

v_1, \dots, v_m and w_1, \dots, w_m lin. independent

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$v_1 + w_1, \dots, v_m + w_m$ lin. independent

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Let $a_1, \dots, a_m \in F$ and suppose $\sum_{i=1}^m a_i (v_i + w_i) = 0$

$$\sum_{i=1}^m a_i (v_i + w_i) = 0 \Leftrightarrow$$

$$\Leftrightarrow \sum_{i=1}^m a_i v_i + \sum_{i=1}^m a_i w_i = 0$$

Assume, $V = \mathbb{R}^1$, $v_1 = 1$, $w_1 = -1$, $a_1 = 1$

$1 \cdot 1 + 1 \cdot -1 = 0 \Rightarrow v_1 + w_1 \dots v_m + w_m$ is NOT

linearly independent.