$\mathcal{L}(V,W)$ is a vector space

Commutativity: Let $T_1, T_2 \in \mathcal{L}(V,W)$, $v \in V$

$$
\begin{aligned}
(T_1+T_2)(v) &= T_1 v + T_2 v \\
&= T_2 v + T_1 v \\
&= (T_2+T_1)(v)
\end{aligned}
$$

Associativity: Let $T_1, T_2, T_3 \in \mathcal{L}(V,W)$, $v \in V$.

$$
\begin{aligned}
(T_1+(T_2+T_3))(v) &= T_1 v + (T_2 v + T_3 v) \\
&= (T_1 v + T_2 v) + T_3 v \\
&= ((T_1+T_2)+T_3)(v)
\end{aligned}
$$

Additive identity:

$\exists\, 0 \in \mathcal{L}(V,W) : 0v = 0$ (zero map)

$(0+T)(v) = 0v + Tv = 0 + Tv = Tv + 0 = Tv \quad \forall T \in \mathcal{L}(V,W)$

$\Rightarrow$ 0 additive identity of $\mathcal{L}(V,W)$

Additive inverse:

Let $(-T)v = -Tv \quad \forall T \in \mathcal{L}(V,W)$

$(T + -T)v = Tv - Tv = 0.$

$\Rightarrow (-T)$ additive inverse of $\mathcal{L}(V,W)$

Multiplicative identity: Let $T \in \mathcal{L}(V,W)$, $v \in V$.

Let $1 \in \mathbb{F}$. $(1T)(v) = 1Tv = Tv. \quad \forall T \in \mathcal{L}(V,W)$

$\Rightarrow 1$ multiplicative identity in $\mathcal{L}(V,W)$

Distributive: Let $T_1, T_2 \in \mathcal{L}(V,W)$, $v \in V$.

Let $\alpha, \beta \in \mathbb{F}$.

$$
\begin{aligned}
\alpha((T_1+T_2)(v)) &= \alpha(T_1 v + T_2 v) \\
&= \alpha T_1 v + \alpha T_2 v \\
&= (\alpha T_1)(v) + (\alpha T_2)(v) \\
&= (\alpha T_1 + \alpha T_2)(v)
\end{aligned}
$$

$$
\begin{aligned}
(\alpha+\beta)(T_1 v) &= \alpha T_1 v + \beta T_1 v \\
&= (\alpha T_1 + \beta T_1)(v)
\end{aligned}
$$