For $\forall k \in \{1, \dots, m\}$: $w_k = v_1 + \dots + v_k$

$\operatorname{span}(v_1 \dots, v_m) = \operatorname{span}(w_1, \dots, w_m)$

$$\left.\begin{cases} w_1 = v_1 \\ w_k = v_1 + \dots + v_k \quad , \forall k \in \{1, \dots, m\} \end{cases}\right\} \Rightarrow$$

$\Rightarrow \forall k \in \{1, \dots, m\}$: w_k is a linear combination of $(v_1, \dots, v_k) \Rightarrow \operatorname{span}(w_1 \dots w_m) \subseteq \operatorname{span}(v_1, \dots v_m)$ (1)

$$\begin{cases} v_1 = w_1 \\ v_k = w_k - v_1 - \dots - v_{k-1} = w_k - (v_1 + \dots + v_{k-1}) = w_k - w_{k-1} \end{cases}$$

$\Rightarrow$ we can define : v_k $\forall k \in \{1, \dots, m\}$ as a linear combination of $(w_1 \dots w_k) \Rightarrow$

$\Rightarrow \operatorname{span}(v_1 \dots v_m) \subseteq \operatorname{span}(w_1 \dots w_m)$ (2)

(1), (2) $\Rightarrow \operatorname{span}(v_1, \dots, v_m) = \operatorname{span}(w_1, \dots, w_m)$