$D = \{ f \in C[0,1] : \int_0^1 f = b \}$, $b \in \mathbb{R}$.

D is a subspace of $\mathbb{R}^{[0,1]} \iff b = 0$

Assume D is a subspace of $\mathbb{R}^{[0,1]} \Rightarrow$

$\Rightarrow \exists f(x) \in D$ s.t. $f(x) = 0 \quad \forall x \in [0,1]$ (add. identity)

$$\int_0^1 f(x) = b = \int_0^1 0 = 0 \Rightarrow b = 0$$

So, if D is a subspace of $\mathbb{R}^{[0,1]} \Rightarrow b = 0$ (1)

Assume $b = 0 \Rightarrow D = \{ f \in C[0,1] : \int_0^1 f = 0 \}$

Let $0(x) \in D$ s.t. $0(x) = 0 \quad \forall x \in [0,1]$

$$\int_0^1 0(x) = \int_0^1 0 = 0 \Rightarrow 0 \in D.$$ (additive identity)

Let $f, g \in D \Rightarrow \int_0^1 f = 0, \int_0^1 g = 0.$

$$\begin{aligned}
\int_0^1 (f+g)(x) &= \int_0^1 f(x) + g(x) \\
&= \int_0^1 f(x) + \int_0^1 g(x) \\
&= 0 + 0 \\
&= 0 \text{ (closed under addition)}
\end{aligned}$$

$\Rightarrow (f+g) \in D.$

Let $f \in D$, $a \in \mathbb{R} \Rightarrow \int_0^1 f = 0.$

$$\int_0^1 (af)(x) = \int_0^1 a f(x)$$
$$= a \int_0^1 f(x)$$
$$= a \cdot 0$$
$$= 0$$

$\Rightarrow af \in D.$ (Closed under scalar multiplication)

So, if $b=0 \Rightarrow D$ is a subspace of $\mathbb{R}^{[0,1]}$ (2)

(1), (2) $\Rightarrow D$ is a subspace of $\mathbb{R}^{[0,1]} \Leftrightarrow b=0.$