$$\dim(X+Y) = \dim X + \dim Y - \dim(X \cap Y)$$

$$\dim(V_1+V_2+V_3) = \dim((V_1+V_2)+V_3)$$
$$= \dim(V_1+V_2) + \dim V_3 - \dim((V_1+V_2)\cap V_3)$$
$$\dim(V_1+V_2+V_3) = \dim((V_2+V_3)+V_1)$$
$$= \dim(V_2+V_3) + \dim V_1 - \dim((V_2+V_3)\cap V_1)$$
$$\dim(V_1+V_2+V_3) = \dim((V_1+V_3)+V_2)$$
$$= \dim(V_1+V_3) + \dim V_2 - \dim((V_1+V_3)\cap V_2)$$

$\Rightarrow$

$$\Rightarrow 3\dim(V_1+V_2+V_3) = \dim V_1 + \dim V_2 + \dim V_3 +$$
$$+ \dim(V_1+V_2) + \dim(V_2+V_3) + \dim(V_1+V_3)$$
$$- (\dim((V_1+V_2)\cap V_3) + \dim((V_2+V_3)\cap V_1 + \dim((V_1+V_3)\cap V_2))$$

$$= \dim V_1 + \dim V_2 + \dim V_3 + \dim V_1 + \dim V_2 - \dim(V_1\cap V_2)$$
$$+ \dim V_2 + \dim V_3 - \dim(V_2\cap V_3) + \dim V_1 + \dim V_3$$
$$- \dim(V_1\cap V_3) - (\dim((V_1+V_2)\cap V_3) + \dim((V_2+V_3)\cap V_1$$
$$+ \dim((V_1+V_3)\cap V_2))$$

$$= 3(\dim V_1 + \dim V_2 + \dim V_3) -$$
$$- (\dim(V_1\cap V_2) + \dim(V_2\cap V_3) + \dim(V_1\cap V_3) -$$

$$- (\dim((V_1+V_2)\cap V_3) + \dim((V_2+V_3)\cap V_1 + \dim((V_1+V_3)\cap V_2))$$

$$\Rightarrow \dim(V_1+V_2+V_3) = \dim V_1 + \dim V_2 + \dim V_3 -$$

$$- \frac{\dim(V_1\cap V_2) + \dim(V_2\cap V_3) + \dim(V_1\cap V_3)}{3}$$

$$- \frac{\dim((V_1+V_2)\cap V_3) + \dim((V_2+V_3)\cap V_1 + \dim((V_1+V_3)\cap V_2))}{3}$$