$\nexists \lambda \in \mathbb{C}$ s.t. $\lambda(2-3i, 5+4i, -6+7i) = (12-5i, 7+22i, -32-9i)$

$\lambda = a+bi$, $a, b \in \mathbb{R}$

$$(a+bi)(2-3i) = 12-5i \Leftrightarrow (2a-3b)+(-3a+2b)i = 12-5i$$
$$\Leftrightarrow \begin{cases} 2a-3b = 12 \\ -3a+2b = -5 \end{cases}$$

$$(a+bi)(5+4i) = 7+22i \Leftrightarrow (5a-4b)+(4a+5b)i = 7+22i$$
$$\Leftrightarrow \begin{cases} 5a-4b = 7 \\ 4a+5b = 22 \end{cases}$$

$$(a+bi)(-6+7i) = -32-9i \Leftrightarrow (-6a-7b)+(7a-6b)i = -32-9i$$
$$\Leftrightarrow \begin{cases} -6a-7b = -32 \\ 7a-6b = -9 \end{cases}$$

$$A = \begin{bmatrix} 2 & -3 \\ -3 & 2 \\ 5 & -4 \\ 4 & 5 \\ -6 & -7 \\ 7 & -6 \end{bmatrix}$$

$(2,-3)$, $(-3,2)$ are linearly independent $\Rightarrow \operatorname{rank}(A) = 2$

$$[A|b] = \left[\begin{array}{cc|c} 2 & -3 & 12 \\ -3 & 2 & -5 \\ 5 & -4 & 7 \\ 4 & 5 & 22 \\ -6 & -7 & -32 \\ 7 & -6 & -9 \end{array}\right]$$

$$\begin{vmatrix} 2 & -3 & 12 \\ -3 & 2 & -5 \\ 5 & -4 & 7 \end{vmatrix} = 24 \neq 0$$

$\Rightarrow \operatorname{rank}([A|b]) = 3.$

$\text{rank}(A) \neq \text{rank}([A|b])$ so by Rouché Capelli the system is inconsistent.

□