

Sample

November 22, 2025

1 Preamble / Introduction

The point (1, -2) is on the graph of f . Describe the following transformations on f , and determine the resulting point. We use

$$\begin{aligned}g(x) &= af(k(x - d)) + c, \\x' &= \frac{x}{k} + d, \\y' &= ay + c.\end{aligned}$$

a) $g(x) = 2f(x) + 3$ The $a = 2$ indicates a vertical stretch by a factor of 2 and the $c = 3$ indicates a vertical translation of 3 units up.

$$\begin{aligned}x' &= \frac{1}{1} + 0 = 1, \\y' &= 2(-2) + 3 = -1.\end{aligned}$$

Therefore, the resulting point is (1, -1).

b) $g(x) = f(x + 1) - 3$ The $d = -1$ (since $x - d = x - (-1) = x + 1$) indicates a horizontal translation of 1 unit to the left and the $c = -3$ indicates a vertical translation of 3 units down.

$$\begin{aligned}x' &= \frac{1}{1} + (-1) = 0, \\y' &= 1(-2) + (-3) = -5.\end{aligned}$$

Therefore, the resulting point is (0, -5).

c) $g(x) = -f(2x)$ The $a = -1$ indicates a reflection in the x-axis and the $k = 2$ indicates a horizontal compression by a factor of 1/2.

$$\begin{aligned}x' &= \frac{1}{2} + 0 = \frac{1}{2}, \\y' &= (-1)(-2) + 0 = 2.\end{aligned}$$

Therefore, the resulting point is $(\frac{1}{2}, 2)$.

d) $g(x) = -f(-x - 1) + 3$ The $a = -1$ indicates a reflection in the x-axis, the $k = -1$ indicates a reflection in the y-axis, the $d = -1$ (from $x - d = x - (-1) = x + 1$) indicates a horizontal translation of 1 unit to the left, and the $c = 3$ indicates a vertical translation of 3 units up.

$$x' = \frac{1}{-1} + (-1) = -2,$$

$$y' = (-1)(-2) + 3 = 5.$$

Therefore, the resulting point is $(-2, 5)$.