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METHODS ARTICLE

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Linking hypothetical extraction, the accumulation of production factors, and the addition of value

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Abstract

In industrial ecology, approaches have been developed to analyze the contribution of specific sectors to environmental impacts within supply chains. In economics, a range of methods addresses the forward linkage (use of output) and backward linkage (dependency on inputs) of sectors, and the analysis of key sectors. This article offers a formal investigation of the relationship between these. It shows that both the analysis of supply chain impacts and of intersectoral linkages can be seen as special cases of a more general hypothetical extraction method (HEM). In HEM, sectors' role is assessed as the effect of their removal on the input–output model's solution. HEM also allows for the (partial) extraction of individual transactions. HEM thus offers a flexible approach to assessing the contribution of one or several sectors, or transactions, or parts thereof, to value added or footprint of any final demand. It can be applied to study the environmental footprints of companies or intermediate products, the contribution of certain inputs to sectors, or the potential impact of disruptions of supply chains on producers and consumers. In this article, the price model for HEM is introduced to identify the contribution of the extracted (target) sectors to the price or unit footprint of a commodity.

KEYWORDS
backward linkage, contribution analysis, forward linkage, Leontief price model, supply chain decomposition, upstream environmental impacts

1 | INTRODUCTION

The hypothetical extraction method (HEM) is a well-established method of input–output economics for investigating how economic sectors influence each other (Miller, 1966; Miller & Blair, 2022, p. 310; Paelinck et al., 1965; Schultz, 1977; Strassert, 1968). It is often framed as investigating how the output of one sector influences the production of another sector (Dietzenbacher & Lahr, 2013; Miller & Blair, 2022; Miller & Lahr, 2001; Nørregaard Rasmussen, 1956) and has been extensively used to quantify the potential impact of catastrophes and terrorist acts (Dietzenbacher et al., 2019; Haddad et al., 2021). To determine the contribution of one or several sectors of the economy to the rest of the economy, in HEM, these sectors are removed from the input–output table and the system is solved anew. The solution of this incomplete system is then compared with that of the complete system, for whatever indicator is of interest, and the difference between the solution to the complete system and the truncated or remaining system is ascribed to the hypothetically extracted sectors. While the initial interest was on whole and individual sectors, HEM can be applied to several sectors at once, a fraction of a sector (Dietzenbacher & Lahr, 2013), or one or several transactions among sectors that do not need to encompass the entire row or column (Hertwich, 2021).

Key sector analysis seeks to measure the importance of sectors to an economy (Miller & Lahr, 2001; Nørregaard Rasmussen, 1956). A sector can be important in its role as a purchaser of intermediate inputs that are produced by other sectors. This is called backward linkage. Or it can be important as a supplier of intermediate inputs to other sectors. This is called forward linkage (Miller & Blair, 2022, Sec. 7.2). Linkage analysis was originally based on manipulations of the coefficient matrices or Leontief/Gosh inverses, where elements of these matrices were summed across columns (backward linkage) or rows (forward linkage).

Cella (1984) introduced hypothetical extraction to formalize the study of linkage by dividing the economy into two groups of sectors and investigating the impact of one group on the other. If quantifying the effect of one individual sector on all other sectors, the total flow concept (Szyrmer, 1986) provides the same results in a computationally simpler manner (Gallego & Lenzen, 2005; Szyrmer, 1992), but it cannot be applied to the combined effect of several sectors or the effect of individual transactions. The use of the hypothetical extraction approach to analyze linkages was further elaborated and applied by others (Clements, 1990; Dietzenbacher & Lahr, 2013; Miller & Lahr, 2001; Temurshoev, 2010). Duarte, Sánchez-Chóliz, and Bielsa (DSB) are credited with the first application to environmental analysis in a study of water use in the Spanish economy (Duarte et al., 2002). DSB's approach inspired subsequent environmentally focused studies (Guerra & Sancho, 2010; Wang et al., 2013; Zhang et al., 2019). Departing from the linkage approach, HEM was used by some of the authors of this article to quantify the carbon footprints of materials and their contribution to the carbon footprint of products and consumption baskets (Hertwich, 2021; Rasul & Hertwich, 2023).

Dente, Aoki-Suzuki, Tanaka, and Hashimoto (DATH) independently quantified the importance of materials to the Japanese economy. To achieve this, they developed an approach to avoid double counting in specifying the contribution of selected inputs to the life cycle impacts of products (Aoki-Suzuki et al., 2021; Dente et al., 2018, 2019). Cabernard, Pfister, and Hellweg (CPH) applied this approach to a wider basket of resources in a global analysis that was used by the International Resource Panel (Cabernard et al., 2019; Oberle et al., 2019). While DSB and Hertwich explicitly acknowledged to use of hypothetical extraction, DATH and CPH present their methods as new methods and without referencing hypothetical extraction. Given that these two approaches achieve the same aim, the question arises how they are related, and whether the method developed by DATH and applied by CPH constitutes a novel approach.

The present article seeks to clarify the relationship between hypothetical extraction, linkage analysis following Cella (1984) and DSB (Duarte et al., 2002), and the supply chain impact method by DATH (Dente et al., 2018, 2019) and CPH (Cabernard et al., 2019). It does so by formally presenting each method and comparing mathematical equations. Both price and quantity models are examined. We demonstrate that the supply chain impact method and the environmentally oriented linkage analysis are similar, except in their treatment of final demand for target-sector products. Results may have been interpreted somewhat differently. Hypothetical extraction is a broader framework of which the two other models can be seen as special cases. In the Supporting Information (Supporting Information Appendix F), we argue that including factor use coefficients, as the Leontief price model suggests, allows for a meaningful interpretation of Cella's linkage measure as the contribution of the cost of production of some sectors to the price and accumulated value in the output of other sectors.

This article addresses the similarity of concepts derived in input–output analysis and industrial ecology. Terms from both fields are introduced and their correspondence is indicated. To increase mutual comprehension, key concepts are introduced, and mathematical derivations are presented, with detailed steps contained in the Supporting Information. The mathematical relationships presented below are generally valid for both single- and multiregional settings but might require small adjustments when used for either case. To limit our use of indexing, we do not show regional detail. This means, for example, that Equation (5) would require the consideration of imports and exports if applied to an open economy.

2 | BACKGROUND

2.1 | The Leontief quantity and price models

A system definition of the economy as depicted in input–output models is shown in Figure 1. The variables are defined in Table 1. Primary factors of production Θ are used by industries together with commodities from other industries Z to produce homogeneous commodities, output volume x. These commodities are distributed by commodity markets to other industries Z (intermediate demand) and final demand by demand category, Y.

The assumption of Leontief models is that there is a linear relationship between the inputs and outputs of industries. Each industry produces only a single commodity, and each commodity is produced by only one industry. The term sector may be used to refer to either of them, depending on if an input–output table is in industry or commodity classification. The flow of a specific commodity to an industry or final demand category is a transaction. The transaction matrices Z and Y have the dimensions (sector, sector) and (sector, final demand category), respectively.

The production recipe of a sector is the quantity of primary factors and intermediate inputs needed to produce a single unit of output, described by the matrices T and A, defined through the equations $T\hat{x} = \Theta$ and $A\hat{x} = Z$. Further, if flows are not traced in monetary units, balancing the inputs and outputs of industries requires the use of factor prices π and commodity prices p. The system depicted in Figure 1 has two unknowns, the prices of commodities p and the production volumes x needed to satisfy a given final demand Y. The system can be uniquely described by two sets of

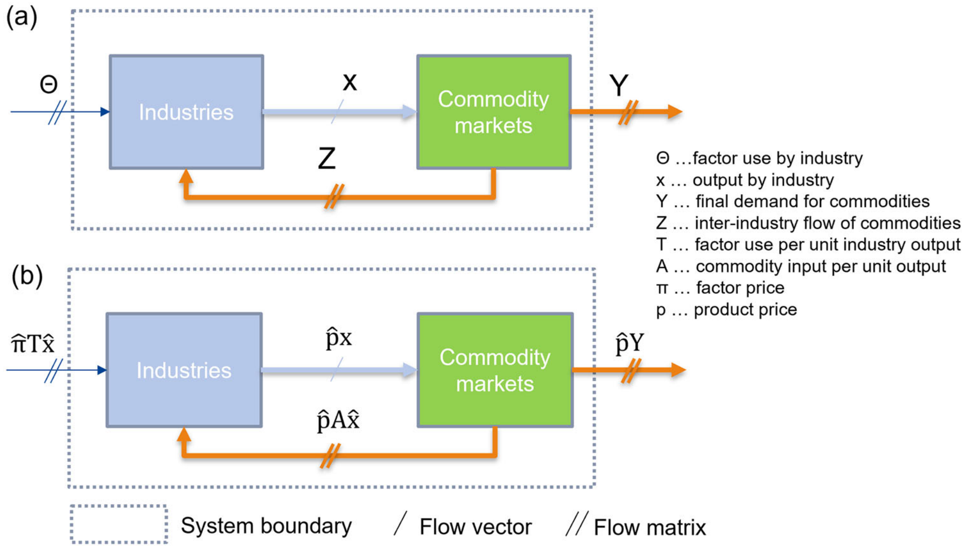

FIGURE 1 (a) System definition of the economy used in input–output economics and its application in industrial ecology. *Source*: Pauliuk et al. (2015) (b) System with flows to a common unit (through multiplication by prices) and inputs to production normalized.

TABLE 1 Symbols and nomenclature used in this article.

Symbol	Dimension	Formula	Name (and synonyms)
Miscellaneous signs			
^			Diagonalization of a vector
'			Transpose of a vector or matrix
−1			Inverse of a matrix
Indices and similar			
c	Letter *or* integer		Commodities *or* number of commodities.
e	Letter *or* integer		Final demand categories *or* number of them
i	Letter *or* integer		Industries *or* number of industries.
w	Letter *or* integer		Value added or environmental pressure categories *or* number of them
Vectors and matrices			
$\boldsymbol{A}$	$c \times c, i \times i$	$\boldsymbol{A} = \boldsymbol{Z}\hat{\boldsymbol{x}}^{-1}$	**Technical** (input–output) **coefficients** (direct requirements matrix)
$\boldsymbol{\Delta}$	2×2		**Contributions to GDP or footprint**, disaggregated by sector groups
$\boldsymbol{I}$	Arbitrary		**Identity matrix** = matrix with ones on the diagonal and zeros elsewhere
$\boldsymbol{i}$	Arbitrary		**Summation vector** = vector of ones of arbitrary length
$\boldsymbol{L}$	$c \times c, i \times i$	$\boldsymbol{L} = (\boldsymbol{I} - \boldsymbol{A})^{-1}$	**Leontief inverse** (total requirements matrix)
$\boldsymbol{\pi}$	$w \times 1$		**Factor prices** or **characterization factors**
$\boldsymbol{p}$	$c \times 1, i \times 1$		**(Commodity/sector) prices** or **environmental footprint**
$\boldsymbol{\Theta}$	$w \times c, w \times i$		**Value added** or **environmental pressures** matrix/block, with $\theta = \boldsymbol{\Theta}' \boldsymbol{i}$
$\boldsymbol{T}$	$w \times c, w \times i$	$\boldsymbol{T} = \boldsymbol{\Theta}\hat{\boldsymbol{x}}^{-1}$	**Value added** or **environmental pressure coefficients** matrix, with $\boldsymbol{t}'$ being a row vector taken from $\boldsymbol{T}$
$\boldsymbol{x}$	$c \times 1, i \times 1$	$\boldsymbol{x} = \boldsymbol{Zi} + \boldsymbol{y} = \boldsymbol{Z}'\boldsymbol{i} + \boldsymbol{\Theta}'\boldsymbol{i} = \boldsymbol{Ly}$	**Total** (commodity/industry) **output**; market and firm balance; input–output model
$\boldsymbol{Y}$	$c \times e, i \times e$		**Final demand** matrix
$\boldsymbol{y}$	$c \times 1, i \times 1$	$\boldsymbol{y} = \boldsymbol{Yi}$	**Final demand** vector
$\boldsymbol{Z}$	$c \times c, i \times i$		**Transactions matrix**

TABLE 2 Correspondence between the Leontief price model, which describes the price as an accumulation of the cost of factor use in the production required to produce a unit of commodity, and the footprint model, which describes the environmental multiplier, aka unit footprint, as the accumulated environmental factor use in the production required to produce a unit of commodity.

Leontief price model	Leontief environmental multiplier model
Θ.. matrix of value added by production factor in each sector π.. vector of prices of production factors p.. vector of the price of goods = accumulated value added in supply chain = multiplier for value added	Θ.. matrix of emissions or resource use for each sector (environmental extensions) π.. vector of characterization factors, e.g., global warming potential for greenhouse gases[4] p.. vector of footprint of goods = accumulated environmental factor use or costs along the supply chain = environmental multiplier

TABLE 3 Leontief price and quantity models (equs. 2, 4), based on market and production balances (eqs. 1, 3).

Leontief quantity model		Leontief price model	
$x = Zi + Yi$	(1)	$p'\hat{x} = \pi'\Theta + p'Z$	(3)
$x = Ax + Yi$		$p'\hat{x} = \pi'T\hat{x} + p'A\hat{x}$	
$(I - A)x = Yi\, x = (I - A)^{-1}\, Yi = LYi$	(2)	$p'\,(I - A) = \pi'T$	
where $A = Z\hat{x}^{-1}$ and $L = (I - A)^{-1}$		$p' = \pi'\, T\,(I - A)^{-1} = \pi'\, TL$	(4)
		where $T = \Theta\hat{x}^{-1}$	

equations, one describing the inputs and outputs of each industry, the production balance, and the other describing the inputs and outputs of each commodity market, the market balance (Table 1). The two balances are connected through the vertical integration of commodity flows, the Leontief inverse $L = (I - A)^{-1}$.

The production balance yields the Leontief price model which determines the price of a commodity i, p_i as the costs of inputs of production factors and intermediate commodities needed to produce a unit of the commodity. This derivation is shown in the right column of Table 3.

The market balance yields the Leontief quantity model, which specifies the activity of industries, measured by their output x, which is needed to deliver any combination y of outputs to final demand (left column of Table 3). In Figure 1b, flows are converted to common units through the application of prices. Matrices expressing the flows are replaced by their normalized versions, and the diagonalization of the p and x vectors is used to maintain the dimensionality of the flows from Figure 1a. In the quantity model, we sum over intermediate and final demand sectors as $x = \hat{x}\, i$ and $y = Yi$.

The most fundamental indicator of economic activity, the gross domestic product (GDP) can be measured in three places in Figure 1, at the beginning as total value added (cost of factor inputs $' = \pi\Theta i$), in the middle as total value of outputs minus cost of intermediate inputs, or at the end as the total value of output delivered to final demand. Equation (5), as written, applies to a single, closed economy and needs to be adjusted for trade in case of an open economy or in a multiregional input–output table.

$$\Delta = \pi'\, Tx = p'\, x - p'Ax = p'\, Yi \tag{5}$$

Equations (1)-(4) can be utilized to confirm that these measures are mathematically identical, even though in practice, measuring each can result in differences given the inevitable uncertainties of such measurement.

In environmental footprint analysis, one is not interested in the value added but in the quantity of resources or pollution associated with the production of a given good. Hence, the input Θ to sectors is of physical factors such as materials or the use by sectors of the environment as receptor of waste residuals such as CO_2. See Table 2 for the correspondence between the economic and environmental variables. π can either be seen as the external cost of those factors or, what industrial ecology calls the characterization factors, such as global warming potentials for greenhouse gases, which are used in life cycle assessment (LCA) to bring different primary flows to a common unit. T then represents the resource use or pollution coefficients (per unit output), and p is the multiplier, or footprint per unit of good delivered to final demand. For environmental analysis, Δ expresses the total resource use or emissions in an economy when Yi represents total final demand; however, Y can be varied to express whatever unit of analysis, the demand for a specific product, the consumption of a household, or the inputs utilized by an enterprise. Footprinting relies on both the Leontief price model (to define multipliers) and the Leontief quantity model to combine multipliers with the quantity demanded. This juxtaposition of footprints and prices shows that the footprint is properly understood as the resource/environmental cost of a commodity, just like the price is its monetary cost.

Please note that the derivation of the price or multiplier presumes that the (monetary or environmental) cost of an intermediate input is equal to that of the same product sold to final demand (Hertwich & Wood, 2018). It is hence meaningful to speak of the footprint (cradle-to-gate impact in the parlor of LCA) of an intermediate input to production. Those who have argued that defining footprints of intermediate inputs results in a double counting (Lenzen, 2008) presume that footprints of products along a supply chain are added up, which they should not be. Rather, the Leontief multiplier model ensures that the direct inputs are added along the value chain in the exact proportion in which they are needed to deliver the desired output to final demand. We will return to this issue when addressing linkages.

2.2 | The hypothetical extraction method and linkages

2.2.1 | Linkages

To assess the importance of certain sectors to an economy by measure of their connectedness, various linkage measures were introduced in the literature (Nørregaard Rasmussen, 1956). These are used to estimate the importance of a sector to other sectors further along or earlier in the value chain. In the first case, one speaks of forward linkages, quantifying the connectedness to sectors using one's output; in the latter case, one speaks of backward linkages, quantifying the connectedness to sectors who's output one depends on for production.

Early measures of direct and total forward/backward linkages included the row/column sums of A and L, respectively (that is, Ai and Li as well as $i'A$ and $i'L$). As the plausibility of forward linkages calculated in this way was questioned repeatedly, a consensus seems to have been reached to quantify them as the row sums of the Ghoshian direct and total distribution coefficients. Miller and Lahr (2001) as well as Miller and Blair (2022) provide a detailed overview of this take. As shown in the Supporting Information, next to backward linkages, forward linkages calculated with the Leontief model are sensible as long as they trace embodied primary factors.

2.2.2 | Basic hypothetical extraction method

HEM was developed as one of the approaches to assess the concept of key sectors and the interindustry linkage between sectors (Miller, 1966; Paelinck et al., 1965; Strassert, 1968). The idea is simple. If you are interested in the contribution of specific sectors[1] (the "targets" of your analysis), you simply remove these from your input–output model, solve the model, and subtract the solution from the original solution. The difference between the two solutions, the full model and the model of the remaining economy (i.e., with target sectors removed), quantifies the role of these target sectors (Miller & Blair, 2022, p. 310). In mathematical form this is simply expressed as:

$$x^o = x - x^* = Ly - L^* y^* \tag{6}$$

Underlying this relationship is the assumption that remaining and extracted portions of direct requirements and final demand sum to the original totals (Hertwich, 2021):

$$A = A^o + A^* \tag{7}$$

$$y = y^o + y^* \tag{8}$$

Superscripts $^\circ$ and * symbolize the targeted sectors and the remaining economy, respectively. The term "target" was introduced by DATH.

These sets of equations provide the mathematical framework to perform different types of sector extractions (i.e., different definitions of L^* and y^*). In fact, using this framework, one can not only extract full sectors but also portions of one-directional transactions between sectors.

Although simple in terms of its formalism, HEM allows different interpretations. This depends largely on how far one would go on the cause–effect chain of *hypothetically* extracting sectoral transactions. HEM may be used for an imputation analysis or an impact analysis. If the former, one would try to map the interlinkages of said transactions as they are. If the latter, one may wish to investigate what would happen if those interlinkages were to change (e.g., be disrupted by extreme events[2]). In the present paper, we limit our exposition to the use of HEM for imputation analysis.

2.2.3 | Linkages via hypothetical extraction method

Cella (1984) introduced the use of HEM for the study of linkages. Cella divided the sectors into two different groups, labeled 1 and 2, to analyze the relationship between those two groups (Figure 2a). That is, for example, matrix A_{11} contains all direct transactions of group 1, which may consist of one or multiple sectors. Direct transactions between group 1 and group 2 as well as within group 2 are contained in respectively named matrices.

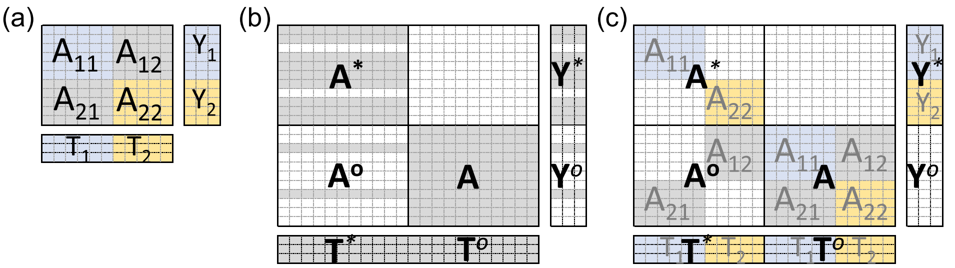

FIGURE 2 (a) Division of an economy into four quadrants, following the approach of Cella. (b) Scheme illustrating the hypothetical extraction of two sectors both in final and intermediate demand, and the purchase of those products from a different instance of the same economy. *Source*: Hertwich (2021). (c) Special case of the extraction of the two off-diagonal quadrants according to Cella's approach, formulated in the general hypothetical expression approach illustrated in (b).

The system of Cella can thus be displayed as

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{11}$$

Cella starts with the market balance,

$$A_{11}x_1 + A_{12}x_2 + y_1 = x_1 \tag{12}$$

$$A_{21}x_1 + A_{22}x_2 + y_2 = x_2 \tag{13}$$

Supporting Information Appendix A retraces the steps of the algebraic solution of this set of equations. It is a general solution of a Leontief equation where the matrix is being divided into four quadrants. For an overview of partitioning matrices and their inverses see appendix A. 10 of Miller and Blair (2022). Relying on such matrix partitioning, and if $H = (I - A_{11} - A_{12}L_{22}A_{21})^{-1}$ and $L_{22} = (I - A_{22})^{-1}$,

$$L = \begin{pmatrix} H & HA_{12}L_{22} \\ L_{22}A_{21}H & (L_{22}A_{21}HA_{12} + I)L_{22} \end{pmatrix} \tag{14}$$

Focusing on linkage analysis, Miller and Lahr (2001) provided an overview of seven types of extractions of elements of the A matrix that can be made using the framework (See table 1 in Miller & Lahr, 2001 or Supporting Information Appendix G in this paper for illustrations of each case). They tabulated the extraction of a single or multiple quadrants (for example A_{12} and A_{21} as shown in the following) of the partitioned A matrix. Removing these blocks results in a differently looking L matrix, where one or multiple quadrants diverge from those depicted in Equation (14). One of these seven cases is the one analyzed by Cella.

Cella investigates the case where off-diagonal quadrants are zero, $A_{12} = A_{21} = 0$, in the hypothetical economy. That is, there is no trade between sectors 1 and sectors 2 (case 2a in Miller & Lahr, 2001). In addition, y is not changed (Figures 2c and 3a). Hence,

$$A^0 = \begin{pmatrix} 0 & A_{12} \\ A_{21} & 0 \end{pmatrix} L^* = \begin{pmatrix} L_{11} & 0 \\ 0 & L_{22} \end{pmatrix} y^o = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{15}$$

Using Equation (14),

$$x^0 = Ly - L^*y = \begin{pmatrix} H - L_{11} & HA_{12}L_{22} \\ L_{22}A_{21}H & L_{22}A_{21}HA_{12}L_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{16}$$

Cella identifies a backward linkage as

$$\mathrm{BL} = \mathrm{i}' \begin{pmatrix} (H - L_{11})\, y_1 \\ L_{22}A_{21}Hy_1 \end{pmatrix} \tag{17}$$

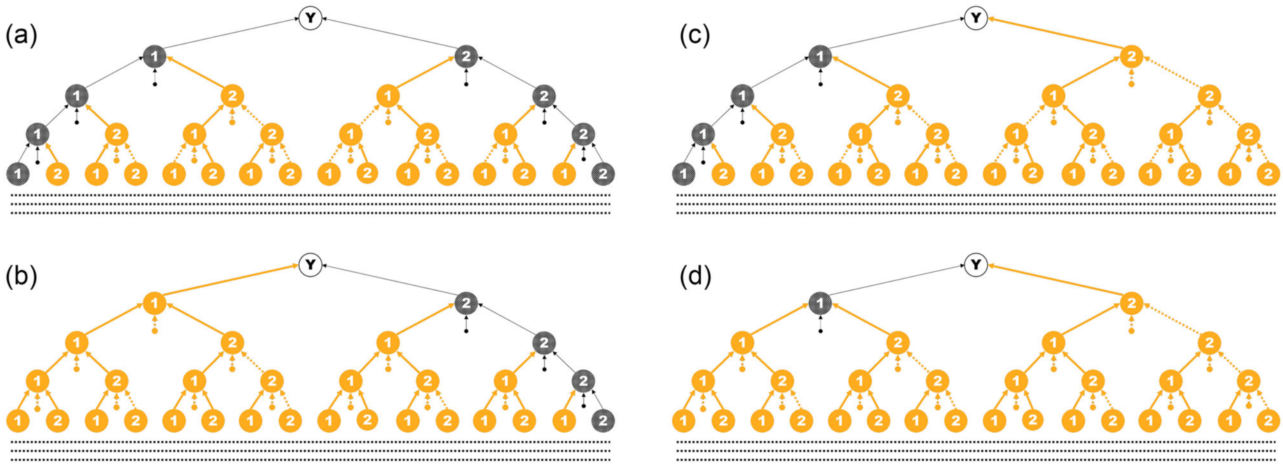

FIGURE 3 Tree diagrams illustrating: (a) case 2a.I, also addressed by Duarte, Sánchez-Chóliz, and Bielsa; (b) case 1.II (and by similarity cases 2a.II, 2c.II, 3a.II) when $\boldsymbol{y^o} = \binom{y_1}{0}$; (c) case 2a.III (and by similarity case 3b.III) when $\boldsymbol{y^o} = \binom{0}{y_2}$; and (d) case 1.III (and by similarity case 2b.III) when $\boldsymbol{y^o} = \binom{0}{y_2}$. Each tree represents the entire economy, partitioned into sectors 1 and 2, consisting of its extracted and remaining portions. Vertices represent sectors which are connected to each other and to a final demand (Y) via edges. Edges connected to only one vertex represent the input of a primary factor. Directly extracted transactions (as per $\boldsymbol{A^o}$) are shown as thick, solid orange edges. Indirectly extracted transactions (due to being cut off in their paths connecting to Y) are shown as thick, dashed orange edges. Remaining transactions are shown as thin, solid black edges. Primary factor inputs take on the style of edges being extracted or remaining. For emphasis, nodes on remaining paths are shown in grey pattern, whereas those on (directly and indirectly) extracted paths are shown in orange. Black dotted lines indicate the infinite nature of inter-industry relationships. More trees for the standard cases of Miller and Lahr (2001) and the modified versions are shown in appendix G of the Supporting Information.

and a forward linkage as

$$\mathrm{FL} = i' \begin{pmatrix} HA_{12}L_{22}y_2 \\ L_{22}A_{21}HA_{12}L_{22}y_2 \end{pmatrix} \tag{18}$$

With the total linkages defined as

$$\mathrm{TL} = \mathrm{BL} + \mathrm{FL} \tag{19}$$

It should be noted that this distinction of forward and backward linkages is not universally accepted. Clements (1990) points out that the lower right quadrant in Equation (16) specifies the amount of production needed to produce the quantity of products 1 serving as intermediate inputs in the production of 2. He argues that $i'L_{22}A_{21}HA_{12}L_{22}y_2$ should be part of backward linkages. This argument shows the difficulty arising when separating the A matrix in four quadrants and extracting both off-diagonal quadrants at the same time. The situation is symmetric. The first quadrant has also been identified as part of the forward linkage of sectors 1. Several subsequent studies associated each quadrant with its own type of linkage, as described in the next section. Hanaka et al. (2022) labeled the off-diagonal second and third quadrants betweenness oriented.

3 | ENVIRONMENTAL FACTORS

3.1 | Environmental linkages

DSB extended the approach by Cella (1984) to environmental impacts (Duarte et al., 2002):

$$\begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix} = \begin{pmatrix} t_1{}' & 0 \\ 0 & t_2{}' \end{pmatrix} \begin{pmatrix} H - L_{11} & HA_{12}L_{22} \\ L_{22}A_{21}H & L_{22}A_{21}HA_{12}L_{22} \end{pmatrix} \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix} \tag{20}$$

where DSB denote Δ_{11} as the mixed effect, Δ_{12} as the net forward linkage, Δ_{21} as the net backward linkage (all relative to the extracted sector 1); their internal effect is given by $t_1' L_{11} y_1$. DSB's extractions correspond to case 2a of Miller and Lahr (2001). DSB investigated the linkages of the water footprint. Other analysts investigated energy (Guerra & Sancho, 2010), CO_2 emissions (Hanaka et al., 2022; Zhang et al., 2019), and other pollution. For example, Zhang et al. (2019) discuss the carbon footprint of the construction sector and the contribution of construction to the carbon footprint of downstream products.

3.2 | Supply chain impacts

Without acknowledging HEM, DATH developed a set of equations to describe the impact of products that are potentially in the supply chain of each other, avoiding double counting. DATH introduced the term target sectors, subscript t, and other sectors, subscript o, which correspond to subscripts 1 and 2 in the approach by Cella. More specifically, the use of the term target sectors implies that all direct transactions of these sectors are subject of analysis, both those among sectors and to final demand. Further, like DSB, DATH use the quadrants of the complete inverse. While they do not completely solve for the components of the Leontief inverse, eqs. 6–9 of Dente et al. (2018) can be shown to be identical to the components in Cella's Leontief inverse displayed in Equation (14) above. Their display of a series expansion of the Leontief inverse is like Figure 3b. In Cella's notation, eqs. 10 and 11 in Dente et al. (2018) are:

$$x_1 = Hy_1 + HA_{12}L_{22}y_2 \tag{21}$$

$$x_2 = L_{22}A_{21}Hy_1 + (L_{22}A_{21}HA_{12} + I)L_{22}y_2 \tag{22}$$

These are just the components of the Leontief inverse. DATH identify H as the factor by which the production of target products needs to be increased to deliver enough target product to non-target sector and final demand while considering the intermediate demand for target products in the production of target products. DATH (Dente et al., 2018, eq. 10) thus define the demand for target products from final demand and other sectors of the economy

$$\xi_1 = y_1 + A_{12}L_{22}y_2\,. \tag{23}$$

DATH (Dente et al., 2018, eq. 11) identify two components of the output of non-target sectors, those that are used upstream of the target sectors (backward linkage) and those that are downstream of the target sectors (forward linkage). Dente (2023) has confirmed that there is a missing element in their equation (11), and the correct equation is

$$x_2 = x_2^b + x_2^f = L_{22}A_{21}H\xi_1 + L_{22}y_2 \tag{24}$$

3.3 | DATH and CPH via HEM

The derivation of the supply chain impact method by DATH (Dente et al., 2018), independent from the related hypothetical extraction and linkage literature, raises the question of how the methods compare. Both set out with the same goal, to describe the simultaneous impacts of several products that are used both as intermediate and final products, with a focus on raw materials and their initial use in the economic supply chain.

DATH (Dente et al., 2018) and CPH (Cabernard et al., 2019) investigate the case where the target products and all their interactions are removed. This may be characterized as a slight variation of case 1 in Miller and Lahr (2001), now also affecting the interactions with final demand.

$$A^0 = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{pmatrix} y^0 = \begin{pmatrix} y_1 \\ 0 \end{pmatrix}$$

In this case, $L^* = \begin{pmatrix} I & 0 \\ 0 & L_{22} \end{pmatrix}$

Using these elements,

$$x^0 = Ly - L^* y^* = \begin{pmatrix} H & HA_{12}L_{22} \\ L_{22}A_{21}H & (L_{22}A_{21}HA_{12} + I)\,L_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} I & 0 \\ 0 & L_{22} \end{pmatrix} \begin{pmatrix} 0 \\ y_2 \end{pmatrix}$$

The equation can be simplified to

$$x^0 = \begin{pmatrix} H \\ L_{22}A_{21}H \end{pmatrix} (y_1 + A_{12}L_{22}y_2) = \begin{pmatrix} H \\ L_{22}A_{21}H \end{pmatrix} \xi_1 \tag{25}$$

The share of value added (or contribution to environmental impacts) of the target products is given by $\Delta^o = \pi' \, Tx^o$. The first matrix in Equation (25) represents the first column of the Leontief inverse in Equation (14). CPH (Cabernard et al., 2019, eq. 9) diagonalize the demand without double counting, ξ_1, and recognize that this diagonal can be used to calculate the upstream impact of each of the individual target sectors. In the classification of DSB, Equation (25) describes the net backward linkage plus mixed and internal effects. The supply chain impact method can be derived from a generic HEM approach, as long as $y_1^* = 0$ (see Supporting Information Appendix E). While the said method is mathematically identical to the earlier decomposition approach under the title "linkages," their interpretation—as the economic activity required to satisfy the demand ξ_1 by the rest of the economy including final demand for target products—is nonetheless instructive.

4 | HEM LINKAGES AND THE EXTRACTION OF FINAL DEMAND

The partitioning-based hypothetical extraction introduced by Cella (1984) and generalized by Miller and Lahr (2001) provides a framework for the extraction of sectors. However, as Dietzenbacher and Lahr (2013) have pointed out, HEM does not require the complete extraction of sectors. Rather, only a fraction of a sector or some transactions can be removed, depending on what is of interest.[3] Hertwich (2021, 2023) expanded the mathematical framework to allow for such extractions. Figure 2b illustrates the conception of treating the (extracted) target transactions as imports from an otherwise identical economy. Satisfying the targeted final demand requires a volume of production calculated by the Leontief inverse. Following Figure 2b, the inputs of targeted products required for the production in the rest of the economy is given by $A^oL^*y^*$. Hence, Equation (6) can also be expressed as

$$x^o = Ly^o + LA^o\,L^*y^* \tag{26}$$

Using Equations (7) and (8), Equation (6) can be derived from Equation (26), as shown in Supporting Information Appendix B. Their equivalence is also suggested by Figure 2b.

In the case where the final demand is left unchanged, $y^* = y$, as in Miller and Lahr (2001), the equation reduces to:

$$x^o = LA^o\,L^*y$$

The extraction for the case investigated by Cella is illustrated in Figure 2c. Equations for cases with modifications of final demand are shown in Table 4. Tree diagrams for instances when cases collapse as well as for the original case of DSB are shown in Figure 3.

Moreover, the framework introduced by Hertwich (2021) allows to calculate the extracted and remaining total outputs through one single equation:

$$\begin{pmatrix} x^* \\ x^o \end{pmatrix} = \left(\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} - \begin{pmatrix} A^* & 0 \\ A^o & A \end{pmatrix} \right)^{-1} \begin{pmatrix} y^* \\ y^o \end{pmatrix} = \begin{pmatrix} L^* & 0 \\ B & L \end{pmatrix} \begin{pmatrix} y^* \\ y^o \end{pmatrix} = \begin{pmatrix} L^* & 0 \\ LA^oL^* & L \end{pmatrix} \begin{pmatrix} y^* \\ y^o \end{pmatrix} \tag{27}$$

where $B = L - L^* = LA^oL^*$. This holds for any extraction of transactions, including the seven cases summarized by Miller and Lahr (2001).

Hertwich (2021) modeled the case where the total intermediate and final demand of the extracted sector is removed. This corresponds to case 2c of Miller and Lahr (2001), yet with the extension of also extracting a portion of final demand.

$$A^0 = \begin{pmatrix} A_{11} & A_{12} \\ 0 & 0 \end{pmatrix} \; y^0 = \begin{pmatrix} y_1 \\ 0 \end{pmatrix}$$

TABLE 4 Equations for seven hypothetical extraction method cases with and without extracted final demand, where $x^o = Ly^o + LA^o L^* y^*$ and $\Phi = (I - A_{12}L_{22}A_{21})^{-1}$. The white and black boxes indicate the extracted (A^o, y^o) and remaining (A^*, y^*) portions of the direct requirements matrix and final demand, respectively. For example, in the right-most column, $y^o = \binom{0}{y_2}$ and $y^* = \binom{y_1}{0}$. Subsequently we label the left column with .I, the middle column with .II, and the right column with .III. Cases 1. I–3c.I are what Miller and Lahr (2001) summarized (see also Fig. S3-S9). Case 2a.I, that is without any final demand extractions, is what Cella (1984) and Duarte, Sánchez-Chóliz, and Bielsa described, respectively. If $y^o = \binom{y_1}{0}$, cases 1.II, 2a.II, 2c.II, and 3a.II have the same solution (see also Fig. S10-S13). If $y^o = \binom{0}{y_2}$, the expressions of x^o for cases 1.III and 2b.III become identical (see also Fig. S14-S15), and so do 2a.III and 3b.III (see also Fig. S16-S17).

Miller & Lahr	A-matrix	x_I^o, if $y = \begin{bmatrix}\blacksquare\\ \blacksquare\end{bmatrix}$	x_{II}^o, if $y = \begin{bmatrix}\square\\ \blacksquare\end{bmatrix}$	x_{III}^o, if $y = \begin{bmatrix}\blacksquare\\ \square\end{bmatrix}$
1	$\begin{bmatrix}\square & \square\\ \square & \blacksquare\end{bmatrix}$	$\begin{pmatrix}H-I & HA_{12}L_{22}\\ L_{22}A_{21}H & L_{22}A_{21}HA_{12}L_{22}\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$	as in modified case 2a.II (used in CPH, Rasul & Hertwich (2023))	$\begin{pmatrix}H & HA_{12}L_{22}\\ L_{22}A_{21}H & (L_{22}A_{21}HA_{12}+I)L_{22}\end{pmatrix}\begin{pmatrix}A_{11}y_1\\ y_2+A_{21}L_{11}y_1\end{pmatrix}$
2a	$\begin{bmatrix}\blacksquare & \square\\ \square & \blacksquare\end{bmatrix}$	$\begin{pmatrix}H-L_{11} & HA_{12}L_{22}\\ L_{22}A_{21}H & L_{22}A_{21}HA_{12}L_{22}\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$	$\begin{pmatrix}H\\ L_{22}A_{21}H\end{pmatrix}(y_1+A_{12}L_{22}y_2)$	$\begin{pmatrix}HA_{12}L_{22}\\ (L_{22}A_{21}HA_{12}+I)L_{22}\end{pmatrix}(y_2+A_{21}L_{11}y_1)$
2b	$\begin{bmatrix}\square & \blacksquare\\ \square & \blacksquare\end{bmatrix}$	$\begin{pmatrix}H-I & (H-I)A_{12}L_{22}\\ L_{22}A_{21}H & L_{22}A_{21}HA_{12}L_{22}\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$	$\begin{pmatrix}H & HA_{12}L_{22}\\ L_{22}A_{21}H & (L_{22}A_{21}HA_{12}+I)L_{22}\end{pmatrix}\begin{pmatrix}A_{11}y_1\\ y_2+A_{21}L_{11}y_1\end{pmatrix}$	as in modified case 1.III
2c	$\begin{bmatrix}\square & \square\\ \blacksquare & \blacksquare\end{bmatrix}$	$\begin{pmatrix}H-I & HA_{12}L_{22}\\ L_{22}A_{21}(H-I) & L_{22}A_{21}HA_{12}L_{22}\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$	as in modified case 2a.II (used in Hertwich (2021))	$\begin{pmatrix}H & HA_{12}L_{22}\\ L_{22}A_{21}H & (L_{22}A_{21}HA_{12}+I)L_{22}\end{pmatrix}\begin{pmatrix}A_{11}y_1+A_{12}L_{22}A_{21}y_1\\ y_2\end{pmatrix}$
3a	$\begin{bmatrix}\blacksquare & \square\\ \blacksquare & \blacksquare\end{bmatrix}$	$\begin{pmatrix}H-L_{11} & HA_{12}L_{22}\\ L_{22}A_{21}(H-L_{11}) & L_{22}A_{21}HA_{12}L_{22}\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$	as in modified case 2a.II	$\begin{pmatrix}H & HA_{12}L_{22}\\ L_{22}A_{21}H & (L_{22}A_{21}HA_{12}+I)L_{22}\end{pmatrix}\begin{pmatrix}A_{12}L_{22}A_{21}L_{11}y_1\\ y_2\end{pmatrix}$
3b	$\begin{bmatrix}\blacksquare & \blacksquare\\ \square & \blacksquare\end{bmatrix}$	$\begin{pmatrix}H-L_{11} & (H-L_{11})A_{12}L_{22}\\ L_{22}A_{21}H & L_{22}A_{21}HA_{12}L_{22}\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$	$\begin{pmatrix}H & HA_{12}L_{22}\\ L_{22}A_{21}H & (L_{22}A_{21}HA_{12}+I)L_{22}\end{pmatrix}\begin{pmatrix}y_1\\ A_{21}L_{11}A_{12}L_{22}y_2\end{pmatrix}$	as in modified case 2a.III
3c	$\begin{bmatrix}\square & \blacksquare\\ \blacksquare & \blacksquare\end{bmatrix}$	$\begin{pmatrix}H-\Phi & (H-\Phi)A_{12}L_{22}\\ L_{22}A_{21}(H-L_{11}) & L_{22}A_{21}(H-\Phi)A_{12}L_{22}\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$	$\begin{pmatrix}H\\ L_{22}A_{21}H\end{pmatrix}(y_1+A_{11}\Phi A_{12}L_{22}y_2)$	$\begin{pmatrix}H & HA_{12}L_{22}\\ L_{22}A_{21}H & (L_{22}A_{21}HA_{12}+I)L_{22}\end{pmatrix}\begin{pmatrix}A_{11}\Phi y_1\\ y_2\end{pmatrix}$

$$L^* = \begin{pmatrix} I & 0 \\ L_{22}A_{21} & L_{22} \end{pmatrix}$$

The solution is the same as the problem investigated by DATH and CPH given by Equation (25).

It can hence be ascertained that Cella's linkages and the supply chain impact method are a special form of the HEM, given that both have been derived here using a more general formulation of HEM represented by Equation (27). HEM offers the same equations and results when applied to the quantification of the demand for target products in the economy and the upstream production activities necessary to produce those target products.

5 | Hypothetical extraction and the price model

The price model provides information on the use of production factors by target and non-target sectors. The price of the target products is defined by the balance of the right half of the columns in Figure 2b.

$$p^{o\prime} = p^{o\prime} A + \pi^{o\prime} T^o$$

The solution of this equation is $p^{o\prime} = \pi^{o\prime} T^o L$, which is the same as Equation (4) given that factor prices and factor coefficients are not modified by the extraction. Please note that this equation defines the costs of all products in the economy if they are needed to produce the target products, as it is the same as for the full model.

The balance of the left half of the columns in Figure 2b indicates that the price of non-target product is comprised of the cost of intermediate inputs of non-target products, the cost of inputs of target products, and the cost of production factors.

$$p^{*\prime} = p^{*\prime} A^* + p^{o\prime} A^o + \pi^* T^* \tag{28}$$

$$p^{*\prime} (I - A^*) = \pi^{o\prime} T^o L A^o + \pi^{*\prime} T^*$$

$$p^{*\prime} = \pi^{o\prime} T^o L A^o L^* + \pi^{*\prime} T^* L^* \tag{29}$$

The solution of Equation (28) for the price of non-target products allows us to distinguish the contribution of the target products to the price of the final goods (first right-hand term in Equation 29) from the contribution of the remaining inputs. Given that $LA^o\, L^* = L - L^*$ as shown in Supporting Information appendix B, and $\pi^{*\prime} = \pi^{o\prime} = \pi'$ and $T^* = T^o = T$ by assumption, we can see that the total price of products is of course $\pi' TL$. The significance of above equation lies in identifying the contribution of intermediate inputs of target products to the price of non-target products. It is the first term of Equation (29) and can also be calculated as $m' = \pi' T(L - L^*)$.

As indicated earlier, there is a correspondence between the input of economic production factors which add up to prices and environmental factors which add up to footprints. Hertwich (2021) utilized Equation (29) to determine the contribution of materials to the carbon footprint of other products.

5.1 | Partitioning the price model

Applying Cella's partitioning to the production balance, the following segments of the price vector can be identified.

$$p_1' A_{11} + p_2' A_{21} + \pi' T_1 = p_1' \tag{30}$$

$$p_1' A_{12} + p_2' A_{22} + \pi' T_2 = p_2' \tag{31}$$

With some steps of algebra, displayed in Supporting Information Appendix C, the prices of product groups 1 and 2 are given by Equation (32).

$$\begin{pmatrix} p_1' & p_2' \end{pmatrix} = \pi' \begin{pmatrix} T_1 & T_2 \end{pmatrix} \begin{pmatrix} H & HA_{21}L_{22} \\ L_{22}A_{21}H & (L_{22}A_{21}HA_{12} + I)L_{22} \end{pmatrix} \tag{32}$$

The expression in the big brackets can be identified as Cella's Leontief inverse, so that the price model is what we would expect it to be.

5.2 | The prices with off-diagonal extractions

The hypothetical extraction of the off-diagonal elements to study linkages as proposed by Cella (case 2a) yields the following expression.

$$A^0 = \begin{pmatrix} 0 & A_{12} \\ A_{21} & 0 \end{pmatrix} L^* = \begin{pmatrix} L_{11} & 0 \\ 0 & L_{22} \end{pmatrix} \tag{33}$$

$$\begin{pmatrix} p_1^{o\prime} & p_2^{o\prime} \end{pmatrix} = \pi' \begin{pmatrix} T_1 & T_2 \end{pmatrix} \begin{pmatrix} H - L_{11} & HA_{21}L_{22} \\ L_{22}A_{21}H & L_{22}A_{21}HA_{12}L_{22} \end{pmatrix} \tag{34}$$

Here we can identify the following elements:

$\pi' T_2 L_{22} A_{21} H$ contribution of value added by non-target sectors to the price of target products (net backward linkage, according to DSB).

$\pi' T_1 (H - L_{11})$ contribution to the price of target products of the value added in the production of target products that serve as intermediate inputs to the production of non-target products required in the production of target products (mixed linkage of target products, according to DSB).

The element that was subtracted through the hypothetical extraction process, $\pi' T_1 L_{11}$ represents the contribution to the price of target products of value added by target sectors either directly or in the part of the supply chain that does not involve non-target sectors (internal effect according to DSB).

Inter alia, $\pi' T_1 H A_{21} L_{22}$ is the contribution of value added by target products to the price of non-target goods (net forward linkage, according to DSB).

$\pi' T_2 L_{22} A_{21} H A_{12} L_{22}$ is the contribution of the value added in production of non-target products used as intermediate inputs to the production of target goods needed to produce non-target goods for final demand (mixed linkage of non-target products).

When combining the price and the quantity model, we get the respective contributions to GDP or footprint, expressed here as Δ. Diagonalizing T and y allows us to keep the four segments, which we can then interpret in the manner above.

$$\begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix} = \begin{pmatrix} \pi' T_1 & 0 \\ 0 & \pi' T_2 \end{pmatrix} \begin{pmatrix} H - L_{11} & H A_{21} L_{22} \\ L_{22} A_{21} H & L_{22} A_{21} H A_{12} L_{22} \end{pmatrix} \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix} \tag{35}$$

We will now use the example of the investigation of the carbon footprint of materials to illustrate the insights and limitations of this approach. Let us assume that index 1 represents all material production processes, but not necessarily their inputs such as mining. T represents the emission factors of various greenhouse gases by the various production processes. π is the global warming potential which measures the contribution to global warming of the different greenhouse gases. y_1 is the final demand for materials and y_2 is the final demand for non-materials.

$\Delta_{11} = \pi' \ T_1 (H - L_{11}) y_1$ represents the carbon footprint of the materials that are needed in non-material-producing processes, such as energy, transport, or insurance, to produce inputs to materials production. $\Delta_{21} = \pi' \ T_2 L_{22} A_{21} H y_1$ represents the GHG emissions occurring in the production of inputs of non-materials needed for materials production. $\Delta_{11} + \Delta_{21}$ is the GHG emissions of all inputs to materials production of non-materials and their total upstream emissions, including of materials (1) and non-materials (2), to satisfy the final demand for materials. It is the backward linkage (or upstream) of materials production. It does, hence, exclude emissions that occur directly in materials production or the production of materials that are used by materials-production processes. These have been subtracted given the L_{11} term.

$\Delta_{12} = \pi' \ T_1 H A_{21} L_{22} y_2$ is the carbon footprint caused in the production of materials needed to produce non-materials to satisfy the final demand for non-materials. $\Delta_{22} = \pi' \ T_2 L_{22} A_{21} H A_{12} L_{22} y_2$ is the carbon footprint of non-materials needed to produce the materials used in the production of non-materials for final demand. Hence, all elements of Equation (31) are part of the supply chains of both materials and of non-materials. Those transactions that either involve only materials or only non-materials have been removed. If one wants to know the carbon footprint of all materials (whether used for intermediate or final consumption) using this approach, one needs to add $\Delta^*_{11} = \pi' \ T_1 L_{11} y_1$ to $\Delta_{11} + \Delta_{12} + \Delta_{21} + \Delta_{22}$. If one wants to know the carbon footprint of non-materials, one needs to add $\Delta^*_{22} = \pi' \ T_2 L_{22} y_2$.

The approaches of DATH and Hertwich result in somewhat different prices and footprints of target products in the remaining system. When only the row of the target products is removed, the cost of the target products as intermediate inputs to the production of target products is broken out. When both rows and columns of the target products are removed, the cost of all intermediate inputs is broken out. See Supporting Information Appendix D for an exhibition of the equations. These differences show that cases, even if they have the same decomposition of x°, do not need to have the same decomposition of p.

The decomposition of the price of target products is of no consequence to the type of analysis conducted by DATH, CPH, and Hertwich, given that in a calculation of footprints, the first columns would be multiplied by a final demand of zero. Different decompositions may be of interest for different questions. In the end, what this exploration shows is that there are various ways to decompose the total to isolate and quantify the role of partial systems within the total. Depending on the research questions, one of the decompositions offered by HEM may give the desired answer.

6 | DISCUSSION

We have shown that the supply chain impact method suggested by DATH and CPH can be derived using HEM and provides results that are very similar to those described by environmental linkage analysis following DSB. The claim that it constitutes a new method is hence not justified. The central difference between DATH and DSB, the extraction of elements of final demand, had to our knowledge not been considered formally in linkage studies and is quite relevant for environmental analysis. At the same time, it is trivial, as the footprint of the final demand for target products is quantified using the standard footprint equations.

The derivation here shows that the equations proposed by DATH, CPH, Cella, and DSB are special cases of the more general hypothetical extraction approach explicated in the section *HEM linkages and the extraction of final demand*. The decomposition and linkage analysis offered by DSB are very similar but not identical to the approach taken by DATH, CPH, and Hertwich, and have been classified as different cases by Miller and Lahr (2001). Further, industrial ecologists have also extracted final demand, a case not considered by economists. The identification of the demand for target products without double counting derived by DATH (Equation 23) was already contained in the solution of Cella. While it can be argued that the two things are different given the exclusion of the direct impacts of the production of target products, the environmental linkage literature following DSB correctly considered and interpreted mixed effect and net backward linkages to exclude the internal effect, which would be added to calculate total impacts of demand for products from sectors 1. For example, Zhang et al. (2019) investigated countries and sectors benefiting from the construction sector. He et al. (2017) identified sectors responsible for air pollution emissions using forward linkages. DATH and Hertwich hence added a formal explication of the final demand for target products, already correctly interpreted in the environmental linkage literature.

Further, applying HEM to the price model, and using the entire system description comprised of price and quantity model, provide new insights into linkages. It shows how the prices, or multipliers, of non-target products can be decomposed to quantify the contribution of target products, and how the contribution of non-target products in the supply chain of target products can be identified.

HEM offers new insights to industrial ecology. It provides a clear framework of how to understand the impact of intermediate inputs to production. According to the Greenhouse Gas Protocol, the scope 1 + 2 + (upstream) 3 emissions constitute the carbon footprint of a company or economic entity. This carbon footprint can be understood as the role of the company in the economy, and its contribution to the carbon footprint of its customers can be quantified using the HEM approach where the company is identified as the target. Thereby, the issue of double counting is resolved.

Most applications of the supply chain impact method and linkage analysis have focused on the role of key sectors, with material production being the most prominent in the work of industrial ecologists. However, emerging applications of HEM use the decomposition to combine results from different input–output models. For example, Rasul et al. (2024) extracted the agricultural sectors from EXIOBASE to quantify the non-agricultural inputs to the production of agricultural products. The impacts of agriculture are obtained from a more highly resolved, physical model of the agricultural system, FABIO (Food and Agriculture Biomass Input–Output model). A combination of those two results then provides a more accurate and detailed picture of how the environmental impacts of food production come about. HEM serves to identify the contribution of non-agricultural sectors to the impact of food products.

Hopefully, the clarifications offered by this article can further the application of HEM in environmental analysis. It may also encourage economists to offer a more explicit, physical interpretation of their linkage analysis results.

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study

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Notes

[1] In the context of inter- and multiregional analyses, the extraction of entire regions or selected sector–region combinations may of course be of interest. To keep things simple, we do not break matrices further down and continue with a generic notation.

[2] Dietzenbacher et al. (2019), focusing on the application of HEM to the study of disasters, argued that HEM could not be applied in a global context because there is no other planet to import from. They hence suggested a modification of HEM for global MRIOs. The mathematical identity above shows that Equation (6) merely identifies the production volume needed to produce the quantity of target product required by the rest of the economy. For an alternative mental image, it can be assumed that in a first instance, an economy produces the extracted product in the required quantity of target products, x°, and in a second instance, it produces the remainder, x^*. HEM is simply the method to determine x°.

[3] The extraction of a single transaction collapses to case 3c of Miller and Lahr (2001). The extraction of several individual transactions cannot be shown explicitly in the partitioned matrix framework employed here (Figure 2c); one may, however, show in Figure 2b not only the extraction of entire rows but instead of a chosen set of individual cells. Of course, one may want to extract also individual paths from the economy. This is possible when extracting corresponding elements from the Neumann series expansion of L. As this is not relevant for our current objective, we do not describe this further.

[4] If multiple impact categories such as global warming potential and eutrophication potential are to be determined simultaneously, we need a matrix of characterization factors, with each column characterizing environmental pressures into one category. If that is done, a footprint matrix instead of a vector results.

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SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

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