V finite dimensional, $\dim V = n \geq 1$

$\exists\, V_1, \dots, V_n \leq V : \dim V_i = 1,\ V = V_1 \oplus \dots \oplus V_n$

Let $v_1, \dots, v_n$ be the basis of V

Consider $V_i = \text{span}(v_i)$ for $i \in \{1 \dots n\}$

Since $V = \text{span}(v_1, \dots, v_n) \Rightarrow \forall\, x \in V : x = \sum_{i=1}^{n} a_i v_i$

$x = \sum_{i=1}^{n} a_i v_i \in V_1 + \dots + V_n$

$\Rightarrow V = V_1 + \dots + V_n \;(1)$

Let $x \in V_1 + \dots + V_n \Rightarrow$

$\Rightarrow x = x_1 + \dots + x_n \quad : x_i \in V_i \quad i \in 1 \dots n$

$x_i \in V_i = \text{span}(v_i) \Rightarrow x_i = a_i v_i$

$\Rightarrow x = a_1 v_1 + a_2 v_2 + \dots + a_n v_n$

Consider $x = a_1 v_1 + a_2 v_2 + \dots + a_n v_n = 0$

Since $v_1, v_2 \dots v_n$ are lin independent (basis of V)

$\Rightarrow a_1 = a_2 = \dots = a_n = 0.$

$\Rightarrow x_1 = x_2 = \dots = x_n = 0$

$\Rightarrow$ The only way to write $x = x_1 + \dots + x_n$, $x_i \in V_i$ in for all $x_i = 0$. (2)

(1), (2) $\Rightarrow V = V_1 \oplus V_2 \oplus \dots \oplus V_n$.