(a) $\{(a,b,c) \in \mathbb{R}^3 : a^3 = b^3\} = V$

V subspace of $\mathbb{R}^3$?

$a^3 = b^3 \Leftrightarrow a = b \quad \forall\, a,b \in \mathbb{R} \Rightarrow$

$\Rightarrow V = \{(a, a, c) \in \mathbb{R}^3\}$

- $0 = (0,0,0) \in V$. (additive identity)
- Let $x = (x_1, x_1, x_2)$, $y = (y_1, y_1, y_2) \in V$

  $x + y = (x_1 + y_1, x_1 + y_1, x_2 + y_2) \in V \Rightarrow$

  $\Rightarrow$ closed under addition
- Let $x = (x_1, x_1, x_2)$, $a \in \mathbb{R}$.

  $ax = (ax_1, ax_1, ax_2) \in V \Rightarrow$ closed under scalar multiplication.

So, V is a subspace of $\mathbb{R}^3$.

(b) $\{(a,b,c) \in \mathbb{C}^3 : a^3 = b^3\} = V$

V subspace of $\mathbb{C}^3$?

No, it's not closed under addition.

Let ω be a primitive root of 1 ($\omega^3 = 1$).

Let $x = (1, 1, 0) \in V$

$y = (1, \omega, 0) \in V$

$x + y = (2, 1+\omega, 0)$

ω - primitive root of $1 \Rightarrow 1 + \omega + \omega^2 = 0 \Rightarrow$

$\Rightarrow 1 + \omega = -\omega^2$

$$(1+\omega)^3 = (-\omega^2)^3$$
$$= -(\omega^3)^2$$
$$= -1 \neq 8 = 2^3.$$