Basic Advanced Functions — Part 1: Communication Problems

Your Name

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Question 1 (6 points)

Rewrite each relationship using function notation. *All given text retained; one "=" per line; equals aligned.*

a)

An airplane must travel 400 km. Let t be the travel time (in hours) and let $s(t)$ denote the speed (in km/h).

$$\begin{aligned} \text{Speed} &= \frac{\text{Distance}}{\text{Time}} \\ s(t) &= \frac{400}{t} \end{aligned} \qquad (t > 0,\ \text{km/h})$$

b)

An ice cream cone starts at 125 mL and loses half its volume every 5 min. Let t be in minutes and $v(t)$ be the volume (mL); the discrete half-life model is

$$\begin{aligned} v(t) &= v_0 \left(\tfrac{1}{2}\right)^{t/T_{1/2}} \\ v(t) &= 125 \left(\tfrac{1}{2}\right)^{t/5} \end{aligned}$$

c)

Scott drives at a constant speed of 50 km/h. If $d(t)$ is the distance (km) after t hours,

$$d(t) = 50t$$

Question 2 (6 points)

Formatting: parts (a) and (b) are side by side with a clean divider; all "=" signs aligned inside each block.

a) $p(r) = 2r^2 + 2r - 1$

$$x = 2y^2 + 2y - 1$$
$$x + 1 = 2(y^2 + y)$$
$$x + 1 = 2\left[\left(y + \tfrac{1}{2}\right)^2 - \tfrac{1}{4}\right]$$
$$x + 1 = 2\left(y + \tfrac{1}{2}\right)^2 - \tfrac{1}{2}$$
$$x + \tfrac{3}{2} = 2\left(y + \tfrac{1}{2}\right)^2$$
$$\frac{x}{2} + \frac{3}{4} = \left(y + \tfrac{1}{2}\right)^2$$
$$y + \tfrac{1}{2} = \pm\sqrt{\tfrac{x}{2} + \tfrac{3}{4}}$$
$$y = -\tfrac{1}{2} \pm \sqrt{\tfrac{x}{2} + \tfrac{3}{4}}$$
$$\boxed{p^{-1}(x) = -\tfrac{1}{2} \pm \sqrt{\tfrac{x}{2} + \tfrac{3}{4}}}$$

b) $3y + 5x = 18$

$$3y + 5x = 18$$
$$3y = -5x + 18$$
$$y = -\tfrac{5}{3}x + 6$$
$$x = -\tfrac{5}{3}y + 6$$
$$x - 6 = -\tfrac{5}{3}y$$
$$y = -\tfrac{3}{5}(x - 6)$$
$$y = -\tfrac{3}{5}x + \tfrac{18}{5}$$
$$\boxed{f^{-1}(x) = -\tfrac{3}{5}x + \tfrac{18}{5}}$$

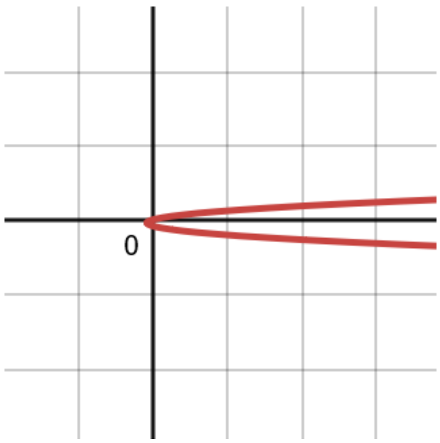
c) $h(t) = -4.9(t + 3)^2 + 45.8$

$$x = -4.9(y + 3)^2 + 45.8$$
$$x - 45.8 = -4.9(y + 3)^2$$
$$45.8 - x = 4.9(y + 3)^2$$
$$\frac{45.8 - x}{4.9} = (y + 3)^2$$
$$y + 3 = \pm\sqrt{\frac{45.8 - x}{4.9}}$$
$$y = -3 \pm \sqrt{\frac{45.8 - x}{4.9}}$$
$$\boxed{h^{-1}(x) = -3 \pm \sqrt{\frac{45.8 - x}{4.9}}}$$

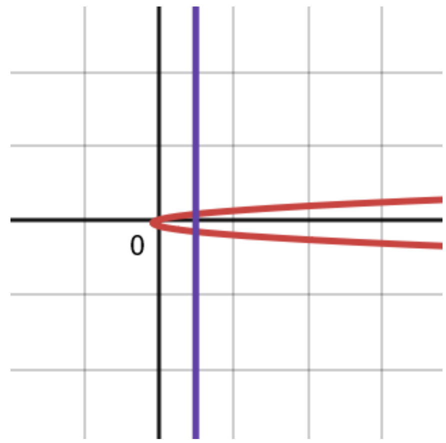
Question 3 (6 points)

Using graphs, decide whether each inverse is a function. Figures are side by side (uniform size) with concise captions. Below each pair, the reasoning lines up the "⇒" arrows and the verdict is boxed.

a) p^{-1}



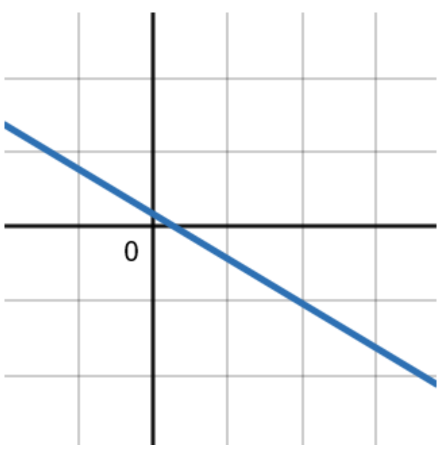
i Inverse (reflection across $y = x$).



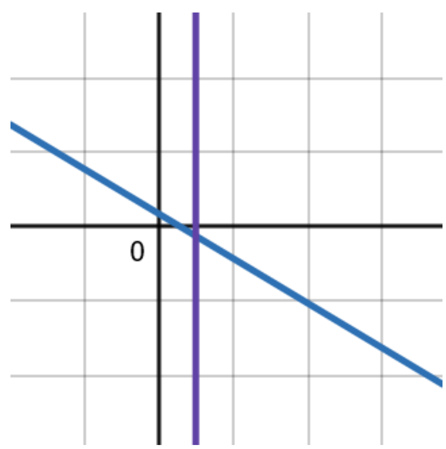
ii Vertical line test: fails.

$$\begin{aligned} \text{Construct inverse: } & \text{reflect graph of } y = p(x) \text{ across } y = x && \Rightarrow \text{ graph of } p^{-1} \\ \text{Apply VLT to } p^{-1}\text{: } & \text{some verticals cut the graph twice} && \Rightarrow \boxed{\textbf{Inverse is } \textbf{\textit{not}} \textbf{ a function}} \\ \text{Domain/Range swap: } & \mathrm{Dom}(p^{-1}) = \mathrm{Ran}(p),\ \mathrm{Ran}(p^{-1}) = \mathrm{Dom}(p) \end{aligned}$$

b) f^{-1}



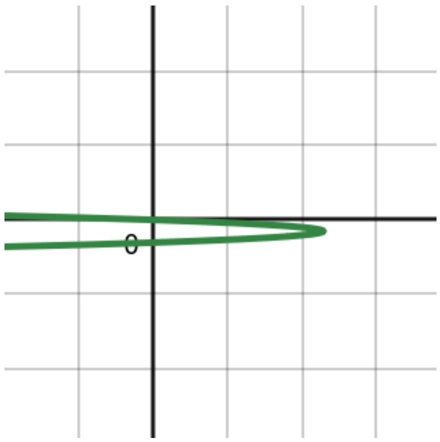
i Inverse of a line (reflection across $y = x$).



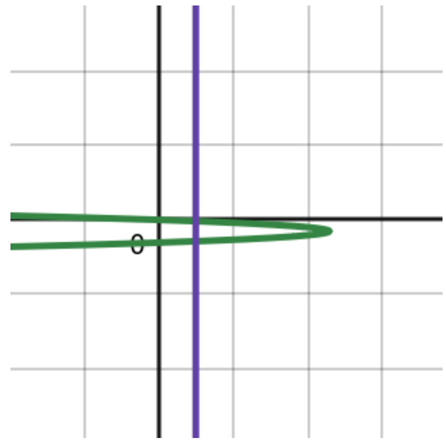
ii Vertical line test: passes.

$$\begin{aligned} \text{Construct inverse: } & \text{reflect non-vertical line across } y = x && \Rightarrow \text{ another non-vertical line} \\ \text{Apply VLT to } f^{-1}\text{: } & \text{each vertical meets at most once} && \Rightarrow \boxed{\textbf{Inverse } \textbf{\textit{is}} \textbf{ a function}} \\ \text{Domain/Range swap: } & \mathrm{Dom}(f^{-1}) = \mathrm{Ran}(f),\ \mathrm{Ran}(f^{-1}) = \mathrm{Dom}(f) \end{aligned}$$

c) h^{-1}



i Inverse (reflection across $y = x$).



ii Vertical line test: fails.

$$\begin{aligned}
\text{Construct inverse}: &\ \text{reflect graph of } y = h(x) \text{ across } y = x && \Rightarrow \text{ relation } h^{-1} \\
\text{Apply VLT to } h^{-1}: &\ \text{fails (some verticals cut twice)} && \Rightarrow \boxed{\textbf{Inverse is } \textbf{\textit{not}} \textbf{ a function}} \\
\text{Domain/Range swap}: &\ \mathrm{Dom}(h^{-1}) = \mathrm{Ran}(h),\ \mathrm{Ran}(h^{-1}) = \mathrm{Dom}(h)
\end{aligned}$$

Question 4 (24 points)

All original answers preserved. Reformatted into three readable "summary cards" (no clipping; full-size math).

(a) $f(x) = 2x^2 - 8$

Domain & Range	Domain : $\{x \in \mathbb{R}\}$ Range : $\{y \in \mathbb{R} \mid y \geq -8\}$
Restrictions	Domain : None Range : $y \geq -8$
Increasing / Decreasing	Decreasing : $(-\infty, 0)$ Increasing : $(0, +\infty)$
x-intercepts (roots)	$f(x) = 0 \quad \Rightarrow (2, 0),\ (-2, 0)$ $0 = 2x^2 - 8$ $\frac{8}{2} = x^2$ $x = \pm 2$
y-intercepts ($x = 0$)	$F(0) = 2(0)^2 - 8 \quad \Rightarrow (0, -8)$ $= -8$
Vertex / Notes	$x = \frac{-b}{2a} \quad \Rightarrow (0, -8)$ $= \frac{-0}{2 \cdot 2}$ $= 0$ $y = 2(0)^2 - 8$ $= -8$

(b) $f(x) = +\sqrt{x - 2}$

Domain & Range	Domain : $\{x \in \mathbb{R} \mid x \geq 2\}$ Range : $\{y \in \mathbb{R} \mid y \geq 0\}$
Restrictions	Domain : $x \geq 2$ Range : $y \geq 0$
Increasing / Decreasing	Decreasing : N/A Increasing : $[2, +\infty)$
x-intercepts (roots)	$f(x) = 0 \quad \Rightarrow (2, 0)$ $0 = +\sqrt{x - 2}$ $x = 2$
y-intercepts ($x = 0$)	$F(0) = +\sqrt{0 - 2} \quad \Rightarrow$ N/A (none)
Vertex / Notes	No vertices.

(c) $f(x) = \dfrac{(x+1)}{(x-1)}$

Domain & Range	Domain : $\{x \in \mathbb{R} \mid x \neq 1\}$ Range : $\{y \in \mathbb{R} \mid y \neq 1\}$
Restrictions	Domain : $x \neq 1$ Range : $y \neq 1$
Increasing / Decreasing	Decreasing : $(-\infty, 1) \cup (1, +\infty)$ Increasing : N/A
x-intercepts (roots)	$f(x) = 0 \quad \Rightarrow (-1, 0)$ $0 = \dfrac{x+1}{x-1}$ $x = -1$
y-intercepts ($x = 0$)	$F(0) = \dfrac{0+1}{0-1} \quad \Rightarrow (0, -1)$ $= -1$
Vertex / Notes	No vertices.

Question 5 (8 points)

The point $(1, -2)$ is on the graph of f. Describe the following transformations on f, and determine the resulting point.

We use

$$g(x) = a\, f\big(k(x-d)\big) + c, \qquad \begin{aligned} x' &= \frac{x}{k} + d, \\ y' &= a\,y + c. \end{aligned}$$

a) $g(x) = 2f(x) + 3$

The $a = 2$ indicates a vertical stretch by a factor of 2 and the $c = 3$ indicates a vertical translation of 3 units up.

$$\begin{aligned} x' &= \frac{x}{k} + d \\ &= \frac{1}{1} + 0 \\ &= 1 \\ y' &= a\,y + c \\ &= 2(-2) + 3 \\ &= -1 \end{aligned}$$

Therefore, the resulting point is $\boxed{(1, -1)}$.

b) $g(x) = f(x+1) - 3$

The $d = -1$ (since $x - d = x - (-1) = x + 1$) indicates a horizontal translation of 1 unit to the left and the $c = -3$ indicates a vertical translation of 3 units down.

$$\begin{aligned} x' &= \frac{x}{k} + d \\ &= \frac{1}{1} + (-1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} y' &= a\,y + c \\ &= 1(-2) + (-3) \\ &= -5 \end{aligned}$$

Therefore, the resulting point is $\boxed{(0, -5)}$.

c) $g(x) = -f(2x)$

The $a = -1$ indicates a reflection in the x-axis and the $k = 2$ indicates a horizontal compression by a factor of 1/2.

$$\begin{aligned} x' &= \frac{x}{k} + d \\ &= \frac{1}{2} + 0 \\ &= \tfrac{1}{2} \end{aligned}$$

$$\begin{aligned} y' &= a\,y + c \\ &= (-1)(-2) + 0 \\ &= 2 \end{aligned}$$

Therefore, the resulting point is $\boxed{(\tfrac{1}{2}, 2)}$.

d) $g(x) = -f(-x-1) + 3$

The $a = -1$ indicates a reflection in the x-axis, the $k = -1$ indicates a reflection in the y-axis, the $d = -1$ (from $x - d = x - (-1) = x + 1$) indicates a horizontal translation of 1 unit to the left, and the $c = 3$ indicates a vertical translation of 3 units up.

$$\begin{aligned} x' &= \frac{x}{k} + d \\ &= \frac{1}{-1} + (-1) \\ &= -2 \\ y' &= a\,y + c \\ &= (-1)(-2) + 3 \\ &= 5 \end{aligned}$$

Therefore, the resulting point is $\boxed{(-2, 5)}$.