Opexpl demonstration

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February 5, 2021

1 In center of mass frame

In the center of mass there is

$$\begin{aligned} m_1 v_{1,m} + m_2 v_{2,m} &= 0 \\ m_1 v_{1,m}^2 + m_2 v_{2,m}^2 &= 2\frac{m_1 m_2^2 gd + m_1^2 m_2 gd}{(m_1+m_2)^2}. \end{aligned}$$

From this we solve

$$\begin{aligned} & m_2^2 v_{2,m}^2/m_1 + m_2 v_{2,m}^2 = 2\frac{m_1 m_2^2 gd + m_1^2 m_2 gd}{(m_1+m_2)^2} \\ \implies & v_{2,m}^2 = \frac{2}{m_2^2/m_1 + m_2}\frac{m_1 m_2^2 gd + m_1^2 m_2 gd}{(m_1+m_2)^2} \\ \overset{(\alpha)}{\implies} & v_{2,m} = \frac{m_1}{m_1+m_2}\sqrt{2gd}\frac{\sqrt{m_1 m_2^2 + m_1^2 m_2}}{\sqrt{m_1 m_2^2 + m_1^2 m_2}} = \frac{m_1}{m_1+m_2}\sqrt{2gd} \\ \implies & v_{2,m} - v_{1,m} = \left(1 + m_2/m_1\right)\frac{m_1}{m_1+m_2}\sqrt{2gd} \\ \implies & v_{2,m} - v_{1,m} = \sqrt{2gd}. \end{aligned}$$

(α): We can't have $v_{2,m} < 0$, because conservation of momentum would imply $v_{1,m} \geq 0 > v_{2,m}$, meaning particle 1 goes right through particle 2. This is not allowed here.

This is the relative speed between the particles.

2 In restframe of particle 2 before collision

We see

$$\begin{aligned} & m_1 v_1^2 + m_2 v_2^2 = 2m_1 gd \\ & m_1 v_1 + m_2 v_2 = m_1\sqrt{2gd} \\ \implies & m_2 v_2^2 + \frac{1}{m_1}\left(m_1\sqrt{2gd} - m_2 v_2\right)^2 = 2m_1 gd \\ \implies & m_1 m_2 v_2^2 - 2m_1 m_2\sqrt{2gd}v_2 + m_2^2 v_2^2 = 0 \\ \implies & (v_2 = 0) \text{ of } \left(v_2(m_1+m_2) = 2m_1\sqrt{2gd}\right) \\ \overset{(\beta)}{\implies} & v_2(m_1+m_2) = 2m_1\sqrt{2gd}. \end{aligned}$$

(β): We need $v_2 > 0$ because of conservation of momentum, and the fact particle 1 can't go right through particle 2.

This yields

$$
\begin{aligned}
v_2 - v_1 &= (1 + m_2/m_1)v_2 - \frac{1}{m_1}\left(m_1 v_1 + m_2 v_2\right) \\
\implies v_2 - v_1 &= (1 + m_2/m_1)v_2 - \sqrt{2gd} \\
\implies v_2 - v_1 &= 2\sqrt{2gd} - \sqrt{2gd} = \sqrt{2gd}.
\end{aligned}
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