

V_1, V_2, U subspaces of V

$V = V_1 \oplus U$ and $V = V_2 \oplus U \Rightarrow V_1 = V_2$

Assume $V = F^2$.

$$V_1 = \{(x, 0) \in F^2 : x \in F\}$$

$$V_2 = \{(0, y) \in F^2 : y \in F\}$$

$$U = \{(a, a) \in F^2 : a \in F\}$$

$$\bullet \left. \begin{aligned} V_1 + U &= \{(x+a, a) \in F^2 : a, x \in F\} = F^2 \\ V_1 \cap U &= \{0\} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow V_1 \oplus U = F^2$$

$$\bullet \left. \begin{aligned} V_2 + U &= \{(a, y+a) \in F^2 : a, y \in F\} = F^2 \\ V_2 \cap U &= \{0\} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow V_2 \oplus U = F^2$$

So - $F^2 = V_1 \oplus U$ and $F^2 = V_2 \oplus U$, but $V_1 \neq V_2$.