

V_1, V_2, V_3 subspaces of V

$$(V_1 + V_2) + V_3 = \{(\alpha_1 + \alpha_2) + \alpha_3 : \alpha_1 \in V_1, \alpha_2 \in V_2, \alpha_3 \in V_3\}$$

$$V_1 + (V_2 + V_3) = \{\alpha_1 + (\alpha_2 + \alpha_3) : \alpha_1 \in V_1, \alpha_2 \in V_2, \alpha_3 \in V_3\}$$

Due to associativity on V :

$$(\alpha_1 + \alpha_2) + \alpha_3 = \alpha_1 + (\alpha_2 + \alpha_3) = \alpha_1 + \alpha_2 + \alpha_3 \Rightarrow$$

$$\Rightarrow (V_1 + V_2) + V_3 = V_1 + (V_2 + V_3) = \left\{ \alpha_1 + \alpha_2 + \alpha_3 : \begin{array}{l} \alpha_1 \in V_1, \\ \alpha_2 \in V_2, \\ \alpha_3 \in V_3 \end{array} \right\}$$