$T \in \mathcal{L}(V, W)$

$v_1, \ldots, v_m \in V$

$Tv_1, \ldots, Tv_m$ lin. independent in W

$$\overline{}$$

$v_1, \ldots, v_m$ lin independent

$$\overline{} \#$$

Let $a_1, \ldots a_m \in F$: $0 = a_1 v_1 + a_2 v_2 + \ldots + a_m v_m$

Apply T: $T(0) = T(a_1 v_1 + a_2 v_2 + \ldots + a_m v_m)$

$(\Longrightarrow)$ $0 = T(a_1 v_1) + T(a_2 v_2) + \ldots + T(a_m v_m)$

$(\Longrightarrow)$ $0 = a_1 T v_1 + a_2 T v_2 + \ldots + a_m T v_m$

Since $T v_1, \ldots, T v_m$ are lin independent $\Longrightarrow a_1 = \ldots = a_m = 0$.

So, $0 = a_1 v_1 + \ldots + a_m v_m$ $(\Longrightarrow)$ $a_1 = \ldots = a_m = 0$.