Duffing oscillator

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1 Explanation

This python code simulates the Duffing oscillator: A damped driven harmonic oscillator in a double well potential:

$$m\frac{d^2x}{dt^2} = -\gamma\frac{dx}{dt} + 2 * a * x - 4.0 * b * x^3 + F_0 \cos \omega t \tag{1}$$

m is the mass (assumed to be 1), x is the position of the particle, γ, a, b, F_0, ω are constants. The second order nonlinear differential equation is solved by Taylor series expansion, computing derivatives up to order 5 and updating the position and velocity at each time step.

The motion is quite different for various values of the parameters. The particle can exhibit sinusoidal motion in one of the wells, visit both wells with a fixed period, or behave chaotically. In the chaotic regime, the motion is strongly dependent on the initial conditions and displays no fixed period of repetition. For example, for γ = 0.1, a = 0.5, b = 1/16, F_0 = 2.5, ω = 2.0, x[0] = 0.5, x'[0] = 0.0, the trajectory of the particle is shown below:

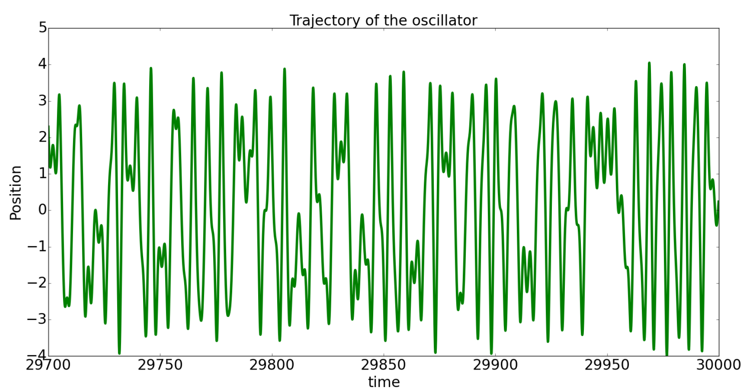

Figure 1: Snapshot of the trajectory of the oscillator. γ = 0.1, a = 0.5, b = 1/16, F_0 = 2.5, ω = 2.0, x[0] = 0.5, x'[0] = 0.0

Figure 1 shows that the particle has no fixed period of oscillation. Further evidence of this can be seen in the phase space plot:

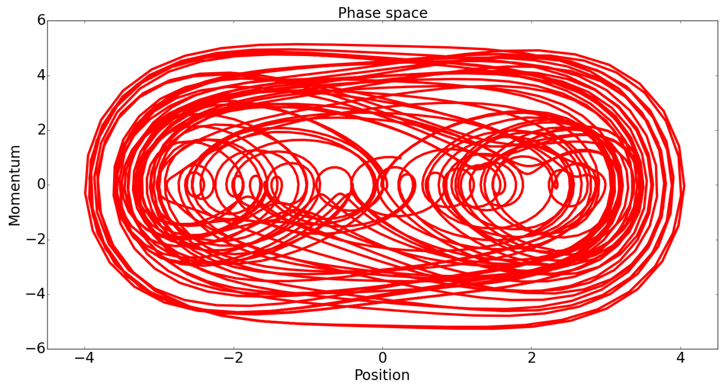

Figure 2: Phase space: γ = 0.1, a = 0.5, b = 1/16, F_0 = 2.5, ω = 2.0, x[0] = 0.5, x'[0] = 0.0

For a particle in harmonic motion in a single well, the phase space plot would be a simple ellipse. Clearly this is not observed. Finally one can plot the Poincare section, which is a snapshot of phase space at fixed multiples of the driving period: $T = \frac{2N\pi}{\omega}$ N = 1,2,3... For simple harmonic motion, this plot would be a single point, or a few discrete points. Instead, a fractal is observed. This is called a 'strange attractor'.

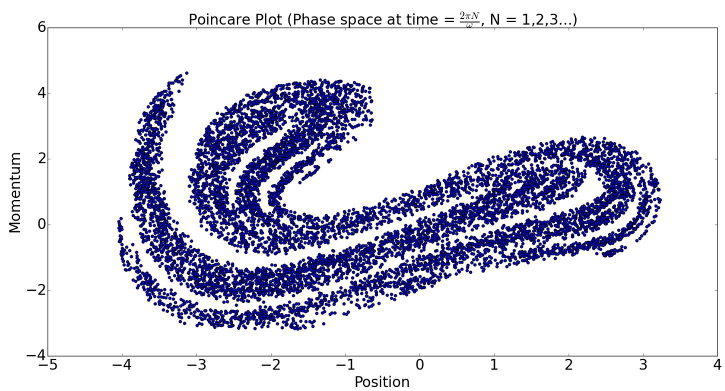

Figure 3: Poincare section: γ = 0.1, a = 0.5, b = 1/16, F_0 = 2.5, ω = 2.0, x[0] = 0.5, x'[0] = 0.0