$\forall \alpha \in \mathbb{C}$, $\alpha \neq 0$ $\exists$ unique $\beta \in \mathbb{C}$ s.t. $\alpha\beta = 1$

Suppose $\alpha = a+bi$, $a,b \in \mathbb{R}$.

Define $\beta = \dfrac{a-bi}{a^2+b^2}$

$$\alpha\beta = (a+bi)\left(\frac{a-bi}{a^2+b^2}\right)$$
$$= \frac{(a+bi)(a-bi)}{a^2+b^2} = \frac{a^2+b^2}{a^2+b^2} = 1.$$

So $\exists$ $\beta \in \mathbb{C}$ s.t. $\alpha\beta = 1$.

Suppose $\exists$ $\lambda_1, \lambda_2 \in \mathbb{C}$ s.t. $\alpha\lambda_1 = 1$ and $\alpha\lambda_2 = 1$.

$\Rightarrow \alpha\lambda_1 = \alpha\lambda_2 \Rightarrow \alpha(\lambda_1 - \lambda_2) = 0$

Since $\alpha \neq 0$, $\alpha(\lambda_1 - \lambda_2) = 0 \Rightarrow \lambda_1 - \lambda_2 = 0 \Rightarrow$
$\underline{\lambda_1 = \lambda_2}$

Hence the inverse is unique.

$\square$