

The list  $(1, z, z^2, z^3, z^4) \in \mathcal{P}_4(F)$  is linearly independent and has len 5.

No list with len  $< 5$  can span  $\mathcal{P}_4(F)$