Basic Advanced Functions — Part 1: Communication Problems

Your Name

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Question 1

(6 points)

a) An airplane must travel 400 km. Let t be the travel time (in hours) and let $s(t)$ denote the speed (in km/h).

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$s(t) = \frac{400}{t} \quad (t > 0,\ \text{km/h})$$

b) An ice cream cone starts at 125 mL and loses half its volume every 5 min. Let t be in minutes and $v(t)$ be the volume (mL); the discrete half-life model is

$$v(t) = v_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$v(t) = 125 \left(\frac{1}{2}\right)^{t/5}$$

c) Scott drives at a constant speed of 50 km/h. If $d(t)$ is the distance (km) after t hours,

$$d(t) = 50t$$

Question 2

(6 points)

a)

$$
\begin{aligned}
p(r) &= 2r^2 + 2r - 1 \\
x &= 2y^2 + 2y - 1 \\
x + 1 &= 2\left(y + 1\right)^2 - 1 \\
x + 1 &= 2\left(\frac{y+1}{2}\right)^2 - 1 \\
x + 1 &= 2\left(\frac{(y+1)^2}{4}\right) - 1 \\
x + 1 &= \frac{(y+1)^2}{2} - 1 \\
x + 3 &= \frac{(y+1)^2}{2} \\
2(x+3) &= (y+1)^2 \\
y + 1 &= \pm\sqrt{2(x+3)} \\
y &= -1 \pm \sqrt{2(x+3)} \\
p^{-1}(x) &= -1 \pm \sqrt{2(x+3)}
\end{aligned}
$$

b)

$$
\begin{aligned}
3y + 5x = 18 \\
3y &= -5x + 18 \\
y &= -\frac{5}{3}x + 6 \\
x &= -\frac{5}{3}y + 6 \\
x - 6 &= -\frac{5}{3}y \\
y &= -\frac{3}{5}(x - 6) \\
y &= -\frac{3}{5}x + \frac{18}{5} \\
f^{-1}(x) &= -\frac{3}{5}x + \frac{18}{5}
\end{aligned}
$$

c)

$$\begin{aligned}
h(t) &= -4.9(t+3)^2 + 45.8 \\
x &= -4.9(y+3)^2 + 45.8 \\
x - 45.8 &= -4.9(y+3)^2 \\
45.8 - x &= 4.9(y+3)^2 \\
\frac{45.8 - x}{4.9} &= (y+3)^2 \\
y + 3 &= \pm\sqrt{\frac{45.8 - x}{4.9}} \\
y &= -3 \pm \sqrt{\frac{45.8 - x}{4.9}} \\
h^{-1}(x) &= -3 \pm \sqrt{\frac{45.8 - x}{4.9}}
\end{aligned}$$

Question 3

(6 points)

a)

- Inverse (reflection across $y = x$).
- Vertical line test: fails.

Construct inverse: reflect graph of $y = p(x)$ across $y = x$

$$\Rightarrow \text{graph of } p^{-1}$$

Apply VLT to p^{-1}: some verticals cut the graph twice

$$\Rightarrow \text{Inverse is not a function}$$

Domain/Range swap: $\mathrm{Dom}(p^{-1}) = \mathrm{Ran}(p)$, $\mathrm{Ran}(p^{-1}) = \mathrm{Dom}(p)$

b)

- Inverse of a line (reflection across $y = x$).
- Vertical line test: passes.

Construct inverse: reflect non-vertical line across $y = x$

$$\Rightarrow \text{another non-vertical line}$$

Apply VLT to f^{-1}: each vertical meets at most once

$$\Rightarrow \text{Inverse is a function}$$

Domain/Range swap: $\mathrm{Dom}(f^{-1}) = \mathrm{Ran}(f)$, $\mathrm{Ran}(f^{-1}) = \mathrm{Dom}(f)$

Question 5

(8 points)

The point $(1, -2)$ is on the graph of f. Describe the following transformations on f, and determine the resulting point.

We use

$$g(x) = af(k(x-d)) + c,$$
$$x' = \frac{x}{k} + d,$$
$$y' = ay + c.$$

a) $g(x) = 2f(x) + 3$

The $a = 2$ indicates a vertical stretch by a factor of 2 and the $c = 3$ indicates a vertical translation of 3 units up.

$$x' = \frac{x}{k} + d = 1 \cdot \frac{1}{1} + 0 = 1$$
$$y' = ay + c = 2(-2) + 3 = -1$$

Therefore, the resulting point is $(1, -1)$.

b) $g(x) = f(x+1) - 3$

The $d = -1$ (since $x - d = x - (-1) = x + 1$) indicates a horizontal translation of 1 unit to the left and the $c = -3$ indicates a vertical translation of 3 units down.

$$x' = \frac{x}{k} + d = 1 \cdot \frac{1}{1} + (-1) = 0$$
$$y' = ay + c = 1(-2) + (-3) = -5$$

Therefore, the resulting point is $(0, -5)$.

c) $g(x) = -f(2x)$

The $a = -1$ indicates a reflection in the x-axis and the $k = 2$ indicates a horizontal compression by a factor of $\frac{1}{2}$.

$$x' = \frac{x}{k} + d = 1 \cdot \frac{1}{2} + 0 = \frac{1}{2}$$
$$y' = ay + c = (-1)(-2) + 0 = 2$$

Therefore, the resulting point is $\left(\frac{1}{2}, 2\right)$.

d) $g(x) = -f(-x-1) + 3$

The $a = -1$ indicates a reflection in the x-axis, the $k = -1$ indicates a reflection in the y-axis, the $d = -1$ (from $x - d = x - (-1) = x + 1$) indicates a horizontal translation of 1 unit to the left, and the $c = 3$ indicates a vertical translation of 3 units up.

$$x' = \frac{x}{k} + d = 1 \cdot \frac{1}{-1} + (-1) = -2$$
$$y' = ay + c = (-1)(-2) + 3 = 5$$

Therefore, the resulting point is $(-2, 5)$.