Let A = arbitrary index set over the collection of all subspaces of V: $\{V_i\}_{i \in A}$

Let $W = \bigcap_{i \in A} V_i$

Additive identity (1)

$\forall V_i \in \{V_i\}_{i \in A}$, $0 \in V_i$ (V_i is a subspace of V) $\Rightarrow$

$\Rightarrow 0 \in W$

Closure under addition (2)

Let $x, y \in W$.

$$\left.\begin{array}{l} \forall V_i \in \{V_i\}_{i \in A} : x, y \in V_i \quad (x, y \in W) \\ \forall V_i \in \{V_i\}_{i \in A} : x + y \in V_i \quad (V_i \text{ subspace of } V) \end{array}\right\} \Rightarrow$$

$\Rightarrow x + y \in W$.

Closure under scalar multiplication (3)

Let $x \in W$, $\alpha \in \mathbb{F}$

$$\left.\begin{array}{l} \forall V_i \in \{V_i\}_{i \in A} : x \in V_i \quad (x \in W) \\ \forall V_i \in \{V_i\}_{i \in A} : \alpha x \in V_i \quad (V_i \text{ subspace of } W) \end{array}\right\} \Rightarrow$$

$\Rightarrow \alpha x \in W$

(1),(2),(3) $\Rightarrow$ the intersection of every collection of subspaces of V is a subspace of V. $\square$