$v_1 \dots v_m \in V$

For $k \in \{1, \dots, m\}$: $w_k = v_1 + \dots + v_k$

$v_1, \dots, v_m$ linearly independent $\iff$ $w_1 \dots w_m$ linearly independent

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($\Rightarrow$) Assume $v_1, \dots, v_m$ lin. independent

Let $a_1, \dots, a_m \in F$.

Consider $a_1 w_1 + a_2 w_2 + \dots + a_m w_m = 0$

$\iff a_1 v_1 + a_2 (v_1 + v_2) + \dots + a_m (v_1 + \dots + v_m) = 0$

$\iff a_1 v_1 + a_2 v_1 + a_2 v_2 + \dots + a_m v_1 + \dots + a_m v_m = 0$

$\iff (a_1 + a_2 + \dots + a_m) v_1 + (a_1 + \dots + a_{m-1}) v_2 + \dots +$

$+ a_m v_m = 0.$

Since $v_1, \dots, v_m$ are lin. independent, this is true only for $a_1 = a_2 = \dots = a_m = 0$, so $w_1, \dots, w_m$ are also linearly independent. (1)

($\Leftarrow$) Assume $w_1, \dots, w_m$ are linearly independent.

$v_1 = w_1$

$v_k = w_k - v_1 - \dots - v_{k-1} = w_k - (v_1 + \dots + v_{k-1}) = w_k - w_{k-1}$

Let $a_1, \dots, a_m \in F$.

Consider $a_1 v_1 + a_2 v_2 + \dots + a_m v_m = 0$

$\Leftrightarrow a_1 w_1 + a_2(w_2 - w_1) + \dots + a_m(w_m - w_{m-1}) = 0$

$\Leftrightarrow (a_1 - a_2) w_1 + \dots + (w_{m-1} - w_m) w_{m-1} + a_m w_m = 0$

Since $w_1, \dots, w_m$ are lin. independent, this is true only for $a_1 = a_2 = \dots = a_m = 0$, so $v_1, \dots, v_m$ are also linearly independent. (2)

(1), (2) $\Rightarrow$ $v_1, \dots, v_m$ linearly independent $\Leftrightarrow$ $w_1, \dots, w_m$ linearly independent