Find two distinct square roots of i

Trying to find $z \in \mathbb{C}$ s.t. $z^2 = i$. Let $z = a + bi$ where $a, b \in \mathbb{R}$

$$(a+bi)^2 = i = 0 + 1i$$
$$a^2 - b^2 + 2abi = 0 + 1i \iff \begin{cases} a^2 - b^2 = 0 \\ 2ab = 1 \end{cases} \iff \begin{cases} a^2 = b^2 \\ ab = \frac{1}{2} \end{cases} \iff \begin{cases} a = \pm b \\ ab = \frac{1}{2} \end{cases}$$

1. $\begin{cases} a = b \\ ab = \frac{1}{2} \end{cases} \iff \begin{cases} a = b \\ b^2 = \frac{1}{2} \end{cases} \iff \begin{cases} a = b \\ b = \pm \frac{1}{\sqrt{2}} \end{cases} \implies \begin{cases} z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \\ z_2 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i \end{cases}$

2. $\begin{cases} a = -b \\ ab = \frac{1}{2} \end{cases} \iff \begin{cases} a = -b \\ -b^2 = \frac{1}{2} \end{cases}$ → Not real