

$v_1, v_2 \dots v_m$ lin. independent

$\lambda \in F, \lambda \neq 0 \Rightarrow \lambda v_1, \lambda v_2 \dots \lambda v_m$ lin. independent

Let $a_1, \dots, a_m \in F$.

Consider $a_1 \lambda v_1 + \dots + a_m \lambda v_m = 0$

$$(\Leftrightarrow) (a_1 \lambda) v_1 + \dots + (a_m \lambda) v_m = 0$$

v_1, \dots, v_m is lin. independent $\Rightarrow \sum_{i=1}^m (a_i \lambda) v_i = 0 \Leftrightarrow$

$$(\Leftrightarrow) a_i = 0 \quad \forall i \in \{1 \dots m\}$$

$$\Rightarrow a_i \lambda = 0 \quad \forall i \in \{1 \dots m\}$$

$$\lambda \neq 0 \Rightarrow a_i = 0 \quad \forall i \in \{1 \dots m\}$$

Hence, if $v_1 \dots v_m$ lin. independent and $\lambda \neq 0 \Rightarrow$
 $\Rightarrow \lambda v_1 \dots \lambda v_m$ lin. independent.