$\dim V = 10$

$V_1, V_2, V_3 \subset V$

$\dim V_1 = \dim V_2 = \dim V_3 = 7$

$V_1 \cap V_2 \cap V_3 \neq \{0\}$

$V_1, V_2 \subseteq V \Rightarrow \dim(V_1 \cap V_2) \geq \dim V_1 + \dim V_2 - \dim V$

$\Leftrightarrow \dim(V_1 \cap V_2) \geq 14 - 10 = 4$

$(V_1 \cap V_2), V_3 \subseteq V \Rightarrow$

$\Rightarrow \dim((V_1 \cap V_2) \cap V_3) \geq \dim(V_1 \cap V_2) + \dim V_3 - \dim V$

$\Leftrightarrow \dim((V_1 \cap V_2) \cap V_3) \geq 4 + 7 - 10 = 1$

$\Rightarrow V_1 \cap V_2 \cap V_3$ has a basis $\neq ()$ $\Rightarrow V_1 \cap V_2 \cap V_3 \neq \{0\}$