V_1, V_2 subspaces of V

$V_1 \cap V_2$ subspace of V

Additive identity (1)

V_1 Subspace of $V \Rightarrow 0 \in V_1$
V_2 subspace of $V \Rightarrow 0 \in V_2$
$\Rightarrow 0 \in V_1 \cap V_2$

Closure under addition (2)

Assume $x, y \in V_1 \cap V_2$.

$x, y \in V_1 \cap V_2 \Rightarrow x, y \in V_1$ and $x, y \in V_2$.

V_1 subspace of $V \Rightarrow x+y \in V_1$
V_2 subspace of $V \Rightarrow x+y \in V_2$
$\Rightarrow$

$\Rightarrow x+y \in V_1 \cap V_2$.

Closure under scalar multiplication (3)

Assume $x \in V_1 \cap V_2$, $\lambda \in F$

$x \in V_1 \cap V_2 \Rightarrow x \in V_1$ and $x \in V_2$

V_1 subspace of $V \Rightarrow \lambda x \in V_1$
V_2 subspace of $V \Rightarrow \lambda x \in V_2$
$\Rightarrow$

$\Rightarrow \forall x \in V_1 \cap V_2$

(1),(2),(3) $\Rightarrow$ if V_1, V_2 subspaces of $V \Rightarrow V_1 \cap V_2$ is a subspace of V.

□