Let W be a subspace of $\mathbb{R}^3$.

$\dim W \leq \dim \mathbb{R}^3 \Rightarrow \dim W \leq 3 \Rightarrow \dim W \in \{0, 1, 2, 3\}$

1) $\dim W = 0 \Rightarrow W = \{0\}$ - valid subspace

2) $\dim W = 1 \Rightarrow \exists\, v$ basis of $W \Rightarrow \operatorname{span}(v) = W$

$\Rightarrow W = \{av : a \in \mathbb{R}\}$

This is a valid subspace and denotes all lines in $\mathbb{R}^3$ containing the origin (proof previous ex.).

3) $\dim W = 3 \Rightarrow W = \mathbb{R}^3$ - valid subspace

4) $\dim W = 2 \Rightarrow \exists\, v, w$ basis of $W \Rightarrow$

$\operatorname{span}(v, w) = W \Rightarrow W = \{av + bw : a, b \in \mathbb{R}\}$

This set denotes all planes in $\mathbb{R}^3$ containing the origin.

This set is also a valid subspace:

- $0 \in W$ for $a = b = 0$.
- Closed under addition:

Let $x = a_1 v + b_1 w \in W$, $y = a_2 v + b_2 w \in W$

$x + y = a_1 v + b_1 w + a_2 v + b_2 w = (a_1 + a_2) v + (b_1 + b_2) w \in W$

- Closed under scalar multiplication:

Let $\alpha \in \mathbb{R}$: $\alpha(av + bw) = \alpha a v + \alpha b w = (\alpha a)v + (\alpha b)w \in W$

$\Rightarrow$ The subspaces of $\mathbb{R}^3$ are precisely $\{0\}$, all lines in $\mathbb{R}^3$ containing the origin, all planes in $\mathbb{R}^3$ containing the origin and $\mathbb{R}^3$.