Homework 5

Due April 27, 19:00

Assignment 5.1

Solve the following tridiagonal system:

$$\begin{aligned}
2x_1 + x_2 &= 3\\
-x_1 + 2x_2 + x_3 &= 2\\
-x_2 + 2x_3 + x_4 &= 2\\
-x_3 + 2x_4 + x_5 &= 2\\
-x_4 + 2x_5 &= 1
\end{aligned}$$

Assignment 5.2

Using Gauss elimination method without pivoting, solve the linear system $Ax = b$, with

$$A = \begin{bmatrix} 3 & -1 & 3 \\ 1 & 2 & -2 \\ 1 & 2 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 11 \\ 2 \\ 0 \end{bmatrix}.$$

Also, give the LU factorization of A.

Assignment 5.3

Consider the following iteration method:

$$\begin{pmatrix} x_1^{(m+1)} \\ x_2^{(m+1)} \\ x_3^{(m+1)} \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} \\ 5 \\ 4 \end{pmatrix} - \frac{1}{10} \begin{pmatrix} 0 & 3 & 1 \\ -2 & 0 & -3 \\ 1 & 4 & 0 \end{pmatrix} \begin{pmatrix} x_1^{(m)} \\ x_2^{(m)} \\ x_3^{(m)} \end{pmatrix}$$

Does $x^{(m)}$ converge as $m \to \infty$ for any choice of the initial guess $x^{(0)}$?

Assignment 5.4

Solve the following set of equations both by means of a direct method and iterative method. Describe the methods used and why you chose them.

$$\begin{aligned}
x_2 + 5x_3 - 7x_4 + 23x_5 - x_6 + 7x_7 + 8x_8 + x_9 - 5x_{10} &= 10\\
17x_1 - 24x_3 - 75x_4 + 100x_5 - 18x_6 + 10x_7 - 8x_8 + 9x_9 - 50x_{10} &= -40\\
3x_1 - 2x_2 + 15x_3 - 78x_5 - 90x_6 - 70x_7 + 18x_8 - 75x_9 + x_{10} &= -17\\
5x_1 + 5x_2 - 10x_3 - 72x_5 - x_6 + 80x_7 - 3x_8 + 10x_9 - 18x_{10} &= 43\\
100x_1 - 4x_2 - 75x_3 - 8x_4 + 83x_6 - 10x_7 - 75x_8 + 3x_9 - 8x_{10} &= -53\\
70x_1 + 85x_2 - 4x_3 - 9x_4 + 2x_5 + 3x_7 - 17x_8 - x_9 - 21x_{10} &= 12\\
x_1 + 15x_2 + 100x_3 - 4x_4 - 23x_5 + 13x_6 + 7x_8 - 3x_9 + 17x_{10} &= -60\\
16x_1 + 2x_2 - 7x_3 + 89x_4 - 17x_5 + 11x_6 - 73x_7 - 8x_9 - 23x_{10} &= 100\\
51x_1 + 47x_2 - 3x_3 + 5x_4 - 10x_5 + 18x_6 - 99x_7 - 18x_8 + 12x_{10} &= 0\\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 &= 100
\end{aligned}$$

Assignment 5.5

Solve the following system of equations by Gauss-Jordan and Gauss-Seidel iteration starting with an initial guess of $x = y = z = 1$.

$$\begin{aligned} 8x + 3y + 2z &= 20.00 \\ 16x + 6y + 4.001z &= 40.02 \\ 4x + 1.501y + z &= 10.01 \end{aligned}$$

Comment on the accuracy of your solution and the relative efficiency of the two methods.

Assignment 5.6

Hooke's law states that the displacement x of a uniform spring is a linear function of the force, y, applied to it. In other words we can write

$$y = a + kx,$$

where the coefficient k is called the **spring constant**.

Suppose that we want to determine the spring constant of some spring by epxperiment. This particular spring has an unstretched measured length of $15.5\,cm$ (i.e., $x = 15.5$ when $y = 0$). Because of experimental error, you want to take more than one measurement.

Using forces of $9\,N$, $18\,N$, and $27\,N$ applied to the spring, you measure the corresponding lengths to be $19.31\,cm$, $22.1\,cm$, and $26.42\,cm$, respectively. Using linear data fit find k. Plot the data together with linear fit to them.

Assignment 5.7

Below are provided some real data:

x	395.1	448.1	517.7	583.3	790.2
y	171.0	289.0	399.0	464.0	620.0

a) Find the quadratic least square data fit. Plot the data together with fitting parabola.

b) Find the cubic least square data fit. Plot the data together with fitting cubic.

c) Which one is better? Argue your answer.