$$\mathcal{U} = \{ f : \mathbb{R} \to \mathbb{R} : \exists p \, f(x) = f(x+p) \ \forall x \in \mathbb{R} \}$$

Let $f(x) = \sin(x) \in \mathcal{U}$ $(p = 2\pi)$

$g(x) = \sin(\sqrt{2}x) \in \mathcal{U}$ $(p = \frac{2\pi}{\sqrt{2}})$

$$(f+g)(x) = f(x) + g(x) = \sin(x) + \sin(\sqrt{2}x)$$

The sum of two periodic functions is periodic if the ratio of their periods is in $\mathbb{Q}$.

For f, g, the ratio of their periods =

$$= \frac{2\pi}{\left(\frac{2\pi}{\sqrt{2}}\right)} = \frac{2\pi\sqrt{2}}{2\pi} = \sqrt{2} \notin \mathbb{Q}. \implies$$

$\implies$ For $f = \sin(x) \in \mathcal{U}$, $g = \sin(\sqrt{2}x) \in \mathcal{U}$,

$f + g \notin \mathcal{U} \implies \mathcal{U}$ is not closed under addition.

So, $\mathcal{U}$ is not a subspace of $\mathbb{R}^{\mathbb{R}}$.