

$$0v = 0 \quad \forall v \in V$$

scalar

additive inverse of v

We'll show that if this condition holds along all other vector space axioms hold (except the additive inverse one), then additive inverses exist for all $v \in V$.

Consider $v \in V$, $(-1) \in F$, the additive inverse of $1 \in F$.

$$\begin{aligned} v + (-1)v &= 1v + (-1)v \\ &= (1 + (-1))v \\ &= 0v \\ &= 0 \end{aligned}$$

$v + (-1)v = 0$, so $(-1)v$ is the additive inverse of v .

So if $0v = 0$ holds, then additive inverses exist in V .