$\text{span}(v_1, v_2, v_3, v_4) = V$

$\text{span}(v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4) = V$?

Let $w_1 = v_1 - v_2$

$w_2 = v_2 - v_3$

$w_3 = v_3 - v_4$

$w_4 = v_4$

Eeach w_1, w_2, w_3, w_4 is a linear combination of $(v_1, v_2, v_3, v_4) \Rightarrow \text{span}(w_1, w_2, w_3, w_4) \subseteq \text{span}(v_1, v_2, v_3, v_4)$ (1)

$$\left.\begin{aligned}
v_4 &= w_4 \\
v_3 &= w_3 + v_4 = w_3 + w_4 \\
v_2 &= w_2 + v_3 = w_2 + w_3 + w_4 \\
v_1 &= w_1 + v_2 = w_1 + w_2 + w_3 + w_4
\end{aligned}\right\} \Rightarrow$$

$\Rightarrow$ Each v_1, v_2, v_3, v_4 is a linear combination of $(w_1, w_2, w_3, w_4) \Rightarrow \text{span}(v_1, v_2, v_3, v_4) \subseteq \text{span}(w_1, w_2, w_3, w_4)$ (2)

(1),(2) $\Rightarrow \text{span}(v_1, v_2, v_3, v_4) = \text{span}(w_1, w_2, w_3, w_4) = V$