$p_0, \dots, p_m \in \mathcal{P}(F)$: $\deg p_k = k$.

$p_0, \dots, p_m$ basis of $\mathcal{P}_m(F)$

Since $p_0, \dots, p_m$ all have different degrees $\Rightarrow$ $p_0, \dots, p_m$ is linearly independent.

Since $\dim \mathcal{P}_m(F) = m+1$ and $p_0, \dots, p_m$ is a lin. independent list of $m+1$ elements $\Rightarrow$ $p_0, \dots, p_m$ is a basis of $\mathcal{P}_m(F)$