

U, W subspaces of \mathbb{R}^8

$$\dim U = 3, \dim W = 5$$

$$U + W = \mathbb{R}^8$$

$$\begin{array}{c} \hline \mathbb{R}^8 = U \oplus W \\ \hline \# \end{array}$$

$$\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$$

$$\Leftrightarrow \dim(\mathbb{R}^8) = 3 + 5 - \dim(U \cap W)$$

$$\Leftrightarrow 8 = 8 - \dim(U \cap W)$$

$$\Leftrightarrow \dim(U \cap W) = 0$$

$$\Leftrightarrow U \cap W = \{0\}$$

Since $U + W = \mathbb{R}^8$ and $U \cap W = \{0\} \Rightarrow \mathbb{R}^8 = U \oplus W$