$V = C([0,1], \mathbb{R})$

V is infinite-dimensional

Consider $S = \{1, z, z^2, \dots : z \in [0,1]\} \subset V$

Take a finite subset of S: $S_n = \{1, z, z^2, \dots, z^n\}$
We want to show S_n is linear independent in V.

$$P(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_n z^n = 0 \quad \forall z \in [0,1]$$

This equation defines a polynomial with deg n.

We know a polynomial with deg n can have at most n roots, so the only way for $P(z) = 0 \; \forall z \in [0,1]$ is if $a_0 = a_1 = \dots = a_n = 0$.

This shows that S_n is linearly independent $\forall n \Rightarrow$
$\Rightarrow \nexists$ a finite list which can span $V \Rightarrow$
$\Rightarrow V$ is infinite-dimensional.