EXAMPLE 3.10 For later reference we consider the special case of $p = 3$. Then

$$\mathbf{R}_{xx} = \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix},$$

where $r = \mathbf{x}^{*(1)\prime}\mathbf{x}^{*(2)}$. Also, from (3.55), we have

$$\begin{aligned} \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} &= \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{x}^{*(1)\prime}\mathbf{Y} \\ \mathbf{x}^{*(2)\prime}\mathbf{Y} \end{pmatrix} \\ &= \frac{1}{1-r^2}\begin{pmatrix} 1 & -r \\ -r & 1 \end{pmatrix} \begin{pmatrix} \mathbf{x}^{*(1)\prime}\mathbf{Y} \\ \mathbf{x}^{*(2)\prime}\mathbf{Y} \end{pmatrix}, \end{aligned}$$

so that

$$\hat{\gamma}_1 = \frac{1}{1-r^2}(\mathbf{x}^{*(1)\prime}\mathbf{Y} - r\mathbf{x}^{*(2)\prime}\mathbf{Y}) \quad \text{and} \quad \hat{\beta}_1 = \hat{\gamma}_1/s_1. \tag{3.56}$$

By interchanging the superscripts (1) and (2), we get

$$\hat{\gamma}_2 = \frac{1}{1-r^2}(\mathbf{x}^{*(2)\prime}\mathbf{Y} - r\mathbf{x}^{*(1)\prime}\mathbf{Y}) \quad \text{and} \quad \hat{\beta}_2 = \hat{\gamma}_2/s_2.$$

Since

$$\tilde{\mathbf{X}} = \mathbf{X}^* \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} = \mathbf{X}^*\mathbf{S}_d,$$

say, it follows that

$$\begin{aligned} \mathbf{P} =& n^{-1}\mathbf{1}_n\mathbf{1}_n' + \mathbf{X}^*\mathbf{S}_d(\mathbf{S}_d\mathbf{X}^{*\prime}\mathbf{X}^*\mathbf{S}_d)^{-1}\mathbf{S}_d\mathbf{X}^{*\prime} \\ =& n^{-1}\mathbf{1}_n\mathbf{1}_n' + \mathbf{X}^*(\mathbf{X}^{*\prime}\mathbf{X}^*)^{-1}\mathbf{X}^{*\prime} \\ =& n^{-1}\mathbf{1}_n\mathbf{1}_n' \\ &+ \frac{1}{1-r^2}(\mathbf{x}^{*(1)}, \mathbf{x}^{*(2)}) \begin{pmatrix} 1 & -r \\ -r & 1 \end{pmatrix} (\mathbf{x}^{*(1)}, \mathbf{x}^{*(2)})' \qquad (3.57) \\ =& n^{-1}\mathbf{1}_n\mathbf{1}_n' + \mathbf{x}^{*(2)}\mathbf{x}^{*(2)\prime} \\ &+ \frac{1}{1-r^2}(\mathbf{x}_*^{(1)} - r\mathbf{x}^{*(2)})(\mathbf{x}^{*(1)} - r\mathbf{x}^{*(2)})'. \qquad (3.58) \end{aligned}$$

$\to P^* = \tilde{P}$

□

EXERCISES 3I

1. If $\tilde{Y}_i = Y_i - \overline{Y}$ and $\tilde{\mathbf{Y}} = (\tilde{Y}_1, \ldots, \tilde{Y}_n)'$, prove from (3.54) that $\text{RSS} = \tilde{\mathbf{Y}}'(\mathbf{I}_n - \tilde{\mathbf{P}})\tilde{\mathbf{Y}}$.
 first demean then show $\tilde{y}^T\tilde{P}\tilde{y} = y^T P y$
2. Suppose that we consider fitting a model in which the Y-data are centered and scaled as well as the x-data. This means that we use $Y_i^* = (Y_i - \overline{Y})/s_y$ instead of Y_i, where $s_y^2 = \sum_i (Y_i - \overline{Y})^2$. Using (3.54), obtain an expression for RSS from this model.

so just need to sub y^* for $\tilde{y}$

as the $\bar{y}^2 1_n^T \tilde{P} 1_n$ ... vanishes by centering.