

$$\{(x, y, z) \in \mathbb{F}^3 : x + y + z = 0\} = \mathcal{U}$$

$$\left. \begin{aligned} \text{Let } v_1 &= (1, 0, -1) \in \mathcal{U} \\ v_2 &= (0, 1, -1) \in \mathcal{U} \\ v_3 &= (2, 0, -2) \in \mathcal{U} \\ v_4 &= (0, 2, -2) \in \mathcal{U} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \text{span}(v_1, v_2, v_3, v_4) \subseteq \mathcal{U} \quad (1)$$

Consider any  $(x, y, z) \in \mathcal{U}$

$$\begin{aligned} (x, y, z) &= (x, y, -x-y) \\ &= x(1, 0, -1) + y(0, 1, -1) \\ &= xv_1 + yv_2 \end{aligned}$$

$$xv_1 + yv_2 \in \text{span}(v_1, v_2) \subseteq \text{span}(v_1, v_2, v_3, v_4)$$

$$\Rightarrow \mathcal{U} \subseteq \text{span}(v_1, v_2, v_3, v_4) \quad (2)$$

$$(1), (2) \Rightarrow \text{span}(v_1, v_2, v_3, v_4) = \mathcal{U}$$

( $v_1, v_2$  already span  $\mathcal{U}$ ,  $v_3, v_4$  are redundant).