Associative: Let T_1, T_2, T_3 be linear maps s.t. T_3 maps into the domain of T_2 and T_2 maps into the domain of T_1.

$$\begin{aligned}((T_1T_2)T_3)(v) &= (T_1T_2)(T_3 v)\\ &= T_1(T_2(T_3 v))\\ &= T_1(T_2T_3(v))\\ &= (T_1(T_2T_3))(v)\end{aligned}$$

Identity: Let $I_V : V \to V$, $I_W : W \to W$, $T \in \mathcal{L}(V, W)$

$I_V(v) = v \;\forall\, v \in V$, $I_W(w) = w \;\forall\, w \in W$.

$$\left.\begin{aligned}(TI_V)(v) &= T(I_V(v)) = Tv\\ (I_WT)(v) &= I_W(T(v)) = Tv\end{aligned}\right\} \Rightarrow$$

$\Rightarrow TI_V = I_WT = T$

Distributive: Let $S_1, S_2 : V \to W$, $T: U \to V$

$$\begin{aligned}\forall\, u \in U : ((S_1+S_2)T)(u) &= (S_1+S_2)(Tu)\\ &= S_1(Tu) + S_2(Tu)\\ &= (S_1T + S_2T)(u)\end{aligned}$$

Let $S: V \to W$, $T_1, T_2 : U \to V$

$$\begin{aligned}\forall\, u \in U : S(T_1+T_2)(u) &= S(T_1u + T_2u)\\ &= (ST_1)(u) + (ST_2)(u)\\ &= (ST_1 + ST_2)(u)\end{aligned}$$