13. $\rho(x) = 99 \mid (\text{begin } (\text{set } x\ 3)\ x) = 3$

$$\dfrac{\dfrac{\dfrac{x \in \text{dom } \rho \quad \rho(x) = 99}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle 99, \xi, \phi, \rho \rangle} \quad \langle \text{LITERAL}(3), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \rangle}{\langle \text{SET}(x,3), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\} \rangle} \qquad \dfrac{x \in \text{dom } \rho \quad \rho(x) = 3}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \rangle}}{\langle \text{BEGIN}(\text{SET}(x,3), \text{VAR}(x)), \xi, \phi, \rho \rangle \Downarrow \langle 3, \xi, \phi, \rho \rangle}$$

14. IF

$\langle IF(VAR(x), VAR(x), LITERAL(0)), \xi, \phi, P\rangle \Downarrow \langle v_1, \xi', \phi, P'\rangle$

and

$\langle VAR(x), \xi, \phi, P\rangle \Downarrow \langle v_2, \xi'', \phi, P''\rangle$

Then Prove

$v_1 = v_2$

~~Assu~~ Let VAR(x) evaluate to true;

Formal Definition of IF

TRUE: $$\frac{\langle e_1, \xi, \phi, P\rangle \Downarrow \langle v_1, \xi', \phi, P'\rangle \quad v_1 \neq 0 \quad \langle e_2, \xi', \phi, P'\rangle \Downarrow \langle v_2, \xi'', \phi, P''\rangle}{\langle IF(e_1, e_2, e_3), \xi, \phi, P\rangle \Downarrow \langle v_2, \xi'', \phi, P''\rangle}$$

FALSE: $$\frac{\langle e_1, \xi, \phi, P\rangle \Downarrow \langle v_1, \xi', \phi, P'\rangle \quad v_1 = 0 \quad \langle e_3, \xi', \phi, P'\rangle \Downarrow \langle v_3, \xi'', \phi, P''\rangle}{\langle IF(e_1, e_2, e_3), \xi, \phi, P\rangle \Downarrow \langle v_3, \xi'', \phi, P''\rangle}$$

The formal Definition of IF requires 3 expressions, if the first expression is true, the evaluate the second expression, If the first expression is false then evaluate the third. If we suppose $e_1 = Var(x)$, $e_2 = Var(x)$ & $e_3 =$ LITERAL(0) and we let e_1 be true, then by definition e_2 must be evaluated. By the Second premise we know Var(x) evaluates to v_2 $\therefore$ if e_1 is true, $Var(x) = v_2$ and $Var(x) = v_1$, so $v_1 = v_2$

Now Suppose e_1 is false, so $e_1 = 0$ and we now evaluate the third expression or e_3. e_3 is the LITERAL(0) so $e_3 = 0$ and so $v_1 = 0$. But since e_1 evaluated to 0 that must mean that $VAR(x) = 0$ and by the second premise again $VAR(x) = v_2 \rightarrow 0 = v_2$

$\therefore v_1 = v_2$ in ~~all~~ Both cases

Q.E.D

21.

$$\frac{x \notin \mathrm{dom}\,P \quad x \notin \mathrm{dom}\,\xi}{\langle \mathrm{VAR}(x), \xi, \phi, P\rangle \Downarrow \langle 0, \xi\{x \mapsto 0\}, \phi, P\rangle}$$

a) Awk-like (Create Global)

$$\frac{x \notin \mathrm{dom}\,P \quad x \notin \mathrm{dom}\,\xi \quad \langle e, \xi, \phi, P\rangle \Downarrow \langle v, \xi', \phi, P\rangle}{\langle \mathrm{SET}(x, e), \xi, \phi, P\rangle \Downarrow \langle v, \xi'\{x \mapsto v\}, \phi, P'\rangle}$$

$$\frac{x \notin \mathrm{dom}\,P \quad x \notin \mathrm{dom}\,\xi}{\langle \mathrm{VAR}(x), \xi, \phi, P\rangle \Downarrow \langle 0, \xi, \phi, P'\{x \mapsto 0\}\rangle}$$

b) Icon-like (Create Locally)

$$\frac{x \notin \mathrm{dom}\,P \quad x \notin \mathrm{dom}\,\xi \quad \langle e, \xi, \phi, P\rangle \Downarrow \langle v, \xi', \phi, P'\rangle}{\langle \mathrm{SET}(x, e), \xi, \phi, P\rangle \Downarrow \langle v, \xi, \phi, P'\{x \mapsto v\}\rangle}$$

c) Even though Global Variables are frowned upon, I think in this case I would prefer the Awk-like change since if you do not yourself specify the domain of the variable, then the variable should be accessable everywhere by default. ~~If you don't specify the domain and~~ If you randomly create a variable in a locally intended domain then you should create it in that domain. Not specifying to me means "this variable already exists outside of this function and I wish to use it here" then if that variable is not found it is created Globally.

20.

$$\langle e_0, \Xi, \phi, P\rangle \Downarrow \langle v_1, \Xi', \phi, P'\rangle$$

$$\langle e_0, \Xi, \phi, P\rangle \Downarrow \langle V_2, \Xi'', \phi, P''\rangle$$

if $e_0 = V_1$ $\quad e_0 = V_2$ $\quad \therefore V_1 = V_2$

$fv(VAR(x)) = \{x\}$

$$\frac{x \in dom P}{\langle VAR(x), \Xi, \phi, P\rangle \Downarrow \langle P(x), \Xi, \phi, P\rangle}$$

$Var(x) = e_0 \quad P(x) = V_1$

Since $x \in dom P$

$fv(VAR(x)) = \{x\} \subseteq dom P \subseteq (dom P \cup dom \Xi)$

So
$$\frac{x \in dom P}{\langle e_0, \Xi, \phi, P\rangle \Downarrow \langle V_2, \Xi, \Phi, P\rangle}$$

$fv(e_0) = \{V_2\} \subseteq dom P \subseteq (dom P \cup dom \Xi)$

Since $x = V_2$ & $x \in dom P$

$\therefore P(x) = V_2$

$e_0 \to P(x) = V_1$

$e_0 \to P(x) = V_2$

$\therefore V_1 = V_2$