$v_1 = (2, 3, 1)$

$v_2 = (1, -1, 2)$

$v_3 = (7, 3, c)$

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v_1, v_2, v_3 linearly dependent $\iff c = 8$

$\Rightarrow)$

Assume v_1, v_2, v_3 linearly dependent $\Rightarrow$

by the lin. dependence lemma : $\exists\, k \in \{1, 2, 3\}$:

$v_k \in \operatorname{span}(v_1, \dots, v_{k-1})$

Take v_1, v_2 : $a v_1 + b v_2 = 0 \iff$

$$\begin{cases} 2a + b = 0 \\ 3a - b = 0 \\ a + 2b = 0 \end{cases} \iff \begin{cases} 2a + b = 0 \\ b = 3a \\ a + 2b = 0 \end{cases} \iff \begin{cases} 5a = 0 \\ b = 3a \end{cases} \iff a = b = 0 \Rightarrow$$

$\Rightarrow v_1, v_2$ are linearly independent $\Rightarrow$

$\Rightarrow v_3 \in \operatorname{span}(v_1, v_2) \Rightarrow \exists\, \alpha, \beta \in F : v_3 = \alpha v_1 + \beta v_2$

$$\begin{cases} 2\alpha + \beta = 7 \\ 3\alpha - \beta = 3 \\ \alpha + 2\beta = c \end{cases} \iff \begin{cases} \beta = 7 - 2\alpha \\ 3\alpha - \beta = 3 \\ \alpha + 2\beta = c \end{cases} \iff \begin{cases} \beta = 7 - 2\alpha \\ 5\alpha = 10 \\ \alpha + 2\beta = c \end{cases}$$

$$\iff \begin{cases} \beta = 3 \\ \alpha = 2 \\ \alpha + 2\beta = c \end{cases} \iff \underline{c = 8}$$

So, v_1, v_2, v_3 lin. independent $\Rightarrow c \neq 8$ (1)

($\Leftarrow$) Assume $c = 8 \Rightarrow v_3 = (7, 3, 8)$

$(7, 3, 8) = 2(2, 3, 1) + 3(1, -1, 2) \Rightarrow$

$\Rightarrow v_3 = 2v_1 + 3v_2$

So, $c = 8 \Rightarrow v_1, v_2, v_3$ linearly dependent. (2)

(1), (2) $\Rightarrow v_1, v_2, v_3$ linearly dependent $\Leftrightarrow c = 8$.