

$$\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta \quad (\forall) \lambda, \alpha, \beta \in \mathbb{C}$$

Suppose  $\alpha = a+bi$ ,  $\beta = c+di$ ,  $\lambda = e+fi$  where  $a, b, c, d, e, f \in \mathbb{R}$

$$\begin{aligned} \lambda(\alpha + \beta) &= (e+fi) \left( \underbrace{(a+c)}_x + \underbrace{(b+d)}_y i \right) \\ &= (e+fi)(x + yi) \\ &= (ex - fy) + (ey + fx)i \\ &= (e(a+c) - f(b+d)) + (e(b+d) + f(a+c))i \\ &= (ea + ec - fb - fd) + (eb + ed + fa + fc)i \quad (1) \end{aligned}$$

$$\begin{aligned} \lambda\alpha + \lambda\beta &= (e+fi)(a+bi) + (e+fi)(c+di) \\ &= ((ea - fb) + (eb + fa)i) + ((ec - fd) + (ed + fc)i) \\ &= (ea - fb + ec - fd) + (eb + fa + ed + fc)i \quad (2) \end{aligned}$$

$$(1), (2) \Rightarrow \lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$$

□