

$\forall z \in \mathbb{C} \quad \exists \text{ unique } \beta \in \mathbb{C} \text{ s.t. } z + \beta = 0$

Suppose  $z = a + bi$  where  $a, b \in \mathbb{R}$

Define  $\beta = -a - bi$

$$\begin{aligned} z + \beta &= (a + bi) + (-a - bi) = (a - a) + (b - b)i \\ &= 0 + 0i = 0. \quad (\text{Existence}) \end{aligned}$$

Suppose  $\exists \lambda \in \mathbb{C}, \lambda \neq \beta$  s.t.  $z + \lambda = 0$ .

$$\lambda = -z \Rightarrow \lambda = -a - bi \Rightarrow \lambda = \beta \quad (\text{Uniqueness}) \quad \square$$