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Question 1:

Suppose that END_1 were decidable. Let M be a Turing Machine that decides it. We can use M to construct a Turing Machine that decides A_{TM} as follows:

Input is $\langle X, w \rangle$ where X is Turing Machine code and w is a string.

Construct new Turing Machine Y as follows:

• Input is a binary string, s.

• If the start of the string is a 0 and the end of the string there is a 1, accept, else reject.

Run M on input $\langle Y \rangle$, if M accepts, accept, if M rejects, reject.

If Y accepts s, we know that s is in the language and vice-versa if Y rejects or loops. Therefore we can say that M decides A_{TM} which is a contradiction since we know that A_{TM} is undecidable and therefore END_1 must be undecidable.

Question 2:

Suppose that $BLANK_{TM}$ were decidable. Let M be a Turing Machine that decides it. We can use M to construct a Turing Machine that decides A_{TM} as follows:

Input is $\langle X, w \rangle$ where X is Turing Machine code and w is a string.

Construct new Turing Machine Y as follows:

- Input is a binary string, s.
- At each character, erase it
- If s ends, accept, else reject.

Run M on input $\langle Y \rangle$, if M accepts, accept, if M rejects, reject.

If Y accepts s, we know that Y finished computation with a completely blank tape and vice-versa if Y rejects or loops. Therefore we can say that M decides A_{TM} which is a contradiction since we know that A_{TM} is undecidable and therefore $BLANK_{TM}$ must be undecidable.

Question 3:

3.1

$S \to XY|YX|\epsilon$
$X \to ZXZ|a|\epsilon$
$Y \to ZYZ|b\epsilon$
$Z \to a|b$

The grammar describes an arbitrary collection of a's and b's in a string w, $w \in \{a, b\}*$.

3.2

The following TM, X, maps from D to our grammer, L:

Input is $\langle M, w\rangle$ where M is a Turing Machine and w is a string in our grammar above:

- Replace each instance of the letter 0 with a, and replace 1 with b. "

What is left on the tape is now $\{a, b\}*$, so we can say D maps to L. However since we know, D is not context-free. Claim 1 must be false.

3.3

The following TM, Y, maps from our grammer, L to D:

Input is $\langle M, w\rangle$ where M is a Turing Machine and w is a string in D:

- Replace each instance of the letter a with 0, and replace 0 with 1. "

What is left on the tape is now $\{0, 1\}*$, so we can say L maps to D. However since we know, L is context-free. Claim 2 must be false.