Sample

November 23, 2025

1 Preamble / Introduction

Basic Advanced Functions — Part 1: Communication Problems Your Name October 28, 2025

Question 01

Question~**??** (6 points)

$$Rewrite each relationship using function notation. All given text retained; one “=" per line; equals$$

aligned.

a) An airplane must travel 400 km. Let t be the travel time (in hours) and let $s(t)$ denote the speed

$$(in km/h). Speed = Distance$$

Time $t > 0$, km/h

$$s(t) = 400$$

b) An ice cream cone starts at 125 mL and loses half its volume every 5 min. Let t be in minutes and $v(t)$ be the volume (mL); the discrete half-life model is

$$v(t) = v_0 \frac{1}{2^{t/T_{1/2}}}$$

where $T_{1/2}$ is the half-life period. Given that $v_0 = 125$:

$$v(t) = 125 \frac{1}{2^{t/5}}$$

c) Scott drives at a constant speed of 50 km/h. If $d(t)$ is the distance (km) after t hours:

$$d(t) = 50t$$

Question 01

Question~**??** (6 points)

$Formatting : parts(a)and(b)aresidebysidewithacleandivider; all`` ='' signsalignedinsideeachblock.$

1 a) $p(r) = 2r^2 + 2r - 1$ b) $3y + 5x = 18$

$$x = 2y^2 + 2y - 1$$
$$3y + 5x = 18$$
$$x + 1 = 2$$
$$y^2 + y$$

$$3y = -5x + 18$$
$$x + 1 = 2$$

y + 1 2 4

$$x = -5$$

2 -1

$$x + 1 = 2$$

y + 1 3y + 6 2 2

$$x - 6 = -5$$
$$2 = 2$$

y + 1 2 3y x + 3 2

$$y = -3$$

5(x -6) x 2 + 3 2

$$4 =$$

y + 1 5x + 18

$$y = -3$$

2 5

$$2 = \pm q$$

y + 1 x 2 + 3

$$f^{-1}(x) = -3 \pm q$$

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Question 03

Question~**??** (6 points)

Using graphs, decide whether each inverse is a function. Figures are side by side (uniform size) with concise captions. Below each pair, the reasoning lines up the "⇒" arrows and the verdict is boxed.

a) p^{-1}

$$iInverse(reflectionacrossy = x).$$

ii Vertical line test: fails. Construct inverse : reflect graph of $y = p(x)$ across $y = x \Rightarrow$ graph of p^{-1} Apply VLT to p^{-1}: some verticals cut the graph twice $\Rightarrow$ Inverse is not a function Domain/Range swap : Domp^{-1} = Ran$p$, Ranp^{-1} = Dom$p$

b) f^{-1}

$$iInverseofaline(reflectionacrossy = x).$$

ii Vertical line test: passes. Construct inverse : reflect non-vertical line across $y = x \Rightarrow$ another non-vertical line Apply VLT to f^{-1}: each vertical meets at most once $\Rightarrow$ Inverse is a function Domain/Range swap : Domf^{-1} = Ran$f$, Ranf^{-1} = Dom$f$

3 c) h^{-1} Construct inverse : reflect graph of $y = h(x)$ across $y = x \Rightarrow$ relation h^{-1} Apply VLT to h^{-1}: fails (some verticals cut twice) $\Rightarrow$ Inverse is not a function Domain/Range swap : Domh^{-1} = Ran$h$, Ranh^{-1} = Dom$h$

Question 04

Question~**??** (24 points)

All original answers preserved. Reformatted into three readable "summary cards" (no clipping; full-size math).

(a) $f(x) = 2x^2 - 8$ 4 Domain & Domain : $\mathbb{R}$ Range Range : $\{y \in \mathbb{R} | y \geq -8\}$ Restrictions Domain : None Range : $y \geq -8$ Increasing / Decreasing : $(-\infty, 0)$ increasing Increasing : $(0, +\infty)$ decreasing x-intercepts $\Rightarrow$ $(2, 0)$, $(-2, 0)$ $f(x) = 0$ roots $0 = 2x^2 - 8$ 8 2 $= x^2$ $x = \pm 2$ $F(0) = 2(0)^2 - 8$ $y - intercepts \Rightarrow (0, -8)$ $(x = 0) = -8$ Vertex / Notes $x = -b \Rightarrow (0, -8)$ $2a = -0$ $2 \cdot 2 = 0$ $y = 2(0)^2 - 8 = -8$

(b) $f(x) = +\sqrt{x} - 2$ Domain & Domain : $\{x \in \mathbb{R} | x \geq 2\}$ Range Range : $\{y \in \mathbb{R} | y \geq 0\}$ Restrictions Domain : $x \geq 2$ Range : $y \geq 0$ Increasing / Decreasing : N/A increasing Increasing : $[2, +\infty)$ x-intercepts $f(x) = 0 \Rightarrow (2, 0)$ $0 = +\sqrt{(roots)}\, x - 2$ $x = 2$ $F(0) = +\sqrt{(roots)}\, y - intercepts$ $0 - 2 \Rightarrow N/A(none)$ $(x = 0)$ $Vertex/Notes\ Novertices.$

(c) $f(x) = \frac{(x+1)}{(x-1)}$ Domain : $\{x \in \mathbb{R} | x \neq 1\}$ Range : $\{y \in \mathbb{R} | y \neq 1\}$ Restrictions Domain : $x \neq 1$ Range : $y \neq 1$ Decreasing : $(-\infty, 1) \cup (1, +\infty)$ Increasing : N/A $\Rightarrow (-1, 0)$ $0 = x + 1$ $x - 1$ $x = -1$ $F(0) = 0 + 1 \Rightarrow (0, -1)$ $0 - 1 = -1$

Question 06

Question~**??** (8 points)

The point (1, -2) is on the graph of f. Describe the following transformations on f, and determine the resulting point.

We use $g(x) = af(k(x - d)) + c$, where $x' = x$ and $k + d$.

a) $g(x) = 2f(x) + 3$ The $a = 2$ indicates a vertical stretch by a factor of 2 and the $c = 3$ indicates a vertical translation of 3 units up. $x' = x$ $k + d = 1$ $1 + 0 = 1$ $y' = ay + c = 2(-2) + 3 = -1$ Therefore, the resulting point is $(1, -1)$.

6 b) $g(x) = f(x+1) - 3$ The $d = -1$ (since $x - d = x - (-1) = x + 1$) indicates a horizontal translation of 1 unit to the left and the $c = -3$ indicates a vertical translation of 3 units down. $x' = x$ $1 + (-1) = 0 = 1(-2) + (-3) = -5$ Therefore, the resulting point is $(0, -5)$.

c) $g(x) = -f(2x)$ The $a = -1$ indicates a reflection in the x-axis and the $k = 2$ indicates a horizontal compression by a factor of $1/2$. $x' = x$ $2 + 0$ $2 = (-1)(-2) + 0 = 2$ $2, 2$. Therefore, the resulting point is $(2, 2)$.

d) $g(x) = -f(-x-1) + 3$ The $a = -1$ indicates a reflection in the x-axis, the $k = -1$ indicates a reflection in the y-axis, the $d = -1$ (from $x - d = x - (-1) = x + 1$) indicates a horizontal translation of 1 unit to the left, and the $c = 3$ indicates a vertical translation of 3 units up. $k + d$ $= 1$ $-1 + (-1) = -2$ $y' = ay + c = (-1)(-2) + 3 = 5$ Therefore, the resulting point is $(-2, 5)$.

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