$v_1, v_2, \dots, v_m$ linearly independent

$5v_1 - 4v_2, v_2, \dots, v_m$ linearly independent

Let $a_1 \dots a_m \in F$:

$a_1(5v_1 - 4v_2) + a_2 v_2 + \dots + a_m v_m = 0$

$5a_1 v_1 - 4a_1 v_2 + a_2 v_2 + \dots + a_m v_m = 0$

$5a_1 v_1 + (-4a_1 + a_2) v_2 + \dots + a_m v_m = 0$

$v_1, \dots, v_m$ lin. independent $\Rightarrow a_1 v_1 + \dots + a_m v_m = 0$

$\Leftrightarrow a_i = 0$ for $i \in \{1 \dots m\}$

$$\Rightarrow \begin{cases} 5a_1 = 0 \\ -4a_1 + a_2 = 0 \\ a_{3 \dots m} = 0 \end{cases} \Rightarrow a_1 = a_2 = \dots = a_m = 0.$$

$\Rightarrow$ if $v_1, \dots, v_m$ lin. independent $\Rightarrow 5v_1 - 4v_2, v_2 \dots v_m$ is lin. independent