$$u(x) + dx^2 = a + bx + (c-d)x^2 + dx^3 + ex^4 + dx^2$$

$$= a + bx + cx^2 + dx^3 + ex^4$$

$$= g(x) \Rightarrow$$

$$\Rightarrow U + W = \mathcal{P}_4(\mathbb{R}) \quad (1)$$

- We show that $U \cap W = \{0\}$

$$\dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W)$$

$$\left.\begin{array}{l} \dim(U+W) = \dim \mathcal{P}_4(\mathbb{R}) = 5 \\ \dim(U) = 4 \\ \dim(W) = 1 \end{array}\right\} \Rightarrow$$

$$\dim(U) + \dim(W) - \dim(U \cap W) = \dim(U) + \dim(W)$$

$$\Rightarrow \dim(U \cap W) = 0$$

$$\Rightarrow U \cap W = \{0\} \quad (2)$$

$$(1), (2) \Rightarrow U \oplus W = \mathcal{P}_4(\mathbb{R})$$