$v_1, \dots, v_m$ lin. independent in V, $w \in V$

$\vdash$

$\dim \operatorname{span}(v_1 + w, \dots, v_m + w) \geq m-1$

$\Vdash$

Let $L = (v_1 + w) - (v_m + w), (v_2 + w) - (v_m + w), \dots$

$\dots, (v_{m-1} + w) - (v_m + w)$

$= v_1 - v_m, v_2 - v_m, \dots, v_{m-1} - v_m$

$L \in \operatorname{span}(v_1 + w, \dots, v_m + w)$

Consider $a_1, \dots, a_{m-1} \in F$ and a linear combination of L:

$a_1(v_1 - v_m) + a_2(v_2 - v_m) + \dots + a_{m-1}(v_{m-1} - v_m) = 0$

$a_1 v_1 - a_1 v_m + a_2 v_2 - a_2 v_m + \dots + a_{m-1} v_{m-1} - a_{m-1} v_m = 0$

$a_1 v_1 + a_2 v_2 + \dots + a_{m-1} v_{m-1} - v_m(a_1 + \dots + a_{m-1}) = 0$

Since $v_1, \dots, v_m$ are lin. independent this is true only for $a_1 = a_2 = \dots = a_{m-1} = 0$. $\Rightarrow$ elements in L are linearly independent. $\Rightarrow$

Since $L \in \operatorname{span}(v_1 + w, \dots, v_m + w)$ and L is a lin. independent list of $m-1$ vectors $\Rightarrow$

$\Rightarrow \dim \operatorname{span}(v_1 + w, \ldots, v_m + w) \geq m-1.$