

## Homework 5

Due April 27, 19:00

### Assignment 5.1

Solve the following tridiagonal system:

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ -x_1 + 2x_2 + x_3 &= 2 \\ -x_2 + 2x_3 + x_4 &= 2 \\ -x_3 + 2x_4 + x_5 &= 2 \\ -x_4 + 2x_5 &= 1 \end{aligned}$$

### Assignment 5.2

Using Gauss elimination method without pivoting, solve the linear system  $Ax = b$ , with

$$A = \begin{bmatrix} 3 & -1 & 3 \\ 1 & 2 & -2 \\ 1 & 2 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 11 \\ 2 \\ 0 \end{bmatrix}.$$

Also, give the  $LU$  factorization of  $A$ .

### Assignment 5.3

Consider the following iteration method:

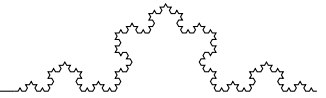
$$\begin{pmatrix} x_1^{(m+1)} \\ x_2^{(m+1)} \\ x_3^{(m+1)} \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} \\ 5 \\ 4 \end{pmatrix} - \frac{1}{10} \begin{pmatrix} 0 & 3 & 1 \\ -2 & 0 & -3 \\ 1 & 4 & 0 \end{pmatrix} \begin{pmatrix} x_1^{(m)} \\ x_2^{(m)} \\ x_3^{(m)} \end{pmatrix}$$

Does  $x^{(m)}$  converge as  $m \rightarrow \infty$  for any choice of the initial guess  $x^{(0)}$ ?

### Assignment 5.4

Solve the following set of equations both by means of a direct method and iterative method. Describe the methods used and why you chose them.

$$\begin{aligned} x_2 + 5x_3 - 7x_4 + 23x_5 - x_6 + 7x_7 + 8x_8 + x_9 - 5x_{10} &= 10 \\ 17x_1 - 24x_3 - 75x_4 + 100x_5 - 18x_6 + 10x_7 - 8x_8 + 9x_9 - 50x_{10} &= -40 \\ 3x_1 - 2x_2 + 15x_3 - 78x_5 - 90x_6 - 70x_7 + 18x_8 - 75x_9 + x_{10} &= -17 \\ 5x_1 + 5x_2 - 10x_3 - 72x_5 - x_6 + 80x_7 - 3x_8 + 10x_9 - 18x_{10} &= 43 \\ 100x_1 - 4x_2 - 75x_3 - 8x_4 + 83x_6 - 10x_7 - 75x_8 + 3x_9 - 8x_{10} &= -53 \\ 70x_1 + 85x_2 - 4x_3 - 9x_4 + 2x_5 + 3x_7 - 17x_8 - x_9 - 21x_{10} &= 12 \\ x_1 + 15x_2 + 100x_3 - 4x_4 - 23x_5 + 13x_6 + 7x_8 - 3x_9 + 17x_{10} &= -60 \\ 16x_1 + 2x_2 - 7x_3 + 89x_4 - 17x_5 + 11x_6 - 73x_7 - 8x_9 - 23x_{10} &= 100 \\ 51x_1 + 47x_2 - 3x_3 + 5x_4 - 10x_5 + 18x_6 - 99x_7 - 18x_8 + 12x_{10} &= 0 \\ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 &= 100 \end{aligned}$$



### Assignment 5.5

Solve the following system of equations by Gauss-Jordan and Gauss-Seidel iteration starting with an initial guess of  $x = y = z = 1$ .

$$\begin{aligned} 8x + 3y + 2z &= 20.00 \\ 16x + 6y + 4.001z &= 40.02 \\ 4x + 1.501y + z &= 10.01 \end{aligned}$$

Comment on the accuracy of your solution and the relative efficiency of the two methods.

### Assignment 5.6

Hooke's law states that the displacement  $x$  of a uniform spring is a linear function of the force,  $y$ , applied to it. In other words we can write

$$y = a + kx,$$

where the coefficient  $k$  is called the **spring constant**.

Suppose that we want to determine the spring constant of some spring by experiment. This particular spring has an unstretched measured length of  $15.5 \text{ cm}$  (i.e.,  $x = 15.5$  when  $y = 0$ ). Because of experimental error, you want to take more than one measurement.

Using forces of  $9 \text{ N}$ ,  $18 \text{ N}$ , and  $27 \text{ N}$  applied to the spring, you measure the corresponding lengths to be  $19.31 \text{ cm}$ ,  $22.1 \text{ cm}$ , and  $26.42 \text{ cm}$ , respectively. Using linear data fit find  $k$ . Plot the data together with linear fit to them.

### Assignment 5.7

Below are provided some real data:

x	395.1	448.1	517.7	583.3	790.2
y	171.0	289.0	399.0	464.0	620.0

- Find the quadratic least square data fit. Plot the data together with fitting parabola.
- Find the cubic least square data fit. Plot the data together with fitting cubic.
- Which one is better? Argue your answer.