Vladimir Hugec

Question 1:

$L_{neq} = \{w|$ w contains a different number of a's, b's, and c's$\}$

Proof. Suppose L_{neq} is a regular language. Let p be the pumping length as defined by the Pumping Lemma. Let $w = a^j b^k c^l$, s.t. $j \neq k \neq l$.

By pumping lemma, all words in the language longer than p can be pumped, $|w| \geq p$. Let $j = 2$, $k = 1$, and $l = 3$ arbitrarily. And so $w = aabccc$.

Divide the string, w into parts xyz s.t. $|xy| \leq p$ as follows: $x|y|z = aa|b|ccc$. Therefore by pumping lemma if $xyz \in L_{neq}$ then $xy^iz \in L_{neq}$ $\forall i \geq 0$.

However, if we let $i = 2$, then we have $xy^2z = aa|bb|ccc$. However $xy^2z \notin L_{neq}$ since there are the same number of a's as well as b's.

There are two other possible ways to divide w in xyz. $x|y|z = a|ab|ccc$ and $x|y|z = aa|bc|cc$. In both cases $xyz \notin L_{neq}$. In the former division, if we let $i = 2$, then we have $xy^2z = a|abab|ccc$, where the string contains an equal number of a's and c's. And in the latter division, if we let $i = 2$ again, then we have $xy^2z = aa|bcbc|cc$, where the string contains an equal number of a's and b's.

Therefore, no matter how the string w is divided into xyz, there exist contradictions to the pumping lemma and therefore L_{neq} is not regular. And since it is know that languages are closed under complement, the complement of L_{neq} is also an irregular language

□

Question 2:

$L_{\geq} = \{w1w2 |\ w2 = b^n, n \geq 1, \text{ and } w1 \in \Sigma^* \text{ contains } m \geq n \text{ a's}\}$
$L_{\leq} = \{w1w2 |\ w2 = b^n, n \geq 1, \text{ and } w1 \in \Sigma^* \text{ contains } m \leq n \text{ a's}\}$

2.1 A

I could not figure out how to do a RegEx for this language. I felt like I've tried everything. Im not sure how on earth you limit the number of succeeding b's to be at most the number of a's present.

2.2 B

Suppose $L_{\leq}$ is a regular language. Let p be the pumping length as defined by the Pumping Lemma.

By pumping lemma, all words in the language longer than p can be pumped, $|w| \geq p$. Let $w = abaabb$ arbitrarily.

Divide the string, w into parts xyz s.t. $|xy| \leq p$ as follows: $x|y|z = ab|a|abb$. Therefore by pumping lemma if $xyz \in L_{\leq}$ then $xy^iz \in L_{\leq}\ \forall i \geq 0$.

However, if we let $i = 3$, then we have $xy^2z = ab|aaa|abb$. However $xy^2z \notin L_{\leq}$ since the number of a's exceeds the number of succeeding b's.

There are other possible ways to divide w in xyz. $x|y|z = aba|ab|b$. If we let $i = 2$ here, we see that, once again, the number of a's exceeds the number of succeeding b's: $x|y^2|z = aba|abab|b$. Similarly, if we divide, $x|y|z = a|baa|bb$ and let $i = 2$ again we see the same problem. No matter how this string gets divided there exist contradictions to the pumping lemma and therefore $L_{\leq}$ is not regular.

Question 3:

$L_* = \{w1 * w2 = w3|$ w1, w2, and w3 are binary numbers, and w3 is the product of w1 and w2$\}$

Proof. Suppose L_* is regular. Let p be the pumping length as defined by the Pumping Lemma.

By pumping lemma, all words in the language longer than p can be pumped, $|w| \geq p$ Let $w = 1 * 10 = 10$ arbitrarily.

Divide the string, w into parts xyz s.t. $|xy| \leq p$: $x|y|z = 1 * |10| = 10$. Therefore by pumping lemma if $xyz \in L_*$ then $xy^iz \in L_*$ $\forall i \geq 0$.

However, if we simply let $i = 2$, then we have $xy^2z = 1 * |1010| = 10$. However $xy^2z \notin L_*$ since this is not a valid binary multiplication.

Therefore, the language L_* is not regular.

$\square$