

$$U = \{(x, y, x+y, x-y, 2x) \in F^5 : x, y \in F\}$$

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$$w_1, w_2, w_3 \in F^5 =? \text{ n.t. } w_1, w_2, w_3 \neq \{0\}, F^5 = U \oplus w_1 \oplus w_2 \oplus w_3$$


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$$\text{Let } w_1 = \{(0, 0, a, 0, 0) \in F^5 : a \in F\}$$

$$w_2 = \{(0, 0, 0, b, 0) \in F^5 : b \in F\}$$

$$w_3 = \{(0, 0, 0, 0, c) \in F^5 : c \in F\}$$

Consider an arbitrary element  $(p, q, r, s, t) \in F^5$ .

$$(p, q, r, s, t) = (p, q, p+q, p-q, 2p) + \left. \begin{aligned} &(0, 0, r-p-q, 0, 0) + \\ &(0, 0, 0, s-p+q, 0) + \\ &(0, 0, 0, 0, t-2p) \end{aligned} \right\} \in U + w_1 + w_2 + w_3$$

$$\Rightarrow F^5 = U + w_1 + w_2 + w_3$$

$$\text{Consider } w_1 = (x, y, x+y, x-y, 2x) \in U$$

$$w_2 = (0, 0, a, 0, 0) \in w_1$$

$$w_3 = (0, 0, 0, b, 0) \in w_2$$

$$w_4 = (0, 0, 0, 0, c) \in w_3$$

$$v_1 + v_2 + v_3 + v_4 = 0 \Leftrightarrow x=y=a=b=c=0.$$

$\Rightarrow$  The only way of creating 0 is to take each element = 0  $\Rightarrow U \oplus W_1 \oplus W_2 \oplus W_3 = F^5$ .