$$T \in \mathcal{L}(F^n, F^m)$$

$$\exists A_{j,k} \in F, \ j = 1..m, \ k = 1..n :$$

$$T(x_1, \dots, x_n) = (A_{1,1}x_1 + \dots + A_{1,n}x_n, \dots, A_{m,1}x_1 + \dots + A_{m,n}x_n)$$

$$\forall (x_1, \dots, x_n) \in F^n$$

Let $(e_1, \dots, e_n)$ be the standard basis of F^n

$$T(e_k) = (A_{1,k}, A_{2,k}, \dots A_{m,k}) \in F^m \quad (k = 1, \dots, n)$$

$$\forall x = (x_1, \dots, x_n) \in F^n : \ x = \sum_{k=1}^{n} x_k e_k$$

$$T(x) = T\left(\sum_{k=1}^{n} x_k e_k\right)$$

$$= \sum_{k=1}^{n} T(x_k e_k) \quad \text{(Additivity)}$$

$$= \sum_{k=1}^{n} x_k T e_k \quad \text{(Homogeneity)}$$

$$= \sum_{k=1}^{n} x_k (A_{1,k}, A_{2,k}, \dots, A_{m,k})$$

$$= (A_{1,1}x_1 + \dots + A_{1,n}x_n, \dots, A_{m,1}x_1 + \dots + A_{m,n}x_n)$$

$$\Rightarrow \exists A_{j,k} \in F : T(x_1, \dots, x_n) = (A_{1,1}x_1 + \dots + A_{1,n}x_n, \dots, A_{m,1}x_1 + \dots + A_{m,n}x_n)$$