    2. Sampling distribution's standard deviation (Standard error) = $\frac{\sigma}{\sqrt{n}}$, where σ is the population's standard deviation and n is the sample size
    3. For n > 30, the sampling distribution becomes a normal distribution

To summarize, let's say we have a sample with sample size n, mean $\bar{X}$ and standard deviation S. Now, the y% confidence interval (i.e. the confidence interval corresponding to y% confidence level) for μ would be given by the range:

Confidence interval = $(\bar{X} - \frac{Z^* S}{\sqrt{n}}, \bar{X} + \frac{Z^* S}{\sqrt{n}})$

Where, Z* is the Z-score associated with a y% confidence level. In other words, the population mean and sample mean differ by a margin of error given by $\frac{Z^* S}{\sqrt{n}}$

b.) We have sample size n = 100, mean $\bar{X}$ = 207, standard deviation S = 65 and confidence level = 95%

Now Z-score associated with a 95% confidence level is ± 1.96

So, margin of error = Z*S / √n

= 1.96*65 / √100 = 12.74

Hence the required range = (207-12.74) to (207+12.74)

= 194.26 to 219.74

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure(with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having α and β as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both α and β values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your choice(Assume that only the values of α and β as mentioned above are provided to you and no other information is available).

Answer:

a.) Defining the null hypothesis H_o: $\mu \leq 200$ sec and The alternate hypothesis H_1: $\mu > 200$ sec

1. **Using Critical Value Method**

   Step 1 : The area of the critical region beyond the only critical point, which is on the right side, is 0.05 So, the cumulative probability of the critical point (the total area till that point) would be (1 – 0.05) = **0.950**.

Step 2: find the Zc, which would basically be the z-score for the value of 0.950. 0.950 is not there in the z-table. So, we will look for the numbers nearest to 0.950. We can see that the z-score for 0.9495 is 1.64 (1.6 on the horizontal bar and 0.04 on the vertical bar), and the z-score for 0.9505 is 1.65. So, taking the average of these two, the z-score for 0.9500 is **1.645**.

Step 3: The critical value can be calculated from $\mu + Zc * (\sigma/\sqrt{N})$.

$= 200 + 1.645(65/\sqrt{100})$

$= 200 + 10.69$

$= 210.69$

Step 4: Since 207 ($\bar{x}$) is less than 210.69, $\bar{x}$ lies in the acceptance region and we **fail to reject the null hypothesis.**

2. **Using p- value Method**

Step 1: We can calculate the z-score for sample mean 207 using the formula: $(\bar{x} - \mu) / (\sigma/\sqrt{n})$. This gives us $(207 - 200) / (65/\sqrt{100}) = 7/6.5 =$ **1.07**. the sample mean lies on the right side of the hypothesized mean of 200, the z-score comes out to be positive.

Step 2: The value in the z-table corresponding to 1.0 on the vertical axis and 0.07 on the horizontal axis is 0.8577. Since the sample mean is on the right side of the distribution and this is a one-tailed test, the p-value would be same 1 - 0.8577 = **0.1423**

Step 3: Since the p-value is greater than the significance level (**0.1423 > 0.05**), we **fail to reject the null hypothesis.**

b.) Examples of a situation

1. Where conducting a hypothesis test having α and β as 0.05 and 0.45 respectively would be preferred over having them both at 0.15 is a criminal trial, a man goes to trial where he is being tried for the murder of his wife. The hypotheses being tested are: H_o: Not Guilty and H_1: Guilty

   Type I error (0.05 or 5%) is committed if we reject H_o when it is true. In other words, did not kill his wife but was found guilty and is punished for a crime he did not really commit.

   Type II error (0.45 or 45%) is committed if we fail to reject when it is false. In other words, if the man did kill his wife but was found not guilty and was not punished.

   Reason for choice : in general not punishing a guilty is preferred more than punishing a not guilty.

2. where conducting the hypothesis test with both α and β values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively is a Culinary Arts Study,

A group of culinary arts students is comparing two methods for preparing asparagus: traditional steaming and a new frying method. They want to know if patrons of their school restaurant prefer their new frying method over the traditional steaming method. A sample of patrons are given asparagus prepared using each method and asked to select their preference. A statistical analysis is performed to determine if more than 50% of participants prefer the new frying method:
H_o: $p = 0.5$ and H_1: $p > 0.5$
Type I error occurs if they reject the null hypothesis and conclude that their new frying method is preferred when in reality is it not.
Type II error occurs if they fail to reject the null hypothesis and conclude that their new method is not superior when in reality it is.
Reason for choice : In both error no catastrophe happen, only the wrong belief will get created.

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer:

A/B testing allows individuals, teams, and companies to make careful changes to their user experiences while collecting data on the results. This allows them to construct hypotheses, and to learn better why certain elements of their experiences impact user behavior. In another way, they can be proven wrong—their opinion about the best experience for a given goal can be proven wrong through an A/B test.

A/B Testing Process

- **Collect Data:** Our analytics will often provide insight into where we can begin optimizing. It helps to begin with high traffic areas of our site or app, as that will allow us to gather data faster. Look for pages with low conversion rates or high drop-off rates that can be improved.

- **Identify Goals:** Our conversion goals are the metrics that we are using to determine whether or not the variation is more successful than the original version. Goals can be anything from clicking a button or link to product purchases and e-mail signups.
- **Generate Hypothesis:** Once we've identified a goal we can begin generating A/B testing ideas and hypotheses for why we think they will be better than the current version. Once we have a list of ideas, prioritize them in terms of expected impact and difficulty of implementation.
- **Create Variations:** Using our A/B testing software (like Optimizely), make the desired taglines
- **Run Experiment:** Kick off our experiment and wait for visitors to participate! At this point, visitors to our site or app will be randomly assigned to either the control or variation of our experience. Their interaction with each experience is measured, counted, and compared to determine how each performs.
- **Analyze Results:** Once our experiment is complete, it's time to analyze the results. Our A/B testing software will present the data from the experiment and show us the difference between how the two versions of our taglines performed, and whether there is a statistically significant difference.