

Counterexample:

Let $V_1 = \text{span}(1, 0)$, $V_2 = \text{span}(0, 1)$, $V_3 = \text{span}(1, 1)$

$$\dim(V_1) = \dim(V_2) = \dim(V_3) = 1$$

$$V_1 \cap V_2 = \{0\}, \quad V_1 \cap V_3 = \{0\}, \quad V_2 \cap V_3 = \{0\}$$

$$V_1 \cap V_2 \cap V_3 = \{0\} \Rightarrow \dim 0.$$

The proposed formula:

$$\begin{aligned} \dim(V_1 + V_2 + V_3) &= \dim V_1 + \dim V_2 + \dim V_3 - \dim(V_1 \cap V_2) - \\ &\quad - \dim(V_1 \cap V_3) - \dim(V_2 \cap V_3) + \\ &\quad + \dim(V_1 \cap V_2 \cap V_3) = 3. \end{aligned}$$

But $V_1 + V_2 + V_3 = \mathbb{R}^2$, so $\dim(V_1 + V_2 + V_3) = 2$.