

PROBLEM M:

Prove: $(o ((\text{curry map}) f) ((\text{curry map}) g)) == ((\text{curry map}) (o f g))$

PROOF:

GIVEN:

$(\text{map } f \text{ '()}) == \text{'()}$; map-nil
$(\text{map } f (\text{cons } y \text{ ys})) == (\text{cons } (f y) (\text{map } f \text{ ys}))$; map-cons
$((\text{curry } f) x) y == (f x y)$; curry
$((\text{uncurry } f) x y) == ((f x) y)$; un-curry
$((o f g) x) == (f (g x))$; apply-compose law

Allow y to be an arbitrary input on the left hand side of the proof equation. If we can show that the left hand side acting on input y , is in reality the same thing as the right hand side acting on y .

> Start with left hand side with addition of input y
 $(o ((\text{curry map}) f) ((\text{curry map}) g) y)$
> apply-compose law
 $(((\text{curry map } f) ((\text{curry map}) g) y)$
> curry law
 $(\text{map } f ((\text{curry map}) g) y)$
> curry Law
 $(\text{map } f (\text{map } g y))$
> apply-compose law
 $(\text{map } (o f g) y)$
> curry law
 $(((\text{curry map}) (o f g))$

Therefore we can see that starting with $(o ((\text{curry map}) f) ((\text{curry map}) g) y)$ and applying just the given 'apply-compose' law and the 'curry' law, we can reach $(((\text{curry map}) (o f g))$ and so we can conclude that $(o ((\text{curry map}) f) ((\text{curry map}) g)) == ((\text{curry map}) (o f g))$