$$V_e = \{ f : f \in \mathbb{R}^{\mathbb{R}},\ f(-x) = f(x) \}$$
$$V_o = \{ f : f \in \mathbb{R}^{\mathbb{R}},\ f(-x) = -f(x) \}$$

$$\mathbb{R}^{\mathbb{R}} = V_e \oplus V_o$$

$$V_e \cap V_o = \{ f(x) = 0 \} = \{0\} \quad (1)$$
$$V_e + V_o = \{ f + g : f, g \in \mathbb{R}^{\mathbb{R}},\ f \text{ even},\ g \text{ odd} \}$$

Can every function in $\mathbb{R}^{\mathbb{R}}$ be decomposed in an even + odd part?

Take arbitrary $h : \mathbb{R} \to \mathbb{R}$.

$$h_e(x) = \frac{h(x) + h(-x)}{2}, \quad h_o(x) = \frac{h(x) - h(-x)}{2}$$

$$h_e(-x) = \frac{h(-x) + h(x)}{2} = \frac{h(x) + h(-x)}{2} = h_e(x) \Rightarrow$$

$\Rightarrow h_e(x) \in V_e$

$$h_o(-x) = \frac{h(-x) - h(x)}{2} = -\frac{h(x) - h(-x)}{2} = -h_o(x)$$

$\Rightarrow h_o(x) \in V_o$

$$h_e(x) + h_o(x) = \frac{h(x)+h(-x)}{2} + \frac{h(x)-h(-x)}{2} = h(x)$$

$\Rightarrow \forall h \in \mathbb{R}^{\mathbb{R}},\ h \in V_e + V_o$.

$$\left.\begin{array}{l} \mathbb{R}^{\mathbb{R}} \subseteq V_e + V_o \\ V_e + V_o \subseteq \mathbb{R}^{\mathbb{R}} \end{array}\right\} \Rightarrow$$

$\Rightarrow \mathbb{R}^{\mathbb{R}} = V_e + V_o \quad (2)$

$(1), (2) \Rightarrow \mathbb{R}^{\mathbb{R}} = V_e \oplus V_o$.