$P_4(F)$ is spanned by $(1, z, z^2, z^3, z^4)$ which

has len 5.

In a finite-dimensional vector space, the len of every lin. independent list of vectors $\leq$ the length of every spanning list, hence there does NOT exist a list of six polynomials linearly independent in $P_4(F)$.