(a) $\mathbb{C}$ vector space over $\mathbb{R} \Rightarrow 1+i,\ 1-i$ lin independent

$1+i,\ 1-i$ lin. independent $\Leftrightarrow \nexists\ a,b \in \mathbb{R},\ a,b \neq 0$:

$a(1+i) + b(1-i) = 0$

$a(1+i) + b(1-i) = 0 \Leftrightarrow$

$\Leftrightarrow a + ai + b - bi = 0$

$\Leftrightarrow (a+b) + (a-b)i = 0 + 0i$

$\Leftrightarrow \begin{cases} a+b = 0 \\ a-b = 0 \end{cases} \Leftrightarrow \begin{cases} 2b = 0 \\ a = b \end{cases} \Leftrightarrow a = b = 0 \Rightarrow$

$\Rightarrow 1+i,\ 1-i$ lin. independent

(b) $\mathbb{C}$ vector space over $\mathbb{C} \Rightarrow 1+i,\ 1-i$ lin. dependent.

$1+i,\ 1-i$ lin. dependent $\Leftrightarrow \exists\ a,b \in \mathbb{C},\ a,b \neq 0$:

$a(1+i) + b(1-i) = 0$

Assume $a = (1+i),\ b = (1-i)$

$$a(1+i) + b(1-i) = (1+i)^2 + (1-i)^2$$
$$= 1 + i^2 + 2i + 1 + i^2 - 2i$$
$$= 0$$

$\Rightarrow \exists\ a,b \in \mathbb{C},\ a,b \neq 0 : a(i+1) + b(1-i) = 0 \Rightarrow$

$\Rightarrow 1+i\ ,\ 1-i$ are lin. dependent