Let W be a Subspace of $\mathbb{R}^2$.

$\dim W \leq \dim \mathbb{R}^2 \Rightarrow \dim W \leq 2. \Rightarrow \dim W \in \{0,1,2\}$

1) $\dim W = 0 \Rightarrow W = \{0\}$ - valid subspace

2) $\dim W = 2 \Rightarrow W = \mathbb{R}^2$ - valid subspace

3) $\dim W = 1 \Rightarrow \exists\, v \in W : \mathrm{Span}(v) = W \Rightarrow$

$\Rightarrow W = \{av : a \in \mathbb{R}\}$

This set describes all lines containing the origin.

This is a valid subspace:

- $0 \in W$ for $a = 0$.
- Set is closed under addition:

  $av + bv = (a+b)v \in W, \forall\, av, bv \in W.$
- Set is closed under scalar multiplication:

  $\alpha(av) = (\alpha a)v \in W, \forall\, \alpha \in \mathbb{R}, av \in W.$

$\Rightarrow$ Subspaces of $\mathbb{R}^2$ are $\{0\}$, all lines containing the origin and $\mathbb{R}^2$.