$U \subseteq V \quad , \quad U \neq V$

$n = \dim V \quad , \quad m = \dim U$

---

$\exists\ U_1, \dots U_{n-m} : \bigcap_{i=1}^{n-m} U_i = U \quad , \dim U_i = n-1$

---

Let $u_1, \dots, u_m$ be a basis of $U$.

Extend $u_1, \dots, u_m$ with $\underbrace{v_{m+1}, \dots, v_n}_{n-m}$ to a basis of $V$.

For each $k \in \{m+1, \dots, n\}$ consider:

$H_k = \text{span}(\{u_1, \dots, u_m, v_{m+1}, \dots, v_n\} \setminus \{v_k\})$

$\dim H_k = n-1$ and $U \subseteq H_k$.

Let $x \in \bigcap_{m+1}^{n} H_k$. Write $x$ as a lin. comb. of the basis:

$$x = \sum_{i=1}^{m} a_i u_i + \sum_{i=m+1}^{n} b_i v_i$$

Since each $H_k$ does not contain $v_k$ and $x \in \bigcap_{m+1}^{n} H_k$

$\Rightarrow$ each $b_i$ must be $0$.

$$\Rightarrow x = \sum_{i=1}^{m} a_i u_i \in U$$

$$\Rightarrow \bigcap_{k=m+1}^{n} H_k = U, \quad \dim H_k = n-1.$$