1 Candidate Working Definitions

D1. Object of analysis. The simulation is meant to analyse how a functional dealer affects a system of debtors organised in a Kalecki ring.

Baseline: a ring of debt obligations among agents (debtors/creditors) that can be simulated both with and without a dealer, holding other primitives fixed.

D2. Functional dealer. A dealer is *functional* if, in the simulation:

1. It quotes both bid and ask prices over time for a security (or set of securities) that are claims on the underlying debts in the ring.
2. The spread is endogenous and time-varying (changes in response to its own state and/or market conditions), not an arbitrary fixed parameter.
3. Over the relevant horizon, the dealer does not lose money in expectation, and ideally is at least break-even or profitable:

   $$\mathbb{E}\left[\Pi_{\text{dealer}}\right] \geq 0$$

   over the run or across runs.
4. The dealer is willing both to:
   - buy securities from traders/debtors (provide liquidity to them), and
   - sell securities to traders/debtors (offer them risky assets with a positive expected return).

D3. Trader/debtor behaviour. Traders/debtors are agents that both owe debts in the ring and can trade securities with the dealer.

They should satisfy:

1. **Profit-seeking motive in securities:**
   - There is a conceivable, realistic possibility for them to earn by buying securities from the dealer (positive expected excess return versus holding cash or immediate repayment).
   - Their demand for securities is driven by this search for returns, not by arbitrary exogenous rules.

2. **Liquidity-on-demand through sales:**
   - Agents holding securities can sell them to the dealer in order to obtain liquidity needed to settle their own debts.
   - Thus, securities are both investment assets and liquidity buffers.
3. **Repayment priority constraint:**

   Agents prioritise their capacity to repay debts. Formally, at each decision point, their strategy must satisfy a constraint of the form:
   - planned securities position at time t must not make it impossible (or too risky) to meet future debt obligations.

   They invest in securities only if, given their beliefs about returns and future obligations, the investment
   - either helps them to be able to repay (by generating income), or
   - at least does not jeopardise their ability to repay.
4. **Continuation condition:**

   They continue holding or investing in securities only as long as the repayment priority constraint is respected.

   When the constraint threatens to bind (liquidity stress, upcoming large payment), they begin to sell down securities or stop new purchases.

If we agree, we can treat D1–D3 as the definitions of "functional dealer" and "trader/debtor behaviour" for this thread.

2 Requirements for what the simulation "has to do"

Given D1–D3, we can formulate the minimal requirements for the simulation as follows:

R1. Two regimes: with and without dealer. The simulation must be able to run the same Kalecki ring:

- same schedule of obligations, same agents, same initial conditions, once without dealer and once with a functional dealer.

This allows identification of the effect of the dealer by comparison.

R2. Explicit dealer micro-structure. The code must specify:

1. A state for the dealer: inventory of each security, cash/liquidity, capital.
2. A pricing rule for bid and ask that:
   - depends on its state and perceived risk of the underlying;
   - produces a varying spread over time.
3. A P&L accounting such that we can measure whether the dealer is
   - loss-making, break-even, or profitable over the relevant horizon.

The "functional dealer" condition is then a requirement on this P&L and on the persistence of two-way quotes.

R3. Securities with genuine return possibilities. The securities that the dealer buys and sells must have a defined payoff structure (e.g. claims on future ring payments or default-contingent payoffs) such that:

- there are states of the world where the realised or expected return for agents buying from the dealer is positive and non-trivial;
- it is not mechanically dominated by immediate repayment or cash hoarding.

Otherwise, traders/debtors would have no rational reason to buy.

R4. Trader/debtor decision rule with repayment priority. The simulation must encode a rule for each trader/debtor that:

- observes (at least) its own:
  - current cash/liquidity,
  - current securities holdings,
  - schedule of future obligations,
  - dealer's quoted bid/ask (and perhaps beliefs about future prices),

- and chooses among:
  - buying securities,
  - selling securities,
  - repaying debt early,
  - holding cash.

This choice is subject to a repayment priority constraint. For example (one possible formalisation):

$$\mathbb{P}\big(\text{default}_i \mid \text{current plan}\big) \leq \bar{p}_i \tag{1}$$

for some tolerated probability $\bar{p}_i$, or

$$\big(\text{Projected cash+saleable securities at bid}\big) - \big(\text{future payments}\big) \geq \text{safety\_margin}_i. \tag{2}$$

The specific form is a modelling choice, but some explicit constraint of this kind is required to encode "do not jeopardise capacity to repay".

R5. Liquidity via sale of securities. The model must implement the operation "sell securities to dealer to obtain cash" and ensure that:

- this operation is available in the relevant states (subject to dealer capacity and price),
- it feeds into the settlement algorithm for ring obligations (i.e. selling securities changes ability to pay).

R6. Output variables to measure the dealer's effect. To answer "in what way a functional dealer affects the system of debtors in a Kalecki ring", the simulation must, at minimum, produce:

- Kalecki ring metrics (per day / period): S_t, $\bar{M}_t$, M_t, v_t, ϕ_t, δ_t, default rates and distribution of default burdens.
- Dealer metrics: bid/ask, spread, inventory, cash, P&L, realised return on inventory.
- Trader/debtor investment metrics:
  - securities positions by agent,
  - realised returns on those positions,
  - instances where securities are sold to meet obligations.

3 Interpreting the 125-pair comparison experiment

We begin by translating what the 125-pair experiment is actually telling us, and then map that back to the "functional dealer in a Kalecki ring of debtors" specification.

3.1 What the current comparison experiment actually measures

The comparison sweep has the following structure:

- 100 agents in a Kalecki ring;
- total nominal dues $Q_1 = 10{,}000$; maturity horizon 10 days;
- parameter grid

  $$\kappa \in \{0.25, 0.5, 1, 2, 4\}, \quad c \in \{0.2, 0.5, 1, 2, 5\}, \quad \mu \in \{0, 0.25, 0.5, 0.75, 1\}.$$

For each triple (κ, c, μ), the experiment runs:

1. a *control* simulation with no dealer;
2. a *treatment* simulation in which the dealer is enabled with the same parameters and random seed.

The core outcome is the default shortfall rate

$$\delta_{\text{total}} = 1 - \phi_{\text{total}},$$

where ϕ_{total} is the value-weighted payoff rate.

For each parameter triple, define

$$\delta_{\text{control}}, \quad \delta_{\text{treatment}},$$

and the *relief ratio*

$$R = \begin{cases} \dfrac{\delta_{\text{control}} - \delta_{\text{treatment}}}{\delta_{\text{control}}}, & \delta_{\text{control}} > 0, \\ 0, & \delta_{\text{control}} = 0. \end{cases}$$

Aggregate statistics from the comparison sweep are:

- 125 parameter pairs in total; all 125 completed (no failures in either arm);
- mean $\delta_{\text{control}} \approx 0.5632$;
- mean $\delta_{\text{treatment}} \approx 0.4706$;
- mean relief ratio $R \approx 0.1904$, i.e. an average reduction in defaults of about 19%;
- 94 out of 125 pairs show strict improvement ($\delta_{\text{treatment}} < \delta_{\text{control}}$);
- 31 out of 125 pairs are unchanged;
- 0 out of 125 pairs worsen.

In systemic terms:

- on this grid, the dealer never increases the default shortfall;
- on average, the default shortfall falls by about 9 percentage points (from roughly 56% to 47%), corresponding to a reduction of about 19% relative to baseline defaults.

The specific configuration ($\kappa = 0.25, c = 0.2, \mu = 0, \text{seed} = 42$)—the "run 42" control/treatment pair—lies among the 31 unchanged cases:

- $\delta_{\text{control}} = \delta_{\text{treatment}} \approx 0.518$, so $R = 0$;
- in this scenario, the dealer has no effect on the default pattern.

The comprehensive comparison also reports that:

- the mean relief ratio over this grid is about 19%;
- 94/125 pairs improve and 0/125 worsen;
- there is a dominant interaction between concentration c and maturity structure μ: the dealer is most helpful when claims are more concentrated and maturities are more spread out; increasing dealer capital does not yield a simple monotone improvement in outcomes.

4 Mapping back to the "functional dealer" specification

Informally, but with reasonably precise model content, the intended *functional dealer* in a Kalecki ring is characterised by the following requirements:

1. **Two-sided market maker with sustainable P&L.** The dealer quotes buy and sell prices (a spread) over time, adjusts that spread, and does not systematically lose money.

2. **Non-pathological investment opportunity for debtors/traders.** Traders have a realistic possibility of earning by buying the dealer's securities. Their search for yield is what motivates them to buy.

3. **Liquidity provision on the sell side.** Debtors holding securities can sell them to the dealer to raise liquidity when needed to settle their obligations.

4. **Priority of debt service in trader behaviour.** Traders invest in securities only if they expect to retain the capacity to meet future obligations, and they continue to hold/invest only so long as this does not jeopardise their repayment capacity.

The current comparison experiment primarily measures the net effect on default rates via δ_{total}, ϕ_{total} and the relief ratio R. It does not, in its summarised form, report:

- dealer P&L or spread dynamics;
- individual trader P&L from trading;
- any explicit repayment-priority metric at the agent level.

The experiment is therefore focused on the question "Does the dealer reduce defaults?" rather than the fuller behavioural specification above.

5 What did happen, relative to the intended mechanism

5.1 System-level effect: dealer as relief mechanism

On the dimension "the dealer makes it easier for debtors to repay", the results are internally coherent:

- on average, the dealer reduces defaults by approximately 19%;
- in 94 out of 125 parameter combinations, introducing the dealer strictly lowers the default rate;
- in 31 out of 125 combinations, the default rate is unchanged;
- there are no parameter combinations in which the dealer worsens defaults.

Interpreted literally, this implies:

- in a large majority of parameter configurations, the dealer reallocates liquidity and/or absorbs claims in a way that relieves default pressure;
- on the tested grid, the dealer never harms systemic default outcomes, which is an important internal consistency check.

With respect to the basic requirement that the dealer should be able to reduce default stress in a Kalecki ring of debtors, the experiment shows that the implemented dealer has this property in a statistically strong sense (the reported effect is associated with a p-value of order 10^{-18}).

5.2 Parameter regions where the expected effect does not materialise

There are two distinct classes of "non-relief" cases:

1. **Baseline zero-default cases.** Roughly 10 of the 125 parameter pairs have $\delta_{\text{control}} = 0$; by construction $R = 0$ in these cases. Here, the system clears all debts even in the control; the dealer cannot improve defaults because there are none. The requirement "dealer reduces defaults" is vacuous in these configurations.

2. **Positive defaults but zero relief (about 21 pairs).** In these cases, $\delta_{\text{control}} > 0$ and $\delta_{\text{treatment}} = \delta_{\text{control}}$, so defaults persist at the same rate even with the dealer. These cases are concentrated in low-μ (maturities bunched near the horizon) and relatively low or moderate concentration c, with one outlier at $\mu = 1$ and very low κ.

The "run 42" example is in this second class:

$$\kappa = 0.25, \quad c = 0.2, \quad \mu = 0, \quad \text{seed} = 42, \quad \delta_{\text{control}} = \delta_{\text{treatment}} \approx 0.518.$$

A provisional economic interpretation of these non-relief cases is:

- with μ near 0, maturities are tightly bunched at the horizon, leaving little time structure for the dealer to exploit; there are fewer intertemporal liquidity imbalances to intermediate;
- with low concentration c, payoffs are diffuse, so the network structure itself can become the binding constraint: some nodes run out of cash even when aggregate liquidity is ample, and the dealer's limited outside capital (set at 25% of system cash) is not sufficient, or not deployed in the right places, to repair those bottlenecks;
- in these configurations, re-trading alone is not enough: for the given level of outside capital and trading rules, the remaining defaults are essentially structural.

Thus, on a first pass, there is a region of roughly one quarter of the tested parameter space in which a dealer of this size and with these heuristics cannot convert remaining defaults into repayments. This is one concrete sense in which the idealised narrative ("dealer plus trading improves repayment capacity") does not hold uniformly across the grid.

6 Behavioural components: missing or misaligned pieces

6.1 Dealer as spread-quoting, non-loss-making market maker

In implementation, the dealer subsystem is configured so that the dealer and value-backed token (VBT) bring in new outside cash (at 25% and 50% of system cash respectively) and start with zero inventory, building inventory by purchasing from traders. Traders retain 100% of their initial receivables. The integration between ring and dealer has been debugged so that claims transfer correctly between traders and dealer, prices scale with face values, and the dealer's internal accounting is consistent.

However, the comparison experiment does not record dealer P&L or spread dynamics in the aggregated outputs. These metrics are only present in per-run metrics files and are not summarised in the 125-pair sweep. One can infer from the micro-specification that the dealer is not designed as a systematic loss-maker, but the current analysis does not explicitly test the non-loss-making requirement.

6.2 Traders earning from buying dealer securities

The intended semantics are:

- traders are pulled into buying by the prospect of earning a return;
- there is a realistic chance that this pays off ex post.

Two implementation facts are relevant:

1. In an earlier phase, buying from the dealer was disabled (to prevent liquidity drain), and with trading fixed and buying disabled, a smaller grid already showed a mean relief ratio of roughly 17% with many improvements.
2. After buying was re-enabled and the full 125-pair grid was run, the analysis shows that:
    - with buying: mean relief ratio $\approx$ 19.0%;
    - without buying: mean relief ratio $\approx$ 19.3%;
    - the conclusion is that buying is slightly counterproductive (about -0.3 percentage points) because it reduces the liquidity of traders who may need cash for their own obligations.

At the system level, allowing traders to invest in dealer securities has a small but clearly negative effect on repayment outcomes relative to a world in which they never buy at all. This indicates that the current buying rule does not fully implement the desired behaviour "invest, but not at the expense of the ability to repay". Traders are allowed to tie up some cash in investments that could have been used to meet obligations.

Moreover, the 125-pair summary does not expose trader-level P&L from these transactions, so one cannot yet determine in which configurations traders actually earn a positive return from buying. All that is visible is that, in aggregate, additional buying marginally harms systemic relief compared to the same dealer with only the sell side active.

6.3 Traders selling to raise liquidity to repay

This is the mechanism most directly supported by the comparison results. Whenever $\delta_{\text{treatment}} < \delta_{\text{control}}$ for a given parameter triple, some obligations that would have defaulted in the control are met in the treatment. The only new channel in the treatment is the dealer-mediated secondary market (plus the dealer's outside capital) through which agents can sell claims for cash.

Thus, in the 94/125 improvement cases, some agents raise additional liquidity via sales to the dealer (directly or indirectly) and use that liquidity to meet obligations that would otherwise have defaulted.

In the 31 unchanged cases, there are two sub-classes:

- baseline zero defaults: selling to the dealer is irrelevant for repayment;
- positive defaults with zero relief: here, either defaulting agents never manage to sell enough to the dealer, or the dealer's limited capital and ticket size mean that even after maximal trading, remaining shortfalls cannot be covered.

The 21 "positive-default but zero-relief" cases are precisely those in which the intended mechanism—debtor liquidation of claims via the dealer to avoid default—does not materialise in outcome space. In these parts of the parameter space, given current capital levels and trading heuristics, the dealer fails to act as a meaningful liquidity backstop.

6.4 Traders prioritising capacity to repay

In the code, traders' access to the dealer is constrained by eligibility rules (lookahead horizon, cash buffers, and so on), intended as a crude approximation of "do not jeopardise ability to repay". Two observations indicate that this is not yet sufficient for the stronger narrative:

1. At the macro level, enabling buying slightly worsens repayment compared to a world where traders never buy. If traders really prioritised repayment capacity in the strong sense described above, buying would not reduce systemic relief.
2. The experiment tracks only δ and ϕ at system level; it does not track per-agent buffer adequacy or coverage of future dues. It is therefore not possible, from the current summary, to identify which agents buy, how their coverage of future obligations evolves after trading, and whether those who default had over-invested.

Thus, the simulation includes some guardrails against reckless behaviour, but it does not yet implement or measure the stronger requirement that agents invest only when they expect to earn enough to repay and stop investing when that capacity is threatened.

7 Provisional explanation in one paragraph

In summary: the implemented dealer is functionally capable of reducing defaults. Across the 125-point grid, the mean default rate falls by about 19%, and there are no cases in which introducing the dealer worsens defaults. However, in roughly a quarter of the parameter space, either defaults are already zero (so the dealer is irrelevant) or the remaining defaults are not alleviated at all, indicating regions where re-trading plus the dealer's fixed outside capital cannot resolve structural liquidity problems. Behaviourally, the current design does not fully realise the narrative of traders seeking yield while always prioritising repayment: in aggregate, enabling buying slightly reduces relief, and key elements of the specification—dealer P&L dynamics, trader returns, and explicit repayment-priority metrics—are not yet measured in the comparison experiment.

8 Measurement and data collection for functional dealer properties

The definitions D1–D3 and requirements R1–R6 specify a *functional dealer* and associated trader behaviour in conceptual terms. To evaluate these properties in concrete simulations, the experiment design must expose a richer set of outputs than the global default metrics δ_t, φ_t and the relief ratio R. This section lists the additional data and summary statistics required.

8.1 Run-level structure and notation

Consider a single simulation run indexed by r, with time periods $t = 0, 1, \ldots, T_r$. For each run we require:

- Ring-level state:
  - aggregate ring metrics per period $(S_t, \bar{M}_t, M_t, v_t, \varphi_t, \delta_t)$ as in R6;
  - counts and sizes of defaults, including the distribution of recovery rates across issuers.
- Dealer and VBT state, per maturity bucket b:
  - dealer inventory $a_t^{(b)}$ (tickets), dealer cash $C_t^{(b)}$;
  - outside mid $M_t^{(b)}$ and spread $O_t^{(b)}$ from the corresponding VBT;
  - inside bid/ask quotes $b_t^{(b)}(x)$, $a_t^{(b)}(x)$ evaluated at the realised inventory $x = a_t^{(b)}$;
  - mark-to-mid equity

    $$E_t^{(b)} \;=\; C_t^{(b)} + M_t^{(b)} a_t^{(b)},$$

    with increments $\Delta E_t^{(b)} := E_t^{(b)} - E_{t-1}^{(b)}$.
- Trader-level state, for each ordinary agent i:
  - cash $C_i(t)$, set of tickets held (issuer, face, maturity);
  - schedule of future obligations (tickets where i is issuer);
  - default indicator and recovery received if a counterparty of i defaults.

In addition, we require a trade log per run:

- for each dealer trade at time t:

  $$\text{record }(\text{time } t, \text{bucket } b, \text{side} \in \{\text{BUY, SELL}\}, \text{agent } i, \text{price } p, \text{issuer, maturity}).$$

These primitives are sufficient to construct the diagnostic statistics below.

8.2 Dealer and VBT profitability

To test the "non-loss-making" part of D2, we measure realised P&L for each dealer (and, analogously, for each VBT) at both per-period and run-level horizons.

[Dealer P&L and profitability] For bucket b in run r, define

$$\Pi_r^{(b)} \;:=\; E_{T_r}^{(b)} - E_0^{(b)} \qquad \text{and} \qquad \pi_r^{(b)} \;:=\; \frac{\Pi_r^{(b)}}{E_0^{(b)}},$$

whenever $E_0^{(b)} > 0$. A dealer is *empirically sustainable* on a set of runs $\mathcal{R}$ if

$$\frac{1}{|\mathcal{R}|}\sum_{r\in\mathcal{R}} \Pi_r^{(b)} \;\geq 0 \quad \text{and} \quad \Pi_r^{(b)} \geq 0 \text{ on a large majority of runs.}$$

The per-trade log allows decomposition of $\Pi_r^{(b)}$ into:

- inside spread income (difference between dealer buy and sell prices);
- outside layoff costs at VBT quotes;
- mark-to-mid revaluation of inventory between periods.

Summaries of these components diagnose whether losses are driven by systematic mispricing, extreme outside layoff frequency, or adverse selection in order flow.

8.3 Trader investment returns and liquidity use

D3 requires that traders have a realistic possibility of earning from buying dealer securities, and that they can use those securities as liquidity buffers when needed. We therefore track realised returns and liquidity uses at the agent level.

Ticket-level realised return. For each trader i and each ticket τ purchased from a dealer at time t_{buy} for price p_{buy}, define its realised payoff X_τ as the sum of:

- any coupon payments received while i holds τ;

- the settlement amount at maturity (if i still holds τ);
- the resale price if i later sells τ to a dealer, evaluated at the dealer bid at the sale time.

The realised return on τ is

$$R_\tau \;:=\; \frac{X_\tau - p_{\text{buy}}}{p_{\text{buy}}}.$$

[Trader investment performance] For trader i in run r, let $\mathcal{T}_i$ be the set of tickets they ever purchase from a dealer. Define

$$\overline{R}_i \;:=\; \frac{1}{|\mathcal{T}_i|} \sum_{\tau \in \mathcal{T}_i} R_\tau.$$

At the run level we record the empirical distribution of $\{\overline{R}_i\}_i$ and the fraction of traders with $\overline{R}_i > 0$.

Liquidity-driven sales and "rescues". From the trade log, we mark a dealer trade as *liquidity-motivated* when the selling trader satisfies the liquidity-policy conditions (shortfall in current obligations) at the time of sale. For each run we then compute:

- the number of such liquidity-driven sales;
- the number of obligations that would have defaulted in the control configuration but are met in the dealer configuration following such sales ("rescue events").

This measures the extent to which D3's "liquidity-on-demand through sales" mechanism is actually used to avoid default.

8.4 Repayment-priority diagnostics

To operationalise the repayment-priority constraint in D3, we define a simple safety-margin functional. For trader i at time t, let

$$A_i(t) \;:=\; C_i(t) + \sum_{\tau \in \mathcal{H}_i(t)} b_t^{(\tau)} S_\tau,$$

where $C_i(t)$ is cash, $\mathcal{H}_i(t)$ is the set of tickets held by i, S_τ is the face amount of τ, and $b_t^{(\tau)}$ is the relevant dealer bid (bucket-specific) at time t. Let

$$D_i(t) \;:=\; \sum_{\tau \in \mathcal{O}_i(t)} S_\tau$$

be the remaining nominal amount of i's future obligations.

[Deterministic safety margin] The deterministic safety margin of trader i at time t is

$$m_i(t) \;:=\; A_i(t) - D_i(t).$$

When trader i buys a ticket at time t, we record $m_i(t^-)$ and $m_i(t^+)$ (before and after the transaction). We can then construct statistics such as:

- the fraction of BUY trades that reduce $m_i(t)$ below zero;
- the distribution of $m_i(t)$ at the time of default for agents that eventually default;
- the distribution of $m_i(t)$ at the times when agents liquidate tickets for liquidity reasons.

These diagnostics tell us whether the simple heuristic trading policies are in fact respecting a strong repayment-priority norm, or whether agents frequently invest in ways that ex post jeopardise their ability to repay.

8.5 Experiment-level summary statistics

Given the above measurements, each parameter triple (κ, c, μ) generates not only a default-relief statistic R, but also:

- dealer and VBT profitability indicators (mean and dispersion of $\Pi_r^{(b)}$ and $\overline{\pi}_r^{(b)}$ over runs);
- trader investment performance (distribution of $\overline{R}_i$);
- repayment-priority compliance (frequency and magnitude of safety-margin violations at BUY times);
- usage of the liquidity channel (counts of rescue events).

A configuration can then be classified as supporting the intended functional dealer narrative only if defaults are reduced *and* dealer profitability, trader investment returns, and repayment-priority metrics are simultaneously in acceptable ranges.

9 Design levers and experimental tools

If the diagnostics in Section 8 show that the functional dealer requirements are not satisfied—for example, buying from the dealer systematically harms traders, or the dealer runs losses while providing relief—we can consider model extensions that preserve the microstructure of the dealer kernel and trader heuristics but change the environment in which they operate. This section outlines three such levers.

Throughout, we assume that dealers and VBTs are profit-seeking and that trader decision rules remain of the simple, rule-based form already implemented.

9.1 Coupon-bearing claims

The baseline specification uses zero-coupon tickets of fixed face value S with a single payoff at maturity and no interest accrual. This limits the scope for genuine investment returns, because any dealer bid $p > 1$ on a claim whose maximum payoff is normalised to 1 cannot be sustained by fundamental cash flows.

To create meaningful earning opportunities while leaving the dealer kernel unchanged, we can introduce a simple coupon structure.

9.1.1 Payment structure

For each underlying debt contract with face value S and maturity τ, define a coupon rate $q \geq 0$ and a fixed coupon date $t_{\text{cp}} < \tau$ (or a set of coupon dates). The ticketised claim associated with this contract then pays

- a coupon qS at each coupon date, provided the issuer has not defaulted; and
- the remaining principal S at maturity, again subject to default.

In the simplest case, $t_{\mathrm{cp}} = \tau$ and the total payoff is $S(1+q)$ at maturity. The outside VBT mid $M_t^{(b)}$ and the dealer midline $p_t^{(b)}(x)$ can then be interpreted as prices relative to this higher fundamental payoff, with no change to the ladder structure or the zero-expected-profit identity for the inside spread.

9.1.2 Measurement and expected effect

With coupon-bearing claims, the realised ticket-level payoff X_τ used in R_τ naturally includes coupon receipts. This should increase the fraction of tickets with positive realised R_τ for traders who buy at moderate discounts to the eventual payoff.

At the same time, dealer P&L $\Pi_r^{(b)}$ will reflect coupon inflows on inventory as well as spread income, making it possible in principle for dealers to remain profitable even when they occasionally bid above par in early periods.

Empirically, we can test whether the introduction of coupons:

- increases the share of traders with $\overline{R}_i > 0$;
- allows dealer profitability to remain non-negative on average;
- does so without weakening the default-relief property (change in R).

If buying remains systematically harmful to traders even with coupons, the problem lies in the trading heuristics rather than the payoff structure.

9.2 Reappearing debts and extended horizon

In the current Kalecki ring environment there is no new debt issuance; all liabilities are specified and ticketised at initialisation, and the simulation stops once these have matured. This makes investment in longer-maturity claims a one-shot affair: agents have limited scope to use accumulated earnings to support future obligations beyond the initial horizon.

To give agents an ongoing reason to earn and reinvest, we can allow debts to reappear (be rolled over) once repaid, thus extending the effective horizon while preserving the ring structure.

9.2.1 Simple rollover rule

Let each agent i have an initial liability L_i maturing at time T_i. When L_i is fully repaid at T_i, we create a new liability L'_i with:

- face value S'_i determined by a simple rule (for example, $S'_i = S_i$ or $S'_i = S_i(1+\gamma)$ for some fixed growth rate γ);
- maturity $T'_i = T_i + H$ for a fixed horizon H;
- the same creditor or a creditor drawn from a simple rule that preserves the ring topology.

The new liability is ticketised in the same way as initial debts and enters the appropriate maturity bucket. Coupons on existing tickets can be designed to align with this rollover frequency.

9.2.2 Measurement and expected effect

With rollover, each agent faces a stream of obligations rather than a one-off payment. The diagnostics in Section 8 then track not only whether agents repay initial debts, but whether they can sustain a sequence of repayments:

- dealer P&L becomes a function of a longer inventory path and more layoff events;
- trader returns R_τ aggregate over multiple cycles of buying and selling;
- safety margins $m_i(t)$ can be evaluated relative to a longer horizon of obligations.

The intended effect of rollover is to create configurations where agents can "earn their way" into meeting future dues by appropriate investment in dealer-traded securities, rather than treating the dealer interaction as a single-period liquidity patch.

If, after introducing rollover, traders still mostly incur losses on their investments and defaults remain concentrated in agents that previously bought tickets, the diagnosis is that the *combination* of payoff structure and decision rules is misaligned with D3.

9.3 Three-way comparison of market architectures

The 125-pair experiment compares only two regimes: a pure Kalecki ring with no large holders and the same ring with a full dealer/VBT microstructure. To disentangle the effects of balance-sheet capacity from those of price-mediated trading, it is useful to insert an intermediate regime with large passive holders.

9.3.1 Definition of regimes

Fix a ring configuration (agents, initial debts, ticketisation, and any coupon/rollover rules). For each parameter triple (κ, c, μ) we define three regimes:

1. **Baseline ring (no big holders).** The original Kalecki ring with no dealer and no VBTs. All tickets are held by the original creditors until maturity or default; there is no secondary market.

2. **Ring with big passive holders.** Introduce one or more large agents that initially hold the same *quantity and distribution* of tickets as the dealer and VBTs would hold in the full microstructure, but impose the rule that these passive holders never trade. They simply bear losses mechanically through defaults.

3. **Ring with full dealer/VBT microstructure.** The current dealer/VBT design: big holders are replaced by dealers and VBTs with identical initial balance sheets, who quote prices and trade with the ring agents according to the specified kernel and trading policies.

The key is that the total outside capital and initial allocation of claims to large balance sheets is held fixed across regimes. Only the *market-making function* changes.

9.3.2 Metrics and identification

For each regime and parameter triple we compute:

- default statistics (δ_t, φ_t) and relief ratios;

- distribution of losses across small agents vs. large holders;

- in the full microstructure regime, dealer and VBT P&L and trader investment metrics as in Section 8.

Comparisons then answer distinct questions:

- *Baseline vs. passive holders* isolates the effect of adding large balance sheets that can absorb losses, without any liquidity provision or price discovery.
- *Passive holders vs. full dealer/VBT* isolates the value added by an active microstructure (two-sided quotes and dynamic inventory management), holding constant the amount of outside capital and its loss-bearing capacity.

If the passive-holder regime already captures most of the default improvement relative to the baseline, the main mechanism is simply loss-absorption by large balance sheets. If, by contrast, the full dealer/VBT regime further reduces defaults or improves trader investment outcomes without making dealers loss-making, this provides evidence that the market-making mechanism itself is contributing to repayment capacity in the sense of D1–D3.

9.3.3 Use as a diagnostic tool

In practice, the three-way comparison serves as a diagnostic:

- If adding passive holders but *not* dealers eliminates most defaults, then failures of the functional dealer narrative in the full regime are likely due to pricing or behavioural rules, not insufficient capital.
- If passive holders have little effect but dealers do, the liquidity and price-formation mechanism is doing real work.
- If neither passive holders nor dealers materially change defaults in some region of parameter space, the remaining defaults are structural with respect to both balance-sheet capacity and the current microstructure.

These distinctions are essential for interpreting any failure to satisfy the functional dealer criteria and for deciding which modelling layer (claim design, rollover, trading heuristics, or dealer kernel) should be modified next.

10 Implementation in the Bilancio codebase

This section describes how to realise the measurement framework of Section 8 and the three levers of Section ?? in the existing Bilancio codebase. The focus is on minimal extensions to the current dealer–Kalecki integration: the `Payable` instrument, the dealer ticket model, the `DealerSubsystem` wrapper, and the experiment harness for Kalecki–ring sweeps.

10.1 Overview of existing integration

The current implementation already provides a bridge between the main Bilancio engine (contracts and payables) and the standalone dealer ring module:

- *Contracts and payables.* Ring debts are represented as `Payable` contracts in `credit.py`, with fields `asset_holder_id`, `liability_issuer_id`, `amount`, `denom`, and `due_day`. A separate field `holder_id` tracks the current secondary–market holder and is used via the `effective_creditor` property at settlement.

- *Dealer subsystem.* The module `dealer_integration.py` defines a `DealerSubsystem` dataclass holding per–bucket dealer and VBT state, trader states, the universe of tickets, and mapping tables between tickets and payables. It exposes entry points to initialise the subsystem, run a dealer trading phase within `run_day()`, and sync trade results back to the main system.

- *Bridge.* Functions in `dealer/bridge.py` convert payables to tickets (`payables_to_tickets`), assign tickets to maturity buckets, group them by owner (`tickets_to_trader_holdings`), and apply executed trades back to payables (`apply_trade_results_to_payables`).

- *Simulation loop.* The main engine's `run_day()` function has an `enable_dealer` flag and inserts a dedicated dealer subphase between the main liquidity and settlement subphases. When enabled, this subphase runs the dealer trading loop for the current day.

Against this backdrop, we now specify concrete extensions needed for: (i) coupon–bearing claims; (ii) reappearing debts and an extended horizon; and (iii) a three–regime comparison harness.

10.2 Coupon–bearing claims

10.2.1 Data model extensions

To implement the coupon structures of Section **??**, we extend the credit and ticket models with optional coupon fields.

Payables. In `credit.py`, augment the `Payable` contract with coupon metadata:

- a coupon rate $q \geq 0$;
- an optional list of coupon dates $t_1^{\mathrm{cp}}, \ldots, t_K^{\mathrm{cp}}$ (with $t_k^{\mathrm{cp}} <$ `due_day`), or a single coupon date t^{cp} in the one–coupon special case;
- a simple convention for zero–coupon payables: $q = 0$ or an empty coupon–date list.

These fields default to the zero–coupon case, so existing scenarios run unchanged.

Tickets. In the dealer module's `Ticket` model, introduce corresponding optional fields:

- `coupon_rate` q;
- `coupon_dates` (relative or absolute time indices).

The bridge `payables_to_tickets()` copies coupon metadata from each `Payable` to its ticketised claims. For scenarios with homogeneous coupons (one rate q and a common t^{cp} per maturity), this can be implemented as a global setting in the YAML scenario, applied at ticketisation time.

10.2.2 Event loop changes

Coupon payments are realised as additional settlement events in the simulation loop:

1. At the start of day t (before dealer trading and before principal settlement):

(a) For each underlying contract and/or ticket with $t \in \{t_k^{\mathrm{cp}}\}$, compute the due coupon qS.

(b) For each issuer i, aggregate coupons on its outstanding liabilities, and attempt payment using current cash $C_i(t)$.

(c) If $C_i(t)$ is sufficient to meet both coupon and any principal due at t, execute coupon payments in full. If not, apply the same proportional recovery rule used for principal to the combined coupon–plus–principal amount, so that coupons and face value share losses proportionally when there is a shortfall.

2. The holder of a ticket (ring agent, dealer, or VBT) receives qS per non–defaulted claim and updates its cash.

In code terms, this can be realised either by:

- extending the settlement logic to treat coupons as a separate "pseudo–maturity" event keyed by `coupon_dates`, or
- representing each coupon as an on–demand `Payable` created at initialisation and maturing at the coupon date; these coupon payables are then settled with the existing machinery.

The dealer kernel and VBT anchor rules remain unchanged. Coupons appear only through cash flows into the cash components of dealer, VBT, and trader states.

10.2.3 Diagnostics wiring

Under this implementation, the diagnostics of Section 8 are obtained as follows:

- Dealer and VBT P&L $\Pi_r^{(b)}$ per bucket b accumulate both coupon receipts on inventory and spread income on trades. Run–level P&L and equity paths $E_t^{(b)}$ are computed from the existing dealer and VBT cash and inventory records.
- Ticket–level realised returns R_τ include coupon cash flows by construction: the payoff measure X_τ used in $R_\tau = (X_\tau - P_0)/P_0$ must aggregate all coupon and principal payments received by the ticket holder.

- The ring–level statistics ϕ_t, δ_t and their treatment–control differences are unaffected, but can now be decomposed by whether obligations were serviced from coupon income, primary cash, or secondary–market realisations.

Empirically, we test whether coupons increase the share of agents with $R_i > 0$, preserve the dealer's non–loss–making property, and do not erode the default–relief property of the dealer.

10.3 Reappearing debts and extended horizon

10.3.1 Rollover rule at the contract level

To implement the rollover mechanism of Section **??**, we introduce a simple, configurable rule at the `Payable` level.

For each original liability L_i of issuer i with face value S_i and maturity T_i, define:

- a rollover horizon $H \geq 1$ (in days);
- a face–value update rule $S_i' = f(S_i)$, with simple cases $S_i' = S_i$ or $S_i' = S_i(1+\gamma)$ for fixed γ;
- a rollover condition "full repayment" at T_i, i.e. a realised recovery rate $R_i(T_i) = 1$ on the principal component.

At the end of day T_i, after settlement:

1. If $R_i(T_i) = 1$, create a new `Payable` L_i' with:
   - issuer i;
   - creditor equal to the last effective creditor of L_i (preserving the ring topology);
   - face value S_i';
   - maturity $T_i' = T_i + H$;
   - coupon structure inherited or updated according to the coupon regime (for instance, the same q and coupon dates shifted by H).
2. Add L_i' to the contract set; the dealer bridge will ticketise it on the next bucket update.

Liabilities that default at T_i (i.e. $R_i(T_i) < 1$) do not roll over; the issuer leaves the ring on that leg.

10.3.2 Changes to the simulation loop

At the end of each day t:

1. Run the usual settlement and default procedures, obtaining recovery rates $R_i(t)$ and updated cash positions $C_i(t^+)$.
2. If t is a maturity date for some L_i, apply the rollover rule above, creating any new L'_i with maturity $t + H$.
3. Extend the global simulation horizon $T_{\max}$ as needed to accommodate new maturities. For example, run until either:
   - a fixed number of periods $T_{\max}$ is reached (e.g. initial horizon plus K rollover cycles); or
   - a stopping condition such as "no active payables remain" or "no rollovers for M consecutive periods".

The dealer subsystem requires only minor adjustments:

- ticketisation is re–invoked for newly created payables;
- the maturity–update logic and bucket assignment already handle tickets with arbitrarily large `maturity_time`;
- the clean–up logic for matured tickets remains in place, but now runs over a longer horizon.

10.3.3 Diagnostics in the rollover regime

With reappearing debts, the diagnostics of Section 8 are computed over a longer path:

- dealer equity $E_t^{(b)}$ and P&L $\Pi_r^{(b)}$ reflect a multi–cycle inventory path, including layoff events on rolled tickets;
- trader returns R_τ can be aggregated per agent over each cycle or across multiple rollovers;

- safety margins $m_i(t)$ can be defined relative to the current horizon of obligations for agent i, including rolled–over debts.

The main empirical question is whether, in this extended environment, agents are able to "earn their way" into meeting future obligations by investment in dealer–traded securities, in the sense of D3.

10.4 Three–regime comparison harness

10.4.1 Regime definitions in the codebase

The three regimes of Section **??** can be implemented as three configurations of the existing simulation and experiment harness:

1. **Baseline ring (no big holders).**
   - Run the Kalecki ring simulation with `enable_dealer = false`.
   - No dealer subsystem is initialised; all tickets remain directly on the balance sheets of ring agents until maturity or default.
2. **Ring with big passive holders.**
   - Extend the agent set with one or more "passive holder" agents. At initialisation, allocate to them the same outside capital and the same ticket inventory that the dealer/VBT complex would hold in the full microstructure regime.
   - Disable dealer trading: either do not initialise the dealer subsystem, or initialise it but set per–period order–flow limits to zero so that no trades occur.
   - Passive holders never initiate trades; they simply bear losses according to the recovery rule when issuers default.
3. **Ring with full dealer/VBT microstructure.**
   - Initialise the `DealerSubsystem` as in the current integration, with the same total outside capital and initial allocation of tickets between dealers, VBTs, and ring agents as in regime 2.
   - Enable dealer trading by turning on the dealer subphase in `run_day()` and running the configured number of one–ticket arrivals per period.

The key requirement is that the total amount of outside capital and the initial allocation of claims to large balance sheets are held fixed across regimes; only the presence or absence of active market–making changes.

10.4.2 Experiment harness

The existing ring experiment runner can be extended to support a three–regime sweep:

- For each parameter triple (κ, c, μ) and seed, instantiate a base scenario describing the ring (agents, payables, ticketisation, coupon and rollover rules).
- For each regime $r \in \{\text{baseline}, \text{passive}, \text{dealer}\}$, derive a scenario variant:
  - toggle `enable_dealer`;
  - choose whether to include passive holder agents and with what initial ticket allocation;
  - select the appropriate `DealerConfig` and `DealerTraderPolicyConfig`.
- Run the three regimes with the same base seed and record:
  - ring–level metrics (δ_t, ϕ_t) and relief ratios;
  - loss distribution between small agents and large holders;
  - in the dealer regime, dealer and VBT P&L, trader investment metrics, and any safety–margin diagnostics.

The comparison between baseline and passive regimes isolates the effect of large balance sheets; the comparison between passive and dealer regimes isolates the effect of active market–making.

10.5 Summary

The proposed implementation plan keeps the dealer kernel and trading heuristics as they are while enriching the payoff structure (via coupons and rollover) and the experiment architecture (via the three–regime harness). This allows the simulation to test, in a way that is tightly coupled to the existing codebase, whether the dealer and VBT can be profit–seeking, whether traders can obtain positive expected returns on securities, and how much of the observed default relief is due to balance–sheet capacity versus price–mediated secondary markets.