(S12)

CCC

SSS

Seminar săpt. 13

Geometrie analitică euclidiană

1. Fie $A(1,2,1)$, $B(2,1,3)$, $C(-2,1,3)$, $D(0,2,0)$

a) $V_{ABCD}=?$ b) $A_{\Delta BCD}=?$ c) $dist(A,(BCD))=?$

Sol.:

a) $V_{ABCD}=\frac{1}{6}|\Delta|$

$$\Delta=\begin{vmatrix}1&2&1&1\\2&1&3&1\\-2&1&3&1\\0&2&0&1\end{vmatrix}=\begin{vmatrix}1&0&1&1\\2&-1&3&1\\-2&-1&3&1\\0&0&0&1\end{vmatrix}=1\cdot\begin{vmatrix}1&0&1\\2&-1&3\\-2&-1&3\end{vmatrix}=$$

$$=1\cdot\begin{vmatrix}1&0&0\\2&-1&1\\-2&-1&5\end{vmatrix}=\begin{vmatrix}-1&1\\-1&5\end{vmatrix}=-4$$

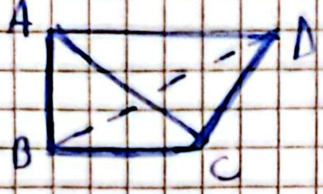
$$V_{ABCD}=\frac{1}{6}\cdot|-4|=\frac{2}{3}$$

b) $A_{\Delta ABC}=\frac{1}{2}\|\overrightarrow{BC}\times\overrightarrow{BD}\|$

$\overrightarrow{BC}=(-4,0,0)$, $\overrightarrow{BD}=(-2,1,-3)$

$$\overrightarrow{BC}\times\overrightarrow{BD}=\begin{vmatrix}e_1&e_2&e_3\\-4&0&0\\-2&1&-3\end{vmatrix}=(0,-12,-4)$$

$$A_{\Delta ABC}=\frac{1}{2}\cdot\sqrt{144+16}=\frac{1}{2}\cdot\sqrt{160}=\frac{4\sqrt{10}}{2}=2\sqrt{10}$$

c) 

$$V_{ABCD} = \frac{1}{3} \cdot d(A,(BCD)) \, A_{\Delta BCD} \Rightarrow d(A,(BCD)) = \frac{\sqrt{10}}{10}$$

Aducerea la formă canonică a conicelor efectuând izometrii

5/6. $(\mathbb{R}^2, (\mathbb{R}^2, g_0), \varphi)$. Fie conica:

$\Gamma: f(x_1, x_2) = \underline{5x_1^2 + 8x_1x_2 + 5x_2^2} - 18x_1 - 18x_2 + 9 = 0$

a) Să se aducă la o formă canonică, efectuând izometrii.

b) Reprezentare grafică

Sol.:

$$A = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} \quad \tilde{A} = \begin{pmatrix} 5 & 4 & -9 \\ 4 & 5 & -9 \\ -9 & -9 & 9 \end{pmatrix}$$

$$\delta = \det A = 9 > 0$$

$$\Delta = \det \tilde{A} = \begin{vmatrix} 0 & 0 & -9 \\ 4 & 5 & -9 \\ -9 & -9 & 9 \end{vmatrix} = (-9)^2 \cdot (-1) = -81 \neq 0$$

$$\left.\begin{array}{l} \delta \neq 0 \Rightarrow \Gamma \text{ are centru unic} \\ \Delta \neq 0 \Rightarrow \Gamma \text{ este nedegenerată} \\ \delta = \det A = \lambda_1 \lambda_2 > 0 \end{array}\right| \Rightarrow \Gamma \text{ este o elipsă}$$

P_0 - centrul unic

$$\begin{cases} \frac{\partial f}{\partial x_1} = 10x_1 + 8x_2 - 18 = 0 \quad |:2 \\ \frac{\partial f}{\partial x_2} = 8x_1 + 10x_2 - 18 = 0 \quad |:2 \end{cases} \Rightarrow \begin{cases} 5x_1 + 4x_2 = 9 \\ 4x_1 + 5x_2 = 9 \end{cases} \Rightarrow \underline{P_0(1,1)}$$

$$\mathcal{R}=\{O;e_1,e_2\}\xrightarrow[\text{translație}]{\theta}\mathcal{R}'=\{P_0;e_1,e_2\}\xrightarrow[\text{rotație}]{\tau}\mathcal{R}''=\{P_0;e_1',e_2'\}$$

$$\theta: X=X'+X_0 \iff \begin{cases} x_1=x_1'+1 \\ x_2=x_2'+1 \end{cases}$$

$$\theta(\Gamma): \underbrace{5x_1'^2+8x_1'x_2'+5x_2'^2}_{Q(x)}+\underset{=-9}{\frac{\Delta}{\delta}}=0$$

$Q:\mathbb{R}^2\to\mathbb{R}$, aplicăm metoda val. proprii

$$P_A(\lambda)=\det(A-\lambda I_2)=\begin{vmatrix} 5-\lambda & 4 \\ 4 & 5-\lambda \end{vmatrix}=0$$

$$(5-\lambda)^2-4^2=(1-\lambda)(5-\lambda)=0 \Rightarrow \lambda_1=1,\ \lambda_2=5$$

$$V_{\lambda_1}=\{x\in\mathbb{R}^2 \mid AX=X\}$$

$$(A-I_2X)=\begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \begin{pmatrix}4&4\\4&4\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Rightarrow x_1=-x_2$$

$$V_{\lambda_1}=\langle\{(1,-1)\}\rangle$$

$e_1'=\frac{1}{\sqrt{2}}(1,-1)$ versor

$$V_{\lambda_2}=\{x\in\mathbb{R}^2 \mid AX=9X\}$$

$$(A-9I_2)X=\begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \begin{pmatrix}-4&4\\4&-4\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}\Rightarrow x_1=x_2$$

$$V_{\lambda_2}=\langle\{(1,1)\}\rangle$$

$$e_2'=\frac{1}{\sqrt{2}}(1,1)$$

$\tau: X' = R \cdot X''$

$R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \in SO(2)$

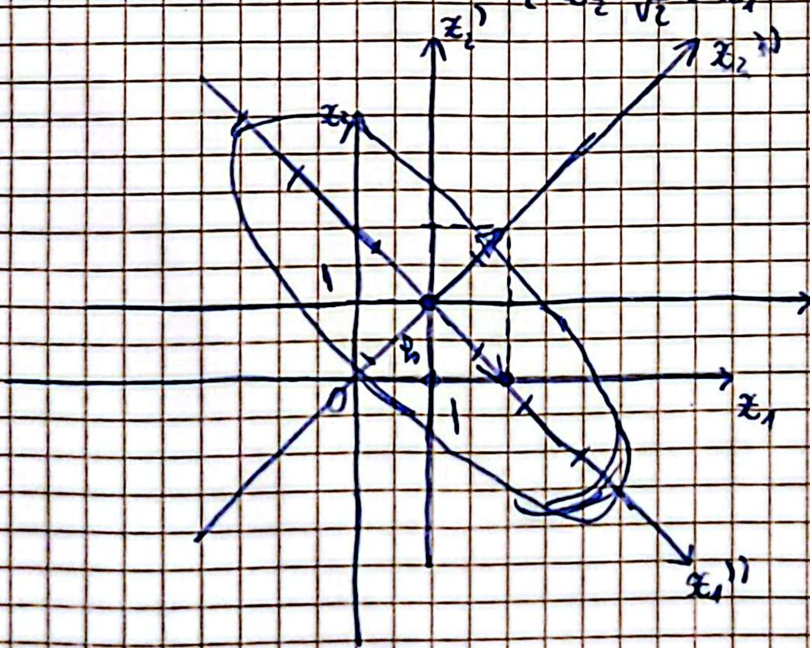
(det R = 1, dacă era −1 schimbam $e_1 \leftrightarrow e_2$)

$\tau(\theta(\Gamma)) = x_1''^2 + 9x_2''^2 - 9 = 0 \Rightarrow$

$\Rightarrow \frac{x_1''^2}{9} + x_2''^2 = 1 \quad \Rightarrow a = 3$

$\tau \circ \theta: X = RX'' + X_0 \Leftrightarrow \begin{cases} x_1 = \frac{1}{\sqrt{2}}(x_1'' + x_2'') + 1 \\ x_2 = \frac{1}{\sqrt{2}}(-x_1'' + x_2'') + 1 \end{cases}$

(d)



$a = 3$

$b = 1$