

$\mathbb{R} \cup \{\infty, -\infty\}$  vector space over  $\mathbb{R}$ ?

Consider any  $t \in \mathbb{R}$ .

$$\left. \begin{aligned} (\infty + (-\infty)) + t &= 0 + t = t. \\ \infty + ((-\infty) + \infty) &= \infty + -\infty = 0. \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow (\infty + (-\infty)) + t \neq \infty + ((-\infty) + \infty)$$

So addition over  $\mathbb{R} \cup \{\infty, -\infty\}$  is not associative,  
thus  $\mathbb{R} \cup \{\infty, -\infty\}$  is not a vector space over  $\mathbb{R}$ .