Applied Digital Signal Processing

Assignment-1

1. Assume an ideal low pass analog filter with cutoff frequency Ω_c. Derive the impulse response of this LPF.

Answer:

The frequency response is given by

$$K(f) = \begin{cases} 1 & if \ -\Omega_c \leq f \leq \Omega_c \\ 0 & otherwise \end{cases}$$

We know that,

$$h(t) = \int_{-\infty}^{\infty} H(f)\, e^{2j\pi f t}\, df$$

$$= \int_{-\Omega_c}^{\Omega_c} H(f)\, e^{2j\pi f t}\, df$$

$$= {}_{-\Omega_c}^{\Omega_c}\left[\frac{e^{2\pi f t}}{2j\pi t}\right]$$

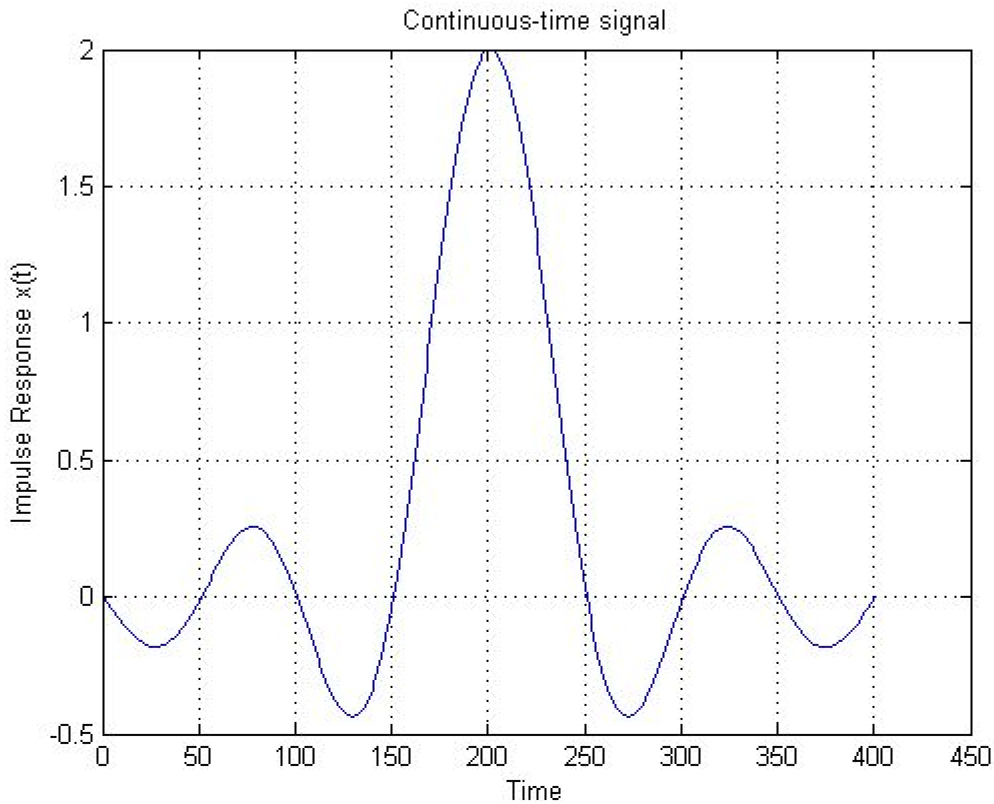
$$= \left[\frac{e^{2\pi\Omega_c t} - e^{-2\pi\Omega_c t}}{2j\pi t}\right]$$

$$= \frac{sin\ (2\ \pi\Omega_c\ t)}{\pi t}$$

$$= \frac{sin\ (2\ \pi\Omega_c\ t)2\Omega_c}{2\Omega_c\pi t}$$

$$h\ (t) = 2\Omega_c sinc(2\Omega_c t)$$

Therefore impulse response of LPF is given by '$2\Omega_c sinc(2\Omega_c t)$'



2. Sample the impulse response using Octave/MATLAB/SCILAB/PYTHON/C, and demonstrate the effect of varying the sampling rate.

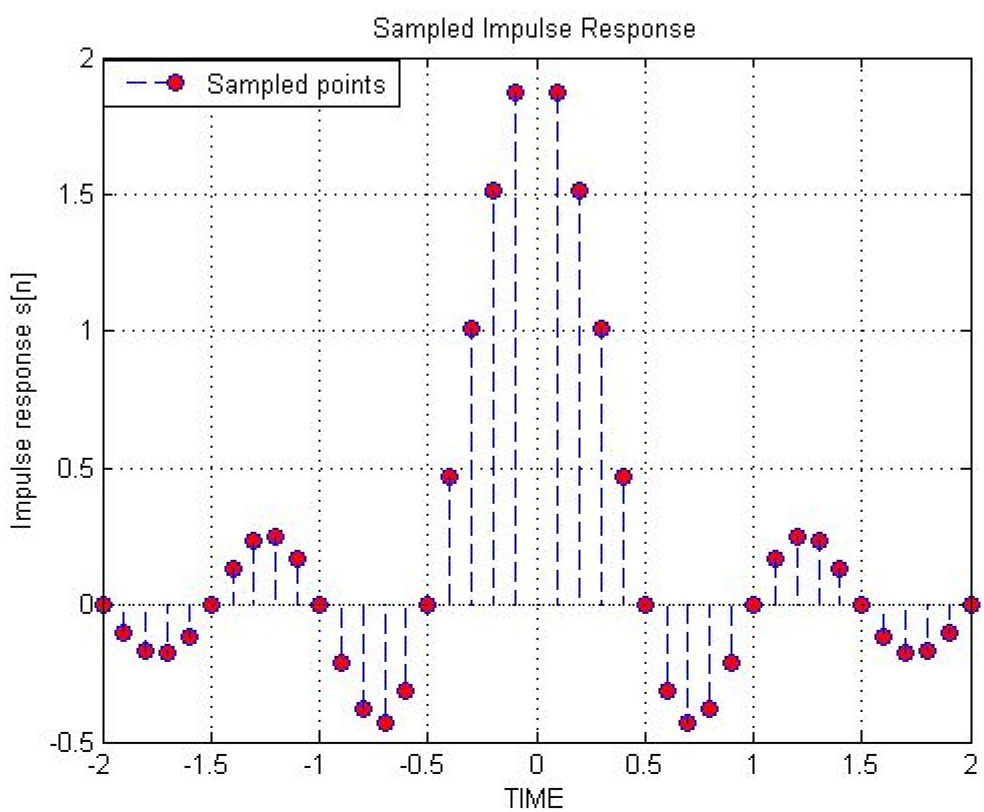
Answer:

Matlab Code:

```
clear

clear all;

k= -2:0.1:2;

c_frequency= 1;

p=2*c_frequency;

s= p*(sin(2*pi*c_frequency*k)./(2*pi*c_frequency*k));

h = stem(k,s,'fill','--');grid on;
```

```
set(get(h,'BaseLine'),'LineStyle',':')

set(h,'MarkerFaceColor','red')

xlabel('TIME');

ylabel('Impulse response s[n]');

title('Sampled Impulse Response');
```

Result:



3. Upsample this response by a factor 5, and demonstrate the discrete spectrum in a simulation.

Answer:

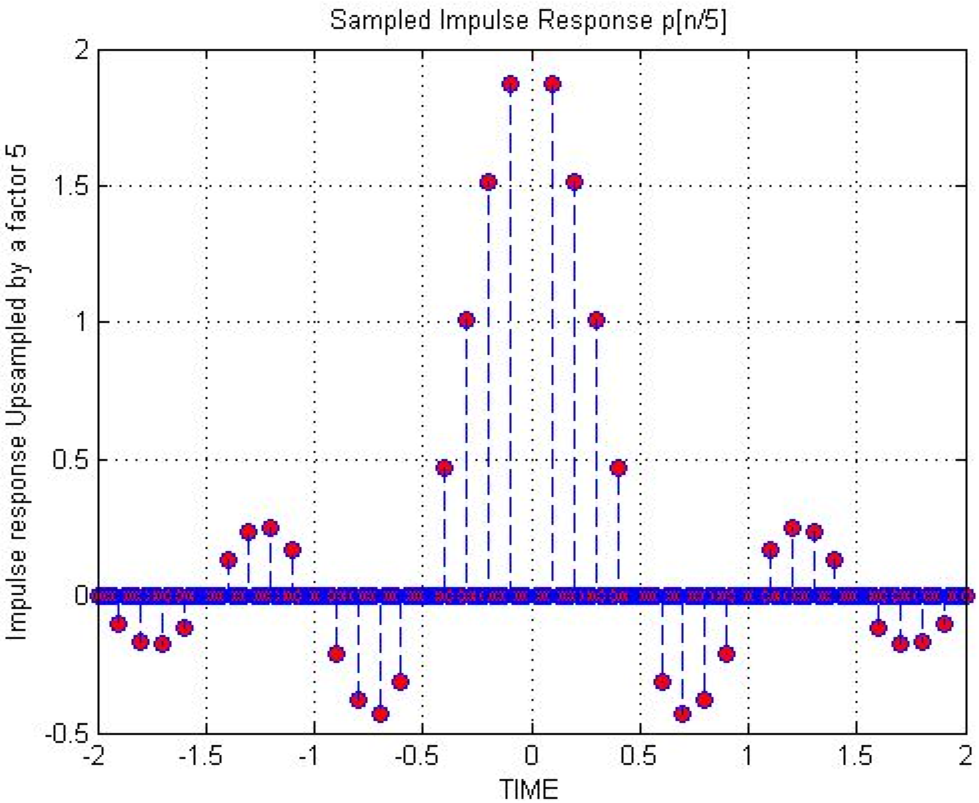
Matlab Code:

```
clear

clear all;

k= -2:0.1:2;

l=zeros(1,4);% Zero matrix of Dimension 1X4

length=size(k,2);% Returns the number of values in the range of k

c_frequency= 1;

p=2*c_frequency;

s= p*(sin(2*pi*c_frequency*k)./(2*pi*c_frequency*k));

Up_sample =[];

for i=1:length-1

  Up_sample = [Up_sample,s(1,i),l];

end

Up_sample = [Up_sample,s(1,length)];

k_new = [-2:2*2/((length-1)*5):2];

h = stem(k_new,Up_sample,'fill','--');grid on;

set(get(h,'BaseLine'),'LineStyle',':')

set(h,'MarkerFaceColor','red')

xlabel('TIME');
```

ylabel('Impulse response Upsampled by a factor 5');

title('Sampled Impulse Response p[n/5]');

Result:



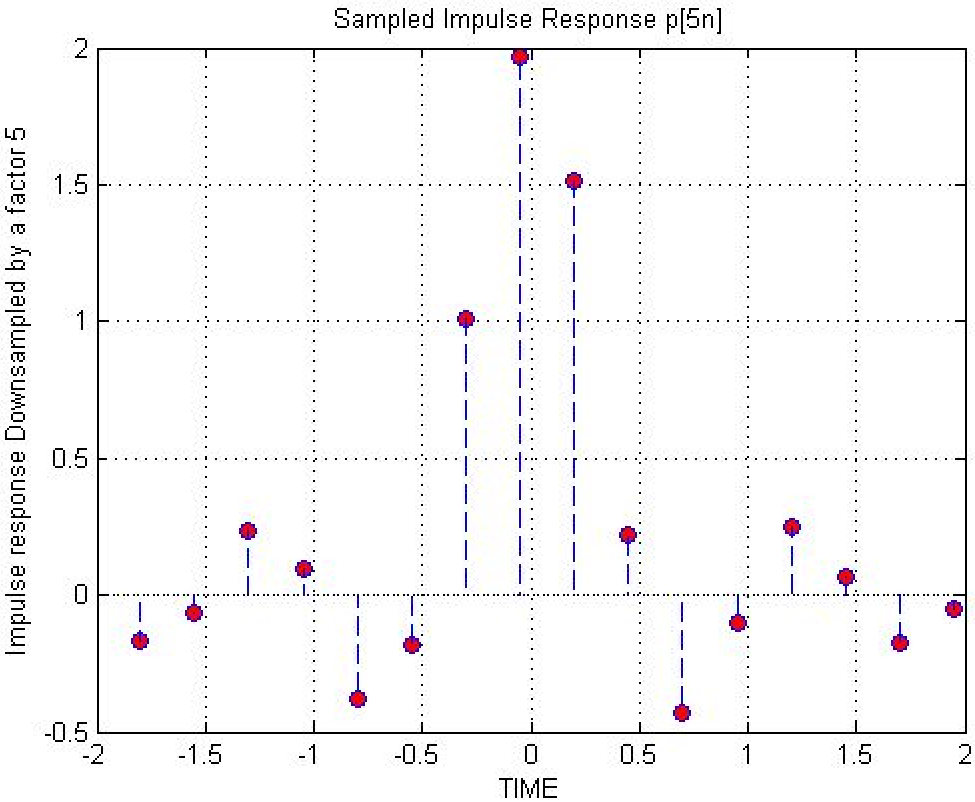
4. Downsample this response by a factor 5, and demonstrate the discrete spectrum in a simulation.

Answer:

Matlab Code:

```
clear
clear all
k=-2 : 0.05 : 2;
c_freq=2;
s= c_freq*(sin(pi*k*c_freq))./(pi*k*c_freq);
l=size(k);
p=ones(l);
[a,b]=size(p);
for i = 1:b
  if mod(i,5)==0 %since multiples of 5 accepts this condition
      p(i)=1;
   else
      p(i)=0;
   end
end
h = stem(k(find(p)),s(find(p)).*p(find(p)),'fill','--');grid on;
set(get(h,'BaseLine'),'LineStyle',':')
set(h,'MarkerFaceColor','red')
xlabel('TIME');
ylabel('Impulse response Downsampled by a factor 5');
title('Sampled Impulse Response p[5n]');
```

Result:



5. Increase the sampling rate of the overall filter by a factor of 1.5. Plot its spectrum in a simulation.

Answer:

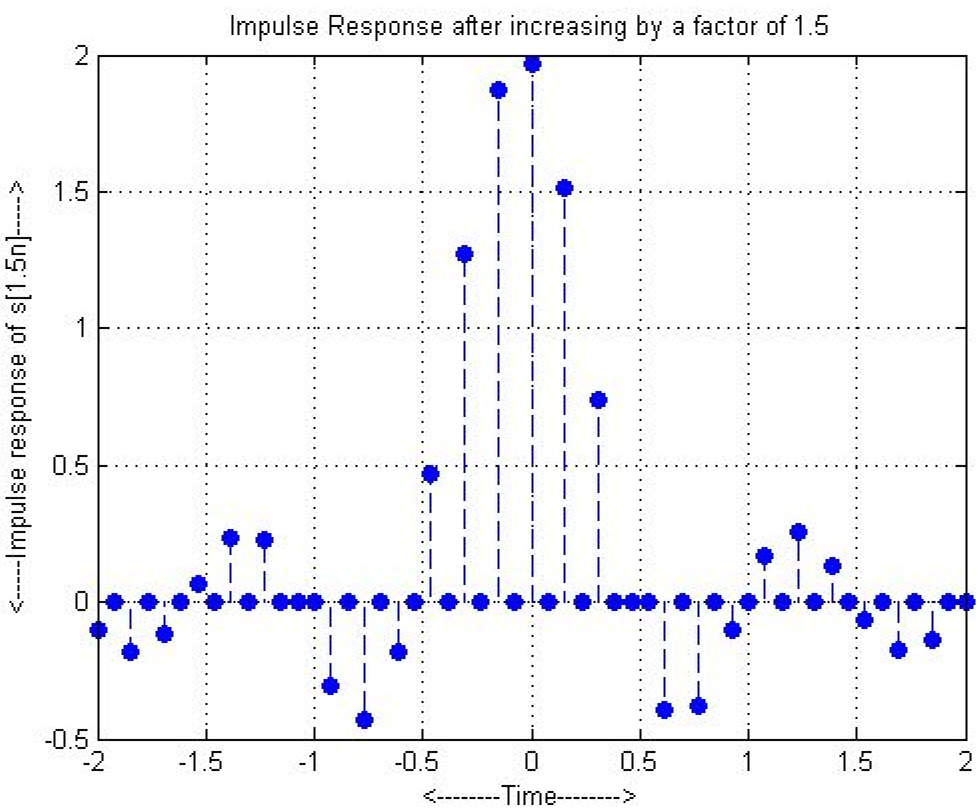
Matlab Code:

```
clear

clear all

k=-2 : 0.05 : 2;
```

```
c_freq=2;
s= c_freq*(sin(pi*k*c_freq))./(pi*k*c_freq);
l=size(k);
y=zeros(1,1);
p=ones(l);
[a,b]=size(p);
for i = 1:b
  if mod(i,3)==0 %since multiples of 3 accepts this condition
      p(i)=1;
   else
      p(i)=0;
   end
end
r= s(find(p)).*p(find(p));
length = size(r,2);
Up_Sample = [];
for i=1:length-1
  Up_Sample = [Up_Sample,r(1,i),y];
end
Up_Sample = [Up_Sample,r(1,length)];
kn = [-2:2/(length-1):2];
h = stem(kn,Up_Sample,'fill','--');grid on;
```

```
set(get(h,'BaseLine'),'LineStyle',':')

set(h,'MarkerFaceColor','Blue')

title('Impulse Response after increasing by a factor of 1.5');

xlabel('<--------Time-------->')

ylabel('<-----Impulse response of s[1.5n]----->')
```

Result:



6.Consider a pulse train defined as follows for a period , with :

$X(t)= \{A \quad |t|<T_1/2$

$\quad\quad 0 \quad \text{Otherwise}$

Derive its exponential Fourier series both theoretically, and by MATLAB simulation. Your

answers by both methods should match. You will not use any toolbox in this exercise, and code it

yourself (even the integral calculation if required).

Ans:

$X(t)= \{A \quad |t| < \frac{\pi}{2}$

$\quad\quad 0 \quad Otherwise$

$$C_0 = a_0 = \frac{1}{T} \int_{\frac{-T_1}{2}}^{\frac{T_1}{2}} x(t)\, dt$$

$$= \frac{1}{T_1} A[\frac{T_1}{2} + \frac{T_2}{2}]$$

$$C_0 = A$$

$$C_n = \frac{1}{T_1} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x(t) e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_1} A \left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{A}{\pi} \left[\frac{e^{-jn\omega_0 \frac{\pi}{2}}}{-jn\omega_0} + \frac{e^{jn\omega_0 \frac{\pi}{2}}}{jn\omega_0}\right]$$

$$= \frac{2A}{\pi} \left[\frac{e^{-jn\omega_0 \frac{\pi}{2}} - e^{jn\omega_0 \frac{\pi}{2}}}{2jn\omega_0}\right]$$

$C_n = \frac{2A}{\pi n\omega_0} sin\ \frac{n\omega_0\pi}{2}$

Therefore the final equation is $C_n = \frac{2A}{\pi n\omega_0} sin\ \frac{n\omega_0\pi}{2}$

7. Derive the necessary, and sufficient condition for a linear-shift invariant system to be BIBO stable.

Answer:

The sufficient and necessary conditions for a linear-shift invariant to be BIBO stable is derived using the following expressions stated below:

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \Leftrightarrow BIBO\ Stability$$

$$We\ need\ to\ prove\ \sum_{n=-\infty}^{\infty} |h(n)| < \infty \Rightarrow BIBO\ Stability$$

$$we\ know\ that$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$we\ have$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty\ and$$

$$Bounded\ Input\ x\ |x(n)| < M\ \forall n$$

$$Take\ modulus\ on\ both\ sides$$

$$|y(n)| = |\sum_{k=-\infty}^{\infty} h(k)x(n-k)|$$

$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)||x(n-k)|$$

$$|y(n)| \leq M \sum_{k=-\infty}^{\infty} |h(k)|$$

$$As\ M\ is\ bounded\ so\ y(n)\ is\ bounded\ \forall n$$

$$case\ :\ 2$$

$$We\ need\ to\ prove\ \sum_{n=-\infty}^{\infty} |h(n)| < \infty \Leftarrow BIBO\ Stability$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \Leftrightarrow BIBO\ Stability$$

$$We\ need\ to\ prove\ \sum_{n=-\infty}^{\infty} |h(n)| < \infty \Rightarrow BIBO\ Stability$$

$we\ know\ that$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$we\ have$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty\ and$$

$Bounded\ Input\ x\ |x(n)| < M\ \forall n$

$Produce\ the\ following\ signal:$

$$x(k) = \begin{cases} h(-k)/|h(k)| & h(-k) \neq 0 \\ 0 & h(-k) = 0 \end{cases}$$

$$x(k) = \{....,1,-1,-1,1,....\}$$

$For\ n = 0$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k)$$

$$y(0) = \sum_{k=-\infty}^{\infty} [h(-k)]^2/|h(-k)|$$

$$y(0) = \sum_{k=-\infty}^{\infty} |h(-k)|$$

$So\ the\ is\ not\ said\ to\ be\ BIBO\ Stable$