$D = \{ f : (-4,4) \to \mathbb{R} \mid f \text{ diff},\ f'(-1) = 3f(2) \}$

D is a subspace of $\mathbb{R}^{(-4,4)}$

Additive identity:

Let $f : (-4,4) \to \mathbb{R}$ s.t. $f(x) = 0$. (zero func)

$f'(x) = 0 \implies f'(-1) = 0$

$f(2) = 0$ (definition)

$f(-1) = 0 = 3 \cdot 0 = 3 \cdot f(2) \implies f \in D$

$\square$

Closure under addition

Let $f, g \in D$, $f'(-1) = 3f(2)$

$g'(-1) = 3g(2)$

$f + g = (f+g)(x) = f(x) + g(x)$

$(f+g)' = f'(x) + g'(x)$

$(f+g)'(-1) = f'(-1) + g'(-1)$

$= 3f(2) + 3g(2)$

$= 3(f(2) + g(2))$

$= 3((f+g)(2)) \implies$

$\Rightarrow f+g \in D.$ $\square$

Closure under scalar multiplication

Let $f \in D$, $a \in \mathbb{R}$. $f'(-1) = 3f(2)$

$af = (af)(x) = af(x)$

$(af)' = (af(x))' = af'(x)$

$(af)'(-1) = af'(-1)$

$= a\,3f(2)$

$= 3(af(2))$

$= 3((af)(2)) \Rightarrow af \in D.$ $\square$

So, D is a subspace of $\mathbb{R}^{(-4,4)}$