F^∞ is infinite-dimensional

$F^\infty = \{(a_1, a_2, \dots) : a_i \in F, \text{ and only finitely many } a_i \neq 0\}$

Let $e_n = (0, \dots, 0, 1, 0) \in F^\infty$ (1 in the n-th position)

Consider $e_1, e_2 \dots \in F^\infty$ and $a_1, a_2 \dots \in F$:

$\sum_{i=1}^{m} a_i e_i = 0 \iff a_i = 0 \ \forall \ i \in \{1 \dots m\} \Rightarrow$

$\Rightarrow$ any sequence of e_i, $i \in \{1, 2 \dots\}$ is linearly independent.

Consider arbitrary $v_1, \dots v_m \in F^\infty$. Let N_k the largest index where $v_k \neq 0$.

Let $N = \max(N_1, \dots, N_m)$. In any linear combination of $v_1, \dots, v_m$, all coordinates past index N are 0.

But, $e_{N+1} \in F^\infty \Rightarrow e_{N+1} \notin \text{span}(v_1, \dots, v_m) \Rightarrow$

There is no finite length list which spans $F^\infty \Rightarrow$

$\Rightarrow F^\infty$ is infinite dimensional.