

# MEST - Assignment 4

Due date: 01/05/2022

(For each solution, show your work through a set of important steps.)

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1. For the  $H_2$  molecule at equilibrium separation, we have following bonding and anti-bonding MOs obtained using the AO basis functions  $\chi_1 = \phi_{1sa}$ , and  $\chi_2 = \phi_{1sb}$ .

$$\Psi_g = c_{11}\chi_1 + c_{12}\chi_2$$

$$\Psi_u = c_{21}\chi_1 + c_{22}\chi_2$$

where,  $c_{11} = c_{12} = 0.554884228$ ,  $c_{21} = -c_{22} = 1.21245192$ . (2+2+3+3+3 pts)

- (a) Calculate ground and excited state orbital energies at the HF level.
- (b) Calculate total HF energy of the  $H_2$  molecule
- (c) Calculate MP2 correction to HF energy. What is the ratio of MP2 correction and HF energy. In this case, will MP2 be a good approximation to correlation energy?
- (d) In this MO basis, construct the CISD (configuration interaction singles doubles) matrix with and without using the Slater-Condon rules, and check that the results are the same. Evaluate each matrix element.
- (e) Calculate CISD ground-state energy. Calculate the ratio of CISD correlation energy and HF total energy?

Use the following one and two electron integrals for calculations.

$$\langle \chi_1 | \hat{h} | \chi_1 \rangle = \langle \chi_2 | \hat{h} | \chi_2 \rangle = -1.12095946, \langle \chi_1 | \hat{h} | \chi_2 \rangle = \langle \chi_2 | \hat{h} | \chi_1 \rangle = -0.95937577$$

$$\langle \chi_1 \chi_1 | | \chi_1 \chi_1 \rangle = \langle \chi_2 \chi_2 | | \chi_2 \chi_2 \rangle = 0.77460594,$$

$$\langle \chi_1 \chi_2 | | \chi_2 \chi_2 \rangle = \langle \chi_2 \chi_1 | | \chi_2 \chi_2 \rangle = \langle \chi_2 \chi_2 | | \chi_1 \chi_2 \rangle = \langle \chi_2 \chi_2 | | \chi_2 \chi_1 \rangle =$$

$$\langle \chi_1 \chi_1 | | \chi_1 \chi_2 \rangle = \langle \chi_1 \chi_1 | | \chi_2 \chi_1 \rangle = \langle \chi_1 \chi_2 | | \chi_1 \chi_1 \rangle = \langle \chi_2 \chi_1 | | \chi_1 \chi_1 \rangle = 0.44459112,$$

$$\langle \chi_1 \chi_2 | | \chi_1 \chi_2 \rangle = \langle \chi_1 \chi_2 | | \chi_2 \chi_1 \rangle = \langle \chi_2 \chi_1 | | \chi_1 \chi_2 \rangle = \langle \chi_2 \chi_1 | | \chi_2 \chi_1 \rangle = 0.29759055,$$

$$\langle \chi_1 \chi_1 | | \chi_2 \chi_2 \rangle = \langle \chi_2 \chi_2 | | \chi_1 \chi_1 \rangle = 0.56999488$$

2. For the ground-state of  $H_2^+$ , the simple MO wavefunction is  $\sigma_g 1s(r_1) = \phi_{1sa}(r_1) + \phi_{1sb}(r_1)$ , where  $a$  and  $b$  denote the two H atoms, and  $\phi_{1sa} = \frac{e^{-ra}}{\sqrt{\pi}}$  is the hydrogen AO. Using confocal elliptical coordinates (see Levine for definition), show that the overlap integral is given by  $S_{ab} = e^{-R}(1 + R + \frac{R^2}{3})$ . Here,  $R$  is the internuclear separation. The volume element in confocal elliptic coordinates is  $dv = \frac{1}{8}R^3(\xi^2 - \eta^2)d\xi d\eta d\phi$ . The following integral formula can be handy (4 pts):

$$\int_t^\infty z^n e^{-az} dz = \frac{n!}{a^{n+1}} e^{-at} \left( 1 + at + \frac{a^2 t^2}{2!} + \dots + \frac{a^n t^n}{n!} \right), n = 0, 1, 2, \dots > 0$$

3. The exact nuclear-electron wavefunction can be expanded in a basis of the BO wavefunctions

$$\Psi(\mathbf{R}, \mathbf{r}) = \sum_{a\alpha} c_{a\alpha} \Phi_a(\mathbf{r}; \mathbf{R}) \chi_{a\alpha}(\mathbf{R})$$

where  $\Phi_a(\mathbf{r}; \mathbf{R})$  is the  $a^{\text{th}}$  eigenfunction of  $\hat{H}^{el}$  and  $\chi_{a\alpha}(\mathbf{R})$  is the  $\alpha^{\text{th}}$  eigenfunction of nuclear Schrodinger equation for electronic surface  $a$  within BO approximation. Starting from the exact energy

$$\langle \Psi | \hat{H} | \Psi \rangle ; \hat{H} = \hat{T}_N + \hat{H}_{el}$$

identify the Hamiltonian elements that should be neglected to obtain the energy expectation used in the Born-Oppenheimer approximation

$$\langle \chi_{a\alpha} | \hat{T}_N + U_a(\mathbf{R}) | \chi_{a\alpha} \rangle ; U_a(\mathbf{R}) = \langle \Phi_a | \hat{H}_{el}(\mathbf{R}) | \Phi_a \rangle . \text{ (4 pts)}$$

4. For  $\text{H}_2$  in a minimal basis-set, but with infinite nuclear separation and using only  $\Phi_0$  and  $\Phi_{1\bar{1}}^{2\bar{2}}$  configurations, derive the CI wavefunctions corresponding to the ground and excited-states, i.e. solve for  $c_0$  and  $c_1$  for  $\Psi = c_0\Phi_0 + c_1\Phi_{1\bar{1}}^{2\bar{2}}$ . Comment on the ionic/covalent nature of the two CI wavefunctions by expanding the MOs of the two Slater determinants in AOs. (4 pts)