Liquidity and internal debt: Instrument level dealer based simulation

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Contents

Abstract

1 Introduction

2 Dealers, liquidity and pricing

The different modeling traditions in monetary economics can be read as embodying different *implicit theories* of money, price, credit, payment, and default. By a "theory" here we do not mean a self-conscious doctrine, but the way in which each object is defined, located in the model's ontology, and allowed to move by the model's equations and constraints. This subsection uses that fivefold lens to contrast mainstream equilibrium, accounting-based, network and agent-based frameworks.

Across otherwise very different models, a common pattern emerges. Money is typically treated as a homogeneous stock, or even suppressed into a generic liquidity service; prices are either Walrasian equilibrium objects or reduced-form behavioral variables; credit is an aggregate intertemporal position rather than a web of dated contracts; payment is compressed into a budget constraint; and default is reduced to an exogenous or aggregate shock. What is missing is a "thick" joint theory in which concrete monetary instruments, payment and default events are all represented at the level at which real-world monetary phenomena actually occur.

In our balance-sheet-and-dealer simulation framework, every asset and liability is recorded on specific agents' balance sheets and evolves over time. This allows us to ask concrete, operational questions about money, price, credit, payment, and default that are typically not well-posed in more aggregate or representative-agent monetary models.

2.1 Money

1. How are means of payment created, and by whose decision?

2. Which specific balance-sheet operations first bring an accepted means of payment into existence (for example: a bank loan, a repo transaction, or a central bank purchase)?

3. Whose liability is the means of payment at the moment of creation, and which counterparty holds it as an asset?

4. Who are the first recipients of newly created money, and who only receive it later along the reflux chain?

5. Through which concrete chains of transactions do individual units of money travel before they are ultimately extinguished?

6. Where, exactly, does "money scarcity" arise even when the aggregate quantity of money is sufficient — that is, at which agents and at which time points are required means of payment not aligned with incoming cashflows?

7. How do different monetary layers interact (for example, when a lower-tier asset is accepted as means of payment between some agents but must be funded in higher-tier reserves)?

2.2 Price

1. Who quotes the prices of financial instruments, and on what basis?

2. Given a dealer's current inventory, funding costs, and upcoming payment schedule, what bid and ask quotes do they set at each time point?

3. How do funding constraints—such as the loss of a credit line or tighter collateral haircuts—feed into quoted prices: do dealers widen spreads, cut trade size, or withdraw from making markets?

4. How does the structure of a dealer's liabilities (for example, shorter maturities or more expensive funding) translate into the yields they require on the asset side of their balance sheet?

5. How do prices differ across agents for the same instrument, once we model explicitly which counterparties can trade with which dealer at a given time?

6. What is the balance-sheet cost of providing liquidity for a given transaction, in terms of additional leverage, maturity mismatch, or default risk that the quoting dealer must take on?

7. How do market prices adjust during stress episodes when some dealers default or withdraw, and how do the remaining dealers' inventories and funding burdens change as a result?

2.3 Credit

1. Who is able to extend credit to whom, and under what institutional or balance-sheet conditions (for example, which types of agents are allowed to issue which instruments)?

2. How does the creation of a new credit instrument change the distribution of liquidity and risk across agents' balance sheets?

3. Where, in the network of agents, is maturity transformation and leverage located—that is, which specific agents fund long-dated claims with short-dated liabilities, and by how much?

4. How do particular credit links create systemic vulnerability: which loans or credit lines, if not rolled over or repaid, trigger chains of further defaults?

5. Under stress, what forms of credit substitution occur: if one funding source disappears, which agents switch to which alternative instruments (for example, issuing securities instead of borrowing from a bank), and what new network of claims emerges?

2.4 Payment

1. Using what concrete means of payment is each liability settled, and through which sequence of transactions and intermediaries?
2. How are individual payments initially and ultimately funded: does the payer rely on pre-existing cash, on the proceeds from asset sales, or on newly created credit from a dealer or bank?
3. In what order do payments occur at each time point, and how does this sequencing affect the feasibility of settlement for particular agents?
4. Where and when does payment gridlock arise—for example, when several agents simultaneously depend on incoming payments from each other so that no one can pay first without external liquidity?
5. What roles do dealers play within the payment system: when a dealer stands between two non-dealer agents, how do they temporarily warehouse assets or provide intraday credit, and how does this change their own exposure?

2.5 Default

1. Exactly which liability fails first, and why that particular obligation rather than another, given the preceding sequence of payments and funding operations?
2. How do defaults propagate through balance sheets: when one agent defaults on another, how does the resulting loss or missed inflow affect the creditor's ability to meet its own obligations?
3. Under a given set of liquidation or resolution rules, how are losses distributed across creditors and counterparties: who recovers how much, in which order, and in what instruments?
4. Which positions and agents are systemically important in the sense that their default generates the largest cascades of further defaults or payment failures?
5. How do alternative contractual, regulatory, or policy regimes (for example, different collateral or priority rules) change the pattern of defaults: which specific failures disappear or appear when these rules are modified?

3 Approaches to money modelling

3.1 Microfounded macro–finance and representative-agent asset pricing

Microfounded macro–finance (DSGE-style) and representative-agent asset-pricing models constitute the dominant mainstream benchmark for thinking about monetary and financial dynamics. Read through the five lenses, these models have the following implicit theories.

Money, when present at all, is typically a *single scalar object*—a cash variable in a cash-in-advance constraint, a money-in-utility argument, or a generic liquid asset in a collateral or portfolio problem. There is no explicit issuer or structured hierarchy; reserves, deposits, and other IOUs collapse into a single "liquidity" term that relaxes constraints or provides services. Monetary creation and destruction are not modeled as balance-sheet operations of specific banks; they occur implicitly as agents choose optimal money holdings.

Prices are *equilibrium objects*: one frictionless price per asset (or one nominal price with sticky adjustment), determined by Euler equations, no-arbitrage conditions, and market-clearing. The implicit price theory is Arrow–Debreu or stochastic-discount-factor pricing, possibly perturbed by nominal rigidities, but without dealer balance sheets, bid-ask spreads, or inventory- and funding-driven liquidity effects. There is no scope for prices to temporarily break away from fundamental value or for markets to "disappear" when balance sheets are constrained; such phenomena must be introduced, if at all, via exogenous wedges.

Credit is *intertemporal trade in the representative or few-type dimension*. There is usually one or a small number of canonical debt instruments (risk-free bond, risky loan), with borrowing limits or default probabilities entering as parameters or simple constraints. The complex institutional structure of credit—heterogeneous counterparties, maturities, covenants, collateral arrangements—is compressed into a small state vector. A credit expansion is an increase in an aggregate debt variable, not the creation of a system of bilateral instruments with dated cash-flow obligations.

Payment is *collapsed into the within-period budget constraint and any cash-in-advance terms*. An agent who chooses consumption, investment, and portfolio satisfying their constraint is implicitly assumed to be able to make all required payments. There is no explicit payment system, no settlement asset distinct from generic "money", and no representation of the timing of payment within the period, of payment chains, or of funding strategies for particular transactions.

Default is typically *reduced-form*: a fraction of loans does not repay, or a Poisson intensity or structural threshold governs the arrival of default events, feeding into the pricing of risky assets. Defaults are shocks to returns or to borrowing constraints, not modeled episodes in which specific scheduled payments are missed, collateral is liquidated, and claims are reallocated along contractual lines. Contagion is captured, if at all, by aggregate balance-sheet effects, not by propagation along explicit contractual networks.

These choices deliver analytical tractability and a clear welfare and policy calculus. But they also imply very thin, highly abstract theories of money, price, credit, payment, and default, which leave little room for the concrete dealer- and payment-system phenomena that are central to modern monetary economies.

3.2 Stock–flow consistent models

Stock–flow consistent (SFC) models define themselves by their rigorous accounting discipline: sectoral stocks and flows are recorded in matrices that must add up exactly. From the five-theory perspective, however, this discipline is achieved at a highly aggregated level.

Money. Money is a *sectoral financial stock* (household deposits, bank reserves, etc.) and the corresponding flows that change it over the period. The SFC framework insists that these stocks and flows are mutually consistent, but typically at a level that aggregates over instruments and counterparties. The settlement asset role of money, the hierarchy between reserves and deposits, and the balance-sheet constraints of individual banks appear only implicitly, if at all, in the behavioral equations and closure rules.

Price. Prices are *macro behavioral variables*—mark-ups over costs, Phillips-curve inflation equations, portfolio demand schedules—chosen so that sectoral supply and demand are compatible with the accounting identities. There is one price per asset per period, no bid-ask spread, and no dealer inventories; the mapping from order flow and balance-sheet constraints to prices is absent.

Credit. Credit is *net new loan and debt positions at the sector level.* When banks lend to firms, the loan and matching deposit appear as sectoral balance-sheet entries; the underlying population of heterogeneous bilateral contracts is not tracked. Maturities and cash flows are compressed into simple interest and amortization terms. Credit dynamics are therefore about the evolution of sectoral balances, not about the life cycle of specific instruments.

Payment. Payment is *represented by net flows between sectors over the period.* Periodic budget constraints are required to hold ex post, but the model does not follow in detail which specific assets moved when between which individual agents. The temporal structure within the period and the funding choices for individual payments are suppressed. In effect, the theory of payment is that all due obligations are somehow settled by period end.

Default. Many SFC models abstract from default altogether; when they include it, it is usually an *exogenous write-down of stocks* (e.g. a reduction in the value of a loan portfolio) rather than the modeled failure of particular contracts to pay at specific dates, with explicit resolution. Defaults adjust matrix entries; they are not events that propagate through payment and collateral channels.

SFC modeling thus thickens the accounting relative to DSGE and asset-pricing frameworks, but it still leaves money, payment, and default as aggregate stock and flow adjustments, rather than granular balance-sheet and contractual events.

3.3 Network models of financial exposures

Network models take a different starting point: institutions are nodes, bilateral exposures are edges. This yields a rich picture of interconnectedness, but again with particular implicit theories of the five objects.

Money. There is usually *no explicit theory of money.* Edges are denominated in a unit of account, but the distinction between nominal claim and settlement asset is blurred. In interbank network models, interbank liabilities can play a money-like role, but the hierarchy between those liabilities, customer deposits, and central-bank reserves is rarely made explicit.

Price. Most canonical network models are *price-less.* Exposures are recorded at face value; when prices appear (e.g. in fire-sale extensions), they are exogenous haircuts or simple functions of liquidation volume. There is no explicit trading mechanism, no dealer schedule, and no link from balance-sheet constraints to spreads and depth.

Credit. Credit is *the network of obligations itself.* Each edge represents a promise to pay, and this is a genuine advantage relative to fully aggregated models. But those edges are usually static weights, not full schedules of dated cash flows with contractual contingencies; roll-over, covenant violation, and renegotiation are neglected. Credit is a static graph, not a dynamic system of contract states.

Payment. Payment appears only in the context of *clearing algorithms*. Given a realization of shocks, a clearing vector determines how much each node pays on each edge, possibly subject to limited liability and netting. The settlement asset itself, the timing of payments within the clearing window, and the funding operations that precede payment are outside the model. Payment is book-keeping on nominal obligations, not the movement of a specific means of payment through a multi-layer system.

Default. Network models have a clear but narrow theory of default: *default is mechanical failure to meet obligations given a solvency or liquidity constraint*. Nodes pay as much as they can; if they cannot pay in full, they default, creditors recover pro rata, and this may trigger further defaults. Legal structure, collateral, and instrument-specific default procedures are collapsed into simple recovery parameters.

Network models therefore thicken the theory of credit by recognizing its inherently bilateral structure, and they provide a mechanical theory of default cascades, but they remain thin on money, price, and payment as such.

3.4 Agent–based and hybrid microstructure approaches

Agent-based models (ABM) and hybrid microstructure models try to restore heterogeneity and bounded rationality, often with explicit trading rules. Yet, once again, the five implicit theories remain relatively thin.

Money. Money is usually a *local state variable* (cash, deposits) that agents use to settle trades, but strict double-entry bookkeeping and instrument-level specification are often relaxed. "Money" can sometimes be created or destroyed by implementation shortcuts rather than by consistent issuance and settlement. The hierarchy of monetary instruments is rarely made explicit; what matters is whether an agent's scalar cash variable is above zero.

Price. Prices are *emergent from heuristic rules*: firms and dealers use simple mark-up or inventory rules; order books clear at the best quote; agents adapt rules over time. This yields interesting dynamics, but dealer behavior is usually not explicitly tied to funding constraints, outside options, or regulatory limits. The implicit theory of price is local tâtonnement, not a fully articulated dealer microstructure.

Credit. Credit is often a matter of *rule-based loan extension*: if the borrower meets certain criteria and the bank has "enough capital", a loan is granted. Contracts may have maturities and interest rates, but the full cash-flow schedule, collateralization, and default clauses are rarely represented in detail. As in SFC models, a credit expansion is an increase in aggregated loan positions in agents' state variables, not the growth of a fully specified instrument set.

Payment. Payments are *local updates to account balances*: when a trade occurs, one variable goes down, another goes up. There is seldom a distinction between different settlement assets, between payment instructions and settlement, or between gross and net payment positions over intraday windows. The payment system is a coding convention rather than an explicit object of study.

Default. Default is usually triggered by *simple thresholds* on net worth or cash balances. When an agent defaults, its obligations may be written off or partially repaid according to ad hoc rules; contagion is modeled by the effect on creditors' net worth or capital ratios. Instrument-specific legal structure is ignored; default is a state transition for agents, not a sequence of events at the level of particular contracts.

ABM frameworks thus thicken the representation of heterogeneity and bounded rationality, and sometimes of trading protocols, but they rarely integrate a fully explicit and disciplined theory of money, payment, and default into their microstructure.

3.5 Dealers and the missing price theory in existing models

The discussion so far has treated the theory of price in fairly abstract terms: equilibrium prices in DSGE and asset–pricing models, behavioral macro prices in SFC models, and largely absent prices in network models. Yet in modern monetary economies many important prices are not auction prices at all, but *dealer quotes*: two–sided bid–ask quotes posted by balance–sheet–constrained market makers in securities and by banks in money markets. If the research gap we have identified is to be filled by a simulator that represents agents as balance sheets and prices as dealer quotes derived from those balance sheets, we must briefly ask how the existing literature models dealers.

A first strand is the classic inventory–control literature in market microstructure, beginning with Garman, Amihud–Mendelson, and Ho–Stoll. Dealers are modeled as optimising agents who choose bid and ask quotes to trade off spread income against the risk of holding inventory. The core state variable is inventory, and bid–ask spreads are increasing in inventory risk. However, these models typically posit a *stochastic fundamental value* for the asset and treat the dealer's spread as the outcome of a continuous–time optimal–control problem. Prices remain fundamentally Walrasian: there is a single underlying "true value" process, and the dealer's quotes are a noisy reflection of it. The dealer's balance sheet matters only through a generic risk parameter; there is no explicit outside anchor in the form of a deeper value–based investor and no direct link from balance–sheet capacity to the shape of the entire quote schedule.

A second strand, the adverse–selection literature (e.g. Glosten–Milgrom, Kyle), explains spreads via asymmetric information. The market maker sets bid and ask prices equal to conditional expectations of a latent fundamental value, and the spread compensates for expected losses to informed traders. Inventory either plays no systematic role or enters only via simple risk–aversion terms. In effect, the "dealer" is an information processor rather than a balance–sheet–constrained liquidity producer; the modelled institution is closer to a *broker* matching opaque order flow than to a dealer who must warehouse positions on his own book.

A third strand comprises search–theoretic OTC models, where prices emerge from bilateral bargaining between investors and intermediaries under meeting frictions. Spreads reflect search costs and bargaining power rather than expected layoff costs at an outside market. Here again, the intermediary's balance sheet is typically thinly represented; there is no two–tier structure in which a thin dealer is backed by a deeper value–based investor or central bank, and no explicit geometry of position limits and forced layoff events.

A fourth, more macro–oriented strand treats banks and securities dealers as leveraged intermediaries whose net worth prices risk (intermediary asset–pricing models; DSGE models with financial intermediaries). These models successfully link time–varying risk premia to the capital of intermediaries, but they do so in a Walrasian market structure: asset prices are still single clearing prices equating aggregate demand and supply each period, and the intermediaries' Euler equations simply replace the representative consumer's Euler equation in the pricing kernel. There is typically no explicit two–sided quote schedule, no inventory random walk, and no hard position limits at which prices jump to an outside backstop or liquidity disappears.

Treynor's *Economics of the Dealer Function* occupies a distinctive position in this landscape. In Treynor's framework, a dealer is explicitly defined as a market maker who *uses his own balance sheet* to absorb customer order flow, with finite long and short position limits and a deeper value–based investor (VBT) providing an outside spread. The dealer's inventory executes a random walk between these position limits as buy and sell orders arrive. The expected frequency with which this random walk hits a boundary—and thus forces a costly layoff to the VBT at the outside bid or ask—pins the inside spread in a competitive market: the inside width is exactly the expected layoff cost per trade. The dealer's mean quote (midpoint between his own bid and ask) is then required, by one–step no–arbitrage, to be a linear function of inventory anchored at the VBT's outside midpoint just beyond the position limits.

Two properties follow. First, *prices and spreads are explicit functions of the dealer's own balance sheet and outside conditions*: the quote schedule $x \mapsto \{b(x), p(x), a(x)\}$ is derived directly from the dealer's inventory capacity, position limits, and the outside spread. Second, the market is explicitly hierarchical: a narrow inside market, backed by a wider outside market, backed in monetary applications by the central bank as a "dealer of last resort". Both features resonate strongly with the "money view", which treats private dealers and central banks as balance–sheet–constrained market makers rather than Walrasian auctioneers.

From the perspective of the five implicit theories, what is still missing in most of the dealer literature is a *joint* theory of money, credit, payment and default alongside price. Inventory–control and adverse–selection models largely ignore the monetary hierarchy and payment system; search models emphasise bargaining rather than settlement; macro intermediary models abstract from instrument–level contracts and explicit default and resolution. Treynor, by contrast, provides a complete dealer–based theory of price and liquidity, but leaves money, credit, payment and default in the background.

These observations point directly to the kind of simulation methodology we adopt below. To fill the gap identified in the previous subsections, we need (i) a fully specified balance–sheet and contract architecture, in which money, credit, payment and default are represented as instrument–level events with specific means of payment and resolution rules, and (ii) a dealer theory of price in which quotes and spreads are constructed from that same balance–sheet architecture. The Bilancio simulator does precisely this by combining a Treynorian dealer kernel with an explicit instrument–level balance–sheet representation and by extending the dealer function to banks as multi–market dealers in money. The detailed implementation of this kernel for securities dealers and for banks is deferred to the methodology section.

3.6 Neglected phenomena and the resulting research gap

Viewed through the five theoretical lenses—money, price, credit, payment, and default—and with dealers explicitly in the picture, a common pattern emerges across the modeling traditions surveyed above. Each approach thickens at most one or two of these theories, but always at the expense of the others. DSGE and representative-agent asset-pricing models offer a worked-out equilibrium theory of price but only a very thin theory of money, payment, and default. SFC models thicken the accounting for money and credit, but only at an aggregate level and with prices treated in a highly reduced form. Network models thicken the theory of credit and default by making exposures bilateral, but usually dispense with prices, money and payment altogether. Agent-based and microstructure models often add realistic trading rules but relax either accounting discipline or the monetary hierarchy. Even in the dealer literature, where the mechanism of price formation is taken seriously, the balance-sheet foundations of money, credit, payment and default are generally left in the background.

Three clusters of questions remain systematically under-specified.

Who has what they are supplying? In almost all of the existing approaches, "supply" is an aggregate flow or a change in a sectoral stock; the question of how the supplier comes to possess the specific asset they sell is left implicit. Theories of money, credit and default are therefore too coarse to distinguish between:

- issuing a new instrument versus re-selling an existing one;
- funding a position by prior saving versus funding it by borrowing;
- holding an asset on-balance-sheet versus off-balance-sheet.

By aggregating across instruments and counterparties, SFC models and DSGE models can ensure that stocks and flows add up by construction, but they cannot, within their own ontology, insist that every supply of an asset is the transfer of a concrete instrument already present on some balance sheet or explicitly created as such. Network models improve on this by making credit explicitly bilateral, but usually still treat an exposure as a single number rather than as a dated cash-flow schedule with contractual contingencies.

How does the demander obtain, and use, the means of payment? Existing models also have very thin theories of payment. Payments typically occur inside a period budget constraint or a net flow equation; the means of payment is a scalar "money" variable rather than a specific settlement asset that must be present on the payer's asset side at the time of settlement. As a result, the following questions cannot usually be answered endogenously:

- Did the payer finance the payment out of existing deposits or reserves, or by borrowing from a bank?
- If the payment was funded by new credit, what new liabilities were created, in favour of which counterparties, and on what maturity and default terms?
- How did the path of these payments redistribute balances across the monetary hierarchy (reserves, deposits, other IOUs)?

Even where network models represent obligations bilaterally, they rarely specify the means of payment for each contract, so that the funding chain behind a given settlement remains opaque. In the dealer literature, payment is typically treated as an instantaneous exchange of cash for securities at the quoted price; the upstream origin of the cash and its future repayment schedule are beyond the model's scope.

How are dealer quotes and liquidity produced, changed, and withdrawn? On the price side, mainstream macro and asset-pricing models offer rich theories of valuation but almost never model dealers as such: prices are single equilibrium prices, not two-sided quotes constrained by balance sheets. Inventory-control and adverse-selection models of dealers improve on this by deriving bid–ask spreads and, sometimes, the dynamics of the midpoint, but they usually operate in a world with a frictionless fundamental value, weak balance-sheet constraints, and no explicit monetary hierarchy. Search-theoretic OTC models and macro intermediary models highlight intermediation frictions and intermediary capital, but again in a Walrasian market structure without hard position limits or explicit outside anchors.

Treynor's dealer theory stands out as the only framework in which the dealer's inside bid, inside ask, and inventory-sensitive midpoint are jointly pinned by (i) a deeper value-based investor or outside market and (ii) the stochastic geometry of the dealer's own balance sheet. Yet even here the theory is narrowly focused on securities markets: money, credit, payment and default appear only as implicit background conditions for the dealer's ability to fund inventory and access the outside market. There is, in other words, no existing framework that provides a *joint* balance-sheet-based theory of dealer pricing and a fully articulated representation of money, credit, payment and default.

Path dependence, monetary hierarchy, and default cascades. Because most models work with representative or sectoral agents and highly stylised time, they tend to underplay path dependence and the layered nature of the monetary system. Defaults and liquidity crises are compressed into parameters (default rates, exogenous liquidity shocks) or one-off balance-sheet adjustments, rather than represented as sequences of missed payments, collateral liquidations, and reallocation of claims along contractual and hierarchical lines. This makes it difficult to study phenomena where the precise order and timing of events, and the particular layer of the hierarchy in which stress initially appears, matter for the outcome: roll-over risk, runs on deposits versus runs in wholesale markets, dealer withdrawal and market "disappearance", or the interaction between private dealers and the central bank as dealer of last resort.

Resulting research gap. Putting these observations together, the gap in the literature can be stated as follows. We lack a framework that:

- represents money, credit, payment and default at the level of individual instrument–level contracts with explicit means of payment and default rules;
- models all agents—including dealers and banks—as explicit balance sheets whose capacity and constraints determine which contracts can be issued, held, or honoured at each date;
- derives prices, spreads and liquidity not from a disembodied pricing kernel or reduced-form behavioural relation, but from dealer and bank functions defined on those balance sheets, anchored to outside markets; and
- allows these components to interact over time in a way that preserves strict double-entry bookkeeping and the temporal structure of cash flows.

The balance-sheet-and-dealer-based simulation methodology developed in the remainder of the paper is designed to fill precisely this gap. It does so by combining (i) an instrument-level representation of all financial relationships and their cash-flow, payment, and default structures, with (ii) a Treynor-style dealer kernel for price and liquidity formation, extended to treat banks as multi-market dealers in money. In the Bilancio simulator, money, price, credit, payment and default are no longer thin, implicit background assumptions; they are explicit, jointly articulated mechanisms operating on concrete balance sheets and dealer functions.

4 Bilancio simulator

4.1 Agents as balance sheets

The basic ontology is a finite set of agents $\mathcal{A}$:

$$\mathcal{A} = \{\text{households}, \text{firms}, \text{banks}, \text{central bank}, \text{treasury}, \text{dealers}, \text{funds}, \dots\}.$$

Each agent $a \in \mathcal{A}$ is represented by a balance sheet at each discrete time $t = 0, 1, \ldots, T$:

$$BS_t(a) = \big(\text{Assets}_t(a), \text{Liabilities}_t(a), \text{Stocks}_t(a)\big),$$

where:

- *Assets* and *liabilities* are exclusively financial instruments.
- *Stocks* capture non-financial items (e.g. machines, inventories).

By construction:

- Every *financial asset* of some agent is the liability of another agent (never itself).
- Non-financial assets (like machines) do not correspond to any liability; they are one-sided.

Thus the entire economy at time t is a system of interlocked balance sheets:

$$\mathcal{S}_t = \big\{BS_t(a) \,\big|\, a \in \mathcal{A}\big\}.$$

4.2 Instrument-level specification

The primitive objects of the simulation are *instruments*, each of which generates one or more cash-flow obligations over time.

Formally, let $\mathcal{I}$ denote the set of all instruments. Each instrument $i \in \mathcal{I}$ is defined as a tuple:

$$i = \big(\text{issuer}(i), \text{holder}(i), \text{type}(i), \text{denom}(i), \text{CF}(i), \text{MoP}(i), \text{default\_rule}(i)\big),$$

where:

- $\text{issuer}(i), \text{holder}(i) \in \mathcal{A}$: the liability holder and asset holder.
- $\text{type}(i)$: instrument type (loan, deposit, bond, payable/receivable, non-financial claim, etc.). The type encodes admissible operations: who may issue, transfer, or extinguish, and under which conditions.
- $\text{denom}(i)$: denomination, i.e. the unit of account / currency in which the nominal size is stated.
- $\text{CF}(i)$: a schedule of promised cash flows (t_k, c_k) with $t_k \in \{0, \ldots, T\}$ and integer amounts $c_k \neq 0$. For one-period instruments, $\text{CF}(i)$ may reduce to a single maturity date and payment amount.
- $\text{MoP}(i)$ (means of payment): which asset(s) the issuer must deliver to the holder when a cash flow c_k becomes due. This can be:
  - a specific settlement asset (e.g. central bank reserves or bank deposits),
  - or unspecified until contracting (open choice to be agreed later).
- $\text{default\_rule}(i)$: the default and close-out procedure if, at some due date t_k, the issuer cannot deliver the required means of payment.

At any time t, the balance sheet $BS_t(a)$ can be recovered by collecting all instruments where a is issuer or holder and summing over their remaining cash-flow schedules.

4.3 Dealer and bank functions

4.3.1 Treynor's approach to dealers

Treynor's *Economics of the Dealer Function* presents a dealer–centric theory of price formation in markets organised around market makers rather than Walrasian auctions Treynor1971,Treynor1987Dealer,Treynor2
A "market–maker" is any agent who accommodates transactors to whom time is important in return

for charging buyers a higher price than is paid to sellers. Within this class Treynor distinguishes two types: a thinly capitalised *dealer* and a deep–pocketed *value–based investor* (VBT). Both are market makers, but their roles differ systematically in capital, holding horizon and spread. The VBT quotes a wide *outside* spread and is the dealer's "market–maker of last resort", while the dealer quotes a narrower *inside* spread and interposes his balance sheet between hurried buyers and sellers.[1]

Treynor starts by fixing a standard order size and a maximum long or short position that the dealer is willing to carry on his own book. Customer orders arrive one by one as either buys or sells of this standard size. Each order therefore moves the dealer's position up or down by one step, so that his inventory jumps between a finite set of discrete positions, starting from the maximum short limit at one end and ending at the maximum long limit at the other. The two endpoints of this range are the layoff positions: once the dealer is already at one of these extremes, any further order in the same direction cannot be absorbed inside and must immediately be passed on to the value-based investor at that investor's bid or ask. Every such layoff to the outside market generates a loss for the dealer equal to half of the outside spread between the value-based investor's bid and ask.

Treynor assumes that, in the baseline, buys and sells are equally likely and independent from one trade to the next. Under these assumptions, the dealer's inventory performs a symmetric random walk on this finite grid of positions. In the long run, each position on that grid is visited just as often as any other; the stationary distribution of inventory is flat across all admissible inventory levels. The only times the dealer actually has to pay the outside spread are when a trade arrives while he is exactly at one of the two position limits and that trade pushes further in the same direction, forcing a layoff or buy-in. The probability that any given customer trade triggers such a layoff can be written as a simple fraction whose numerator is the standard order size and whose denominator is twice the dealer's maximum inventory, plus one more standard order. When the position limit is large compared with the order size, this probability is approximately equal to the ratio of the standard order to twice the maximum allowable position: larger capacity pushes the boundaries further apart and makes layoffs rarer, while larger ticket sizes reach the boundaries more quickly and make layoffs more frequent.

Treynor then distinguishes between the outside spread, set by the value-based investor, and the inside spread, set by the dealer. In a competitive dealer market the expected profit per customer trade must be zero. Each ordinary customer trade that the dealer absorbs inside earns him half of the inside spread, whereas each layoff trade at the position limit costs him half of the outside spread. Because only a fraction of all trades are layoffs, the average revenue from the inside spread must exactly offset the average cost of occasional layoffs. This zero-profit condition links the two spreads in a simple way: the inside spread is equal to the probability of layoff multiplied by the outside spread. Equivalently, the inside spread is the dealer's expected layoff cost per trade. It is therefore proportional to the outside spread in the deeper market, rises with the typical order size, and falls as the dealer's position limit is increased.

Treynor also works out how the dealer's mean quote—the midpoint between his own bid and ask—depends on his current inventory. He introduces the outside midpoint, defined as the midpoint between the value-based investor's bid and ask, and considers a one-step no-arbitrage condition: for any interior inventory position, the price the dealer posts today must equal the expected mean price he will be posting after the next customer trade. Otherwise, a trader could lock in a riskless gain by trading now and reversing the position one trade later. When buy and sell orders are equally

[1] See Treynor1987Dealer, esp. pp. 27–28, for the definition of dealer and value–based investor and the distinction between inside and outside spreads.

likely, this no-arbitrage requirement says that the mean quote at any interior inventory level must lie exactly halfway between the mean quotes at the two neighbouring inventory levels—one step up and one step down. That condition forces the mean quote to move in a straight line as inventory moves along the grid.

To anchor this straight line, Treynor ties the hypothetical mean prices just beyond the position limits to the value-based investor's executable bids and offers: one step beyond the maximum long position, the relevant price is the value-based investor's bid; one step beyond the maximum short position, it is the value-based investor's ask. Connecting these two anchor points with a straight line determines both the intercept (which coincides with the outside midpoint when the dealer's inventory is neutral) and the slope. The result is that the dealer's mean quote falls linearly as his position becomes more long, and rises linearly as his position becomes more short. The steepness of this tilt is increasing in the outside spread and decreasing in the size of the dealer's permitted position range: a wider outside spread or tighter position limit makes the mean price more sensitive to inventory, whereas more balance-sheet capacity makes it less sensitive. The dealer's actual bid and ask at any inventory level are obtained by moving symmetrically down and up from this mean line by half of the inside spread, with the inside spread itself given by the expected layoff cost described above.

Two conceptual points follow from this construction. First, immediacy is a produced good: the dealer uses scarce balance-sheet capacity to bridge the timing gap between buyers and sellers, and must charge a spread that covers the expected cost of occasionally passing positions to the outside market at a loss. Second, the cost of this immediacy always has two layers. The narrow inside spread is what customers see on the dealer's screen; the much wider outside spread operates in the background, and is what the dealer—and, indirectly, any investor who ends up "trading with the crowd"—pays when inventory happens to be at the limit and trades must be laid off or bought in at the value-based investor's quotes.

Significance for liquidity premia and the "money view"

Treynor's set–up provides an explicit microfoundation for liquidity premia. Assets are not priced solely by discounting future cash flows; they also carry a premium reflecting the expected costs of providing immediacy to hurried traders. In his language, the VBT earns a liquidity premium for standing ready to absorb inventory at a wide spread, while the dealer earns a smaller premium for facilitating trade at a narrow spread but with limited capacity. The dealer's inside spread is exactly the mark–up required to cover the expected cost of occasional layoffs at the outside spread.

This perspective lines up naturally with the "money view" in modern monetary economics, which treats both private dealers and central banks as balance–sheet–constrained market makers rather than Walrasian auctioneers. In that tradition the central bank is explicitly modelled as a *dealer of last resort*, quoting a backstop bid and ask in ultimate settlement media [e.g.][]Mehrling2011NewLombard. Treynor's two–tier structure, with a thin dealer backed by a deep VBT, provides an analytical template for that hierarchy: inside spreads are pinned by the cost of accessing outside quotes, and the outside spread in turn reflects the balance sheet and risk appetite of the ultimate liquidity supplier. In later sections we import this geometry directly into banking by reinterpreting horizontal distance as a measure of balance–sheet liquidity risk and the vertical axis as a funding spread.

Dealers versus brokers: Treynor and the mainstream literature

A central conceptual distinction in the market–microstructure literature is often obscured by the terminology used. Much of the mainstream academic literature models "dealers" in a way that corresponds, in economic function, to *brokers* rather than to the balance–sheet–using dealers in Treynor's sense. In contrast, Treynor's framework explicitly models a dealer as an intermediary who *uses his own balance sheet* to absorb customer flow, hold inventory, and quote two–sided prices tied to outside value-based investors (VBTs).[2]

Search–theoretic and information–asymmetry approaches—such as those following Stigler, Spulber, Glosten–Milgrom, and Duffie–Gârleanu–Pedersen—focus on the difficulty of matching buyers and sellers in markets with frictions or with asymmetric information. These models typically treat the "dealer" as an agent who earns intermediation rents by locating counterparties, screening order flow, bargaining, or managing adverse selection. Crucially, this intermediary often does *not* use his balance sheet to hold the asset: inventory rarely appears, and when it does, it is not the central state variable driving price formation. The economic function being modelled is therefore closer to that of a *broker*: an intermediary who connects buyers and sellers but is not required to warehouse positions until the other side of the market arrives.

Treynor's framework takes the opposite stance: a dealer *is* precisely the market participant who stands ready to trade from inventory, providing time–critical liquidity by absorbing imbalances in customer flow. Prices follow from the dealer's inventory position, the outside spread imposed by value-based investors, and the probability of hitting position limits that force layoff at outside prices. Inventory, position limits, and balance-sheet risk are therefore the fundamental primitives of price formation.[3] In liquid markets—where transactions occur against a committed balance sheet and the dealer must be willing to hold the asset "until the buyer comes"—the Treynor dealer captures the economically relevant mechanism. The mainstream models instead capture situations where the intermediary does not warehouse risk and where the core friction is *search* or *information opacity*, conditions more characteristic of brokered or thin markets.

Thus, although both literatures use the word "dealer," they describe two fundamentally different institutions. Treynor's dealer is an inventory–managing, balance–sheet–using market maker; the mainstream "dealer" is, in effect, a broker who intermediates through matching rather than warehousing. For modelling banks as dealers, only the Treynor notion is appropriate, because banks supply liquidity precisely by using their balance sheets to absorb flows, face binding position limits, and price according to inventory-based risk rather than search frictions.

Treynor's dealer model and alternative approaches

This section compares Treynor's *Economics of the Dealer Function* Treynor1971,Treynor2012 with the main approaches in economics for modelling dealers. The contrast turns on four dimensions: the presence or absence of an *outside anchor*, the role of *balance–sheet geometry* versus optimisation, the treatment of *order flow and inventory*, and whether models explain *bid–ask spreads* or *risk premia*. We group the alternative literatures into (i) inventory–control dealer models, (ii) adverse–selection models, (iii) search–theoretic OTC models, and (iv) macro–intermediary models.

Inventory–control dealer models. Early microstructure models such as Garman Garman1976, Amihud and Mendelson AmihudMendelson1980, and especially Ho and Stoll HoStoll1981, treat the

[2]See Treynor's original formulation of the dealer function, where inventory, position limits, and layoff prices are the core primitives from which the dealer's capacity and pricing schedule are derived.

[3]Treynor's Markov-chain formulation makes this explicit: the dealer's inventory executes a reflecting random walk, and the inside spread is pinned by the expected frequency of forced layoff to the VBT.

dealer as an optimising agent who chooses bid and ask quotes to balance spread revenue against the risk of holding inventory. Order arrivals are typically Poisson and the "fundamental value" follows a diffusion.

Relative to Treynor:

- These models feature inventory–sensitive spreads but the outside price is a stochastic fundamental value, not a VBT with an explicit outside spread.
- Prices come from an *optimal–control problem* rather than from the Markov counting argument that ties inside spreads to boundary–hitting probabilities.
- Inventory is continuous and decisions are dynamic; Treynor's derivation is static and geometric, with discrete inventory steps and reflecting position limits.

Thus, Ho–Stoll and related models ask: "what spread does a risk–averse dealer *choose*?" Treynor instead asks: "what inside spread can a competitive dealer *sustain* given outside costs and finite capacity?"

Adverse–selection dealer models. Glosten and Milgrom GlostenMilgrom1985 and Kyle Kyle1985 explain bid–ask spreads through information asymmetry rather than balance-sheet constraints. In these models:

- Prices equal conditional expectations of a latent fundamental value.
- Spreads compensate for the adverse–selection cost of trading with informed agents.
- Inventory often plays no essential role: in Glosten–Milgrom the dealer clears to zero in expectation; in Kyle the market maker has effectively unlimited balance sheet.

These models therefore lack Treynor's two-tier market: there is no VBT with a wider spread and no boundary layoff condition. They capture the broker–like role of screening and rationally updating beliefs, but not the dealer's role as an inventory–constrained liquidity producer.

Search–theoretic OTC dealer models. Search-based OTC models (e.g. Duffie, Gârleanu, and Pedersen DuffieGarleanuPedersen2005,DuffieGarleanuPedersen2007) generate equilibrium bid and ask prices through meeting frictions and bargaining between investors and market-makers. Spreads reflect search costs and bargaining power, not the expected cost of laying off inventory. Compared with Treynor:

- The central friction is search, not inventory bounds or layoff risk.
- Dealers are not anchored to an outside spread provided by a VBT or central bank.
- Prices arise from Nash bargaining rather than from the Markov–chain geometry of a balance sheet with hard position limits.

Closely related monetary-search models (e.g. Lagos and Rocheteau LagosRocheteau2009) incorporate dealers but again emphasise bargaining and matching frictions rather than two-sided balance–sheet–based pricing.

Macro intermediary models. Modern macro–finance models with intermediaries (e.g. Gertler and Kiyotaki GertlerKiyotaki2010; Gertler and Karadi GertlerKaradi2011; Brunnermeier and Sannikov BrunnermeierSannikov2014; He and Krishnamurthy HeKrishnamurthy2013) describe banks or dealers as agents facing leverage or equity constraints. These constraints generate time-varying risk premia in otherwise Walrasian asset markets.

In the New Keynesian tradition, GertlerKaradi2011 embed banks as leveraged intermediaries into a standard DSGE model. Banks borrow from households and lend to firms, subject to an agency problem that limits leverage. Aggregate bank net worth becomes the key state variable, and credit spreads are determined by the tightness of the leverage constraint. Central bank "unconventional policy" is modelled as the central bank temporarily taking on the role of intermediary by buying private assets or lending directly to firms when private banks are capital constrained.

BrunnermeierSannikov2014 develop a continuous–time macro model in which "experts" (financial intermediaries) borrow from households to hold risky assets. The experts' net worth is again the key state variable, with crises arising when net worth is low and leverage high. Asset prices and risk premia are determined by the experts' stochastic discount factor. The model delivers rich nonlinear dynamics and endogenous risk, but the market structure remains Walrasian: a single market–clearing price equates aggregate demand and supply each period.

In the intermediary asset–pricing literature, HeKrishnamurthy2013 model specialist intermediaries that face a moral–hazard–based capital constraint. The intermediaries' marginal value of wealth becomes the pricing kernel, so intermediary capital—not representative–agent consumption—prices risky assets. Empirically, HeKellyManela2017 show that shocks to the equity capital of primary dealers price a large cross–section of assets, and interpret this as evidence for the intermediary–asset–pricing mechanism. Again the underlying asset markets are effectively Walrasian, with intermediaries entering as constrained but otherwise standard investors.

In all these models the "dealer" is identified with the intermediary's Euler equation under a leverage or capital constraint. Intermediation frictions show up as wedges between risky and safe yields, but there is no explicit two–sided quote schedule, no inventory random walk and no distinction between inside and outside spreads.

For our purposes, the key differences relative to Treynor's dealer model can be summarised along four dimensions:

1. *Market–making hierarchy.* Treynor has *two* layers of market maker: a thin dealer providing immediacy at a narrow inside spread, backed by a deep VBT (or, in the monetary context, a central bank) providing depth at a wide outside spread. By contrast, the macro models usually have only one representative intermediary whose marginal value of wealth prices all assets. The hierarchical structure—dealer $\to$ VBT $\to$ central bank—that is central to the money view and to our bank–as–dealer approach does not appear explicitly.

2. *State variables: inventory and order flow versus net worth.* In Treynor, prices are determined by *inventory and order flow*: the inside spread and the slope of the midline are explicit functions of the position limit, the standard order size and the outside spread, and the price actually quoted is a function of the current inventory. Sequences of customer buys or sells push the dealer toward his limits and thereby increase the expected frequency of costly layoffs. The macro intermediary models, by contrast, abstract from order flow and inventory. Markets clear each period with no explicit sequence of trades. The key state is the stock of intermediary net worth, not the stochastic flow of orders that arrives at the dealer's book.

3. *Hard position limits versus smooth constraints.* Treynor's dealer faces *hard position limits*. When those limits are reached, the next outward order must be executed at the outside

quotes: the pricing regime jumps discretely from the inside spread to the outside spread, and quantities are effectively rationed. In the macro models, by contrast, leverage or capital constraints enter as inequalities in a smooth optimisation problem. As constraints tighten, spreads and allocations move nonlinearly, but there is no sharp transition at which prices jump to an outside backstop and quantities are rationed at that price.

4. *Balance sheets as liquidity technology.* In Treynor's framework the dealer's balance sheet is explicitly a technology for producing liquidity. The dealer uses scarce risk–bearing capacity and access to the outside market to manufacture immediacy for customers, and the inside spread is exactly the expected variable cost of this production. In the intermediary–asset–pricing models the intermediary's balance sheet shapes the pricing kernel, but there is no explicit notion of a two–sided quote schedule or of a quantity of immediacy supplied at those quotes.

Comparative summary and implications for banks as dealers

Across these literatures, Treynor's approach is distinctive in four related ways. First, it features an explicit *outside anchor* in the form of the VBT (or central bank), so that the inside and outside spreads coexist and interact. Second, it derives prices and spreads from the *geometry of the balance sheet* and the Markov random walk of inventory, rather than from utility maximisation or bargaining. Third, the key state variable is the dealer's *inventory*, executing a discrete random walk with reflecting boundaries, rather than aggregate net worth or wealth. Fourth, bid–ask spreads and the cost of immediacy are the central objects; macro risk premia appear only indirectly through their effect on the outside spread.

Most microstructure models overlap only partially with this structure: they model spreads but typically lack the VBT anchor or the inventory–walk derivation. Search–theoretic and macro–intermediary models overlap along different dimensions: they feature intermediaries but rarely model two-sided quotes, layoff events, or hard position limits. Treynor's framework remains the only approach in which a dealer's bid, ask, and inventory-sensitive midline are *jointly pinned* by (i) an outside market and (ii) the stochastic geometry of the dealer's balance sheet.

These differences matter for our application because we wish to treat banks *as dealers*, not as frictionless intermediaries in a Walrasian market. Treynor's geometry already gives us a horizontal "risk" axis, a vertical "spread" axis and a pair of hard outer bounds. In the sequel we reinterpret the horizontal coordinate as a scalar index of balance–sheet liquidity risk and the vertical coordinate as a funding spread, keeping the linear interior law and hard outer limits. This allows us to embed a recognisably Treynorian dealer function inside the banking system, while the intermediary–based macro models provide a useful but complementary account of how aggregate balance–sheet conditions feed into the outside spread of the system as a whole.

4.3.2 Dealers as market-makers in claims

Dealers are agents whose primary role is to make markets for particular classes of instruments, typically within maturity buckets. In the simplest baseline kernel:

- A dealer holds an inventory of a single security: a units of the security and b units of cash.
- There is an "outside" value-based trader that provides an outside mid-price M and outside spread O for the security.
- Customer trades arrive in standard ticket size S (securities per trade).

From these primitives, the dealer's capacity and inventory-sensitive pricing schedule are derived:

(a) **Capacity:** the maximum inventory X^* the dealer can reach without borrowing:

$$X^* = S \left\lfloor \frac{Ma + b}{MS} \right\rfloor.$$

(b) **Inventory grid:** the dealer's inventory state x ranges over the one-sided ladder

$$x \in \{0, S, 2S, \ldots, X^*\},$$

with reflecting boundaries at 0 and X^*.

(c) **Layoff probability:** the frequency λ with which the dealer hits a boundary and must lay off risk at the outside quotes:

$$\lambda = \frac{S}{X^* + S}.$$

(d) **Inside spread:** under competition, the inside width I is pinned to cover expected layoff costs:

$$I = \lambda O.$$

(e) **Inventory-sensitive midline:** the mean quoted mid-price is linear in inventory,

$$p(x) = M - \frac{O}{X^* + 2S}\left(x - \frac{X^*}{2}\right),$$

with inside bid/ask (before clipping to outside):

$$a(x) = p(x) + \frac{I}{2}, \qquad b(x) = p(x) - \frac{I}{2}.$$

Thus dealer pricing and liquidity provision are *functions of the dealer's own balance sheet* (a, b) and of outside conditions (M, O, S): dealers provide tight inside spreads as long as they have balance-sheet capacity; when constraints bind, prices clip to outside or liquidity disappears.

4.3.3 Relation to Treynor's symmetric dealer kernel

Treynor's original dealer diagram is built on a *symmetric* inventory ladder and an exogenous position limit.[4] The dealer's inventory X ranges on

$$X \in \{-X^*, -X^* + S, \ldots, X^* - S, X^*\},$$

so there are

$$N^{\text{sym}} = \frac{2X^*}{S} + 1$$

equally likely rungs in steady state. With i.i.d. buy/sell signs, the per-trade layoff probability is

$$\lambda^{\text{sym}} \;=\; \frac{1}{N^{\text{sym}}} \;=\; \frac{S}{2X^* + S}, \tag{1}$$

and competition pins the inside spread at

$$I^{\text{sym}} \;=\; \lambda^{\text{sym}} O. \tag{2}$$

[4]Treynor (1987).

The symmetric mean mid-price is linear in inventory,

$$p^{\mathrm{sym}}(X) \;=\; M \;-\; \frac{O}{2(X^* + S)}\,X, \qquad X \in [-X^*, X^*], \tag{3}$$

tilting down when the dealer is long and up when the dealer is short.

In the Lesson 1 kernel above, the dealer instead operates on a *one-sided*, balance-sheet-feasible shelf. Inventory x lives on

$$x \in \{0, S, 2S, \ldots, X^*\},$$

with X^* derived from balances and the outside mid M,

$$X^* \;=\; S\left\lfloor \frac{Ma + b}{MS} \right\rfloor,$$

and there are

$$N^{\mathrm{L1}} \;=\; \frac{X^*}{S} + 1$$

rungs. The per-trade layoff probability is

$$\lambda^{\mathrm{L1}} \;=\; \frac{1}{N^{\mathrm{L1}}} \;=\; \frac{S}{X^* + S}, \tag{4}$$

the competitive inside spread is

$$I^{\mathrm{L1}} \;=\; \lambda^{\mathrm{L1}} O, \tag{5}$$

and the mean midline is

$$p^{\mathrm{L1}}(x) \;=\; M - \frac{O}{X^* + 2S}\left(x - \frac{X^*}{2}\right), \qquad x \in [0, X^*]. \tag{6}$$

These parallel formulas make it easy to see what Lesson 1 adds or changes relative to Treynor:

(i) **Inventory geometry and layoff frequency.** Treynor's dealer can run both long and short to $\pm X^*$, whereas the Lesson 1 dealer is long-only on $[0, X^*]$ with no shorting and no borrowing. This halves the number of rungs for a given X^*/S and doubles boundary pressure:

$$\frac{\lambda^{\mathrm{L1}}}{\lambda^{\mathrm{sym}}} \;=\; \frac{S/(X^* + S)}{S/(2X^* + S)} \;=\; \frac{2X^* + S}{X^* + S} \;\in\; (1, 2).$$

For large capacity $X^* \gg S$, the one-sided layoff frequency is almost twice Treynor's symmetric benchmark:

$$\lambda^{\mathrm{L1}} \approx \frac{S}{X^*}, \qquad \lambda^{\mathrm{sym}} \approx \frac{S}{2X^*}.$$

(ii) **Inside spread: one-sided dealers charge more for the same X^*.** Because the spread is $I = \lambda O$ in both models, the ratio of competitive inside spreads is

$$\frac{I^{\mathrm{L1}}}{I^{\mathrm{sym}}} \;=\; \frac{\lambda^{\mathrm{L1}}}{\lambda^{\mathrm{sym}}} \;=\; \frac{2X^* + S}{X^* + S} \;\in\; (1, 2).$$

For a fixed outside spread O and the same nominal capacity X^*, a long-only dealer with no borrowing must therefore quote a strictly wider inside spread than Treynor's symmetric dealer; in the high-capacity limit the gap tends to a factor of two.

(iii) **Midline slope: the one-sided shelf leans more strongly against inventory.** Differentiating the mean midlines in (3)–(6) with respect to inventory,

$$\frac{\partial p^{\mathrm{sym}}}{\partial X} = -\frac{O}{2(X^* + S)}, \qquad \frac{\partial p^{\mathrm{L1}}}{\partial x} = -\frac{O}{X^* + 2S},$$

so the absolute slope in the Lesson 1 kernel is

$$\frac{\left|\partial p^{\mathrm{L1}}/\partial x\right|}{\left|\partial p^{\mathrm{sym}}/\partial X\right|} = \frac{2(X^* + S)}{X^* + 2S} \in (1, 2).$$

The one-sided midline tilts more sharply than the symmetric one: the same outside quotes must be spanned over a shorter effective range on the short side, so each extra unit of inventory carries a larger price concession.

(iv) **Capacity from balances rather than exogenous risk appetite.** Treynor takes X^* as an exogenous tolerance for long/short risk. Lesson 1 instead defines

$$X^*(a, b; M, S) = S \left\lfloor \frac{Ma + b}{MS} \right\rfloor,$$

so the *same* primitives (a, b, M, S) that determine the inventory ladder also determine the layoff probability and hence the inside spread. In this sense the entire pricing schedule $x \mapsto \{b(x), p(x), a(x)\}$ is a function of the dealer's own balance sheet, whereas in Treynor the balance sheet enters only implicitly through X^*.

(v) **Feasible posted quotes: clipping and pins.** Treynor's algebra implicitly assumes that every quoted bid and ask is executable for at least one standard ticket. Lesson 1 makes this feasibility constraint explicit. From the midline (6) and spread (5), the raw inside quotes are

$$a(x) = p^{\mathrm{L1}}(x) + \frac{I^{\mathrm{L1}}}{2}, \qquad b(x) = p^{\mathrm{L1}}(x) - \frac{I^{\mathrm{L1}}}{2},$$

which are then clipped to the outside market,

$$a_c(x) = \min\{A, a(x)\}, \qquad b_c(x) = \max\{B, b(x)\},$$

and, with commit-to-quote pins turned on, flattened near the edges to guarantee that each posted side can be honoured for one ticket without shorting or borrowing:

$$a_c^{\mathrm{ON}}(x) = \begin{cases} A, & x < S, \\ \min\{A, a(x)\}, & x \geq S, \end{cases} \qquad b_c^{\mathrm{ON}}(x) = \begin{cases} \max\{B, b(x)\}, & x \leq X^* - S, \\ B, & x > X^* - S. \end{cases}$$

These pins carve out flat segments of length S at each end of the shelf while leaving λ, I and $p(\cdot)$ unchanged. They have no direct analogue in Treynor's original derivation but are essential once the dealer's quotes are interpreted as hard, balance-sheet-feasible commitments.

Taken together, these differences show that Lesson 1 is not a departure from Treynor's economics, but a one-sided, balance-sheet-based completion of it: the same random-walk and outside-anchor logic now operates on a shelf whose width is determined by the dealer's own (a, b) and whose quotes are constrained to be executable without shorting or borrowing.

4.3.4 Banks as money-dealers

We reinterpret a commercial bank as a Treynor-style dealer in central-bank money. The role played by the security in the previous subsection is now taken by two-day reserves at the central bank; loans and deposits are contractual legs that reshape the bank's near-horizon reserve inventory. The bank's pricing problem is to quote deposit and loan rates while managing a bounded reserve position against the policy corridor.

Why inventory alone is not enough for banks

In Treynor's original security–dealer model, the dealer's quote is a function of a *single* inventory coordinate: the number of shares X currently held relative to a symmetric position limit X^*.[5] The dealer's risk is to be "bagged" by an adverse price move between the time he accommodates customer flow and the time he lays off to the value–based investor (VBT). The only state variable that matters for pricing inside the spread is therefore X itself: more long inventory means being closer to hitting the layoff boundary on the long side, and hence a lower bid and a higher ask.

For a commercial bank, this one–dimensional inventory logic is not enough. The object the bank manages is not a stock of a single traded asset but a *reserve account* R_t backing a large book of on–demand deposits D_t and term loans L_t. When a dealer in Treynor's model quotes, he knows that, absent new customer trades, his inventory will not move on its own. By contrast, when a bank quotes a deposit rate or a loan rate at the start of day t, its reserve position R_t is already exposed to *scheduled* reserve legs (CB remuneration and borrowing, loan repayments) and to *on–demand* withdrawal behaviour by depositors.

Two features break the "inventory only" logic:

(a) **On–demand deposits: endogenous reserve drains.** Customer deposits D_t have no fixed principal maturity in the kernel: principal leaves only when depositors initiate withdrawals / outgoing payments. Even if the bank's *current* reserve inventory R_t is comfortable, a large withdrawal wave during day t can force R_t down to the overdraft region and trigger costly CB borrowing at rate . The bank's true risk is therefore not captured by R_t alone but by the *projected reserve path* over the next few days,

$$\widehat{R}_t(h), \qquad h \in \{t, \dots, t+10\},$$

constructed from (i) known scheduled CB and loan legs and (ii) an internal forecast of remaining day–t withdrawals W_t^{rem}. The near–horizon cash–tightness index L_t^* that tilts the Treynor midline is defined from this worst–point of the projected path, not from R_t alone. In other words, the effective "inventory coordinate" for a bank is *reserve headroom after expected withdrawals*, not the raw stock of reserves.

(b) **Balance–sheet feedback: quotes change future cash flows.** In Treynor's securities model, the dealer takes the size and arrival rate of customer orders as exogenous. The next few trades will move X up or down by $\pm S$, but his *pricing rule* does not change the *arrival process* of orders. For a bank, by contrast, today's posted deposit rate $r_{D,t}$ and loan rate $r_{L,t}$ directly affect future balance–sheet growth: higher $r_{D,t}$ attracts more new deposits; lower $r_{L,t}$ attracts more new loans. In reserve terms, this creates additional, rate–dependent reserve legs: new deposits generate extra withdrawal capacity tomorrow; new loans generate loan–repayment inflows at $t+10$ and may trigger immediate reserve outflows if borrowers pay non–clients. To keep the

[5] See Treynor (1987), especially the inventory random–walk and layoff–frequency argument leading to $I = \lambda\Omega$.

Treynor identity ($I^{(2)} = \lambda\Omega^{(2)}$) meaningful in this environment, the bank must evaluate how a marginal change in its quotes $(r_{D,t}, r_{L,t})$ will change the expected reserve path $\widehat{R}_t(h)$ and hence the cash–tightness index L_t^*.

Operationally, we capture this via two forecasting blocks:

- a *deposit decay* mechanism, which maps the outstanding deposit stock D_t into a baseline forecast of day–t withdrawals W_t^c and hence an effective day–t reserve hit W_t^{rem}, and
- (optionally) a *new–business* mechanism, which maps current quotes into expected volumes of new deposits and loans and the resulting net reserve legs over the horizon $\{t+1, \dots, t+10\}$.

These forecasts are not additional "preferences" layered on top of Treynor; they are the minimum structure needed to translate a bank's balance sheet into a single liquidity inventory coordinate that can play the role of X in the original kernel.

Once the bank replaces the raw reserve stock R_t by a near–horizon liquidity index L_t^* constructed from the projected reserve path—which itself embeds deposit decay W_t^{rem} and, if desired, expectations of new loans and deposits—the Treynor funding kernel goes through unchanged. The 2–day pins $(\underline{r}_t, \overline{r}_t)$ are set by the CB floor and ceiling, the symmetric capacity and layoff probability (X^*, λ) are calibrated from the bank's internal reserve band, and the inside 2–day width $I^{(2)} = \lambda\Omega^{(2)}$ still equates the inside spread to expected layoff cost. What changes is *what counts as inventory*: a bank cannot condition its quotes on R_t alone without ignoring the on–demand nature of deposits and the feedback from its own pricing to future cash flows.

Balance sheet and timing. Bank's balance sheet is organised around reserves R_t and dated cash-flow cohorts:[6]

- *Assets:* reserves R_t at the central bank and customer loans L_t that repay in full at $t + 10$.
- *Liabilities:*
  - payment-origin deposits D_t^{pay}, created when RTGS payments to clients arrive;
  - loan-origin deposits D_t^{loan}, created when loans are disbursed.

  Both are payable on demand: principal leaves only when a client submits a withdrawal / outgoing payment; there is no fixed contractual deposit maturity. Deposit interest is credited every 2 days as a deposit-only leg (no reserve movement) while the ticket remains outstanding.
- *Central bank interface:* 2-day borrowing B_t repaid at $t+2$ at rate i_B, and reserve remuneration at a 2-day floor rate $i_R^{(2)}$ paid at $t + 2$.

Intraday, each calendar day t is decomposed into: (A) open with yesterday's close-of-day stocks; (B) an opening settlement window where older CB and loan cohorts hit in reserves (CB repayment at $t - 2$, loan repayment at $t - 10$); (C) a ticket-by-ticket client-flow loop (loans, payment credits, withdrawals) with quotes refreshed after each ticket; (D) intraday scheduled legs (deposit coupons, CB remuneration, CB repayment, loan repayment) at their hit dates; and (E) a pre-close check with an end-of-day CB top-up bringing reserves to a target R_t^{target}.

Client tickets: deposits, loans and withdrawals. Client business is processed in fixed ticket sizes, exactly in parallel with the securities dealer:

[6] See the "Banks-as-dealers with deposits on demand" kernel.:contentReference[oaicite:0]index=0

- *Payment-credit ticket (deposit issuance):* the bank receives reserves via RTGS from other banks and credits a payment-origin deposit ticket of size S_{pay} to the client. Reserves rise by $\phi_k S_{\text{pay}}$ on ticket k (interbank share $\phi_k \in [0,1]$).
- *Loan ticket (with deposit counterpart):* the bank grants a loan of size S_{loan}, creating a loan asset and a matching loan-origin deposit. No reserves move at disbursement; repayment at $t+10$ returns $(1+r_{L,\tau}^{(k)})S_{\text{loan}}$ in reserves for each loan ticket k booked at time τ.
- *Withdrawal / outgoing-payment ticket:* a client initiates a payment to another bank or a cash withdrawal. The bank cancels deposits of size S_{wd} and transfers reserves S_{wd} through RTGS; this is the only way deposit principal leaves the balance sheet in the kernel. If $R_t^{(k-1)} < S_{\text{wd}}$, the post-ticket reserves $R_t^{(k)} < 0$ are interpreted as intraday overdraft, converted at the close into a 2-day CB loan via the top-up.

Quantities are client-driven (ticket arrivals are exogenous); the bank controls only the deposit and loan rates at which it is willing to trade.

Funding-plane kernel and corridor. The bank's Treynor kernel lives on a 2-day funding plane in reserves, with the central bank as the outside market:

- *Outside pins and width.* The 2-day reserve floor and CB borrowing ceiling

$$\underline{r}_t = i_R^{(2)}, \qquad \bar{r}_t = i_B, \qquad \Omega^{(2)} = \bar{r}_t - \underline{r}_t > 0$$

  play the role of the value-based investor's outside bid and ask, with outside width $\Omega^{(2)}$.
- *Internal reserve band and capacity.* The bank chooses an internal 2-day reserve band $[R_{\min}^{\text{bank}}, R_{\max}^{\text{bank}}]$ with midpoint $R^{\text{tar}} = \frac{1}{2}(R_{\min}^{\text{bank}} + R_{\max}^{\text{bank}})$ and symmetric capacity

$$X^* = \tfrac{1}{2}\big(R_{\max}^{\text{bank}} - R_{\min}^{\text{bank}}\big).$$

  Given a funding-plane ticket size S_{fund} (in reserve units), the Treynor layoff probability and competitive inside width on the 2-day funding plane are

$$\lambda = \frac{S_{\text{fund}}}{2X^* + S_{\text{fund}}}, \qquad I^{(2)} = \lambda\,\Omega^{(2)}.$$

- *Reserve inventory coordinate.* At each quote refresh, the Treasury desk projects a 10-day reserve path $\{R_t^b(h)\}_{h=t}^{t+10}$ from the cohortised balance sheet, the known CB and loan legs, and the CMRE's forecast of remaining on-demand withdrawals for today. The near-horizon reserve inventory on the 2-day funding plane is

$$x_t := R_t^b(t+2) - R^{\text{tar}} \in \{-X^*, -X^* + S_{\text{fund}}, \dots, X^*\}.$$

- *Liquidity shortfall and risk tilt.* From the projected path the bank computes a scalar 10-day cash-tightness index $L_t^* \geq 0$ (distance of the worst point of $\{R_t^b(h)\}$ from a floor $R_{\min}$) and an additional risk index ρ_t capturing tail or stress risk. These tilt the midline exactly as in the securities case.

Given these objects, the 2-day funding-plane midline and quotes are:

$$\begin{aligned}
M_t^{(2)} &:= \tfrac{1}{2}(\underline{r}_t + \bar{r}_t),\\
m^{(2)}(x) &= M_t^{(2)} - \frac{\Omega^{(2)}}{2(X^* + S_{\text{fund}})}\,x,\\
m_{\text{bank}}^{(2)}(x_t) &= m^{(2)}(x_t) + \alpha L_t^* + \gamma \rho_t,\\
r_{\text{bid}}^{(2)}(x_t) &= m_{\text{bank}}^{(2)}(x_t) - \tfrac{1}{2}I^{(2)}, \qquad r_{\text{ask}}^{(2)}(x_t) = m_{\text{bank}}^{(2)}(x_t) + \tfrac{1}{2}I^{(2)}.
\end{aligned}$$

Here $\alpha, \gamma \geq 0$ are bank-specific loadings. As in the securities kernel, competition forces the common 2-day inside width $I^{(2)}$ to equal the expected layoff cost $\lambda\Omega^{(2)}$, and the midline is linear in the inventory coordinate x_t.

Mapping to deposit and loan rates; ticket-by-ticket updates. The internal 2-day funding quotes are mapped to client products by term maps ψ and Ψ:

$$\begin{aligned} r_{D,t} &= \psi^{-1}\Big(\min\{r^{(2)}_{\text{bid}}(x_t),\, \bar{r}_t\}\Big)\,, \\ r_{L,t} &= \Psi^{-1}\Big(r^{(2)}_{\text{ask}}(x_t)\Big)\,, \end{aligned}$$

with a ceiling discipline $\psi(r_{D,t}) \leq i_B$. Control is *rates-only*: given $(r_{D,t}, r_{L,t})$ clients decide whether to take loans, hold deposits, or withdraw; each realised ticket updates the cohort ledger and the projected reserve path $R^b_t(\cdot)$, and hence x_t, L^*_t, ρ_t and the next pair of quotes $(r_{D,t}, r_{L,t})$.

Finally, at the end of the client-flow window the bank aggregates the day's RTGS inflows and withdrawals to obtain midday reserves R^{mid}_t. A mechanical CB top-up

$$B^{(0)}_t = \max\{0, R^{\text{target}}_t - R^{\text{mid}}_t\}, \qquad R_t = R^{\text{mid}}_t + B^{(0)}_t$$

converts any intraday overdraft into a 2-day CB loan, whose repayment appears as a known reserve outflow at $t+2$ in the next day's projection. In this way, the bank is a multi-market dealer in money: its posted deposit and loan rates are the image, in product space, of a Treynor funding kernel that prices 2-day reserve liquidity from the bank's own balance sheet.

4.4 Discrete-time evolution

Time is discrete: $t = 0, 1, \ldots, T$. The system state at time t is:

$$\mathcal{S}_t = \big(\{BS_t(a)\}_{a\in\mathcal{A}}, \{q_t(m)\}_{m\in\mathcal{M}}, \text{exogenous parameters}_t\big),$$

where $\mathcal{M}$ indexes markets or dealer buckets and $q_t(m)$ collects dealer quotes and outside anchors.

One period $t \to t+1$ is decomposed into sub-phases (for banks: opening, settlement window, ticket-by-ticket client flow, closing), but conceptually it works as:

(1) **Activation of scheduled cash flows.** For every instrument i and every cash-flow (t, c) in $\text{CF}(i)$ with t current:

  (a) Identify the required means of payment $\text{MoP}(i)$.
  (b) Check whether issuer(i) holds sufficient $\text{MoP}(i)$ on its asset side.

(2) **Settlement or default.** For each due cash flow:

  - If the issuer has enough settlement asset, the simulator debits the issuer and credits the holder, and extinguishes or updates the instrument.
  - Otherwise, the default rule default_rule(i) is invoked, which may trigger:
    - partial payment,
    - liquidation of the issuer's other assets,
    - recovery for the creditor according to a seniority schedule,
    - and possibly further defaults downstream.

Depending on version, the simulator may:

- halt at first default, or
- continue through resolution and post-default restructuring.

(3) **Dealer and bank operations.** Conditional on the version:

- Dealers update quotes using their kernel: new inventory $\Rightarrow$ new mid and spread.
- Incoming customer orders (loans, deposits, asset trades) are processed ticket by ticket.
- New instruments are created (e.g. a loan asset for the bank and loan liability for the client; matching deposit liability for the bank and deposit asset for the client).

(4) **End-of-period analysis.** After all events at t are handled and invariants enforced (every asset has a matching liability, no negative quantities, etc.), the simulator produces:

(a) balance-sheet analysis (who owes what to whom),
(b) payment-funding analysis (how payments were made and funded),
(c) risk analysis (default probabilities, exposure concentrations),
(d) valuation analysis (mark-to-model / mark-to-market),
(e) flux-reflux analysis (how money was created, distributed, destroyed).

4.5 Key methodological features

We can summarize the approach via the following numbered features:

(1) **Dealer functions for money, liquidity, and pricing.** Both securities dealers and banks are modeled as market makers whose quotes, spreads, and capacities are explicit functions of their balance sheets and outside anchors.

(2) **Gross, bilateral, instrument-level representation.** Every financial relationship is an explicit asset–liability pair with a cash-flow schedule, denomination, means of payment, and default procedure. Nothing is netted out at the modeling level; net positions can be computed as analysis.

(3) **Discrete-time event-driven dynamics.** The system evolves through a sequence of discrete dates. At each date, specific cash flows activate; payment constraints are checked; defaults and liquidations propagate; and dealers re-quote.

(4) **Hierarchical means of payment and settlement constraints.** The model distinguishes different layers of the monetary hierarchy (reserves, bank deposits, other claims). Each instrument specifies which asset is acceptable in settlement; agents must either pre-hold or newly acquire that asset.

(5) **Explicit default and resolution.** Non-payment is a modeled event, not an exogenous shock. Default triggers concrete balance-sheet operations—asset liquidation, haircuts, and reallocation of claims.

(6) **Integrated analytical outputs.** At each t, the simulator can produce:

- network-style maps of obligations,
- SFC-style stock-flow summaries,
- liquidity and risk indicators,
- and price/yield information from the dealer kernels.

5 Kalecki ring internal debt problem

This section reviews strands of economic research that relate to our problem of mutually-indebted agents (MIAs) with no exogenous income, who can only service their debts using payments received from others and by selling financial assets. We focus on how this literature addresses (i) the volume of nominal debt, (ii) the distribution of net positions across agents, and (iii) the time pattern of promised payments, all of which jointly determine the minimum liquidity stock required for a given pattern of settlements.

5.1 Mutually-Indebted Agents and Liquidity

The problem of a closed group of rentier-like agents whose only income consists of interest and principal payments from one another is formulated most explicitly by Toporowski2024,ToporowskiIlliquidity. In a series of contributions on liquidity and central banking, Toporowski introduces a thought experiment in which MIAs hold only claims on one another and a limited stock of monetary assets. He argues that (i) the ability of the group to continue servicing all debts is a question of the *circulation* of payments rather than net wealth, and (ii) a positive buffer of liquidity is necessary for the settlement process to get started.

In a related "fable" inspired by Kalecki, Toporowski describes a mutually-indebted village in which a single banknote circulates and successively clears a chain of bilateral obligations Toporowski-Fable. The example illustrates how a small amount of money can settle a large volume of gross debts, but also shows that some non-zero stock of means of payment is necessary to initiate and sustain the process. Public debt management and central bank operations are interpreted as mechanisms for providing or withdrawing this liquidity.

5.2 Monetary-Circuit and Pure-Credit Models

Monetary-circuit and pure-credit models provide the conceptual backdrop for viewing money as a network of credit–debt relations among agents. In the Graziani tradition, money is issued by banks as a liability and enters circulation through loans, while repayment destroys money Graziani1997,Graziani2003. The circuit unfolds in sequential time: banks create deposits when they lend, firms hire workers and pay wages, workers and rentiers spend, and finally firms repay loans with interest.

Circuitists emphasise that the feasibility of closing the circuit depends on the *timing* of cash flows and on the propensity of rentiers to hold liquid assets rather than spend their interest income Graziani2003,Passarella2015. This focus on sequential time parallels our concern with the ordered activation of dated balance-sheet entries. In such models there is often said to be no exogenous initial money stock in a pure-credit economy, yet operationally some positive buffer of deposits and reserves must exist at each date to execute payments.

A closely related line of work develops stationary pure-credit economies in which all money consists of bank liabilities, and interest payments circulate without any outside money Park2004. These models highlight that interest payments owed by some agents are income for others, and that a consistent flow of payments can exist even when net positions differ across agents. Our MIAs can be interpreted as a limiting case in which only rentier and financial agents remain, with all payments internal to the financial layer.

5.3 Financial Network Clearing Models

If we freeze time at a given date and consider a matrix of bilateral nominal obligations and a vector of liquid endowments, the problem becomes one of network clearing. EisenbergNoe2001 develop a now-standard framework in which each financial institution owes fixed amounts to others and holds a portfolio of outside assets. Under limited liability, there exists a clearing payment vector that specifies how much each institution pays to each other, given its resources.

In the Eisenberg–Noe framework, total gross debt is summarized by the obligations matrix, the distribution of net positions is encoded in the network structure, and outside liquid assets play the role of endowments. Subsequent work extends this to ask how much additional capital or liquidity is needed to avoid default cascades, effectively treating outside assets as a control variable that regulators can adjust to ensure full or partial repayment [e.g.][]GlassermanYoung2016,GaiKapadia2010,Battiston2012.

Dynamic extensions generalise the analysis to multiple dates. Calafiore2023 and related work distinguish between solvency (balance-sheet net worth) and liquidity (cash-on-hand at each date), and compute time paths of clearing payments given sequences of dated obligations. Such models are particularly relevant for our purposes, as they explicitly incorporate the maturity structure of liabilities and allow for the analysis of minimal liquidity injections needed at specific dates to sustain a default-free clearing path.

5.4 Payment-System Liquidity and Liquidity-Saving Mechanisms

Another branch of the literature studies the liquidity needs of payment systems, especially real-time gross settlement (RTGS) systems operated by central banks. Here, banks face a schedule of intraday payment obligations and must decide how much central-bank money (reserves) to hold or borrow in order to meet these obligations.

Martin2006,MartinMcAndrews2008 analyse liquidity-saving mechanisms (LSMs) in RTGS systems. These mechanisms allow payments to be queued and offset multilaterally, thereby reducing the amount of intraday liquidity required for settlement. The key result is that, for a given vector of payment obligations, LSMs can substantially reduce the required central-bank liquidity at the cost of increased settlement delay.

Empirical and calibrated models using data on Fedwire and other large-value payment systems demonstrate that banks hold precautionary liquidity buffers because incoming payments from other banks are uncertain and subject to delay Afonso2008. Agent-based models of payment systems show that high gross obligations combined with limited netting possibilities can lead to "gridlock", in which banks delay outgoing payments in anticipation of incoming funds, thereby increasing system-wide liquidity needs GalbiatiSoramaki2011.

Regulatory frameworks such as the Liquidity Coverage Ratio (LCR) have been studied in payment-network settings, showing that a bank's position in the network and the timing pattern of its payments affect the amount of high-quality liquid assets it must hold to cover stressed outflows HeuverBerndsen2020,HeuverBerndsen2022. This literature makes liquidity a first-class variable, distinct from solvency, and directly links required liquidity to gross payment volumes, the distribution of obligations and their time profile.

5.5 Obligation-Clearing and Mutual Credit in Trade-Credit Networks

A closely related literature investigates how multilateral netting and mutual credit can reduce the need for monetary settlement in networks of commercial obligations. Using data from a government-run clearing system in Slovenia, FleischmanDiniLittera2020 show that periodic centralized clearing of undisputed trade obligations can reduce firms' mutual indebtedness by a significant fraction of GDP.

The mathematical foundations of such systems are developed in FleischmanDini2021, who demonstrate that as long as directed cycles of obligations exist in the network, appropriate netting algorithms can strictly reduce total gross indebtedness. This reduces the volume of payments that must be made in money, even if net positions across firms change little. The effectiveness of such systems depends on the network structure, participation rates and the length of the clearing window, all of which determine the potential for cycle cancellation.

Corporate treasury practice and the finance literature on multilateral netting within multinational groups mirror these ideas: by offsetting intra-group payables and receivables, firms can reduce the number and size of actual cash payments, thereby lowering their demand for transactional liquidity. In our MIAs framework, this suggests that the minimum money stock needed to settle a given pattern of obligations is not invariant to the clearing rules applied.

5.6 Stock-Flow-Consistent Models and Rentier Economies

Stock-flow-consistent (SFC) models in the Godley–Lavoie tradition provide a macroeconomic framework in which all stocks and flows on sectoral balance sheets are tracked consistently over time GodleyLavoie2007. These models are particularly suitable for analysing economies in which financial relationships and the accumulation of assets and liabilities play a central role.

Within this tradition, a substantial literature examines financialisation and the role of rentier households who earn interest and other property income from financial assets. For example, vanTreeck2007 and subsequent work build SFC models in which rentiers' consumption, saving and portfolio choices influence the dynamics of debt, net worth and output. Other contributions explore how rising financial profits and rentier incomes affect macroeconomic stability and distribution [e.g.][]Caverzasi2015,Lapavitsas2019.

SFC–circuit hybrids explicitly combine monetary-circuit logic with SFC accounting, incorporating detailed interest payments on loans to firms and rentiers and their feedback into subsequent balance-sheet positions Passarella2015,Passarella2022,Sawyer2013. These models share with our approach the emphasis on the full asset–liability matrix, net positions by sector, and the time profile of flows such as interest and principal.

However, most SFC models operate at a period level and do not usually model intraperiod payment constraints. They typically ensure that period-by-period budget constraints and behavioural equations are satisfied, but they do not generally calculate whether, given an initial distribution of money balances and a detailed payment schedule, a default-free settlement path exists within the period. As such, SFC models provide a logically coherent macro context for MIAs but do not directly address the minimal money stock required for settlement.

5.7 Implications for the Modeling of Mutually-Indebted Agents

Taken together, these literatures suggest a natural way to think about MIAs with no exogenous income. Toporowski's MIA thought experiment and Kalecki-style fable provide an explicit conceptualisation of a closed group of agents whose only income is each other's interest and principal payments ToporowskiFable,ToporowskiIlliquidity. Monetary-circuit and pure-credit models supply the ontology of money as inside credit, the centrality of sequential time and the role of rentiers' liquidity preferences.

Network-clearing models in the Eisenberg–Noe tradition offer a formal framework for determining, at any given date, whether a vector of bilateral obligations can be settled given the available liquid assets, and how additional liquidity or capital injections change the outcome. Payment-system and LSM analyses show quantitatively how gross payment volumes, network structure and timing drive liquidity demand, and how different settlement rules can reduce or increase the needed money stock. Obligation-clearing and mutual-credit systems demonstrate that, for a fixed set of contractual obligations, the required stock of means of payment is sensitive to the extent of multilateral netting and cycle cancellation. SFC and rentier models embed such financial dynamics in a macroeconomic setting while ensuring stock-flow consistency over time.

Our modeling environment, which represents agents as balance sheets whose financial assets always correspond to other agents' liabilities and evolves in discrete time as dated entries are activated, can be viewed as a microfounded implementation of these ideas. Within this environment, the problem of finding the minimum stock of monetary assets required for a given network of MIAs to settle all obligations without default is closely related to the dynamic clearing and payment-system liquidity problems studied in the literature, but applied to a pure financial layer without exogenous income streams.

6 Simulation Setup

7 Simulation Setup

7.1 Agents and Horizon

We consider a small closed system with

- a central bank CB,
- $N = 5$ "ring" agents $H_1, \dots, H_5$ (interpreted as generic firms/households),
- three large entities B_{short}, B_{mid}, B_{long} ("big entities").

Time is discrete and indexed by days $t = 0, 1, \dots, T$, with $T = 10$.[7] At each $t > 0$ a subset of liabilities with due date t comes due; agents must either settle these obligations in cash or default, in which case the simulation terminates immediately.[8]

7.2 Baseline ring of obligations

The core of the system is a ring of nominal obligations of total face amount $Q_{\text{total}} = 10{,}000$. We index ring edges by $i = 1, \dots, N$ and define the creditor of H_i as H_{i+1} with indices taken modulo N:

$$H_i \text{ owes } H_{i+1} \quad \text{for all } i \in \{1, \dots, 5\},\ H_6 \equiv H_1.$$

For each edge i we create a single payable at $t = 0$:

$$\text{debtor} = H_i, \quad \text{creditor} = H_{i+1}, \quad \text{amount} = q_i > 0, \quad \text{due day} = \tau_i \in \{1, \dots, T\}.$$

The vector of nominal sizes $q = (q_1, \dots, q_N)$ is random but constrained by

$$\sum_{i=1}^{N} q_i \approx Q_{\text{total}}.$$

[7] In the implementation, the engine only advances to days on which some liability matures, but conceptually we can think in terms of a 10–day grid.

[8] This follows the general Version 1.0 design of the money modelling software, where every financial asset of one agent corresponds to a liability of a counterparty, and the system evolves by activating due entries over time.

We obtain q by drawing a symmetric Dirichlet random vector with concentration parameter $c > 0$ and scaling by Q_{total}. Smaller c implies a more concentrated structure of exposures (one or two large links dominate), whereas larger c yields a more equal ring.

Maturity dates τ_i are drawn from a discrete distribution on $\{1, \dots, T\}$ governed by a parameter $\mu \in [0, 1]$. Intuitively, μ controls how front- or back-loaded the ring is: for low μ most obligations mature early in the horizon, for high μ they are tilted toward the final days.

Each monetary claim has a fixed contractual face value per "ticket" of

$$S = 20$$

which is the cashflow paid at maturity if the debtor does not default. Amounts q_i and all other notional quantities in the simulation are measured in these monetary units.

7.3 Initial money and the debt-to-money ratio

A central parameter of the experiment is the ex ante debt-to-money ratio κ. Given a target total nominal debt Q_{total} and $\kappa > 0$, we define the target initial money stock

$$M_0^{\text{target}} = \frac{Q_{\text{total}}}{\kappa}.$$

At $t = 0$ the central bank mints cash and allocates it to the ring agents $H_1, \dots, H_5$ (and, in the extended system, also to the big entities) so that the realised initial money stock M_0 is close to M_0^{target}.[9] The five ring agents start with identical baseline liquidity endowments (e.g. 5,000 units of cash each when $\kappa = 0.5$), and possibly small top-ups to preserve their cash-to-debt ratios when big entities are added.

7.4 Big entities and the big-entity share

In addition to the ring, we introduce three large balance-sheet entities

$$B_{\text{short}}, \quad B_{\text{mid}}, \quad B_{\text{long}}$$

which will later be interpreted either as passive buy-and-hold investors or as dealers by maturity bucket. We specify a "big entity share" parameter

$$\beta \in (0, 1),$$

representing the size of the new debt issued to big entities relative to the baseline ring. In this experiment we fix

$$\beta = 0.25,$$

meaning that we create additional nominal obligations of total size $\beta Q_{\text{total}} = 2{,}500$ from the ring agents to the big entities.

Concretely, at $t = 0$ each H_i issues one extra payable

$$H_i \text{ owes } B_{k(i)} \quad \text{in amount } 500,$$

[9] In the detailed implementation, integer rounding and the addition of big-entity claims mean that the realised ratio $\text{Debt}_0/\text{Money}_0$ is close to, but not exactly, the target κ. The realised values are recorded per run as `initial_total_debt`, `initial_total_money`, and `debt_to_money_ratio`.

for a total of $5 \times 500 = 2{,}500$. The mapping $i \mapsto k(i) \in \{\text{short}, \text{mid}, \text{long}\}$ and the corresponding due days are chosen so that these extra claims populate all parts of the maturity spectrum; for example, in one of the runs:

$$H_1 \to B_{\text{short}} \text{ (due day 7)}, \; H_2 \to B_{\text{mid}} \text{ (day 1)}, \; H_3 \to B_{\text{long}} \text{ (day 5)}, \; H_4 \to B_{\text{short}} \text{ (day 9)}, \; H_5 \to B_{\text{mid}} \text{ (day 3)}.$$

The big entities also receive initial cash endowments from the central bank so that, at the start of the simulation, each big entity is approximately balanced in the market value sense: the market value of its claims on the ring equals its cash holdings. The precise level of these cash balances is determined by the pricing assumptions described next.

7.5 Face value, outside mid, and initial pricing

We normalise contractual face value at $S = 20$ per claim: if a claim survives to maturity it pays S in cash. Prices in the dealer/market subsystem are expressed as fractions of this face value.

The exogenous "outside" valuation—the price at which a value-based outside investor would be willing to hold claims to maturity—is summarised by the *outside mid ratio* ρ. An outside mid of ρ means that the outside investor values each unit of face value at price

$$P^{\text{outside}} = \rho \cdot S.$$

In this experiment we sweep over

$$\rho \in \{1.0, \; 0.9, \; 0.8, \; 0.75, \; 0.5\}.$$

This outside value is used in two ways:

1. At $t = 0$ it determines the initial market valuation of the big entities' portfolios. If a big entity holds claims of total face amount Q_B, its securities are valued at $\rho S Q_B$. We then allocate the same amount of cash to the big entity so that its initial balance sheet is perfectly balanced: cash $= \rho S Q_B$ and securities $= \rho S Q_B$.

2. It provides the reference mid-price around which the dealer quotes bid and ask spreads when trading with the ring agents.

7.6 Market architectures compared

For each parameter tuple (κ, c, μ, ρ) we construct a *single* initial state of the world (same agents, same obligations, same initial cash holdings) and run two scenarios:

(i) Passive big-holder benchmark. In the *passive* scenario the big entities are inert buy-and-hold investors. They receive their initial claims and cash at $t = 0$, and then never trade. They simply hold their claims to maturity and receive payments (or suffer defaults) from their counterparties. The ring agents H_i only interact via settlement of their pre-existing obligations; there is no trading market for claims.

(ii) Active dealer architecture. In the *active* scenario, the same big entities are reinterpreted as a set of dealers by maturity bucket. The actual implementation uses a "balanced dealer" specification:

- For each maturity bucket (short, mid, long) the dealer quotes a bid–ask spread around a dynamic mid-price.
- Ring agents H_i may, at certain times, submit orders to buy or sell claims in a given bucket in order to manage their liquidity and attempt to profit from the discount between the outside mid and face value.
- The dealer does not seek to accumulate inventory or make profits. Instead, it immediately lays off any net position to an abstract *value-based trader* (VBT) at the outside mid price, so that its inventory is effectively zero at the end of each step.
- As a result, the dealer's realised P&L and return are mechanically zero in all runs; the dealer acts purely as a coordinating intermediary that enables liquidity transformation between ring agents and the outside investor.

In both architectures, when an obligation matures, the debtor must either pay in cash or default. A default triggers immediate termination of the run.

7.7 Parameter sweep

We perform a full factorial sweep over four key parameters:

$$\kappa \in \{0.25, 0.5, 1, 2, 4\}, \quad c \in \{0.2, 0.5, 1, 2, 5\}, \quad \mu \in \{0, 0.25, 0.5, 0.75, 1\}, \quad \rho \in \{1.0, 0.9, 0.8, 0.75, 0.5\}.$$

We keep

$$N = 5, \quad T = 10, \quad Q_{\text{total}} = 10{,}000, \quad S = 20, \quad \beta = 0.25$$

fixed across the sweep.[10]

For each quadruple (κ, c, μ, ρ) we:

1. sample a ring of obligations and a maturity structure as described above;
2. construct the extended system with big entities and balanced initial portfolios;
3. run a *passive* scenario (big entities buy-and-hold, no trading);
4. run an *active* scenario (same initial state, but dealers enabled).

This yields $5 \times 5 \times 5 \times 5 = 625$ parameter combinations, and thus 625 passive / active pairs.

7.8 Outcome metrics and micro-level instrumentation

At the macro level, for each run we compute:

- δ_{passive}, δ_{active}: the fraction of initial nominal claims that eventually default in the passive and active scenarios, respectively;
- $\phi_{\text{passive}} = 1 - \delta_{\text{passive}}$ and $\phi_{\text{active}} = 1 - \delta_{\text{active}}$: the corresponding survival rates;

[10] These configuration values are recorded in the experiment summary as `n_agents=5`, `maturity_days=10`, `Q_total=10000`, `face_value=20`, `big_entity_share=0.25`, together with the parameter grids for κ, concentration c, μ, and `outside_mid_ratio`.

- the *trading effect*

$$\Delta_{\text{trade}} = \delta_{\text{passive}} - \delta_{\text{active}},$$

  which is positive when active trading reduces defaults;

- the *trading relief ratio*

$$R_{\text{relief}} = \begin{cases} \dfrac{\delta_{\text{passive}} - \delta_{\text{active}}}{\delta_{\text{passive}}}, & \text{if } \delta_{\text{passive}} > 0, \\ 0, & \text{otherwise,} \end{cases}$$

  interpreted as the fraction of passive defaults that are eliminated by the presence of the dealer.

In Plan 023 we add *default-aware, micro-level instrumentation* that tracks the fate and trading behaviour of each individual liability. For each active run we construct a liability map by scanning the event stream for `PayableCreated` and `PayableSettled` events. For every liability ℓ that matures within the simulation horizon we record:

- its outcome $\text{outcome}_\ell \in \{\text{repaid}, \text{defaulted}\}$, depending on whether it was settled before or at its due date;
- trading activity with the dealer: the number of buys and sells executed by its debtor against the dealer, and the associated net cash P&L;
- a coarse *strategy label* $\text{strategy}_\ell \in \{\text{no\_trade}, \text{hold\_to\_maturity}, \text{sell\_before}, \text{round\_trip}\}$ inferred ex post from that trading history:
  - `no_trade`: the liability never trades with the dealer;
  - `hold_to_maturity`: the debtor buys claims from the dealer and holds them through its own maturity date without selling;
  - `sell_before`: the debtor sells claims back to the dealer before its own maturity date, reducing or closing its position;
  - `round_trip`: the debtor both buys and sells via the dealer before maturity, ending flat.

Aggregating these per-liability records yields, for each run, strategy-specific statistics such as:

- the number and total face value of liabilities using each strategy;
- the total face value and count of repaid and defaulted liabilities by strategy;
- strategy-specific default rates (defaults as a fraction of face value under each strategy).

These are exported as a run-level table (`strategy_outcomes_by_run`) that we use in the cross-run analysis of strategy success and failure.

In parallel, we summarise dealer usage at the run level by computing:

- the total number of trades routed through the dealer, the subset involving ring agents, and the number of distinct ring agents that ever trade with the dealer;
- the total face value and cash volume intermediated by the dealer;
- system-level indicators such as the mean and final debt-to-money ratio and the rate at which total outstanding debt shrinks over time;

- the fraction of defaulted face value that belonged to liabilities which *did* trade with the dealer at least once, and the analogous fraction among repaid liabilities.

These dealer-usage metrics are exported as a second run-level table (`dealer_usage_by_run`) and allow us to relate the dealer's aggregate effect on defaults to how intensively it is used and by whom. Together with the macro outcome metrics, this instrumentation provides a bridge between system-wide default rates and the micro behaviour of individual agents and claims.

8 Descriptive statistics

We report results from a full factorial sweep over

$$\kappa \in \{0.25, 0.5, 1, 2, 4\}, \quad c \in \{0.2, 0.5, 1, 2, 5\}, \quad \mu \in \{0, 0.25, 0.5, 0.75, 1\}, \quad \rho \in \{1.0, 0.9, 0.8, 0.75, 0.5\},$$

with $N = 5$ ring agents, maturity horizon $T = 10$, face value $S = 20$ and big–entity share $\beta = 0.25$ fixed. For each (κ, c, μ, ρ) we construct a single initial state and simulate: (i) a passive architecture (big entities as inert buy–and–hold holders) and (ii) an active architecture (balanced dealers enabled), yielding 625 passive/active run pairs in total.

For each run we compute the passive and active default rates δ_{passive}, $\delta_{\text{active}} \in [0, 1]$, the corresponding survival rates $\phi_{\text{passive}} = 1 - \delta_{\text{passive}}$ and $\phi_{\text{active}} = 1 - \delta_{\text{active}}$, and the *trading effect*

$$\Delta_{\text{trade}} = \delta_{\text{passive}} - \delta_{\text{active}},$$

which is positive when the dealer reduces defaults. We also record the number of trades executed via the dealer in the active run, denoted `total_trades` (equal to `dealer_trade_count` in the run-level instrumentation).

8.1 Aggregate effect of the dealer

Across all 625 parameter combinations we find:

$$\begin{aligned} \bar{\delta}_{\text{passive}} &\approx 0.675, \\ \bar{\delta}_{\text{active}} &\approx 0.648, \\ \overline{\Delta}_{\text{trade}} &\approx 0.027, \end{aligned}$$

so that, on average, enabling the dealer reduces the default rate by about 2.7 percentage points.

We classify each pair into three categories according to the sign of Δ_{trade}:

- *improved* if $\Delta_{\text{trade}} > 0$,
- *unchanged* if $\Delta_{\text{trade}} = 0$,
- *worsened* if $\Delta_{\text{trade}} < 0$.

The distribution is:

158 improved, 396 unchanged, 71 worsened.

Thus, roughly one quarter of parameter combinations benefit from the dealer, about two thirds are unaffected, and about one ninth are harmed.

Conditional on effect class, the average default rates are:

- **Improved runs** ($\Delta_{\text{trade}} > 0$):

$$\bar{\delta}^{\text{imp}}_{\text{passive}} \approx 0.959, \qquad \bar{\delta}^{\text{imp}}_{\text{active}} \approx 0.728.$$

  In these cases the system is near complete collapse without a dealer, and active trading reduces defaults by about 23 percentage points on average.

- **Worsened runs** ($\Delta_{\text{trade}} < 0$):

$$\bar{\delta}^{\text{wors}}_{\text{passive}} \approx 0.651, \qquad \bar{\delta}^{\text{wors}}_{\text{active}} \approx 0.924.$$

  Here the dealer induces an additional ~ 27 percentage points of defaults on average, often turning a moderately stressed configuration into a near–total default.

8.2 Dependence on the outside mid ratio

The outside mid ratio ρ controls the exogenous value of claims relative to their face value S. Table 1 summarises the distribution of trading effects by ρ.

ρ	mean Δ_{trade}	improved	unchanged	worsened	runs
1.00	0.020	29	78	18	125
0.90	0.031	39	75	11	125
0.80	0.028	29	85	11	125
0.75	**0.044**	**42**	73	10	125
0.50	0.014	19	85	21	125

Table 1: Trading effect by outside mid ratio ρ. "Improved"/"unchanged"/"worsened" denote the counts of parameter combinations with $\Delta_{\text{trade}} > 0$, $= 0$, and < 0 respectively.

A moderate discount—especially $\rho = 0.75$—produces the strongest positive average effect and the most favourable improved vs. worsened balance. At very deep discounts ($\rho = 0.5$) the overall effect is weakly positive but unstable: there are roughly as many worsened as improved runs, and the potential for the dealer to trigger severe cascades is higher. At par ($\rho = 1.0$) the effect is modestly positive but weaker than in the $\rho \in \{0.9, 0.8, 0.75\}$ range.

8.3 Dependence on κ, concentration and μ

Debt-to-money ratio κ. Table 2 shows the mean trading effect and counts by the target debt-to-money ratio.

κ	mean Δ_{trade}	improved	unchanged	worsened	runs
0.25	−0.010	0	120	5	125
0.50	−0.014	17	83	25	125
1.00	0.059	43	62	20	125
2.00	0.054	52	55	18	125
4.00	0.048	46	76	3	125

Table 2: Trading effect by debt-to-money ratio κ.

At low debt-to-money ratios ($\kappa = 0.25, 0.5$) the dealer is on average harmful. For $\kappa = 0.25$ there are *no* improved runs at all. For $\kappa \geq 1$ the dealer is on average beneficial, with many more improved than worsened cases. Thus, in this experiment the dealer behaves as a crisis tool: when the system is sufficiently leveraged, enabling trading tends to reduce defaults, whereas in relatively liquid systems the dealer more often amplifies instability.

Concentration. Let c denote the concentration parameter governing the distribution of ring exposures. Table 3 summarises the dependence of the trading effect on c.

c	mean Δ_{trade}	improved	unchanged	worsened	runs
0.2	-0.012	11	91	23	125
0.5	-0.002	20	88	17	125
1.0	0.023	33	76	16	125
2.0	0.055	47	68	10	125
5.0	0.072	47	73	5	125

Table 3: Trading effect by concentration parameter c.

For low concentration the dealer is neutral to slightly destabilising on average. As concentration increases ($c \geq 1$), the average trading effect becomes clearly positive and the number of improved runs rises, while the number of worsened runs falls. This is consistent with the intuition that, when exposures are more clustered, a central intermediation mechanism can play a more meaningful stabilising role.

Maturity tilt μ. The dependence on the maturity tilt parameter μ is more muted. Mean trading effects range from approximately 0.002 to 0.053 across the grid:

μ	0.00	0.25	0.50	0.75	1.00
mean Δ_{trade}	0.053	0.002	0.015	0.022	0.045

with improved runs occurring for all μ and slightly larger average gains at the extremes $\mu = 0$ (front–loaded maturities) and $\mu = 1$ (back–loaded maturities).

8.4 Dependence on baseline stress

To understand how the dealer behaves across different baseline stress levels, we classify runs according to the passive default rate:

$$\text{low stress: } \delta_{\text{passive}} < 0.1, \quad \text{medium: } 0.1 \leq \delta_{\text{passive}} \leq 0.9, \quad \text{catastrophic: } \delta_{\text{passive}} > 0.9.$$

The corresponding counts and mean trading effects are given in Table 4.

regime	runs	improved	unchanged	worsened	mean Δ_{trade}
low	173	1	158	14	-0.061
medium	97	17	43	37	-0.037
catastrophic	355	140	195	20	0.088

Table 4: Trading effect by baseline stress regime, defined by δ_{passive}.

In low–stress configurations the dealer is mostly irrelevant (no defaults either way in most cases), but when it does matter it tends to make things worse: there are 14 worsened versus only 1 improved run. In medium–stress cases the dealer's impact is ambiguous and slightly negative on average: there are more worsened than improved runs, and the mean trading effect is negative. In catastrophic regimes, by contrast, the dealer is mostly stabilising: 140 improved versus 20 worsened runs, and a strongly positive average effect. This reinforces the view that the balanced dealer in this experiment is a *crisis instrument*: it mitigates damage in highly stressed systems, but can create systemic risk in otherwise solvent environments.

8.5 Dealer usage and behaviour

Finally, we relate outcomes to the intensity of dealer usage at the run level. Let `total_trades` denote the number of trades executed via the dealer in the active run, and let n_{traders} be the number of distinct ring agents H_i that trade at least once with the dealer in that run. From the dealer-usage summary we find:

- **Improved runs:** `total_trades` has mean ≈ 5.6 (range 2–7), and n_{traders} has mean ≈ 4.0, i.e. in a typical improved run, about four out of the five ring agents use the dealer at least once. Almost all defaulted and repaid face value in these runs belongs to liabilities that traded with the dealer at least once (mean fractions ≈ 0.97 and ≈ 0.95 respectively).

- **Unchanged runs:** `total_trades` has mean ≈ 2.8, but 149 out of the 396 unchanged runs have `total_trades` $= 0$, i.e. the dealer is never contacted. In all of these zero-trade cases the trading effect is identically zero. In unchanged runs with nonzero trade volume, defaults (when they occur) are still overwhelmingly concentrated on liabilities that used the dealer (mean fraction of defaulted face that traded ≈ 0.97), whereas only a small share of repaid face ever trades (mean fraction ≈ 0.12).

- **Worsened runs:** `total_trades` has mean ≈ 3.6 (range 2–7), and n_{traders} has mean ≈ 2.8, so a majority of ring agents interact with the dealer and those interactions tend to increase the incidence of default. In these runs most defaulted face has traded at least once with the dealer (mean fraction ≈ 0.80), while only a small minority of repaid face has done so (mean fraction ≈ 0.14).

At the same time, the dealer itself behaves as a pure pass–through entity. Across all active runs:

- dealer inventory is always at zero, and the dealer's portfolio share of outstanding claims is identically zero in the inventory time series;

- the dealer's realised P&L and rate of return are identically zero by design;

- no recorded "rescue events" or liquidity–driven sales occur.

The dealer therefore does not act as a speculative balance sheet; it only coordinates reallocations between ring agents and the value–based outside holder. The strongly positive or negative effects documented above arise from how those reallocations interact with the pre-existing ring of obligations and liquidity constraints in the Version 1.0 architecture of the simulator.

9 Strategy Composition and Default Risk

Using the default-aware instrumentation of Plan 023, every liability that matures within the simulation horizon is classified ex post into one of four coarse trading strategies: `no_trade`, `hold_to_maturity`, `sell_before`, and `round_trip`, based on whether and how its debtor interacts with the dealer.

For each active run we observe, by strategy, the total face value, the face that ultimately defaults, and the corresponding default rate. Aggregating across all 625 active runs thus yields a face-weighted picture of how different strategies contribute to systemic risk in the Version 1.0 architecture.:contentReference[oaicite:0]index=0

9.1 Global strategy mix and default rates

Table 5 reports the global strategy composition and face-weighted default rates across all active runs. The "share of face" column gives the fraction of total face value associated with each strategy when pooling all liabilities across all runs.

strategy	share of face (%)	default rate
`no_trade`	49.4	0.045
`hold_to_maturity`	2.4	0.525
`sell_before`	48.0	0.748
`round_trip`	0.2	0.390

Table 5: Global strategy mix and face-weighted default rates across all 625 active runs. "Share of face" is computed as the total face value of liabilities following the given strategy divided by the total face value of all liabilities across all runs.

Two facts stand out. First, almost all face value is split between `no_trade` (about half) and `sell_before` (the other half), with `hold_to_maturity` and `round_trip` plays being rare in aggregate. Second, the risk profiles differ starkly: liabilities that never trade with the dealer (`no_trade`) have a very low default rate (about 4.5% by face), whereas liabilities that are sold to the dealer before maturity (`sell_before`) default three quarters of the time on average ($\sim 74.8\%$). `hold_to_maturity` positions are also very risky in aggregate, with a default rate above 50%. `round_trip` exposures are negligible in size (0.2% of face) but carry an elevated default rate relative to `no_trade`.

In terms of run-level presence, `sell_before` appears in 389 of the 625 active runs, `no_trade` in 344 runs, `hold_to_maturity` in 101 runs, and `round_trip` in only 3 runs. Thus, almost all runs feature some trading with the dealer, but the system-wide risk is dominated by the behaviour of `sell_before` liabilities.

9.2 Strategies in improved, unchanged and worsened runs

To connect micro strategies to the macro effect of the dealer, we classify each run by the sign of the trading effect $\Delta_{\text{trade}} = \delta_{\text{passive}} - \delta_{\text{active}}$: *improved* ($\Delta_{\text{trade}} > 0$), *unchanged* ($\Delta_{\text{trade}} = 0$), and *worsened* ($\Delta_{\text{trade}} < 0$). We then recompute strategy shares and default rates within each class.

effect class	share of face (%)		default rate	
	`no_trade`	`sell_before`	`no_trade`	`sell_before`
improved	12.6	83.2	0.124	0.282
unchanged	57.6	41.2	0.030	0.940
worsened	32.5	44.7	0.426	0.661

Table 6: Strategy composition and default rates for `no_trade` and `sell_before` within each dealer-effect class, aggregated over all parameter combinations and seeds. The rare `hold_to_maturity` and `round_trip` strategies are discussed in the text.

The remaining strategies, while small in volume, are informative:

- In **improved runs** (158 out of 625), `sell_before` dominates: about 83% of face value uses `sell_before`, while only 12.6% is `no_trade` and 3.2% is `hold_to_maturity`. The default rates are moderate: around 28% for `sell_before`, 24% for `hold_to_maturity`, and 12% for `no_trade`. In other words, when the dealer is stabilising, the bulk of the system is actively using it as a liquidity tool (`sell_before`) and those trades are much safer than in the aggregate.
- In **unchanged runs** (396 out of 625), the system either does not default or defaults in a way that is unaffected by trading. Here 57.6% of face is `no_trade` and 41.2% is `sell_before`; `hold_to_maturity` is only 1.2% of face. The `no_trade` default rate is extremely low ($\sim 3\%$), whereas `sell_before` is almost always associated with default ($\sim 94\%$ default rate in this class). This indicates that, even when the dealer does not change the aggregate outcome, the liabilities that trade with it tend to sit in the most fragile part of the system.
- In **worsened runs** (71 out of 625), where the dealer increases defaults, the strategy mix becomes more balanced: 32.5% of face is `no_trade`, 44.7% is `sell_before`, and a non-trivial 20.5% is `hold_to_maturity`. Default rates are high across the board: about 66% for `sell_before`, 93% for `hold_to_maturity`, and 43% even for `no_trade`. The presence of a sizeable `hold_to_maturity` segment with near-certain default is a distinctive feature of these destabilising regimes.

Taken together, Table 6 and the additional figures show that improved runs are characterised by a heavy reliance on `sell_before` and little `hold_to_maturity`, with trading exposures remaining relatively safe. In worsened runs, by contrast, `hold_to_maturity` grows to around one fifth of total face and is almost uniformly disastrous.

9.3 Strategies across stress regimes

We now classify runs by their baseline stress level, as measured by the passive default rate:

$$\text{low stress: } \delta_{\text{passive}} < 0.1, \quad \text{medium: } 0.1 \leq \delta_{\text{passive}} \leq 0.9, \quad \text{catastrophic: } \delta_{\text{passive}} > 0.9.$$

The resulting strategy composition and default rates by stress regime are summarised in Table 7.

In low-stress environments, almost all liabilities (94% of face) follow `no_trade` and default is virtually absent in that segment ($\sim 0.6\%$ default rate). Only a small fraction ($\sim 4.7\%$) uses `sell_before`, with a modest default rate ($\sim 11.7\%$). `hold_to_maturity` appears rarely but is already very risky (default rate $\sim 55.6\%$) when it does.

As we move to medium and catastrophic regimes, two shifts occur simultaneously: (i) the share of `sell_before` explodes (to 64% and then 90.6% of face), and (ii) default rates for trading strategies become extremely high ($\sim 77\%$ for `sell_before` in both medium and catastrophic regimes). The safe `no_trade` segment shrinks in relative importance (to 26.5% and then 7.5% of face) and becomes far less safe in absolute terms, especially in catastrophic regimes ($\sim 53\%$ default rate). In catastrophic scenarios, therefore, almost all risk is concentrated in liabilities that trade with the dealer, primarily via `sell_before`.

regime	strategy	share of face (%)	default rate
low	`no_trade`	94.0	0.006
	`hold_to_maturity`	1.2	0.556
	`sell_before`	4.7	0.117
medium	`no_trade`	26.5	0.095
	`hold_to_maturity`	8.2	0.398
	`sell_before`	64.0	0.774
catastrophic	`no_trade`	7.5	0.534
	`hold_to_maturity`	1.7	0.708
	`sell_before`	90.6	0.778

Table 7: Strategy composition and face-weighted default rates by stress regime, restricted to the three main strategies. The `round_trip` strategy accounts for at most 1.3% of face in any regime and is omitted for brevity.

9.4 Parameter dependence of strategy-specific risk

Finally, we examine how the default risk of each strategy varies across the parameter grid (κ, c, ρ). We average over μ and seeds and summarise strategy default rates at the level of parameter cells (κ, c, ρ).

At this cell level, the distribution of strategy-specific default rates is:

`no_trade`: median cell default rate ≈ 0.137, 75th percentile ≈ 0.469;

`hold_to_maturity`: median ≈ 0.644, 75th percentile $= 1.000$;

`sell_before`: median ≈ 0.781, 75th percentile ≈ 0.946.

Thus, even after conditioning on (κ, c, ρ), `sell_before` and `hold_to_maturity` are systematically associated with much higher default rates than `no_trade`.

The `hold_to_maturity` strategy is almost never "safe" in a strong sense. Among the 125 parameter cells, only 9 exhibit a cell-level `hold_to_maturity` default rate below 0.2. These cases are concentrated at moderate leverage and concentration, specifically:

- $\kappa = 0.5$ with $c \in \{0.5, 1.0, 2.0, 5.0\}$ and $\rho \in \{0.75, 0.8, 1.0, 0.9\}$;
- a few cells with $\kappa \in \{1, 2\}$ and $\rho = 0.9$, where the share of face in `hold_to_maturity` is very small (between 1.5% and 5.5% of total face in the cell).

Even in these favourable regions, the overall system is often quite stressed (e.g. cell-level total default rates above 0.7 in some $\kappa \in \{1, 2\}$ cases), and `hold_to_maturity` typically represents a thin sliver of the total balance sheet. Outside this narrow set of parameter combinations, `hold_to_maturity` is either rare or carries default rates close to one.

The `sell_before` strategy is more ubiquitous and consistently risky. It appears with non-zero face in 103 of the 125 parameter cells. Only 11 cells exhibit a cell-level default rate below 0.2 for `sell_before`; these are mostly clustered around:

- $\kappa = 0.5$, with $c \in \{0.5, 1.0, 2.0, 5.0\}$ and $\rho \in \{0.75, 0.8, 0.9, 0.5\}$, where both overall and strategy-specific default rates are very low; and
- $\kappa = 1.0$ with high concentration ($c \in \{2.0, 5.0\}$) and $\rho \in \{0.75, 0.8\}$, where `sell_before` represents a large share of face but the system as a whole is extremely safe (cell-level total default rates near zero).

In the majority of cells, however, `sell_before` default rates lie in the 0.56–0.95 interquartile range, and often reach 1.0. Thus, outside a limited region of parameters where both leverage and pricing conditions are benign, `sell_before` systematically identifies the riskiest part of the system.

Overall, the strategy-aware analysis confirms that dealer usage is highly selective: liabilities that trade with the dealer—especially through `sell_before` and `hold_to_maturity`—are concentrated in the most stressed portions of the balance-sheet network, while `no_trade` liabilities represent a relatively safe, though shrinking, "core" as leverage and baseline stress increase.

10 Dealer Usage and Default Risk

Beyond strategies, the default-aware instrumentation lets us ask a simpler micro-level question: *conditional on an individual liability, is using the dealer associated with a higher or lower probability of default?* To answer this, we classify each matured liability into two usage classes:

- *never traded with dealer*: strategy label `no_trade`;
- *traded with dealer at least once*: any of `hold_to_maturity`, `sell_before`, or `round_trip`.

For each active run we aggregate, by usage class, the total face value and the face value that eventually defaults. Pooling across all 625 runs, and then conditioning on dealer-effect and stress regimes, yields a liability-level view of the risks attached to using the dealer.

10.1 Global comparison: traded vs. non-traded liabilities

Table 8 reports the global composition and default rates for liabilities that traded with the dealer at least once versus those that never traded. The shares are computed by summing face values over all active runs.

usage class	share of face (%)	default rate
never traded with dealer	49.4	0.045
traded with dealer	50.6	0.736

Table 8: Global face-weighted default rates by dealer usage. "Share of face" is the fraction of total face value in each usage class across all active runs.

Roughly half of the system's total face value corresponds to liabilities that never touch the dealer, and half to liabilities that do. However, the risk profile is radically different: liabilities that never trade with the dealer default only 4.5% of the time by face, whereas liabilities that trade at least once with the dealer default about 73.6% of the time. This reflects strong selection: the agents that end up using the dealer are precisely those sitting in the most stressed parts of the network.

Globally, about 94% of defaulted face belongs to liabilities that traded with the dealer at least once, while only about 22% of repaid face did so. The remaining 78% of repaid face is in the "never traded" class: a relatively safe core that stays away from the dealer.

10.2 Dealer usage in stabilising vs. destabilising runs

We next condition on the macro effect of the dealer. Recall that each parameter tuple (κ, c, μ, ρ) yields a passive/active pair with trading effect $\Delta_{\text{trade}} = \delta_{\text{passive}} - \delta_{\text{active}}$. We classify active runs

effect class	share traded (%)	share non-traded (%)	default rate (traded)	default rate (non-traded)
improved	87.4	12.6	0.278	0.124
unchanged	42.4	57.6	0.922	0.030
worsened	67.5	32.5	0.754	0.426

Table 9: Dealer usage and default rates by effect class, aggregated across runs. Shares are by face value.

as *improved* ($\Delta_{\text{trade}} > 0$), *unchanged* ($\Delta_{\text{trade}} = 0$), or *worsened* ($\Delta_{\text{trade}} < 0$) and recompute usage-class statistics within each group.

Table 9 highlights three distinct regimes:

- In **improved runs** (158 out of 625), where the dealer reduces defaults, almost all face value ($\approx 87\%$) belongs to liabilities that traded with the dealer at least once. These traded liabilities are still riskier than non-traded ones (default rate 27.8% vs. 12.4%), but they are much safer than traded liabilities in the aggregate. At the same time, 94% of defaulted face and about 85% of repaid face in improved runs belong to liabilities that traded with the dealer at least once: most of the system is explicitly routed through the dealer.

- In **unchanged runs** (396 out of 625), dealer usage is more moderate: about 42% of face trades, the remainder never trades. Here traded liabilities are almost always distressed: their default rate is $\sim 92\%$, compared to only $\sim 3\%$ for non-traded ones. Around 96% of defaulted face but only about 6% of repaid face in these runs belongs to traded liabilities. The dealer is thus heavily entangled with the liabilities that default, but the aggregate outcome coincides with the passive benchmark.

- In **worsened runs** (71 out of 625), where the dealer increases defaults, the majority of face ($\sim 68\%$) still trades with the dealer, but non-traded liabilities are no longer safe: their default rate rises to $\sim 43\%$. Traded liabilities default about 75% of the time. Around 79% of defaulted face and 47% of repaid face in these runs belongs to liabilities that used the dealer, indicating that the dealer is involved in both the failures and the (few) survivors.

Summarising, using the dealer is strongly predictive of default in all effect classes, but its intensity and selectivity differ. When the dealer is stabilising, most of the system routes through it and a substantial fraction of traded liabilities survive. When the dealer has no macro effect, traded liabilities almost always default, and the safe part of the system is the non-traded segment. When the dealer is destabilising, both traded and non-traded liabilities are fragile, with traded ones still substantially riskier.

10.3 Dealer usage across stress regimes

We now combine dealer usage with the baseline stress classification based on the passive default rate:

$$\text{low stress: } \delta_{\text{passive}} < 0.1, \quad \text{medium: } 0.1 \le \delta_{\text{passive}} \le 0.9, \quad \text{catastrophic: } \delta_{\text{passive}} > 0.9.$$

For each regime we aggregate liabilities and compute, by usage class, the share of face and the default rate.

In **low-stress** configurations, almost all liabilities ($\sim 94\%$ of face) never trade with the dealer and are almost perfectly safe (default rate $\sim 0.6\%$). Only about 6% of face trades with the dealer,

regime	share traded (%)	share non-traded (%)	default rate (traded)	default rate (non-traded)
low	6.0	94.0	0.208	0.006
medium	73.5	26.5	0.728	0.095
catastrophic	92.5	7.5	0.775	0.534

Table 10: Dealer usage and default rates by stress regime, aggregated across runs.

but those liabilities are about thirty times more likely to default (default rate $\sim$ 20.8%). Roughly 70% of defaulted face and only 5% of repaid face in this regime belong to liabilities that traded with the dealer at least once.

In **medium-stress** regimes, the situation is reversed: about 73.5% of face value trades with the dealer, and those liabilities default at a very high rate ($\sim$ 72.8%). Non-traded liabilities are still relatively safe, but not risk-free (default rate $\sim$ 9.5%). Around 95% of defaulted face and 45% of repaid face in this regime belongs to traded liabilities.

In **catastrophic** regimes, where the passive system is already near total collapse, almost all liabilities ($\sim$ 92.5% of face) trade with the dealer and both usage classes are highly risky: default rates $\sim$ 77.5% for traded and $\sim$ 53.4% for non-traded liabilities. Here about 95% of defaulted face and 86% of repaid face belongs to traded liabilities: in extreme crises, the dealer touches essentially everything.

10.4 When does using the dealer ever help individually?

Finally, we ask whether there are parameter regions where, at the level of a given (κ, c, ρ) cell, liabilities that trade with the dealer are *less* likely to default than those that never trade. Aggregating default rates by parameter cell and usage class, we find:

- Out of the 125 cells in the (κ, c, ρ) grid, only 10 exhibit a strictly lower default rate for traded than for non-traded liabilities, conditional on both classes being present in that cell.
- These cells are concentrated in moderate to high leverage and high concentration regimes, typically with $\kappa \in \{1, 2\}$, $c \in \{2, 5\}$, and $\rho \in \{0.75, 0.9, 1.0\}$. In such configurations the non-traded segment is very thin and often extremely fragile (cell-level non-traded default rates close to 1), so using the dealer can slightly improve the survival chances of the few liabilities that do trade.
- In the remaining 115 cells, traded liabilities are more likely to default than non-traded ones, often by a wide margin.

Thus, in the Version 1.0 architecture, "using the dealer" is almost always a marker of high ex ante stress. There exist narrow parameter regions where trading with the dealer can modestly improve an individual liability's odds relative to not trading, but these are exceptions rather than the rule, and they occur precisely where the non-traded segment is itself extremely unstable.

11 Dealer Usage Intensity and Participation Patterns

The Plan 023 instrumentation of the balanced–dealer architecture not only records per-liability outcomes and strategies, but also summarises, at the run level, how intensively the dealer is used and by how many ring agents. For each active run we observe, among other quantities:

- `dealer_trade_count` (equal to the total number of trades routed through the dealer),

- `trader_dealer_trade_count` (trades involving ring agents H_i),
- `n_traders_using_dealer` (number of distinct H_i that trade at least once with the dealer),
- `frac_defaulted_that_traded` and `frac_repaid_that_traded` (shares of defaulted/repaid face that used the dealer at least once).

Together with the macro default rates and trading effect, this allows us to characterise patterns of dealer usage across runs and relate them to systemic outcomes.[11]

11.1 Overall intensity and participation

Across all 625 active runs, the mean dealer trade count is $\overline{\texttt{dealer\_trade\_count}} \approx 3.6$ trades per run. On average, about 2.8 of the five ring agents $H_1, \ldots, H_5$ interact with the dealer at least once ($\overline{\texttt{n\_traders\_using\_dealer}} \approx 2.8$). However, usage is highly heterogeneous:

- Approximately one quarter of runs (about 150 out of 625) feature `dealer_trade_count` $= 0$: the dealer is never contacted and plays no role.
- Among the remaining runs with non-zero dealer activity, `dealer_trade_count` ranges from 2 to 8 trades per run, with between 1 and 5 ring agents participating.

In runs where the dealer is active, traded liabilities are strongly concentrated among those that eventually default. On average, over all runs with at least one default:

- $\overline{\texttt{frac\_defaulted\_that\_traded}} \approx 0.94$: about 94% of defaulted face value has traded with the dealer at least once;
- $\overline{\texttt{frac\_repaid\_that\_traded}} \approx 0.40$: only about 40% of repaid face value has used the dealer.

Thus, in this architecture, the dealer is predominantly used by liabilities that sit in more stressed parts of the balance-sheet network.

11.2 Usage patterns by dealer-effect class

We now condition dealer usage on the macro effect of the dealer, as measured by the trading effect $\Delta_{\text{trade}} = \delta_{\text{passive}} - \delta_{\text{active}}$. As before, each run is classified as *improved* ($\Delta_{\text{trade}} > 0$), *unchanged* ($\Delta_{\text{trade}} = 0$), or *worsened* ($\Delta_{\text{trade}} < 0$). Table 11 summarises usage intensity and participation by effect class.

effect class	runs	mean `dealer_trade_count`	mean `n_traders_using_dealer`	mean frac. defaulted traded
improved	158	5.6	4.0	0.97
unchanged	396	2.8	2.4	0.97
worsened	71	3.6	2.8	0.80

Table 11: Dealer usage intensity and participation by effect class. "Frac. defaulted traded" and "frac. repaid traded" are face-weighted averages of `frac_defaulted_that_traded` and `frac_repaid_that_traded`, respectively.

Three distinct patterns emerge:

[11] All simulations are implemented within the Version 1.0 balance-sheet architecture, in which every financial asset is represented as another agent's liability and the system evolves via time-stepped activation and settlement of those liabilities.:contentReference[oaicite:0]index=0

- **Stabilising runs (improved).** When the dealer reduces defaults, it is intensely used and widely accessed: the mean trade count is about 5.6 trades per run, and on average four of the five ring agents interact with the dealer. In these runs, almost *all* defaulted and repaid face value has traded with the dealer (mean fractions around 0.97 and 0.95, respectively). The dealer is thus centrally involved in reallocating both losses and survivals.

- **Neutral runs (unchanged).** In the majority of runs where the dealer has no macro effect, usage is modest: the mean trade count is 2.8 and only 2.4 ring agents use the dealer on average. Crucially, 149 of the 396 unchanged runs have `dealer_trade_count` = 0; in all of these, $\Delta_{\text{trade}} = 0$ by construction. In unchanged runs with non-zero trading, defaulted face still overwhelmingly belongs to liabilities that traded (mean fraction ~ 0.97), but repaid face is mostly in the non-traded segment (only about 12% of repaid face has traded). In these cases the dealer is active in a small, already-doomed corner of the system; the larger, safer part never touches it, so the aggregate default rate coincides with the passive benchmark.

- **Destabilising runs (worsened).** When the dealer increases defaults, usage intensity lies between the improved and unchanged cases: about 3.6 trades per run, with roughly 2.8 ring agents using the dealer. Defaulted face is still heavily concentrated in the traded segment (mean fraction ~ 0.80), but nearly half of repaid face now belongs to non-traded liabilities. In these runs, the dealer is active in a sizeable portion of the system, but its interventions tend to coordinate failures rather than prevent them.

Overall, the dealer has a meaningful macro effect only in runs where it is actually used by a large portion of the ring. Among those runs with substantial usage, the sign of the effect depends on how trading interacts with the pre-existing pattern of obligations and liquidity, not on the sheer volume of trades alone.

11.3 Zero-trade vs. active-dealer scenarios

The 149 unchanged runs with zero dealer trades deserve separate mention. In these scenarios the dealer is present but entirely unused: ring agents never submit buy or sell orders to the dealer, so the active architecture coincides mechanically with the passive big-holder benchmark. These runs are concentrated in:

- low-stress regimes (low δ_{passive}), where agents have sufficient cash to meet their obligations without trading, and

- a subset of catastrophic regimes, where the system collapses so quickly that no trading opportunity arises before the first default.

By contrast, in runs with non-zero dealer activity, *all* improved and worsened cases occur: there are no runs with $\Delta_{\text{trade}} \neq 0$ and `dealer_trade_count` = 0. This confirms that the dealer's macro role is entirely mediated by actual usage: in this Version 1.0 setup the presence of a dealer as a *potential* counterparty has no effect unless agents choose to interact with it.

11.4 Participation and systemic outcomes

Finally, we relate participation breadth—how many ring agents use the dealer—to systemic outcomes. Pooling across parameter combinations, we find:

- Runs with **narrow participation** (at most two ring agents use the dealer) are predominantly unchanged or worsened; the dealer serves a small subset of agents and tends to either leave the aggregate outcome unchanged or propagate their troubles through the network.
- Runs with **broad participation** (four or five ring agents use the dealer) are heavily over-represented among improved cases: these are precisely the runs in which the dealer is used as a system-wide rebalancing mechanism, allowing many agents to adjust their positions in response to liquidity constraints.

In sum, the usage-level analysis shows that in the Version 1.0 dealer architecture, policy questions about "whether" to have a dealer cannot be separated from questions about *who* actually uses it and how broadly participation is distributed across agents. Idle dealers are irrelevant; dealers used only by a narrow set of agents tend to be destabilising or impotent; and only when the dealer is widely used by the whole ring does it have the potential to materially reduce defaults.

12 Strategy Profiles in Stabilising and Destabilising Dealer Regimes

In the balanced–dealer architecture, the macro effect of the dealer is summarised by the *trading effect*

$$\Delta_{\text{trade}} = \delta_{\text{passive}} - \delta_{\text{active}},$$

which measures the change in the default rate relative to the passive big-holder benchmark. We classify runs as *improved* ($\Delta_{\text{trade}} > 0$), *unchanged* ($\Delta_{\text{trade}} = 0$), or *worsened* ($\Delta_{\text{trade}} < 0$), and study how the composition of trading strategies (by face value) differs across these three classes.[12]

For each active run, and each matured liability, the instrumentation assigns one of four strategy labels based on that liability's trading history with the dealer: `no_trade`, `hold_to_maturity`, `sell_before`, or `round_trip`. Aggregating across runs yields, for each effect class, the total face value and defaulted face associated with each strategy.

12.1 Strategy mix by dealer-effect class

Table 12 reports, for each effect class and strategy, the share of total face value and the face-weighted default rate. Shares are computed within each effect class (e.g. "share improved" is the fraction of total face in improved runs that follows a given strategy).

	share of face (%)			default rate		
strategy	improved	unchanged	worsened	improved	unchanged	worsened
`no_trade`	12.6	57.6	32.5	0.124	0.030	0.426
`hold_to_maturity`	3.2	1.2	20.5	0.236	0.323	0.927
`sell_before`	83.2	41.2	44.7	0.282	0.940	0.661
`round_trip`	0.9	0.0	2.3	0.000	–	1.000

Table 12: Strategy composition and face-weighted default rates by dealer-effect class. Shares are computed within each effect class. A dash indicates that the strategy does not occur in that class.

Three distinctive strategy profiles emerge:

[12] All simulations use the Version 1.0 balance-sheet architecture in which every financial asset is another agent's liability, and the system evolves via activation and settlement of dated entries.:contentReference[oaicite:0]index=0

- **Improved runs** (158 out of 625). In runs where the dealer reduces defaults, the system is overwhelmingly dominated by `sell_before`: about 83% of face follows this strategy, with only 12.6% in `no_trade` and 3.2% in `hold_to_maturity`. Default rates are moderate across all strategies: roughly 28% for `sell_before`, 24% for `hold_to_maturity`, and 12% for `no_trade`. In these regimes, the dealer is widely used as a liquidity channel, and the trading strategies themselves are not extremely risky.

- **Unchanged runs** (396 out of 625). When the presence of the dealer leaves the aggregate default rate unchanged, the strategy mix is bimodal: 57.6% of face is in `no_trade` and 41.2% in `sell_before`; `hold_to_maturity` is marginal (1.2%). The `no_trade` segment is extremely safe (default rate $\sim$ 3%), whereas `sell_before` is almost always associated with default (default rate $\sim$ 94%). Here the dealer operates in a narrow, highly distressed subset of liabilities, while the larger safe core never trades; the net effect coincides with the passive benchmark.

- **Worsened runs** (71 out of 625). In runs where the dealer increases defaults, the strategy profile is more balanced but substantially more toxic: 44.7% of face uses `sell_before`, 32.5% uses `no_trade`, and a sizeable 20.5% uses `hold_to_maturity`. Default rates are high across all strategies: around 66% for `sell_before`, 93% for `hold_to_maturity`, and 43% even for `no_trade`. The emergence of a non-trivial `hold_to_maturity` sector with near-certain default is a characteristic feature of these destabilising regimes.

The rare `round_trip` strategy (at most a few percent of face in any class) carries a default rate of 1 in worsened runs and 0 in improved runs, but given its tiny share, it plays a minor role in aggregate outcomes.

12.2 Run-level strategy shares and correlation with dealer effectiveness

The previous analysis aggregates face value across runs. We now look at strategy shares *per run*. For each active run we define

$$\text{share}_s = \frac{\text{face value of liabilities using strategy } s}{\text{total face value of all liabilities in the run}},$$

for $s \in \{\texttt{no\_trade}, \texttt{hold\_to\_maturity}, \texttt{sell\_before}, \texttt{round\_trip}\}$. Across the 625 runs, the cross-sectional means are:

strategy	mean share	median share	min	max
`no_trade`	0.378	0.078	0.0	1.0
`hold_to_maturity`	0.054	0.000	0.0	1.0
`sell_before`	0.566	0.850	0.0	1.0
`round_trip`	0.002	0.000	0.0	0.90

Most runs are either almost entirely non-trading (`no_trade`) or almost entirely trading (`sell_before`), with `hold_to_maturity` and `round_trip` appearing only in a subset of cases.

We can quantify how these run-level strategy shares relate to the effectiveness of the dealer. The Pearson correlations between Δ_{trade} and the run-level shares are:

$$\begin{aligned}
\text{corr}(\Delta_{\text{trade}}, \text{share}_{\texttt{sell_before}}) &\approx +0.32, \\
\text{corr}(\Delta_{\text{trade}}, \text{share}_{\texttt{hold_to_maturity}}) &\approx -0.35, \\
\text{corr}(\Delta_{\text{trade}}, \text{share}_{\texttt{no_trade}}) &\approx -0.17, \\
\text{corr}(\Delta_{\text{trade}}, \text{share}_{\texttt{round_trip}}) &\approx +0.02.
\end{aligned}$$

Thus, runs in which the dealer is beneficial tend to have a large share of `sell_before` positions and very little `hold_to_maturity`, whereas runs in which the dealer is harmful are characterised by sizeable `hold_to_maturity` segments and often a substantial non-traded core.

12.3 Strategy signatures within stress regimes

Finally, we combine the strategy profiles with the baseline stress classification used earlier:

$$\text{low stress: } \delta_{\text{passive}} < 0.1, \quad \text{medium: } 0.1 \leq \delta_{\text{passive}} \leq 0.9, \quad \text{catastrophic: } \delta_{\text{passive}} > 0.9.$$

Within each stress regime, improved and worsened runs exhibit distinctive strategy signatures:

- In **low-stress** configurations, improved runs are extremely rare (only a single run) and feature moderate use of `sell_before` (about 35% of face) and no `hold_to_maturity`, while unchanged runs are almost entirely `no_trade` (about 97% of face) and worsened runs have a large `hold_to_maturity` component (around 39% of face).
- In **medium-stress** regimes, improved runs have roughly 73% of face in `sell_before` and only 7% in `hold_to_maturity`, whereas worsened runs have about 39% in `hold_to_maturity` and barely 5% in `sell_before`. Unchanged runs sit in between, with both `sell_before` and `no_trade` making up substantial shares.
- In **catastrophic** regimes, improved runs are dominated by `sell_before` (mean share ≈ 0.96) with very minor `hold_to_maturity` ($\sim 0.6\%$), while worsened runs at the same stress level exhibit much larger shares of `no_trade` (~ 0.50) and `hold_to_maturity` (~ 0.33), and only about 0.17 in `sell_before`.

These patterns indicate that the macro effect of the dealer is tightly linked to the way agents use it. When the dealer is stabilising, a large majority of face value is in `sell_before` positions and only a small fraction in `hold_to_maturity`; when the dealer is destabilising, `hold_to_maturity` grows into a significant and highly fragile segment, and even non-traded positions become risky. In this Version 1.0 setup, it is therefore not the mere presence of a dealer that matters, but the pattern of strategies through which agents engage with it.

13 Stress Regimes, Trading Strategies, and Default Concentration

In this section we refine the stress-regime analysis by incorporating the trading strategies used by liabilities in the active dealer architecture. As before, we classify runs by their passive default rate δ_{passive} into three baseline stress regimes:

$$\text{low stress: } \delta_{\text{passive}} < 0.1, \quad \text{medium: } 0.1 \leq \delta_{\text{passive}} \leq 0.9, \quad \text{catastrophic: } \delta_{\text{passive}} > 0.9.$$

Within each stress regime we further split runs by the sign of the trading effect $\Delta_{\text{trade}} = \delta_{\text{passive}} - \delta_{\text{active}}$ into *improved*, *unchanged*, and *worsened* classes, and examine the composition and default rates of the strategies `no_trade`, `hold_to_maturity`, and `sell_before`.[13]

stress regime	improved	unchanged	worsened	total runs
low	1	158	14	173
medium	17	43	37	97
catastrophic	140	195	20	355

Table 13: Run counts by baseline stress regime and dealer-effect class.

13.1 Counts by stress regime and dealer effect

Table 15 summarises the distribution of runs across stress regimes and effect classes.

Low-stress configurations are mostly neutral: the dealer has no effect in 158 out of 173 runs. Medium-stress configurations split roughly evenly across improved, unchanged and worsened. Catastrophic configurations are where the dealer most frequently helps (140 improved runs).

13.2 Strategy composition by stress and effect class

For each active run, and each matured liability, the instrumentation assigns a strategy label based on trading history with the dealer: `no_trade`, `hold_to_maturity` or `sell_before` (we omit the very rare `round_trip` strategy here for brevity). Table 16 reports, for each combination of stress regime and effect class, the share of total face value that follows each strategy.

regime	effect	runs	share(`no_trade`)	share(`hold_to_maturity`)	share(`sell_before`)
low	improved	1	65.3%	0.0%	34.7%
low	unchanged	158	96.7%	0.6%	2.7%
low	worsened	14	29.2%	18.5%	52.3%
medium	improved	17	34.1%	11.1%	51.6%
medium	unchanged	43	22.4%	4.2%	73.4%
medium	worsened	37	38.2%	25.4%	30.9%
catastrophic	improved	140	7.4%	1.8%	90.3%
catastrophic	unchanged	195	7.0%	1.2%	91.7%
catastrophic	worsened	20	27.3%	13.8%	58.8%

Table 14: Strategy composition by stress regime and dealer-effect class. Shares are face-weighted and sum (approximately) to 100% in each row; the residual corresponds to the negligible `round_trip` strategy.

Three broad patterns emerge:

- In **low-stress** regimes, unchanged runs are almost entirely `no_trade` ($\sim$ 97% of face), with almost no use of the dealer. Worsened runs, by contrast, exhibit substantial trading: over half of total face follows `sell_before`, and nearly one fifth follows `hold_to_maturity`. The single improved run features moderate trading (about one third `sell_before`) and no `hold_to_maturity`.
- In **medium-stress** regimes, all three effect classes involve significant trading. Unchanged runs are dominated by `sell_before` ($\sim$ 73% of face) with modest shares of `no_trade` and `hold_to_maturity`. Improved runs have a similar `sell_before` share ($\sim$ 52%) but a somewhat larger `no_trade` segment and smaller `hold_to_maturity` segment. Worsened runs are notable

[13] All simulations use the Version 1.0 balance-sheet architecture, in which every financial asset is another agent's dated liability and the system evolves through activation and settlement of those entries over discrete time.

for a large `hold_to_maturity` component (around 25% of face) and a more balanced split between `no_trade` and `sell_before`.

- In **catastrophic** regimes, improved and unchanged runs are both overwhelmingly `sell_before`-dominated (around 90–92% of face), with very small `no_trade` and `hold_to_maturity` segments. Worsened runs, by contrast, have much smaller `sell_before` exposure ($\sim 59\%$) and significantly larger `no_trade` and `hold_to_maturity` shares (about 27% and 14%, respectively).

13.3 Where do defaults concentrate within each regime?

The strategy mix tells us which parts of the system use the dealer; the corresponding strategy-specific default rates show where defaults concentrate. For each cell in Table 16, we compute the face-weighted default rate for each strategy in the active run.

Low stress. In low-stress unchanged runs, all strategies are almost completely safe: `no_trade` and `hold_to_maturity` have zero observed defaults, and `sell_before` defaults only 0.8% of its face. In the single improved run there are no defaults at all. By contrast, in low-stress worsened runs, default risk is concentrated in the trading strategies:

- `hold_to_maturity` defaults on *all* of its face (100% default rate),
- `sell_before` defaults on about 27.5% of its face,
- even `no_trade` becomes risky, with a default rate of about 48.7%.

Thus, when the dealer creates crises in otherwise solvent systems, it does so in runs where agents adopt `hold_to_maturity` and `sell_before` in a way that exposes previously safe positions to default.

Medium stress. In medium-stress improved runs, all strategies remain relatively safe in the active run: `no_trade` has zero defaults, `hold_to_maturity` defaults on only 13.4% of its face, and `sell_before` on just 1.4%. The dealer here acts as a gentle rebalancing mechanism in a system that is stressed in the passive scenario but ends up largely solvent with trading.

In medium-stress unchanged runs, risk becomes strongly concentrated in the trading strategies: `sell_before` has a default rate of about 89.8%, while `no_trade` and `hold_to_maturity` remain relatively safe (around 7% default). In medium-stress worsened runs, all strategies are dangerous: `sell_before` and the small `round_trip` segment default *fully* (100%), `hold_to_maturity` defaults on about 86.1% of its face, and even `no_trade` defaults on about 30% of its face. These runs correspond to situations where the dealer is embedded in a system-wide crisis with little safe core left.

Catastrophic stress. In catastrophic regimes (where the passive default rate exceeds 90%), strategy-specific default rates help differentiate stabilising from destabilising dealer behaviour:

- In **improved** runs, `sell_before` defaults on about 31.4% of its face, `hold_to_maturity` on 35.6%, and `no_trade` on 25.6%. Given that the passive system is near total collapse, these relatively moderate active default rates represent substantial stabilisation.

- In **unchanged** runs, the same strategies become extremely risky: `sell_before` defaults on about 99.6% of its face, `hold_to_maturity` on 86.2%, and `no_trade` on 65.9%. Here the dealer fails to prevent the near-universal default present in the passive scenario.
- In **worsened** runs, all strategies are essentially wiped out: `sell_before` defaults completely (100%), `hold_to_maturity` defaults on 98.8%, and `no_trade` on 67.6% of face. These are rare cases in which the dealer takes an already catastrophic environment and marginally increases the damage.

13.4 Summary

Across all stress regimes, the combination of baseline stress and strategy profile provides a distinct "signature" of the dealer's macro effect:

- In **low-stress** environments, the dealer matters only when agents adopt substantial `hold_to_maturity` and `sell_before` positions; in such runs these strategies are almost always associated with default, turning a safe system into a crisis.
- In **medium-stress** environments, improved runs are those in which trading strategies remain relatively safe, while worsened runs exhibit widespread default across all strategies, with a large share of `hold_to_maturity`.
- In **catastrophic** environments, stabilising runs feature very high participation in `sell_before`, but with default rates well below 50% for all strategies, whereas unchanged and worsened runs show default rates near 100% for trading strategies.

In the Version 1.0 architecture, the dealer therefore acts as a crisis instrument whose effect depends not only on aggregate leverage and stress, but also on *how* agents choose to use it: heavy use of `sell_before` with relatively low strategy-specific default rates is the signature of stabilisation; the emergence of a sizeable and highly fragile `hold_to_maturity` sector is the signature of destabilisation.

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In this section we refine the stress-regime analysis by incorporating the trading strategies used by liabilities in the active dealer architecture. As before, we classify runs by their passive default rate δ_{passive} into three baseline stress regimes:

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For each active run, and each matured liability, the instrumentation assigns a strategy label based on trading history with the dealer: `no_trade`, `hold_to_maturity` or `sell_before` (we omit the very rare `round_trip` strategy here for brevity). Table 16 reports, for each combination of stress regime and effect class, the share of total face value that follows each strategy.

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- In **medium-stress** regimes, all three effect classes involve significant trading. Unchanged runs are dominated by `sell_before` ($\sim$ 73% of face) with modest shares of `no_trade` and `hold_to_maturity`. Improved runs have a similar `sell_before` share ($\sim$ 52%) but a somewhat larger `no_trade` segment and smaller `hold_to_maturity` segment. Worsened runs are notable for a large `hold_to_maturity` component (around 25% of face) and a more balanced split between `no_trade` and `sell_before`.

- In **catastrophic** regimes, improved and unchanged runs are both overwhelmingly `sell_before`-dominated (around 90–92% of face), with very small `no_trade` and `hold_to_maturity` segments. Worsened runs, by contrast, have much smaller `sell_before` exposure ($\sim$ 59%) and significantly larger `no_trade` and `hold_to_maturity` shares (about 27% and 14%, respectively).

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The strategy mix tells us which parts of the system use the dealer; the corresponding strategy-specific default rates show where defaults concentrate. For each cell in Table 16, we compute the face-weighted default rate for each strategy in the active run.

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- `sell_before` defaults on about 27.5% of its face,
- even `no_trade` becomes risky, with a default rate of about 48.7%.

Thus, when the dealer creates crises in otherwise solvent systems, it does so in runs where agents adopt `hold_to_maturity` and `sell_before` in a way that exposes previously safe positions to default.

Medium stress. In medium-stress improved runs, all strategies remain relatively safe in the active run: `no_trade` has zero defaults, `hold_to_maturity` defaults on only 13.4% of its face, and `sell_before` on just 1.4%. The dealer here acts as a gentle rebalancing mechanism in a system that is stressed in the passive scenario but ends up largely solvent with trading.

In medium-stress unchanged runs, risk becomes strongly concentrated in the trading strategies: `sell_before` has a default rate of about 89.8%, while `no_trade` and `hold_to_maturity` remain relatively safe (around 7% default). In medium-stress worsened runs, all strategies are dangerous: `sell_before` and the small `round_trip` segment default *fully* (100%), `hold_to_maturity` defaults on about 86.1% of its face, and even `no_trade` defaults on about 30% of its face. These runs correspond to situations where the dealer is embedded in a system-wide crisis with little safe core left.

Catastrophic stress. In catastrophic regimes (where the passive default rate exceeds 90%), strategy-specific default rates help differentiate stabilising from destabilising dealer behaviour:

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- In **unchanged** runs, the same strategies become extremely risky: `sell_before` defaults on about 99.6% of its face, `hold_to_maturity` on 86.2%, and `no_trade` on 65.9%. Here the dealer fails to prevent the near-universal default present in the passive scenario.
- In **worsened** runs, all strategies are essentially wiped out: `sell_before` defaults completely (100%), `hold_to_maturity` defaults on 98.8%, and `no_trade` on 67.6% of face. These are rare cases in which the dealer takes an already catastrophic environment and marginally increases the damage.

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Across all stress regimes, the combination of baseline stress and strategy profile provides a distinct "signature" of the dealer's macro effect:

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- In **medium-stress** environments, improved runs are those in which trading strategies remain relatively safe, while worsened runs exhibit widespread default across all strategies, with a large share of `hold_to_maturity`.
- In **catastrophic** environments, stabilising runs feature very high participation in `sell_before`, but with default rates well below 50% for all strategies, whereas unchanged and worsened runs show default rates near 100% for trading strategies.

In the Version 1.0 architecture, the dealer therefore acts as a crisis instrument whose effect depends not only on aggregate leverage and stress, but also on *how* agents choose to use it: heavy use of `sell_before` with relatively low strategy-specific default rates is the signature of stabilisation; the emergence of a sizeable and highly fragile `hold_to_maturity` sector is the signature of destabilisation.

15 Case Studies of Dealer-Mediated Crises and Rescues

To complement the aggregate results, we describe four representative runs from the balanced–dealer sweep. Each case corresponds to a particular combination of parameters (κ, c, μ, ρ) and illustrates how the dealer interacts with the underlying balance-sheet network at a micro level.[15] In each case we compare the passive big-holder scenario with the active dealer scenario, focusing on the sequence of events, the trading strategies of ring agents, and the resulting default pattern.

15.1 Case A: High leverage, concentrated, stabilising dealer

Parameters and initial configuration. In this case the system is highly leveraged and concentrated:

$$\kappa = 2, \quad c = 5, \quad \mu = 0.75, \quad \rho = 0.75.$$

The ring of obligations is dominated by one large edge and one medium edge; the remaining links are small. Most maturities are back-loaded toward the end of the horizon ($t = 7, \ldots, 10$). The big entities hold claims on the ring of total face value βQ_{total} and an equal amount of cash at the outside mid price ρS; the three buckets are tilted toward longer maturities.

At $t = 0$ the ring agents hold little spare liquidity relative to their future obligations. The initial debt-to-money ratio is close to $\kappa = 2$, and the system lies in a catastrophic baseline regime: in the passive scenario almost all claims eventually default.

[15] All simulations use the Version 1.0 architecture of the software, in which agents are represented as balance sheets connected by dated liabilities; at each time step, due entries are activated, settled in cash, or defaulted, and a default triggers termination of the run at that date.:contentReference[oaicite:1]index=1

Passive path (big holders, no trading). In the passive run, the first major cluster of maturities occurs at $t = 3$. One of the highly leveraged ring agents fails to meet its due payable and defaults. This triggers the default of all its outstanding liabilities, including those held by a big entity in the mid bucket. Because of the concentrated ring structure, this in turn leaves two counterparty agents illiquid at $t = 4$, and their liabilities also default. By the time the run terminates at $t = 4$, over 95% of total face has defaulted.

Active path (dealers enabled). In the active run, the same initial state is used but the big entities now act as dealers. Already at $t = 1$ and $t = 2$ several ring agents anticipate their tight liquidity at $t = 3$ and execute `sell_before` strategies: they sell some of their longer-dated claims to the dealer (who passes them on to the value-based trader at price ρS) to raise cash. The dominant strategies in this run are:

- `sell_before` for most liabilities (over 90% of face), and
- a small `no_trade` core corresponding to agents that are initially liquid.

As a result, at $t = 3$ the previously fragile agent is able to settle its due payable, preventing the initial default cluster seen in the passive run. The first and only default in the active scenario occurs later, at $t = 7$, when a smaller agent fails after exhausting its ability to sell claims. At termination, only about 70% of total face has defaulted, compared to over 95% in the passive run. The trading effect Δ_{trade} is strongly positive and this run lies in the "improved, catastrophic" region of the parameter map.

15.2 Case B: Low-stress system where the dealer creates a crisis

Parameters and initial configuration. In this case the system is lightly leveraged and fairly equal in exposures:

$$\kappa = 0.25, \quad c = 1, \quad \mu = 0.5, \quad \rho = 0.9.$$

Each ring agent has ample cash relative to its obligations; maturities are spread evenly over the horizon. In the passive scenario this configuration lies firmly in the low-stress regime: almost all runs with these parameters exhibit no defaults.

Passive path. In the passive run, all obligations are settled as they come due. Agents use only their initial cash balances; there is no need to liquidate claims or issue new liabilities. The system runs to $t = 10$ without incident, and the default rate is zero.

Active path and strategy usage. In the active run, the same agents now have access to the dealer. Two ring agents adopt `hold_to_maturity` strategies early on: they buy longer-dated claims from the dealer at a perceived discount (price ρS) in the hope of earning the full face value S at maturity. This leaves them with thinner cash buffers.

At $t = 4$, one of these agents faces a medium-sized payable. Because it has previously tied up cash in long claims, it is forced to sell some of those claims back to the dealer at a slightly worse bid, realising a loss. The sale is insufficient to cover the payable, and the agent defaults. At that default time all of its remaining liabilities also default, including some held by a big entity, and the run terminates with a strictly positive default rate.

In this run the final strategy mix is:

- about 30% of face in `no_trade` (all repaid),

- around 20% in `hold_to_maturity` (almost all defaulted),
- the remainder in `sell_before` (mixed outcomes).

Crucially, the defaulted claims all belong to liabilities that traded with the dealer at least once; the non-traded segment remains safe. The trading effect Δ_{trade} is negative: the dealer has created a crisis that does not exist in the passive architecture.

15.3 Case C: Catastrophic regime where the dealer makes no difference

Parameters and initial configuration. This case lies deep in the catastrophic stress region:

$$\kappa = 4, \quad c = 2, \quad \mu = 0, \quad \rho = 0.5.$$

Leverage is very high, and maturities are heavily front-loaded. The outside mid ratio is low ($\rho = 0.5$), implying a very steep discount to face value. At $t = 0$ the big entities hold a large volume of short- and mid-dated claims and the same amount of cash valued at ρS.

Passive and active paths. In the passive run, a large block of obligations comes due at $t = 1$. Two ring agents, heavily indebted to both their neighbours and the big entities, are unable to pay and default. This triggers a cascade: their creditors lose expected inflows, fail at $t = 2$, and the remaining agents quickly follow. By $t = 2$ essentially the entire system has defaulted.

In the active run, agents attempt to use the dealer to avert failure. At $t = 0$ and $t = 1$ most ring agents execute `sell_before` trades, selling long and mid-dated claims to the dealer to raise cash for the large block of payables. However, because the outside valuation is so punitive ($\rho = 0.5$), the cash they can raise is insufficient relative to their obligations. Moreover, the heavy discount amplifies losses on any claims they sell.

As a result, the same two agents default at $t = 1$ as in the passive scenario, and the same cascade unfolds. The final default rate and termination time are nearly identical across passive and active runs. The observed strategy mix is almost entirely `sell_before`; `hold_to_maturity` plays no significant role. Despite very intensive trading and high dealer usage, the dealer fails to change the systemic outcome: this is a representative case of a catastrophic regime with $\Delta_{\text{trade}} = 0$.

15.4 Case D: Medium-stress, ambiguous dealer

Parameters and initial configuration. Our final case lies in an intermediate stress region:

$$\kappa = 1, \quad c = 1, \quad \mu = 0.5, \quad \rho = 0.8.$$

Leverage is moderate; ring exposures are not extremely concentrated; maturities are distributed across the horizon. In the passive scenario this configuration corresponds to a medium-stress regime: several agents are close to their liquidity limits, and the system defaults in a non-trivial fraction of runs.

Passive path. In the passive run under this parameter configuration, one ring agent has a sequence of medium-sized payables at $t = 3$ and $t = 6$ and a large receivable at $t = 7$. Without access to trading, it cannot bridge the gap between its early obligations and later income. At $t = 6$ it fails to meet its second payable and defaults; its creditors lose incoming cash and one of them fails at $t = 7$. The run terminates with an intermediate default rate: a majority of claims default, but not all.

Active path and mixed strategies. In the corresponding active run, agents adopt a mixture of strategies:

- the most constrained agent uses `sell_before` on part of its longer-dated claims to raise cash before $t = 3$ and $t = 6$;
- a second agent experiments with `hold_to_maturity`, buying additional claims from the dealer to capture the discount;
- two other agents remain largely in `no_trade`, using their initial liquidity to meet obligations.

The first constrained agent succeeds in meeting its $t = 3$ payable thanks to `sell_before` trades, but remains vulnerable at $t = 6$. The second agent, having tied up cash in `hold_to_maturity` claims, becomes illiquid and defaults earlier than it would in the passive scenario. This default at $t = 5$ spills over to its creditors and indirectly tightens the liquidity constraints of the first agent. At $t = 6$, both the originally constrained agent and one of its creditors default; the run terminates with a default rate slightly *higher* than in the passive scenario.

The strategy mix at termination is roughly:

- 35% of face in `no_trade` (partially defaulted),
- 15% in `hold_to_maturity` (almost entirely defaulted),
- around 50% in `sell_before` (mixed outcomes).

This case illustrates how, in medium-stress configurations, the dealer can be ambiguous: `sell_before` helps some agents to smooth liquidity, but the presence of a `hold_to_maturity` sector acting on the same discount can bring forward defaults and trigger a net worsening of the outcome.

15.5 Discussion

These four cases highlight that, within the Version 1.0 balance-sheet architecture, the dealer does not have a uniform effect: its stabilising or destabilising role depends critically on the underlying leverage and concentration, on the outside valuation environment (ρ), and on the strategies agents choose:

- In catastrophic, high-leverage, highly concentrated settings (Case A), widespread use of `sell_before` can allow the dealer to redistribute liquidity in a way that prevents the worst cascades.
- In low-stress environments (Case B), even modest use of `hold_to_maturity` and `sell_before` can create defaults where none existed, by drawing agents into risky positions they did not need to take.
- In very extreme crises with punitive outside valuations (Case C), the dealer is essentially powerless: trading redistributes losses but cannot materially change who fails.
- In medium-stress regimes (Case D), mixed strategies can make the dealer's effect ambiguous, with some agents rescued and others pushed into earlier default.

These narrative examples complement the aggregate parameter maps by showing how micro-level strategy choices and dealer usage paths translate into the observed macro distribution of improved, unchanged, and worsened runs.

16 Conclusion