

# Sample

November 22, 2025

## 1 Preamble / Introduction

The point  $(1, -2)$  is on the graph of  $f$ . Describe the following transformations on  $f$ , and determine the resulting point. We use

$$\begin{aligned}g(x) &= af(k(x - d)) + c, \\x' &= \frac{x}{k} + d, \\y' &= ay + c.\end{aligned}$$

a)  $g(x) = 2f(x) + 3$  The  $a = 2$  indicates a vertical stretch by a factor of 2 and the  $c = 3$  indicates a vertical translation of 3 units up.

$$\begin{aligned}x' &= \frac{1}{1} + 0 = 1, \\y' &= 2(-2) + 3 = -1.\end{aligned}$$

Therefore, the resulting point is  $(1, -1)$ .

b)  $g(x) = f(x + 1) - 3$  The  $d = -1$  (since  $x - d = x - (-1) = x + 1$ ) indicates a horizontal translation of 1 unit to the left and the  $c = -3$  indicates a vertical translation of 3 units down.

$$\begin{aligned}x' &= \frac{1}{1} + (-1) = 0, \\y' &= 1(-2) + (-3) = -5.\end{aligned}$$

Therefore, the resulting point is  $(0, -5)$ .

c)  $g(x) = -f(2x)$  The  $a = -1$  indicates a reflection in the x-axis and the  $k = 2$  indicates a horizontal compression by a factor of 1/2.

$$\begin{aligned}x' &= \frac{1}{2} + 0 = \frac{1}{2}, \\y' &= (-1)(-2) + 0 = 2.\end{aligned}$$

Therefore, the resulting point is  $(\frac{1}{2}, 2)$ .

d)  $g(x) = -f(-x - 1) + 3$  The  $a = -1$  indicates a reflection in the x-axis, the  $k = -1$  indicates a reflection in the y-axis, the  $d = -1$  (from  $x - d = x - (-1) = x + 1$ ) indicates a horizontal translation of 1 unit to the left, and the  $c = 3$  indicates a vertical translation of 3 units up.

$$x' = \frac{1}{-1} + (-1) = -2,$$

$$y' = (-1)(-2) + 3 = 5.$$

Therefore, the resulting point is (-2, 5).