

$$(x+y) + z = x + (y+z) \quad (\forall) x, y, z \in F^n$$

$$\begin{aligned} (x+y) + z &= ((x_1, \dots, x_m) + (y_1, \dots, y_m)) + (z_1, \dots, z_m) \\ &= (x_1 + y_1, \dots, x_m + y_m) + (z_1, \dots, z_m) \\ &= (x_1 + y_1 + z_1, \dots, x_m + y_m + z_m) \quad (1) \end{aligned}$$

$$\begin{aligned} x + (y+z) &= (x_1, \dots, x_m) + ((y_1, \dots, y_m) + (z_1, \dots, z_m)) \\ &= (x_1, \dots, x_m) + (y_1 + z_1, \dots, y_m + z_m) \\ &= (x_1 + y_1 + z_1, \dots, x_m + y_m + z_m) \quad (2) \end{aligned}$$

$$(1), (2) \Rightarrow (x+y) + z = x + (y+z) \quad (\forall) x, y, z \in F^n$$

□