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Modeling non-linearities in real effective exchange rates

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Abstract

The aim of this paper is to test for and model non-linearities in the real effective exchange rates of 10 major industrial countries (the G-10). To exploit non-linear dependencies in exchange rates, we apply the STAR (Smooth Transition Autoregressive) family of models. These non-linear models imply the existence of two distinct regimes in exchange rates, with potentially different dynamic properties, but the transition between the regimes is smooth. Tests reject linearity for eight exchange rates. The real exchange rate process is cyclical in both regimes for almost all countries, and there appears to be some evidence of asymmetry. STAR models outperform Hamilton's Markov regime-switching model in an out-of-sample forecasting contest.

JEL classifications: F31; C5

Keywords: Non-linearities; Cycles; Real effective exchange rates; STAR models

1. Introduction

A number of empirical studies (e.g. Hsieh, 1989; De Grauwe et al., 1993; Brooks, 1996; Drunat et al., 1996) have uncovered significant non-linearities in nominal

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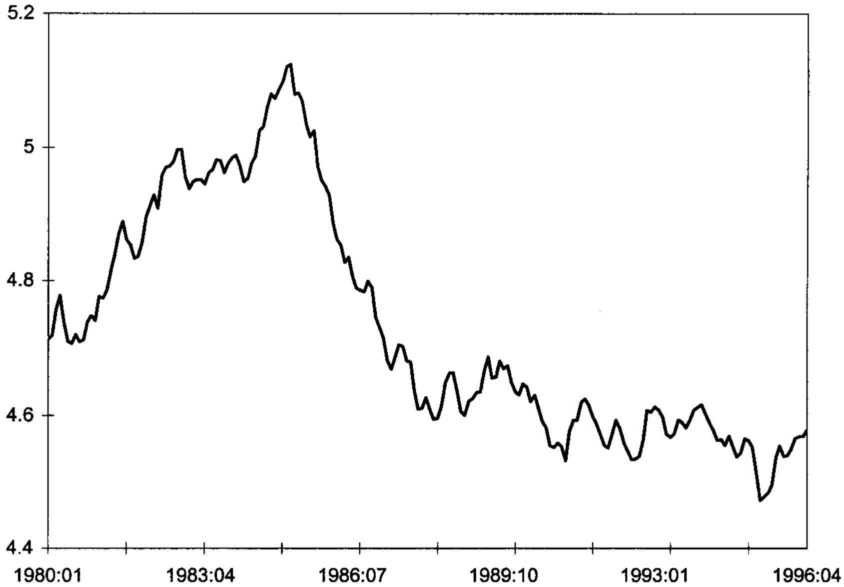

Fig. 1. USA: (log) real effective exchange rate.

bilateral exchange rates, and some authors have estimated non-linear models for these nominal rates.[2] To our knowledge, however, there has been no attempt to investigate and model non-linearities in real exchange rates and particularly in effective exchange rates. An extensive survey by De Grauwe (1996) shows that real bilateral exchange rates for the major industrial countries during the 1962–1994 period have exhibited very long cycles and substantial drifts. Similar patterns are evident in the real effective exchange rates of the G-5 industrial countries during the 1980s and 1990s, shown in Figs. 1–5. The real dollar effective rate exhibits a long cycle during the 1980s, followed by shorter swings in the 1990s. The yen and D-mark real effective exchange rates show cyclical movements (particularly the yen) around a strong upward drift. On the other hand, the sterling and F-franc real effective rates exhibit substantial cyclical movements around a downward drift.[3]

Economic theory offers a number of potential explanations for the presence of non-linearities and cycles in exchange rates. Heterogeneity of participants in the foreign exchange market is often cited as the major source of non-linearities in the exchange rate process. Using an extended version of the Dornbusch sticky-price

[2] For example, Engle and Hamilton (1990) and Engle (1994) applied the Markov regime-switching model, while Diebold and Nason (1990), Meese and Rose (1991) and Mizrach (1992) employed various non-parametric procedures.

[3] A similar pattern of swings and drifts is observed in the real effective exchange rates of the other countries, but the graphs of these rates are not shown for space reasons.

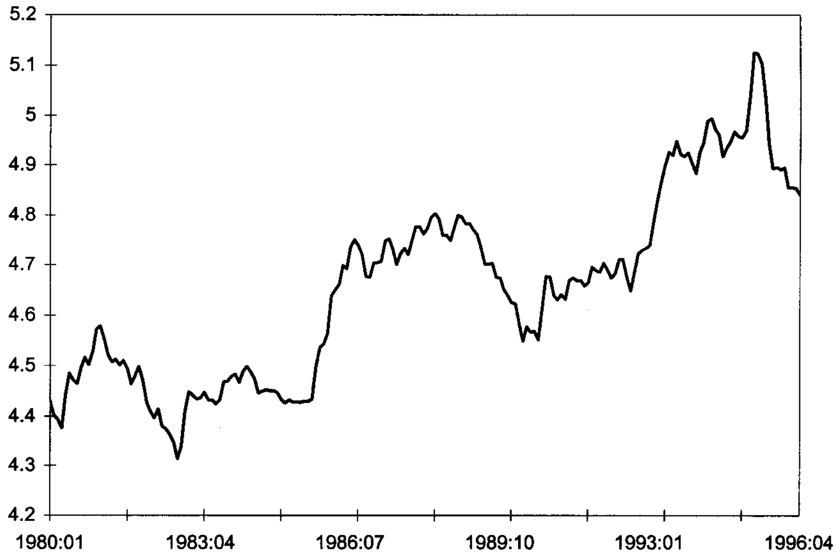

Fig. 2. Japan: (log) real effective exchange rate.

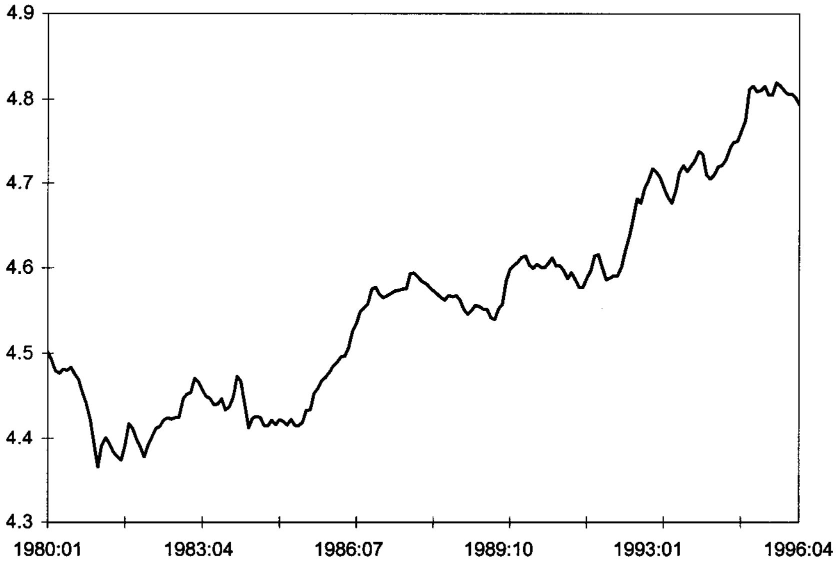

Fig. 3. Germany: (log) real effective exchange rate.

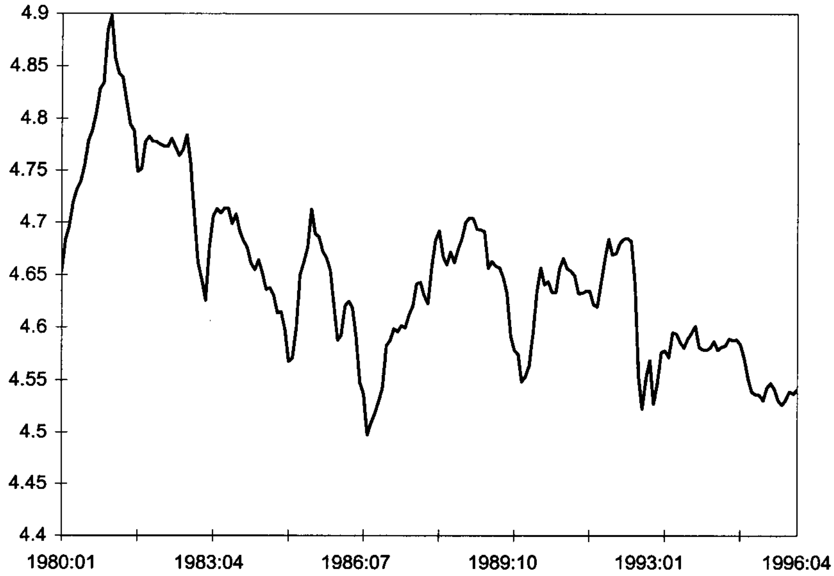

Fig. 4. UK: (log) real effective exchange rate.

exchange rate model, De Grauwe et al. (1993) show that the interaction of fundamentalists and chartists can generate chaotic exchange rate dynamics for a

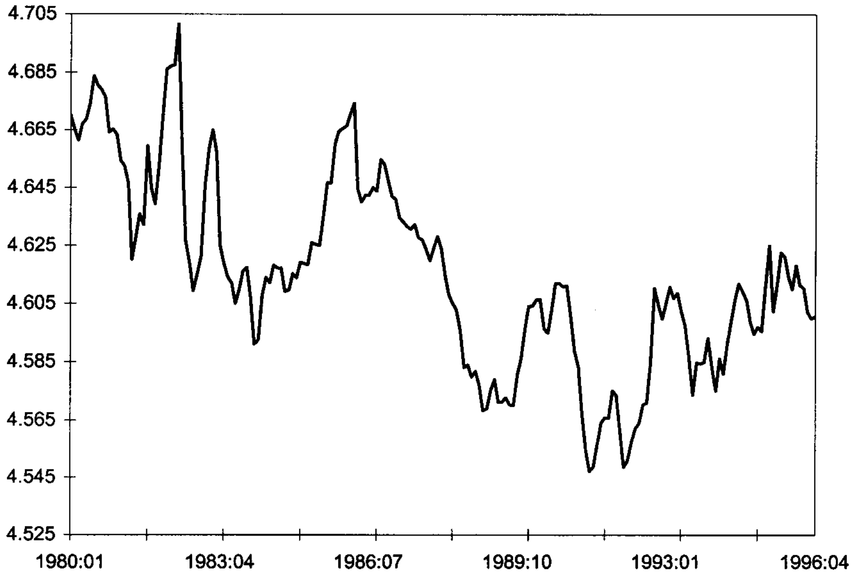

Fig. 5. France: (log) real effective exchange rate.

wide range of parameter values. Brock and Hommes (1996) set out an asset pricing model with heterogeneous expectations, but with traders moving between different beliefs or predictors of future prices according to a 'performance' indicator based on the profitability of these predictors. This structural non-linear model generates complex asset price dynamics varying from pitchfork and Hopf bifurcations to near homoclinic orbits.

While the above studies stress heterogeneity in agents' expectations formation, Peters (1994) and Guillaume et al. (1995) emphasize heterogeneity in investors' objectives arising from different investment horizons, geographical location, and various types of risk profiles and institutional constraints. It is argued that this type of heterogeneity explains why investors often respond differently to the same set of news, and it is shown to generate a non-linear exchange rate process.

Dumas (1992) analyzes the dynamic process of the real exchange rate within a general equilibrium model of the international capital market with spatially separated countries and proportional shipping costs. It is shown that the model produces endogenously a non-linear real exchange rate process. Although the real exchange rate displays mean reversion, the probability of a move away from the parity value of unity is greater than that of a move towards parity. This asymmetric behavior suggests that the real exchange rate exhibits very long cycles.

The aim of this paper is to test for and model non-linearities in the real effective exchange rates of 10 major industrial countries (the G-10). To exploit non-linear dependencies in exchange rates, we apply the STAR (Smooth Transition Autoregressive) family of models. These models were originally developed by Teräsvirta and Anderson (1992) for modeling non-linearities over the business cycle, and their statistical properties and estimation are examined in Granger and Teräsvirta (1993) and Teräsvirta (1994). Their application, however, has been rather limited so far. Granger et al. (1993) have used them to analyze a non-linear relationship between GNP growth and leading indicators, Öcal and Osborn (1997) employ STAR models to investigate non-linearities in UK consumption and industrial production, Leybourne and Mizen (1997) apply the smooth transition model to consumer prices, while Skalin and Teräsvirta (1998) use STAR models to examine Swedish business cycles. To our knowledge, the only application of STAR models to foreign exchange rates is a recent paper by Michael et al. (1997).[4]

The STAR models are flexible and have interesting properties. These parametric

[4] Michael et al. (1997) adopt a different approach from us. The authors test for cointegration in the Purchasing Power Parity (PPP) relationship for a small number of bilateral sterling and US dollar exchange rates. They subsequently apply the ESTAR model to the residuals from the cointegrating relationships to investigate the adjustment process towards PPP. We feel that this approach is questionable. If the residuals of the PPP relationship follow a non-linear process, the validity of the linear cointegration tests and interpretation of these residuals are doubtful. Indeed the concept of equilibrium in non-linear models with endogenously generated cycles may be different from that of linear models. These problems can be avoided by applying the STAR models directly to the real exchange rate, and then investigate the dynamic properties of the exchange rate process using well established mathematical criteria. After all, the Dumas (1992) model which is used by the authors as a basis for their empirical work, analyzes directly the dynamics of the real exchange rate process.

non-linear models contrast with the threshold autoregressive (TAR) and the Hamilton (1989) Markov regime-switching models which assume a sharp switch between regimes. In foreign exchange markets with a large number of investors, each switching at different times (probably due to heterogeneous beliefs, varying learning speeds, and different investment horizons), a smooth transition between regimes seems more appropriate. Another important feature of these models is that they nest the linear regression [AR(p)] model.[5] We can therefore develop Lagrange multiplier tests, which are shown to be more powerful than the BDS test for small samples,[6] for testing the null of linearity before fitting any non-linear model and for choosing between the alternative STAR specifications (see Granger and Teräsvirta, 1993).

The remainder of the paper is organized as follows. Section 2 outlines the specification and estimation methodology. Section 3 analyzes the empirical results for the G-10 countries. The final section draws up the main conclusions.

2. Specification and estimation of STAR models

The STAR model of order k, for variable e_t, has the following specification:

$$e_t = \beta_0 + \boldsymbol{\beta}_1{}'\mathbf{x}_t + (\theta_0 + \boldsymbol{\theta}_1{}'\mathbf{x}_t)F(e_{t-d}) + u_t, \tag{1}$$

where $\mathbf{x}_t = (e_{t-1}, e_{t-2}, \ldots e_{t-k})'$, $\boldsymbol{\beta}_1 = (\beta_1, \beta_2, \ldots \beta_k)'$, $\boldsymbol{\theta}_1 = (\theta_1, \theta_2, \ldots \theta_k)'$, $u_t \sim \mathrm{nid}(0,\sigma^2)$, F is the transition function, e_{t-d} is the transition variable, and d is the delay parameter. In these models, non-linearities arise through conditioning on lagged exchange rates.

Following Teräsvirta and Anderson (1992), we consider two alternative definitions of the transition function, $F(..)$. First, the logistic function

$$F(e_{t-d}) = [1 + \exp\{-\gamma(e_{t-d} - c)\}]^{-1}, \tag{2}$$

and second, the exponential function

$$F(e_{t-d}) = 1 - \exp\left\{-\gamma(e_{t-d} - c)^2\right\}, \tag{3}$$

[5] Granger and Teräsvirta (1993) stress the importance of testing for linearity prior to applying any non-linear model. Arbitrary choice of such models runs the serious risk of spurious fit. A criticism levied against the Hamilton (1989) regime-switching model and various non-parametric models is that they do not test the linearity hypothesis.

[6] It is well known that the BDS test requires large samples (measured in hundreds of observations) to achieve good power. Simulations by Lee et al. (1993) show that in samples up to 200 observations the BDS test has low power and is inferior to LM-type tests. In addition, the null hypothesis of the BDS test is that observations are *i.i.d.*, so rejection of the null does not tell us anything about the parametric non-linear model that we should apply. Granger and Teräsvirta (1993) point out that the LM-tests are more powerful when testing against specific parametric non-linear models, and hence more useful for modeling non-linearities.

where γ measures the speed of transition from one regime to the other, and c indicates the half-way point between the two regimes.

Eq. (1) combined with Eq. (2) yields the LSTAR model, while Eq. (1) accompanied by Eq. (3) defines the ESTAR model. These models imply that there are two distinct regimes in the foreign exchange market, say an 'appreciating' and 'depreciating' regime. The LSTAR and ESTAR models, however, describe quite different types of dynamic exchange rate behavior. The LSTAR model implies that the appreciating and depreciating regimes may have different dynamics, with the transition from one to the other being smooth. The ESTAR model, on the other hand, suggests that the two regimes have rather similar dynamics, while the transition period can have different dynamics. Hence both models are capable of describing asymmetric behavior in exchange rates.

The estimation of STAR models consists of three stages [see Granger and Teräsvirta, 1993, pp. 113–124; and Teräsvirta, 1994]:

(a) Specification of a linear AR (autoregressive) model. We estimate AR models of different orders and the maximum value of k is chosen on the basis of the AIC criterion and the Ljung–Box statistic for autocorrelation.

(b) Testing linearity, for different values of the delay parameter d, against STAR models using the linear model specified in (a) as the null. To carry out this test, we estimate the auxiliary regression

$$v_t = \beta_0 + \boldsymbol{\beta}_1'\mathbf{x}_t + \boldsymbol{\beta}_2'\mathbf{x}_t e_{t-d} + \boldsymbol{\beta}_3'\mathbf{x}_t e_{t-d}^2 + \boldsymbol{\beta}_4'\mathbf{x}_t e_{t-d}^3 + w_t, \tag{4}$$

where v_t is the residual of the AR model.

The linearity test is H_0: $\boldsymbol{\beta}'_2 = \boldsymbol{\beta}'_3 = \boldsymbol{\beta}'_4 = 0$. To specify the value of the delay parameter d, the estimation of (4) is carried out for a wide range of values, $1 \le d \le D$. In cases where linearity is rejected for more than one value of d, d is chosen by $d = \arg\min\ P(d)$ for $1 \le d \le D$, where $P(d)$ is the P-value of the linearity test [see Teräsvirta and Anderson (1992) and Teräsvirta (1994)].

(c) Choosing between LSTAR and ESTAR models for those exchange rates where linearity is rejected. This is done by the following sequence of nested tests:

$$H_{04}: \quad \boldsymbol{\beta}_4 = 0, \tag{5}$$

$$H_{03}: \quad \boldsymbol{\beta}_3 = 0/\boldsymbol{\beta}_4 = 0, \tag{6}$$

$$H_{02}: \quad \boldsymbol{\beta}_2 = 0/\boldsymbol{\beta}_3 = \boldsymbol{\beta}_4 = 0, \tag{7}$$

Rejection of (5) implies selecting the LSTAR model. If we accept (5) and reject (6), we choose the ESTAR model. Accepting (5) and (6) and rejecting (7) leads to an LSTAR model. Granger and Teräsvirta (1993) and Teräsvirta (1994) argue that strict application of this sequence of tests may lead to wrong conclusions, because the higher-order terms of the Taylor expansion used in deriving these tests are disregarded. They therefore recommend that one should compute the P-values for all F-tests of (5)–(7) and make the choice of the STAR model on the basis of the lowest P-value.

3. Empirical results

3.1. The data

We use monthly data on the real effective exchange rates of 10 major industrial countries, over the period 1980:1–1996:4. We choose effective rather than bilateral exchange rates because the former measure a country's international competitiveness against all its trade partners. The use of monthly data provides us with a reasonably large sample and hence meets the requirement of the linearity tests for many degrees of freedom. The real effective exchange rates are based on unit labor costs (1990 = 100) and were obtained from the *IMF International Financial Statistics*.

Application of the linearity tests and of the STAR models requires stationary time series. The Phillips–Peron unit root tests for the level and first difference of the real effective exchange rates, measured in logarithms, are shown in Table 1. These results indicate that all time series are clearly integrated of order 1 at both 5% and 1% significance levels, except for Belgium and Netherlands which are I(1) stochastic processes only at the 1% significance level. Hence the variable (e_t) used in all estimations is the first logarithmic difference of real effective exchange rates.

3.2. Tests for linearity and STAR model selection

The linearity tests, together with the maximum lag and the Ljung–Box statistic (LB) are displayed in Table 2. The LB(..) statistic suggests white noise residuals for all AR models. In carrying out linearity tests we have considered values for the delay parameter d over the range $1 \leq d \leq 8$, and calculated the P-values for the linearity test in each case. The estimate of d is chosen by the lowest P-value. With the exception of France, the delay parameter for all countries is relatively short.

Table 1
Phillips–Perron unit root tests for the (log) real effective exchange rates

Country	Level	First difference
Belgium	−3.250	−8.224
Canada	−1.557	−8.740
France	−2.288	−10.666
Germany	0.294	−9.071
Italy	−0.907	−8.828
Japan	−1.331	−9.254
Netherlands	−3.260	−8.265
Switzerland	−0.385	−9.601
UK	−1.761	−8.762
USA	−0.920	−10.133

Note. The 1% and 5% critical values are −3.465 and −2.876, respectively. The lag truncation for Bartlett kernel is 12.

Using 0.05 as a threshold P-value, the test fails to reject linearity in the real effective exchange rates of Netherlands and Switzerland. However, for all other countries the test classifies the time series as non-linear.[7] We can therefore proceed to build non-linear models for these exchange rates. The tests for the choice between LSTAR and ESTAR models are shown in Table 3. It is interesting to notice that the effective exchange rates of Germany, France and Belgium, three main members of the European Exchange Rate Mechanism, are specified as LSTAR models, suggesting that the expansion and contraction phases of the exchange rates in these countries may have different dynamics. In contrast, the exchange rates of all other countries are classified as ESTAR models, implying that exchange rates move from high or low levels towards the middle ground (or normal level) in a similar fashion.

3.3. Estimates of the non-linear models

The LSTAR and ESTAR models are estimated by non-linear least-squares, using the Marquardt algorithm. Granger and Teräsvirta (1993, pp. 123–124) and Teräsvirta (1994) point out that estimation of the parameter γ may cause particular problems (e.g. slow convergence, overestimation). We have therefore followed their recommendation in scaling the argument of the transition function $F(..)$ by dividing it by the standard deviation of e_t, $\sigma(e)$, in the case of the LSTAR model, and by $\sigma^2(e)$ for the ESTAR model. Hence, the scaled transition functions used in the estimation of LSTAR and ESTAR models are, respectively:

$$F(e_{t-d}) = [1 + \exp\{-\gamma(1/\sigma(e))(e_{t-d} - c)\}]^{-1}, \tag{8}$$

$$F(e_{t-d}) = 1 - \exp\left\{-\gamma(1/\sigma^2(e))(e_{t-d} - c)^2\right\}. \tag{9}$$

On the basis of this scaling, we have used $\gamma = 1$ as an initial value, and the sample mean as a starting value for the parameter c. The estimates of the AR model are used as initial values for the $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$ parameters.

The parameter estimates together with diagnostic statistics are reported in Table 4. The Jargue–Bera normality test for the initial estimates of some countries indicated the presence of few outliers. To cheque whether the non-linearities are due to the presence of these outliers, we have used dummies to filter them out. Surprisingly, all parameter estimates and other diagnostic statistics have remained almost the same, except that the residuals now become normal. Hence the non-linearities in the exchange rate series are not the outcome of any outliers. Since we are using monthly data, residuals are tested for 12th-order autocorrelation and ARCH effects. The P-values (using 0.05 as the threshold) reject serial correlation and the presence of ARCH non-linearity in the residuals for all

[7] Although the P-value for the USA is slightly higher than 0.05, estimation of an ESTAR model is undertaken on the basis of the evidence from tests (5)–(7); i.e. the coefficients on the term $x_t e_{t-2}^2$ in Eq. (4) were highly significant.

Table 2
Linearity tests (minimum P-values)

Delay (d)	Belgium $k = 2$ LB = 8.4	Canada $k = 2$ LB = 13.3	France $k = 4$ LB = 15.9	Germany $k = 2$ LB = 6.66	Italy $k = 3$ LB = 11.1	Japan $k = 2$ LB = 11.1	Netherlands $k = 2$ LB = 15.0	Switzerland $k = 3$ LB = 5.26	UK $k = 2$ LB = 11.3	USA $k = 2$ LB = 5.64
1	0.0000001*	0.00516*	0.057383	0.00891	0.000001	0.03836*	0.19198	0.33582	0.96226	0.98783
2	0.000006	0.02958	0.48006	0.00165*	0.0000001*	0.83400	0.13671*	0.13395*	0.42848	0.07165*
3	0.00005	0.27021	0.64299	0.22482	0.000001	0.26600	0.57871	0.55140	0.52009	0.81435
4	0.33564	0.73581	0.14069	0.27302	0.01514	0.05400	0.37755	0.87381	0.01084*	0.47076
5	0.00016	0.17695	0.13306	0.08605	0.00134	0.05100	0.33073	0.76403	0.20682	0.96300
6	0.00425	0.01182	0.15357	0.40654	0.00039	0.74900	0.16530	0.14272	0.11284	0.77416
7	0.00053	0.18459	0.00920*	0.17691	0.04499	0.57900	0.60632	0.17181	0.86438	0.96500
8	0.01171	0.26464	0.05299	0.44106	0.000003	0.73553	0.83686	0.76590	0.31801	0.86500

Note. The asterisk (*) indicates the minimum P-value over the interval $1 \leq d \leq 8$. The selection of the maximum lag, k, of the linear AR model was made using the AIC statistic. LB is the Ljung–Box statistic for 12th order autocorrelation in the AR model.

Table 3
Specification of the non-linear model

	Delay (d)	H_{04}: $\boldsymbol{\beta}_4 = 0$	H_{03}: $\boldsymbol{\beta}_3 = 0/\boldsymbol{\beta}_4 = 0$	H_{02}: $\boldsymbol{\beta}_2 = 0/\boldsymbol{\beta}_3 = \boldsymbol{\beta}_4 = 0$	Type of model
Belgium	1	0.000030*	0.22496	0.000034	LSTAR
Canada	1	0.74146	0.00021*	0.94004	ESTAR
France	7	0.13462	0.61745	0.00062*	LSTAR
Germany	2	0.01419*	0.07740	0.02671	LSTAR
Italy	2	0.05089	0.00444*	0.15098	ESTAR
Japan	1	0.24300	0.00900*	0.63400	ESTAR
Netherlands	–	–	–	–	Linear
Switzerland	–	–	–	–	Linear
UK	4	0.52630	0.01200*	0.62200	ESTAR
USA	2	0.48691	0.02817*	0.80130	ESTAR

Note. The values for the nested tests H_{03}, H_{02} and H_{01} are *P*-values. An asterisk indicates the lowest *P*-value for the three tests. The threshold value for the linearity tests and the specification of the STAR model is 0.05.

countries. In order to examine the stability of the parameter estimates, we have considered three break points: 1988:1, 1990:1 and 1993:1. The Chow forecast statistics fail to detect any significant parameter instability.

The critical parameters in STAR models are γ and c. The estimates of γ have the anticipated negative sign in all countries, though they are not always significant, reflecting the difficulties in estimating this parameter. One interesting observation, however, is the relative small value of γ in all countries, especially in Belgium, Canada, Germany, Italy, Japan, and the USA.[8] This evidence suggests that the transition from one regime to the other is rather slow, contrary to Hamilton's regime-switching and the TAR models which assume a sharp switch.

Parameter c indicates the halfway point between expansion (appreciation) and contraction (depreciation) phases of the exchange rate. In all countries the estimates of c are small,[9] but significant in six out of eight countries. The values of c are negative in Belgium and Italy, and positive in all other countries. These values are in the neighborhood of the sample mean for the individual exchange rates, so we are led to believe that observations are distributed roughly equally between the left-hand and the right-hand tails of their respective logistic and exponential functions.

3.4. Dynamic behavior

An important feature of STAR models is their dynamic properties. To examine the dynamic behavior of the estimated models, we compute the characteristic roots by solving the equation

$$\lambda^k - \sum_{j=1}^{k} (\beta_{1j} + \theta_{1j}F)\lambda^{k-j} = 0. \tag{10}$$

We consider two regimes: first, $F = 0$, which corresponds to the lower (falling e) regime in the LSTAR model, and the middle regime in the ESTAR model. Second, $F = 1$, which corresponds to the upper (rising e) regime in the LSTAR model, and the outer regime in the ESTAR model.

Table 5 shows the most prominent characteristic roots for each regime. All regimes except three are characterised by complex roots, which implies that real effective exchange rates display cyclical movements during both expansionary and contraction phases. The cycles have a period of 5–6 months on average, except for the cycle of the US exchange rate which has a cycle of just over 1 year in the middle region.[10] The LSTAR model for Belgium shows strong asymmetric behav-

[8] Notice that the scaling of $(e_{t-d} - c)$ in the transition function makes it possible to judge the size of γ (Granger and Teräsvirta, 1993, pp. 123, 153).

[9] It should be borne in mind that the variable we model is the monthly (logarithmic) difference of the exchange rate, so the small absolute values of c are in line with the small numbers of the dependent variable.

[10] The periods of cycles reported in Table 5 refer to the growth rate of real effective exchange rates and should not be confused with the cycles in the level of exchange rates exhibited in Figs. 1–5.

Table 4
Estimates of the LSTAR/ESTAR models

Belgium: LSTAR

$$e_t = -\underset{(0.07)}{0.3591} - \underset{(2.25)}{9.7239}e_{t-1} + \underset{(4.17)}{9.40835}e_{t-2} + (\underset{(0.07)}{0.3589} + \underset{(2.28)}{10.1048}e_{t-1} - \underset{(4.18)}{9.5291}e_{t-2})$$
$$\times[1 + \exp\{-\underset{(0.29)}{1.7617}(1/\sigma(e))(e_{t-1} + \underset{(0.01)}{0.0440})\}]^{-1}$$

S = 0.007, AUTO(12) = 0.995, ARCH(12) = 0.683, NORM(2) = 0.140, STB(88:1) = 0.453, STB(90:1) = 0.118, STB(93:1) = 0.240, S/S_L = 0.746

Canada: ESTAR

$$e_t = \underset{(0.001)}{0.0004} + \underset{(0.16)}{0.0987}e_{t-1} - \underset{(0.10)}{0.1958}e_{t-2} + (\underset{(0.01)}{0.0018} + \underset{(0.30)}{1.0582}e_{t-1} - \underset{(0.25)}{0.6291}e_{t-2})$$
$$\times[1 - \exp\{-\underset{(0.28)}{0.324}(1/\sigma^2(e))(e_{t-1} - \underset{(0.004)}{0.0047})^2\}]$$

S = 0.013, AUTO(12) = 0.167, ARCH(12) = 0.387, NORM(2) = 0.617, STB(88:1) = 0.338, STB(90:1) = 0.415, STB(93:1) = 0.556, S/S_L = 0.929

France: LSTAR

$$e_t = -\underset{(0.001)}{0.0008} + \underset{(0.11)}{0.0702}e_{t-1} - \underset{(0.11)}{0.1878}e_{t-2} + \underset{(0.12)}{0.3313}e_{t-3} - \underset{(0.12)}{0.4782}e_{t-4} + (\underset{(0.001)}{0.0015}$$
$$+ \underset{(0.15)}{0.3833}e_{t-1} + \underset{(0.15)}{0.1065}e_{t-2} - \underset{(0.16)}{0.2665}e_{t-3} + \underset{(0.15)}{0.4680}e_{t-4})$$
$$\times[1 + \exp\{-\underset{(7.32)}{8.00}(1/\sigma(e))(e_{t-7} + \underset{(0.001)}{0.0008})\}]^{-1}$$

S = 0.008, AUTO(12) = 0.097, ARCH(12) = 0.012, NORM(2) = 0.133 STB(88:1) = 0.762, STB(90:1) = 0.403, STB(93:1) = 0.438, S/S_L = 0.876

Germany: LSTAR

$$e_t = -\underset{(0.001)}{0.0248} + \underset{(0.07)}{0.3849}e_{t-1} - \underset{(0.16)}{1.2113}e_{t-2} + (\underset{(0.20)}{0.0559} + \underset{(0.13)}{0.0376}e_{t-1} + \underset{(0.25)}{0.0719}e_{t-2})$$
$$\times[1 + \exp\{-\underset{(0.20)}{0.9816}(1/\sigma(e))(e_{t-2} - \underset{(0.01)}{0.002})\}]^{-1}$$

S = 0.009, AUTO(12) = 0.717, ARCH(12) = 0.368, NORM(2) = 0.275, STB(88:1) = 0.875, STB(90:1) = 0.479, STB(93:1) = 0.408, S/S_L = 0.960

Italy: ESTAR

$$e_t = \underset{(0.06)}{0.0167} + \underset{(0.87)}{1.6696}e_{t-1} + \underset{(2.75)}{0.5949}e_{t-2} - \underset{(0.50)}{1.1958}e_{t-3} + (-\underset{(0.06)}{0.0163} - \underset{(0.87)}{1.2275}e_{t-1}$$
$$- \underset{(2.75)}{1.0482}e_{t-2} + \underset{(0.51)}{1.4334}e_{t-3}) \times [1 - \exp\{-\underset{(1.14)}{3.0418}(1/\sigma^2(e))(e_{t-2} + \underset{(0.002)}{0.0286})^2\}]$$

S = 0.009, AUTO(12) = 0.162, ARCH(12) = 0.585, NORM(2) = 0.784, S/S_L = 0.641

Japan: ESTAR

$$e_t = -\underset{(0.002)}{0.0002} + \underset{(0.13)}{0.2256}e_{t-1} - \underset{(0.08)}{0.1112}e_{t-2} + (\underset{(0.006)}{0.0009} + \underset{(0.30)}{0.4340}e_{t-1} - \underset{(0.19)}{0.3380}e_{t-2})$$
$$\times[1 - \exp\{-\underset{(0.41)}{0.3432}(1/\sigma^2(e))(e_{t-1} - \underset{(0.008)}{0.003})^2\}]$$

S = 0.022, AUTO(12) = 0.530, ARCH(12) = 0.737, NORM(2) = 0.242, STB(88:1) = 0.511, STB(90:1) = 0.723, STB(93:1) = 0.379, S/S_L = 0.902

Table 4 (*Continued*)

UK: ESTAR

$$e_t = \underset{(0.003)}{-\,0.0022} + \underset{(0.18)}{0.4360}e_{t-1} + \underset{(0.25)}{0.4137}e_{t-2} + (\underset{(0.003)}{0.0019} - \underset{(0.22)}{0.0525}e_{t-1} - \underset{(0.26)}{0.6918}e_{t-2})$$
$$\times[1 - \exp\{-\underset{(7.75)}{10.115}(1/\sigma^2(e))(e_{t-4} - \underset{(0.001)}{0.0086})^2\}]$$

S = 0.016, AUTO(12) = 0.742, ARCH(12) = 0.144, NORM(2) = 0.120, STB(88:1) = 0.930, STB(90:1) = 0.962, STB(93:1) = 0.981, S/S_L = 0.876

USA: ESTAR

$$e_t = \underset{(0.002)}{0.0007} + \underset{(0.11)}{0.4817}e_{t-1} - \underset{(0.25)}{0.0734}e_{t-2} + (\underset{(0.004)}{-\,0.0038} - \underset{(0.24)}{0.4155}e_{t-1} + \underset{(0.30)}{0.0300}e_{t-2})$$
$$\times[1 - \exp\{-\underset{(-)}{1.000}(1/\sigma^2(e))(e_{t-2} - \underset{(0.002)}{0.004})^2\}]$$

S = 0.018, AUTO(12) = 0.854, ARCH(12) = 0.904, NORM(2) = 0.585, STB(88:1) = 0.993, STB(90:1) = 0.994, STB(93:1) = 0.988, S/S_L = 0.947

Note. Values under regression coefficients are standard errors. S is the standard error for the non-linear regression. AUTO(12) is the *P*-value for 12th order autocorrelation (Breusch–Godfrey *F*-test). ARCH(12) is the *P*-value for 12th order autoregressive conditional heteroscedasticity (Engle *F*-test). NORM(2) is the *P*-value for the Jargue–Bera normality test. STB(..) is the *P*-value for the Chow forecast *F*-test for breaks in 1988:1, 1990:1 and 1993:1, respectively. S/S_L is the ratio of standard errors for the non-linear and linear models. We were unable to calculate the Chow forecast statistics for Italy because of a near singular matrix.

ior. The lower regime contains a large explosive root, so the real exchange rate moves aggressively from a low level into a higher growth rate. On the other hand, the upper regime is stable, which suggests that once the exchange rate is in the expansionary phase it will tend to stay there. The LSTAR model for France has a stable upper regime, but a modulus for the lower regime close to unit circle (though still stable). The German exchange rate is explosive in both the lower and upper regimes, which implies that the real effective exchange rate moves from a low into a high rate of appreciation and from a high growth rate into a contraction very rapidly.

The ESTAR model for Italy indicates strong asymmetric behavior between the middle and outer regimes. The former has a single explosive root so the growth of the Italian real exchange rate passes the mid-phase quickly on the way up or down. The outer regime, on the other hand, is stable. The ESTAR models for Canada, Japan, UK, and USA are stable both in the middle and outer regimes, though the largest modules for the outer regime in Canada and the mid-regime for the UK are not far from the unit circle.

3.5. Out-of-sample forecasting performance

To evaluate the performance of the estimated non-linear STAR models, we compare their out-of-sample forecasts with those of the linear AR models, as well as with Hamilton's (Hamilton, 1989) Markov regime-switching model which provides the main alternative non-linear formulation.[11] The forecasting performance

Table 5
Most prominent characteristic roots in each regime

Country	Regime	Most prominent roots	Modulus	Period
Belgium (LSTAR):	L	−10.611	10.611	
	U	0.190 ± 0.291i	0.348	6.34
Canada (ESTAR):	M	0.049 ± 0.440i	0.443	4.30
	O	0.578 ± 0.700i	0.908	7.14
France (LSTAR):	L	−0.535 ± 0.738i	0.911	6.66
	U	0.474	0.474	
Germany (LSTAR):	L	0.192 ± 1.084i	1.101	4.50
	U	0.211 ± 1.046i	1.067	4.57
Italy (ESTAR):	M	1.559	1.559	
	O	−0.026 ± 0.692i	0.693	4.10
Japan (ESTAR):	M	0.113 ± 0.314i	0.333	5.13
	O	0.330 ± 0.583i	0.670	5.95
UK (ESTAR):	M	0.897	0.897	
	O	0.192 ± 0.491i	0.527	5.25
USA (ESTAR):	M	0.241 ± 0.124i	0.271	13.22
	O	0.033 ± 0.057i	0.066	6.01

Note. L, lower regime; U, upper regime; M, middle regime; O, outer regime.

of the three models is also compared to that of the random walk which normally provides the benchmark for all econometric exchange rate models. The forecasts were generated as follows. First, all models were re-estimated up to 1992:12 and these estimates were used to generate multi-step ahead forecasts for 1993:1–1993:12. Second, the models were re-estimated up to 1993:12 and then multi-step ahead forecasts were generated for 1994:1–1994:12. Finally, the models were re-estimated up to 1994:12 and these were then used to compute multi-step ahead forecasts for the period 1995:1–1996:4. These prediction periods followed the collapse of the ERM system in the Autumn of 1992 and were characterised by frequent turbulence in currency markets. They therefore provide a strenuous forecast test for the models.

Table 6 presents the root mean squared errors for the computed forecasts of the non-linear and linear models. The RMSEs for both the STAR and AR models are smaller than the standard deviations of the growth rates of real effective exchange rates for most countries during the 1993:1–1994:12 prediction period, and in all countries but one (Germany) during the 1995:1–1996:4 prediction period. We also

[11] It is worth noticing that with the exception of Teräsvirta and Anderson (1992), none of the previous applications of STAR models, including the paper by Michael et al. (1997) on exchange rates, evaluate the forecasting performance of STAR models.

Table 6
Root mean squared forecast errors

Country	Forecast period, 1993:1−1993:12				Forecast period, 1994:1−1994:12				Forecast period, 1995:1−1996:4			
	STAR	Linear	Hamilton	Random walk	STAR	Linear	Hamilton	Random walk	STAR	Linear	Hamilton	Random walk
Belgium	0.0101	0.0094	0.0117	0.0100	0.0072	0.0079	0.0106	0.0066	0.0062	0.0080	0.0121	0.0081
Canada	0.0149	0.0150	0.0273	0.0150	0.0144	0.0135	0.0259	0.0141	0.0125	0.0121	0.0440	0.0125
France	0.0069	0.0080	0.0072	0.0072	0.0063	0.0072	0.0074	0.0074	0.0093	0.0092	0.0094	0.0095
Germany	0.0116	0.0107	0.0109	0.0114	0.0417	0.0095	0.0096	0.0100	0.0365	0.0116	0.0111	0.0118
Italy	0.0169	0.0221	0.0232	0.0243	0.0095	0.0116	0.0118	0.0119	0.0278	0.0336	0.0336	0.0336
Japan	0.0273	0.0269	0.0268	0.0278	0.0243	0.0246	0.0256	0.0259	0.0378	0.0441	0.0443	0.0447
UK	0.0189	0.0187	0.0185	0.0182	0.0080	0.0070	0.0072	0.0072	0.0083	0.0084	0.0083	0.0086
USA	0.0128	0.0124	0.0122	0.0123	0.0134	0.0129	0.0133	0.0141	0.0196	0.0201	0.0204	0.0200

notice that at least one econometric model produces more accurate forecasts than the random walk model in seven out of eight countries during the 1993:1–1994:12 period, and in all countries during the 1995:1–1996:4 period. This shows that the forecasting accuracy of the econometric models is quite satisfactory despite the turbulence of the currency markets during the prediction periods.

The STAR model outperforms the random walk model in approximately half the cases during the 1993:1–1994:12 period, but produces more accurate forecasts than the random walk model in six out of eight countries during the 1995:1–1996:12 period. Taken together, the evidence for the three forecast periods suggests that the STAR model can improve on the random walk, though the degree of improvement is not always unambiguous.

Comparing the forecasting performance of the STAR and linear models, we observe that the linear model outperforms the non-linear model in five out of eight countries during the 1993:1–1993:12 period, each model wins the forecasting contest in half of the cases over the 1994:1–1994:12 period, while the non-linear model improves on the linear model in five out of eight countries during the 1995:1–1996:12 prediction period. It therefore appears that the out-of-sample predictive performance of the STAR models improves over time during the 1993–1996 period, but over the three forecasting periods both models share the number of lowest RMSEs. On the basis of these forecast results, one cannot make a strong case in favor of the STAR models.

In comparing the forecasting performance of the two non-linear models, we notice that each model has the lowest RMSE in half of the cases during the 1993:1–1993:12 period. The STAR model, however, produces more accurate forecasts than the Markov regime-switching model in 12 out of 16 cases during the 1994:1–1996:4 prediction period.

4. Conclusions

Our tests have rejected the linearity hypothesis for the real effective exchange rate in eight industrial countries (the G-8) during the 1980s and 1990s. These exchange rates are classified as logistic STAR models in three ERM countries, and as exponential STAR models in the other countries. The estimated STAR models pass all the main diagnostic tests and provide a satisfactory description of the non-linearity found in the real effective exchange rates.

The estimates of the transition parameter indicate that the speed of transition from one exchange rate regime to another is quite slow for all countries, contrary to the Markov regime-switching and the TAR models which assume a sharp regime switch. The exchange rates are characterised by cyclical movements in almost all regimes. Four regimes (in Belgium, Germany and Italy) display explosive roots, with three more having a modulus close to unity. The exchange rate process is strongly asymmetric in Belgium and Italy, and close to asymmetry in Canada and France. In general, the evidence on the dynamic properties and the transition parameter is in line with the observation of large swings in real exchange rates and the extremely slow convergence to long-run PPP reported by long-horizon data studies (Rogoff, 1996).

With regards to the out-of-sample forecasting performance, there is not much to choose between the STAR and linear models. The STAR models do, however, outperform the alternative non-linear formulation provided by the Markov regime-switching model.

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