v_1, v_2, v_3, v_4 basis of V

$v_1 + v_2,\ v_2 + v_3,\ v_3 + v_4,\ v_4$

Linear independence: Let $a_1, a_2, a_3, a_4 \in F$

Consider $a_1(v_1+v_2) + a_2(v_2+v_3) + a_3(v_3+v_4) + a_4 v_4 = 0$

$\Longleftrightarrow a_1 v_1 + a_1 v_2 + a_2 v_2 + a_2 v_3 + a_3 v_3 + a_3 v_4 + a_4 v_4 = 0$

$\Longleftrightarrow a_1 v_1 + (a_1 + a_2) v_2 + (a_2 + a_3) v_3 + (a_3 + a_4) v_4 = 0$

Since v_1, v_2, v_3, v_4 is a basis of V and thus lin. independent, this is true $\Longleftrightarrow a_1 = a_2 = a_3 = a_4 = 0$.

$\Longrightarrow v_1 + v_2,\ v_2 + v_3,\ v_3 + v_4,\ v_4$ are lin. independent

Spanning: Let $v \in V$, $a_1, a_2, a_3, a_4 \in F$. Consider:

$v = a_1(v_1+v_2) + a_2(v_2+v_3) + a_3(v_3+v_4) + a_4 v_4$

$\Longleftrightarrow v = a_1 v_1 + (a_1 + a_2) v + (a_2 + a_3) v_3 + (a_3 + a_4) v_4$

Since v_1, v_2, v_3, v_4 is a basis of V and thus spans V this is true $\Longrightarrow \exists\, a_1, a_2, a_3, a_4 \in F$ s.t. we can define any arbitrary $v \in V$ as a lin. combination of $v_1 + v_2,\ v_2 + v_3,\ v_3 + v_4,\ v_4$. So these vectors span V.