$\forall\ \alpha \in \mathbb{C}\quad \exists$ unique $\beta \in \mathbb{C}$ s.t. $\alpha + \beta = 0$

Suppose $\alpha = a + bi$ where $a, b \in \mathbb{R}$

Define $\beta = -a - bi$

$$\alpha + \beta = (a + bi) + (-a - bi) = (a - a) + (b - b)i$$
$$= 0 + 0i = 0. \text{ (Existence)}$$

Suppose $\exists\ \lambda \in \mathbb{C},\ \lambda \neq \beta$ s.t. $\alpha + \lambda = 0$.

$\lambda = -\alpha \Rightarrow \lambda = -a - bi \Rightarrow \lambda = \beta$ (Uniqueness)

$\square$