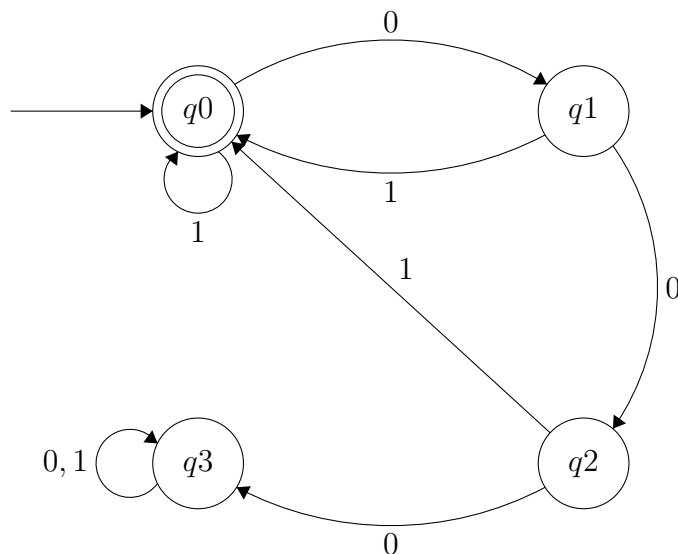


Question 1:

$$L_1 = \{w \mid w \text{ does not contain the substring } 000\}$$

1.1 Construct a DFA



1.2 Full Formal Specifications of the Machine

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{0, 1\}$$

$$\delta : Q \times \Sigma \rightarrow Q$$

$$\delta(q_0, 1) = q_0$$

$$\delta(q_0, 0) = q_1$$

$$\delta(q_1, 1) = q_0$$

$$\delta(q_1, 0) = q_2$$

$$\delta(q_2, 1) = q_0$$

$$\delta(q_2, 0) = q_3$$

$$\delta(q_3, 0) = q_3$$

$$\delta(q_3, 1) = q_3$$

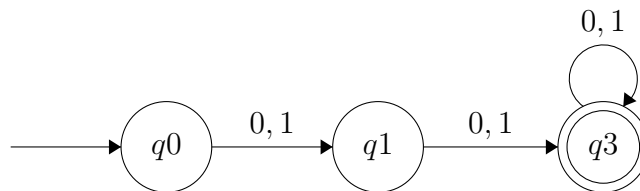
$$q_0 \in Q \text{ is initial state}$$

$$F = q_0$$

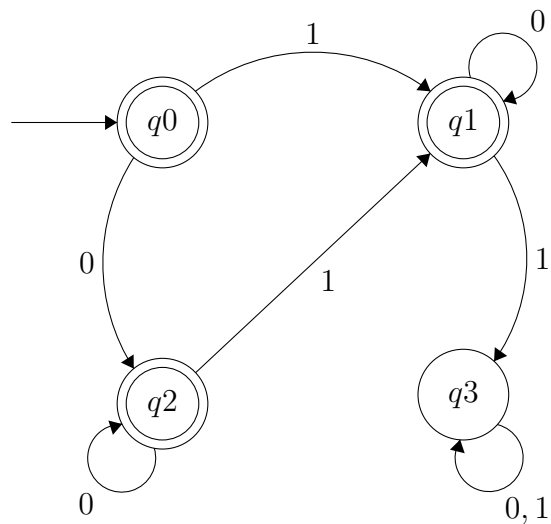
Question 2:

$$L_2 = \{w \mid |w| \geq 2, \text{ but } w \text{ contains less than two 1's}\}$$

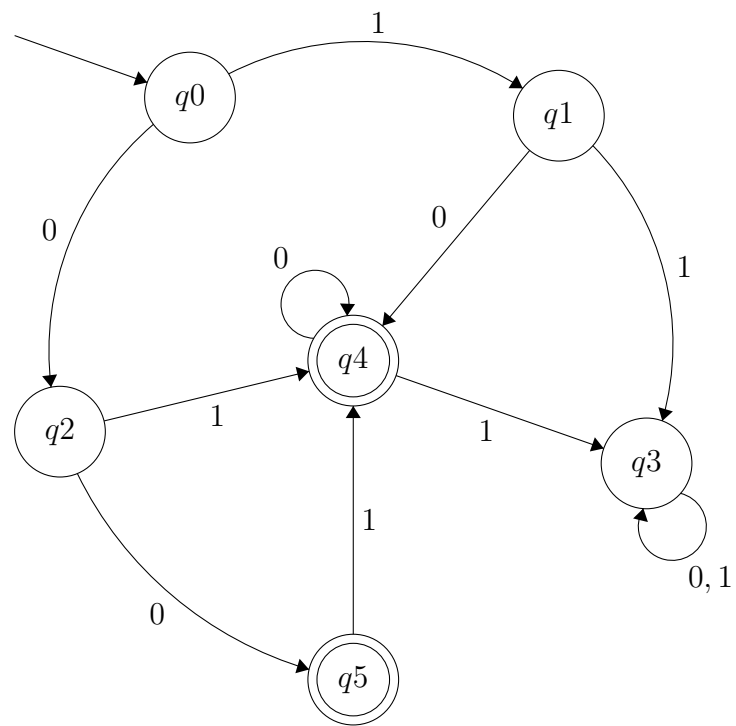
2.1 DFA for $L_2 = \{w \mid |w| \geq 2\}$



2.2 DFA for $L_2 = \{w \mid w \text{ contains less than two 1's}\}$



2.3 DFA for combination of both other DFA's

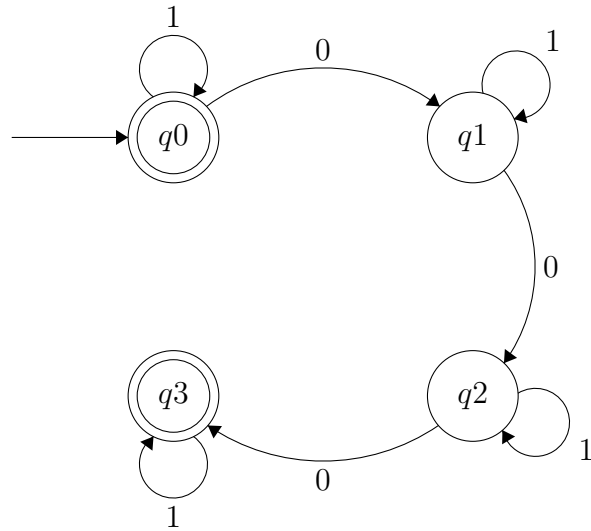


Question 3:

Prove that the following language is regular:

$$L_3 = \{w \mid \text{if } w \text{ contains any 0's, then it contains at least three of them}\}$$

Proof. A language is regular if one can construct a DFA which can recognize it. The following DFA can recognize L_3 :



□

3.1 Full Formal Specifications of the Machine

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{0, 1\}$$

$$\delta : Q \times \Sigma \rightarrow Q$$

$$\delta(q_0, 1) = q_0$$

$$\delta(q_0, 0) = q_1$$

$$\delta(q_1, 1) = q_1$$

$$\delta(q_1, 0) = q_2$$

$$\delta(q_2, 1) = q_2$$

$$\delta(q_2, 0) = q_3$$

$$\delta(q_3, 1) = q_3$$

$$q_0 \in Q \text{ is initial state}$$

$$F = \{q_0, q_3\}$$