$v, w \in V$

$\exists$ unique $x \in V$ s.t. $v + 3x = w$

Existence: $v + 3x = w$

$$3x = w - v$$

$$x = \frac{1}{3}(w - v)$$

$w - v$ exists in V (vector addition + additive inverse)

$\frac{1}{3}(w - v)$ exists in V (scalar multiplication)

$\Rightarrow x$ exists in V.

Uniqueness: Suppose there $\exists\, x_1, x_2 \in V, x_1 \neq x_2$ s.t. $\begin{cases} v + 3x_1 = w \\ v + 3x_2 = w \end{cases}$

$$\begin{cases} v + 3x_1 = w \\ v + 3x_2 = w \end{cases}$$

substract

$$\Leftrightarrow 3x_1 - 3x_2 = 0 \Leftrightarrow 3(x_1 - x_2) = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$\Rightarrow x$ is unique

□