

$$U = \{(x, x, y, y) \in F^4 : x, y \in F\}$$

$$\underline{W = ? \text{ s.t. } F^4 = U \oplus W}$$

$$\text{Let } W = \{(0, a, 0, b) \in F^4 : a, b \in F\}$$

$$\text{Let any } (p, q, r, s) \in F^4$$

$$(p, q, r, s) = (p, p, r, r) + (0, q-p, 0, s-r) \in U + W$$

$$\text{Hence, } U + W = F^4 \quad (1)$$

$$\text{Let arbitrary } (x, x, y, y) \in U \text{ and } (0, a, 0, b) \in W.$$

$$(x, x, y, y) = (0, a, 0, b) \implies \begin{cases} x = a = 0 \\ y = b = 0 \end{cases}$$

$$\implies U \cap W = \{0\} \quad (2)$$

$$(1), (2) \implies U \oplus W = F^4.$$