$V_1, \dots, V_m$ fin. dimensional $\subseteq V$

$V_1 + \dots + V_m$ fin. dimensional and $\dim(V_1 + \dots + V_m) \leq \dim V_1 + \dots + \dim V_m$

Let B_i be a basis of V_i for each $i \in \{1, \dots, m\}$

Each B_i has len. $\dim V_i$.

Let $x \in V_1 + \dots + V_m \Rightarrow x = v_1 + \dots + v_m : v_i \in V_i$.

$$x = \sum_{i=1}^{m} \sum_{j=1}^{\dim V_j} a_j b_j$$

x can be written as a linear combination of vectors from $B = \bigcup_{i=1}^{m} B_i \Rightarrow V_1 + \dots + V_m = \operatorname{span}(B)$

$\Rightarrow V_1 + \dots + V_m$ is spanned by a finite set $\Rightarrow$ is finite dimensional.

Since B spans $V_1 + \dots + V_m \Rightarrow$

$$\left.\begin{array}{l} \Rightarrow \dim(V_1 + \dots + V_m) \leq |B| \\ B = \bigcup_{i=1}^{m} B_i \Rightarrow |B| \leq \sum_{i=1}^{m} |B_i| \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} \Rightarrow \dim(V_1 + \dots + V_m) \leq \displaystyle\sum_{i=1}^{m} |B_i| \\ B_i \text{ is a basis of } V_i \Rightarrow |B_i| = \dim V_i \end{array}\right\} \Rightarrow$$

$$\Rightarrow \dim(V_1 + \dots + V_m) \leq \dim V_1 + \dots + \dim V_m$$