

Question 1:

1.1 A

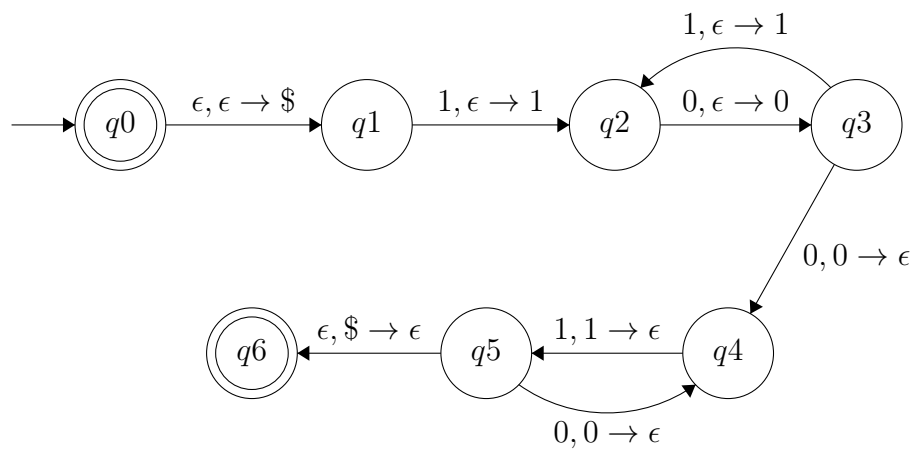
RegEx for:

$$M_A = (10)^*$$

$$M_B = (01)^*$$

1.2 B

PDA for $L_{A||B} = \{uv | u \in L(M_A) \cap v \in L(M_B) \cap |u| = |v|\}$:



1.3 C

Parsing Input: 10100101

Input:															
Stack:	\$	€													

 $[(q_0 \rightarrow q_1) : \epsilon, \epsilon \rightarrow \$]$

Input:	1														
Stack:	1	\$	€												

 $[(q_1 \rightarrow q_2) : 1, \epsilon \rightarrow 1]$

Input:	1	0													
Stack:	0	1	\$	€											

 $[(q_2 \rightarrow q_3) : 0, \epsilon \rightarrow 0]$

Input:	1	0	1												
Stack:	1	0	1	\$	€										

 $[(q_3 \rightarrow q_2) : 1, \epsilon \rightarrow 1]$

Input:	1	0	1	0											
Stack:	0	1	0	1	\$	€									

 $[(q_2 \rightarrow q_3) : 0, \epsilon \rightarrow 0]$

Input:	1	0	1	0	0										
Stack:	1	0	1	\$	€										

 $[(q_3 \rightarrow q_4) : 0, 0 \rightarrow \epsilon]$

Input:	1	0	1	0	0	1	0								
Stack:	0	1	\$	€											

 $[(q_4 \rightarrow q_5) : 1, 1 \rightarrow \epsilon]$

Input:	1	0	1	0	0	1	0	0							
Stack:	1	\$	€												

 $[(q_5 \rightarrow q_4) : 0, 0 \rightarrow \epsilon]$

Input:	1	0	1	0	0	1	0	1							
Stack:	\$	€													

 $[(q_4 \rightarrow q_5) : 1, 1 \rightarrow \epsilon]$

Input:	1	0	1	0	0	1	0	1							
Stack:	€														

 $[(q_5 \rightarrow q_6) : \epsilon, \$ \rightarrow \epsilon]$

Input Accepted.

Question 2:

2.1 A

Grammar for $L_1 = \{1^n 0^{2n} | n \geq 0\}$:

$$S \rightarrow 1S00 | \epsilon$$

2.2 B

Considering the string $w = 1^p 0^{2p} \in L_1$:

For a split into the parts $uvxyz$, the pump-able segment v must consist of only 1, so $v = 1$ and segment y must consist of only 00, $y = 00$. So when you pump both segments as follows: $uv^i xy^i z \forall i \geq 0$ the condition that there are two times as many 0's as 1's will hold.

2.3 C

Considering the string $w = 1^p 0^{2p} 1^p \in L_1$:

There are a number of possible cases for a split into the parts $uvxyz$.

One is that the pump-able segment v would consist of 1, so $v = 1$ and segment y would consist of 00, so $y = 00$. This is not pump-able since the succeeding number of 1's won't equal the preceding number.

Another is that the pump-able segment v would consist of 10, so $v = 10$ and segment y would consist of 1, so $y = 1$. This is not pump-able since the succeeding number of 0's won't be twice the number of preceding and succeeding 1's.

Another is that the pump-able segment v would consist of 1, so $v = 1$ and segment y would consist of 01, so $y = 01$. This is not pump-able since the number of 0's won't be twice the number of preceding and succeeding 1's and they would be out of order.

Another is that the pump-able segment v would consist of 10, so $v = 10$ and segment y would consist of 01, so $y = 01$. This is not pump-able since the values would be out of order.

So in any condition, if you pump both segments as follows: $uv^i xy^i z \forall i \geq 0$ the condition that there are n 1's followed by $2n$ 0's followed by n 1's again can't possibly hold.