

$$\dim(X+Y) = \dim X + \dim Y - \dim(X \cap Y)$$

$$\begin{aligned} \dim(V_1+V_2+V_3) &= \dim((V_1+V_2)+V_3) \\ &= \dim(V_1+V_2) + \dim V_3 - \dim((V_1+V_2) \cap V_3) \\ \dim(V_1+V_2+V_3) &= \dim((V_2+V_3)+V_1) \\ &= \dim(V_2+V_3) + \dim V_1 - \dim((V_2+V_3) \cap V_1) \\ \dim(V_1+V_2+V_3) &= \dim((V_1+V_3)+V_2) \\ &= \dim(V_1+V_3) + \dim V_2 - \dim((V_1+V_3) \cap V_2) \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \Rightarrow$$

$$\begin{aligned} \Rightarrow 3 \dim(V_1+V_2+V_3) &= \dim V_1 + \dim V_2 + \dim V_3 + \\ &+ \dim(V_1+V_2) + \dim(V_2+V_3) + \dim(V_1+V_3) \\ &- (\dim((V_1+V_2) \cap V_3) + \dim((V_2+V_3) \cap V_1) + \dim((V_1+V_3) \cap V_2)) \\ &= \dim V_1 + \dim V_2 + \dim V_3 + \dim V_1 + \dim V_2 - \dim(V_1 \cap V_2) \\ &+ \dim V_2 + \dim V_3 - \dim(V_2 \cap V_3) + \dim V_1 + \dim V_3 \\ &- \dim(V_1 \cap V_3) - (\dim((V_1+V_2) \cap V_3) + \dim((V_2+V_3) \cap V_1) \\ &+ \dim((V_1+V_3) \cap V_2)) \\ &= 3(\dim V_1 + \dim V_2 + \dim V_3) - \\ &- (\dim(V_1 \cap V_2) + \dim(V_2 \cap V_3) + \dim(V_1 \cap V_3)) - \end{aligned}$$

$$- (\dim((V_1 + V_2) \cap V_3) + \dim((V_2 + V_3) \cap V_1) + \dim((V_1 + V_3) \cap V_2))$$

$$\Rightarrow \dim(V_1 + V_2 + V_3) = \dim V_1 + \dim V_2 + \dim V_3 -$$

$$- \frac{\dim(V_1 \cap V_2) + \dim(V_2 \cap V_3) + \dim(V_1 \cap V_3)}{3}$$

$$- \frac{\dim((V_1 + V_2) \cap V_3) + \dim((V_2 + V_3) \cap V_1) + \dim((V_1 + V_3) \cap V_2)}{3}$$