v_1, v_2, v_3, v_4 linearly independent in V

$w_1 = (v_1 - v_2)$, $w_2 = (v_2 - v_3)$, $w_3 = (v_3 - v_4)$, $w_4 = v_4$

w_1, w_2, w_3, w_4 also lin. independent

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v_1, v_2, v_3, v_4 linearly independent $\Rightarrow$

$\Rightarrow a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 = 0 \iff a_1 = a_2 = a_3 = a_4 = 0.$

Consider $a_1 w_1 + a_2 w_2 + a_3 w_3 + a_4 w_4 = 0$, $a_1, a_2, a_3, a_4 \in F$

$\iff a_1(v_1 - v_2) + a_2(v_2 - v_3) + a_3(v_3 - v_4) + a_4 v_4 = 0$

$\iff a_1 v_1 - a_1 v_2 + a_2 v_2 - a_2 v_3 + a_3 v_3 - a_3 v_4 + a_4 v_4 = 0$

$\iff a_1 v_1 + (-a_1 + a_2) v_2 + (-a_2 + a_3) v_3 + (-a_3 + a_4) v_4 = 0$

Because v_1, v_2, v_3, v_4 are lin. independent, this is true $(\Rightarrow)$

$$\begin{cases} a_1 = 0 \\ -a_1 + a_2 = 0 \\ -a_2 + a_3 = 0 \\ -a_3 + a_4 = 0 \end{cases} \iff a_1 = a_2 = a_3 = a_4 = 0 \Rightarrow$$

$\Rightarrow$ If v_1, v_2, v_3, v_4 lin. independent $\Rightarrow w_1, w_2, w_3, w_4$ lin. independent.