Problem 1:

$$LIST(A) = \{'()\} \cup \{(\text{cons } a \text{ as}) \mid a \in A \wedge as \in LIST(A)\}$$

Let $ATOM = BOOL \cup NUM \cup SYM \cup \{'()\}$

$$SEXP_{FG} = ATOM \cup \{(\text{cons } v_1\ v_2) \mid v_1 \in SEXP_{FG} \wedge v_2 \in SEXP_{FG}\}$$

Let $A = SEXP_{FG}$

$$\rightarrow LIST(SEXP_{FG}) = \{'()\} \cup \{(\text{cons } a \text{ as}) \mid a \in SEXP_{FG} \wedge as \in LIST(SEXP_{FG})\}$$

Since $ATOM \in SEXP_{FG}$ & $\{'()\} \in ATOM$

Therefore the two sets, $SEXP_{FG}$ and $LIST(SEXP_{FG})$

$$LIST(SEXP_{FG}) = ATOM \cup \{(\text{Cons } v_1\ v_2) \mid v_1 \in SEXP_{FG} \wedge v_2 \in SEXP_{FG}\}$$

are identical, they are the same set

$\therefore LIST(SEXP_{FG}) \subseteq SEXP_{FG}$

Q.E.D

Problem 37:

Prove (append (append xs ys) zs) = (append zs (append xs ys))

let ~~ls~~ ls = (append xs ys)

(append ls zs) = (append zs ls)

Given: (append (cons p ps) gs) = (cons p (append ps gs))

let ls = (cons p ps) = (append ps gs)

(append ls gs) = (cons p (append ps gs))

$\therefore$ (append (append ps gs) p) = (append p (append ps gs))

Q.E.D

Problem A:

A. $(cdr\ (cons\ x\ xs)) = xs$

let ~~[illegible]~~
$(y) = (y_1\, y_2 \cdots y_n)$
then
$(cdr\ y) = (y_2 \cdots y_n)$ $\quad y_1 \notin (cdr\ y)$

$(xs) = (x_1\, x_2\, x_3 \ldots x_n)$
$(x) = x$

$(cons\ x\ xs) = \{x\} \cup \{xs\} = (x\ x_1\, x_2\, x_3 \cdots x_n)$

$(cdr\,(cons\ x\ xs)) = (x_1\, x_2\, x_3 \cdots x_n) = xS$
Q.E.D

DERIVATIONS

B. CDR

$$\frac{\langle e, \rho, \sigma_0\rangle \Downarrow \langle PRIMATIVE(car), \sigma_1\rangle \qquad \langle e_1, \rho, \sigma_1\rangle \Downarrow \langle PAIR(l_1, l_2), \sigma_2\rangle}{\langle apply(e, e_1), \rho, \sigma_0\rangle \Downarrow \langle \sigma_2(l_2), \sigma_2\rangle}$$

ONS

$$\frac{\begin{array}{c}\langle e, \rho, \sigma_0\rangle \Downarrow \langle PRIMATIVE(cons), \sigma_1\rangle \\ \langle e_1, \rho, \sigma_1\rangle \Downarrow \langle v_1, \sigma_2\rangle \\ \langle e_2, \rho, \sigma_2\rangle \Downarrow \langle v_2, \sigma_3\rangle \\ l_1 \notin dom\,\sigma_3 \quad l_2 \notin dom\,\sigma_3 \quad l_1 \neq l_2\end{array}}{\langle apply(e, e_1, e_2), \rho, \sigma_0\rangle \Downarrow \langle PAIR\langle l_1, l_2\rangle, \sigma_3\{l_1 \mapsto v_1, l_2 \mapsto v_2\}}$$

let $e_1 = CONS$

$$\frac{\langle e_1, \rho, \sigma_1\rangle \Downarrow \langle PAIR(l_1, l_2)\, \rho_2\rangle}{\langle apply(e, e_1), \rho, \sigma_0\rangle \Downarrow \langle \sigma_2(l_2), \sigma_2\rangle}$$

from the Cons derivation we get
$(cdr\ (PAIR\langle l_1, l_2\rangle) = \sigma_2(l_2)$
$\therefore$ for all l_1, l_2
$\therefore (cdr\,(cons\ l_1\ l_2)) = l_2$