$v_1, \dots, v_m$ linearly independent in V, $w \in V$

$v_1 + w, \dots, v_m + w$ linearly dependent $\Rightarrow w \in \operatorname{span}(v_1 \dots v_m)$

$v_1 + w, \dots, v_m + w$ linearly dependent $\Rightarrow \exists\, a_1 \dots a_m$, not all 0 s.t.:

$$\sum_{i=1}^{m} a_i (v_i + w) = 0 \iff \sum_{i=1}^{m} a_i v_i + a_i w = 0$$

$$\iff \sum_{i=1}^{m} a_i v_i + \left(\sum_{i=1}^{m} a_i\right) w = 0$$

Suppose, for the sake of contradiction, $\sum_{i=1}^{m} a_i = 0$.

$$\sum_{i=1}^{m} a_i v_i + \left(\sum_{i=1}^{m} a_i\right) w = 0 \iff$$

$$\left.\begin{array}{l} \iff \displaystyle\sum_{i=1}^{m} a_i v_i = 0 \\ v_1 \dots v_m \text{ are lin. independent} \end{array}\right\} \Rightarrow a_i = \dots = a_m = 0$$

This is a contradiction - $\exists\, a_1 \dots a_m$ not all 0.

Thus, $\sum_{i=1}^{m} a_i \neq 0$.

$$\sum_{i=1}^{m} a_i v_i + \left(\sum_{i=1}^{m} a_i\right) w = 0 \iff$$

$$\iff w = -\frac{1}{\sum_{i=1}^{m} a_i} \left(\sum_{i=1}^{m} a_i v_i\right) \implies w \in \operatorname{span}(v_1 \dots v_m)$$