$(p_0, p_1, p_2, p_3) \in \mathcal{P}_3(F)$. None have deg 2.

(p_0, p_1, p_2, p_3) not a basis ?

Let $p_0 = 1$, $p_1 = z$, $p_2 = z^3$, $p_3 = z^2 + z^3$

None have deg 2. $(0, 1, 3, 3)$

Lin. independence: Consider $a_0 p_0 + a_1 p_1 + a_2 p_2 + a_3 p_3 = 0$

$$\iff a_0 + a_1 z + a_2 z^3 + a_3(z^2 + z^3) = 0$$

$$\iff a_0 + a_1 z + a_2 z^3 + a_3 z^2 + a_3 z^3 = 0$$

$$\iff a_0 + a_1 z + a_3 z^2 + (a_2 + a_3) z^3 = 0$$

$$\iff a_0 = a_1 = a_3 = 0 \iff a_2 = 0$$

$\Rightarrow p_0, p_1, p_2, p_3$ lin. independent (1)

Since $(1, z, z^2, t^3)$ has length 4 and spans $\mathcal{P}_3(F)$ and (p_0, p_1, p_2, p_3) has length 4 and is lin. independent in $\mathcal{P}_3(F) \Rightarrow (p_0, p_1, p_2, p_3)$ is also spanning $\mathcal{P}_3(F)$ (2)

(1), (2) $\Rightarrow$ p_0, p_1, p_2, p_3 is a basis in $\mathcal{P}_3(F)$ so the statement is false.