Sample

November 27, 2025

1 Preamble / Introduction

Basic Advanced Functions — Part 1: Communication Problems Your Name October 28, 2025 Question 1 (6 points) Rewrite each relationship using function notation. All given text retained; one "=" per line; equals aligned. a) An airplane must travel 400 km. Let t be the travel time (in hours) and let s(t) denote the speed

$$(in km/h).$$

Speed = Distance Time

$$\begin{aligned} (t > 0, km/h) \\ s(t) = 400 \end{aligned}$$

t b) An ice cream cone starts at 125 mL and loses half its volume every 5 min. Let t be in minutes and v(t) be the volume (mL); the discrete half-life model is

$$\begin{aligned} t/T1/2 \\ v(t) = v0 \end{aligned}$$

1 2

$$\begin{aligned} t/5 \\ v(t) = 125 \end{aligned}$$

1 2 c) Scott drives at a constant speed of 50 km/h. If d(t) is the distance (km) after t hours,

$$d(t) = 50t$$

Question 2 (6 points) Formatting: parts (a) and (b) are side by side with a clean divider; all "=" signs aligned inside each block. 1

$$a)p(r) = 2r2 + 2r - 1$$
$$b)3y + 5x = 18$$
$$x = 2y2 + 2y - 1$$
$$3y + 5x = 18$$
$$x + 1 = 2$$
$$y2 + y$$

$$3y = -5x + 18$$

2 -1

$$x + 1 = 2$$

$$y + 1$$

$$y = -5$$
$$3x + 6$$

2 4 2 -1

$$x = -5$$
$$3y + 6$$
$$x + 1 = 2$$

$$y + 1$$

2 2

$$x - 6 = -5$$

3y 2

$$2 = 2$$

$$y + 1$$
$$x + 3$$

2

$$y = -3$$

5(x -6) x

$$2 + 3$$

2

$$4 =$$

$$y + 1$$
$$5x + 18$$
$$y = -3$$

2 5

$$2 = \pm$$

q x

$$2 + 3$$
$$y + 1$$
$$f - 1(x) = -3$$
$$5x + 18$$

4 5 2 ± q x

$$2 + 3$$
$$y = -1$$

4 2 ± q

$$p - 1(x) = -1$$

x

$$2 + 3$$

$$
\begin{aligned}
c)h(t) &= -4.9(t+3)2 + 45.8 \\
x &= -4.9(y+3)2 + 45.8 \\
x - 45.8 &= -4.9(y+3)2 \\
45.8 - x &= 4.9(y+3)2
\end{aligned}
$$

45.8 -x 4.9

$$= (y+3)2$$

r45.8 -x

$$y + 3 = \pm$$

4.9 r45.8 -x

$$y = -3\pm$$

4.9 r45.8 -x

$$h - 1(x) = -3\pm$$

4.9 2 Question 3 (6 points) Using graphs, decide whether each inverse is a function. Figures are side by side (uniform size) with concise captions. Below each pair, the reasoning lines up the "⇒" arrows and the verdict is boxed. a) p-1 i Inverse (reflection across y = x). ii Vertical line test: fails. Construct inverse : reflect graph of y = p(x) across y = x ⇒ graph of p-1 Apply VLT to p-1 : some verticals cut the graph twice ⇒ Inverse is not a function

$$Domain/Rangeswap : Dom(p-1) = Ran(p), Ran(p-1) = Dom(p)$$

b) f -1 i Inverse of a line (reflection across y = x). ii Vertical line test: passes. Construct inverse : reflect non-vertical line across y = x ⇒ another non-vertical line Apply VLT to f-1 : each vertical meets at most once ⇒ Inverse is a function

$$Domain/Rangeswap : Dom(f-1) = Ran(f), Ran(f-1) = Dom(f)$$

3 c) h-1 Construct inverse : reflect graph of y = h(x) across y = x ⇒ relation h-1 Apply VLT to h-1 : fails (some verticals cut twice) ⇒ Inverse is not a function

$$Domain/Rangeswap : Dom(h-1) = Ran(h), Ran(h-1) = Dom(h)$$

Question 4 (24 points) All original answers preserved. Reformatted into three readable "summary cards" (no clipping; full-size math). (a)

$$f(x) = 2x2 - 8$$

4 Domain : x ∈ ℝ Domain & Range Range : y ∈ ℝ | y ≥-8 Restrictions Domain : None Range : y ≥-8 Decreasing : (-∞, 0) Increasing / De- creasing

$$Increasing : (0, +\infty)$$

⇒(2, 0), (-2, 0) x-intercepts

$$f(x) = 0$$
$$0 = 2x2 - 8$$

8

$$2 = x2$$

(roots)

$$x = \pm 2$$
$$F(0) = 2(0)2 - 8$$

⇒(0, -8) y-intercepts

$$(x = 0)$$
$$= -8$$

Vertex / Notes

$$x = -b$$

⇒(0, -8) 2a

$$= -0$$

2 · 2

$$= 0$$
$$y = 2(0)2 - 8$$
$$= -8$$

(b)

$$f(x) = +\sqrt{x} - 2$$

Domain : x ∈ ℝ | x ≥2 Domain & Range Range : y ∈ ℝ | y ≥0 Restrictions Domain : x ≥2 Range : y ≥0 Decreasing : N/A Increasing / De- creasing

$$Increasing : [2, +\infty)$$

⇒(2, 0) x-intercepts

$$f(x) = 0$$
$$0 = +$$

√ (roots) x -2

$$x = 2$$
$$F(0) = +$$

√ 0 -2 ⇒ N/A (none) y-intercepts

$$(x = 0)$$

Vertex / Notes No vertices. 5 (c)

$$f(x) = (x + 1)$$

(x -1)

$$Domain : x \in \mathbb{R} | x = 1$$
$$Range : y \in \mathbb{R} | y = 1$$

Restrictions

$$Domain : x = 1$$
$$Range : y = 1$$
$$Decreasing : (-\infty, 1) \cup (1, +\infty)$$

Increasing : N/A ⇒(-1, 0)

$$0 = x + 1$$

x -1

$$x = -1$$
$$F(0) = 0 + 1$$

⇒(0, -1) 0 -1

$$= -1$$

Question 5 (8 points) The point (1, -2) is on the graph of f. Describe the following transformations on f, and determine the resulting point. We use

$$g(x) = af$$

k(x -d)

$$+c,$$
$$x' = x$$
$$k + d,$$
$$y' = ay + c.$$
$$a)g(x) = 2f(x) + 3$$

The a = 2 indicates a vertical stretch by a factor of 2 and the c = 3 indicates a vertical translation of 3 units up.

$$x' = x$$
$$k + d$$
$$= 1$$
$$1 + 0$$
$$= 1$$
$$y' = ay + c$$
$$= 2(-2) + 3$$
$$= -1$$

Therefore, the resulting point is (1, -1) . 6

$$b)g(x) = f(x + 1) - 3$$
$$The d = -1 (since x - d = x - (-1) = x + 1) indicates a horizontal translation of 1 unit to the left$$

and the c = -3 indicates a vertical translation of 3 units down.

$$x' = x$$
$$1 + (-1)$$
$$= 0$$
$$= 1(-2) + (-3)$$
$$= -5$$

Therefore, the resulting point is (0, -5) .

$$c)g(x) = -f(2x)$$

The a = -1 indicates a reflection in the x-axis and the k = 2 indicates a horizontal compression by

$$a factor of 1/2.$$
$$x' = x$$
$$2 + 0$$

2

$$= (-1)(-2) + 0$$
$$= 2$$

2, 2 . Therefore, the resulting point is 1 7

$$d)g(x) = -f(-x - 1) + 3$$

The a = -1 indicates a reflection in the x-axis, the k = -1 indicates a reflection in the y-axis, the

$$d = -1(fromx - d = x - (-1) = x + 1)indicatesahorizontaltranslationof1unittotheleft, and$$

the c = 3 indicates a vertical translation of 3 units up.

$$k + d$$
$$= 1$$
$$-1 + (-1)$$
$$= -2$$
$$y' = ay + c$$
$$= (-1)(-2) + 3$$
$$= 5$$

Therefore, the resulting point is (-2, 5) . 8