(a) $\{(x_1, x_2, x_3) \in F^3 : x_1 + 2x_2 + 3x_3 = 0\} = U$

- Additive identity : $0 + 2(0) + 3(0) = 0 \Rightarrow 0 \in U$ $\square$
- Closed under addition

Let $u = (u_1, u_2, u_3) \in U$, $v = (v_1, v_2, v_3) \in U$.

$u_1 + 2u_2 + 3u_3 = 0$

$v_1 + 2v_2 + 3v_3 = 0$

$u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$

$(u_1 + v_1) + 2(u_2 + v_2) + 3(u_3 + v_3) =$

$= (u_1 + 2u_2 + 3u_3) + (v_1 + 2v_2 + 3v_3)$

$= 0 + 0$

$= 0. \Rightarrow u + v \in U$ $\square$

- Closed under scalar multiplication

Let $u = (u_1, u_2, u_3) \in U$, $a \in F$.

$u_1 + 2u_2 + 3u_3 = 0$

$au = (au_1, au_2, au_3)$

$au_1 + 2au_2 + 3au_3 = a(u_1 + 2u_2 + 3u_3) = a0 = 0.$

$\Rightarrow au \in U.$ $\square$

U is a subspace of F^3.

(b) $\{(x_1, x_2, x_3) \in F^3 : x_1 + 2x_2 + 3x_3 = 4\} = U$

- Additive identity:

$0 \notin U \quad (0 + 2(0) + 3(0) \neq 4)$

U is NOT a subspace of F^3.

(c) $\{(x_1, x_2, x_3) \in F^3 : x_1 x_2 x_3 = 0\} = U$

- Additive identity

$0 \in U \quad (000 = 0) \quad \square$

- Closed under addition

Let $u = (1, 0, 0) \in U$, $v = (0, 1, 1) \in U$.

$u + v = (1, 1, 1) \notin U \quad (111 \neq 0)$

U is NOT a subspace of F^3.

(d) $\{(x_1, x_2, x_3) \in F^3 : x_1 = 5x_3\} = U$

- Additive identity: $0 \in U \quad (0 = 5 \cdot 0)$
- Closed under addition:

Let $u = (u_1, u_2, u_3) \in U$, $v = (v_1, v_2, v_3) \in U$

$u_1 = 5u_3$, $v_1 = 5v_3$

$u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$

$u_1 + v_1 = 5u_3 + 5v_3 = 5(u_3 + v_3) \Rightarrow u + v \in U.$ $\square$

- Closed under scalar multiplication

Let $u = (u_1, u_2, u_3) \in U$, $a \in F$

$u_1 = 5u_3$

$au = (au_1, au_2, au_3)$

$au_1 = a(5u_3) = 5(au_3) \Rightarrow au \in U$ $\square$

U is a subspace of F^3.