$(\alpha \beta) \lambda = \alpha (\beta \lambda) \quad \forall \quad \alpha, \beta, \lambda \in \mathbb{C}$

$\alpha = a + bi \quad , \quad \beta = c + di \quad , \quad \lambda = e + fi \quad a, b, c, d, e, f \in \mathbb{R}$

$(\alpha \beta) \lambda = ((a + bi)(c + di))(e + fi)$

$= (\underbrace{(ac - bd)}_{x} + \underbrace{(ad + bc)}_{y}i)(e + fi)$

$= (x + gi)(e + fi)$

$= (xe - gf) + (xf + ge)i$

$= ((ac - bd)e - (ad + bc)f) + ((ac - bd)f + (ad + bc)e)i$

$= (ace - bde - adf - bcf) + (acf - bdf + ade + bce)i \quad (1)$

$\alpha (\beta \lambda) = (a + bi)((c + di)(e + fi))$

$= (a + bi)((\underbrace{ce - df}_{g}) + (\underbrace{cf + de}_{h})i)$

$= (a + bi)(g + hi)$

$= (ag - bh) + (ah + bg)i$

$= (a(ce - df) - b(cf + de)) + (a(cf + de) + b(ce - df))i$

$= (ace - adf - bcf - bde) + (acf + ade + bce - bdf)i \quad (2)$

$(1), (2) \Rightarrow (\alpha \beta) \lambda = \alpha (\beta \lambda)$ ☐