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Evaluation of Analytical Methods for Parameter Extraction of PV modules

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Abstract

A review and evaluation of the main analytical techniques for parameters extraction of photovoltaic (PV) modules with due account taken of their applications in modelling photovoltaic systems is presented. Six prevalent analytical methods are investigated and assessed using software tools, which have been developed to extract the required parameters of some commercially available PV modules using these methods. The results were subsequently compared with those obtained using well-established numerical methods. It is shown that, despite the fact that analytical methods can involve a fair amount of approximations, some analytical methods can compete in terms of accuracy with their numerical counterparts with much reduced computational complexity.

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1. Introduction

A mathematical model of any PV system requires a set of lumped circuit parameters of its PV modules to enable assessment of its performance under varying operating conditions. Unfortunately, such parameters are not always explicitly or fully provided by the manufacturers of PV module. The problem is further compounded due to effects of partial shading [1, 2]. Consequently, many different methods of parameter extraction with different levels of complexity and accuracy have been proposed in the literature [3, 4, 5]. These methods are, generally, classified as either numerical or analytical. Numerical methods, typically, develop a set of equations whose solution can be achieved using numerical or iterative algorithms [4, 5], whilst others use information provided by manufacturers' datasheets [6, 7]. In contrast, analytical methods use simple and fast parameter extraction procedures by making some approximations or simplifying assumptions without sacrificing accuracy [8]. This is done, for example, by neglecting some model parameters or assigning some approximate values for some parameters [9, 10]. This paper

presents a review of and a comparison between major analytical methods of parameters extraction of the crystalline silicon PV solar module, which are discussed in section 3. Section 2 provides the theoretical analysis, which underpins the PV modelling process. Section 4 presents a comparison between the analytical methods in terms of their accuracy and the results are compared against those obtained using well-known numerical techniques. Finally, section 5 concludes the paper.

Nomenclature

a	modified diode ideality factor ($a = n N_s V_{th}$).
a_{STC}	modified diode ideality factor at standard test conditions (STC).
$\alpha_{I_{sc}}$	thermal coefficient of the short-circuit current (A/°C).
$\beta_{V_{oc}}$	thermal coefficient of the open-circuit voltage (V/°C).
G	solar irradiance (W/m ²).
G_{STC}	solar irradiance at standard test conditions (1000 W/m ²).
I	Output current of a module (A).
I_m	current at the maximum power point (A).
I_{mSTC}	current at maximum power point under standard test conditions (A).
I_{ph}	photocurrent (A).
I_{sat}	reverse saturation current (A).
I_{satSTC}	reverse saturation current at standard test conditions (A).
I_{sc}	short-circuit current of the module (A).
$I_{sc,STC}$	short-circuit current of the module at standard test conditions (A).
k	Boltzmann constant (1.381×10^{-23} J/K).
n	diode ideality factor.
N_s	number of the series-connected cells per module.
P	Output power of the PV module (W).
q	electronic charge (-1.602×10^{-19} C).
R_s	series resistance of the PV module (Ω).
R_{so}	reciprocal of slope of the I-V characteristic at $V=V_{oc}$. (Ω).
R_{sh}	shunt resistance of the PV module (Ω).
R_{sho}	reciprocal of slope of the I-V characteristic at $I=I_{sc}$. (Ω).
T	temperature of the PV cell (K).
T_{STC}	temperature of a module at standard test conditions (25°C or 298.15K).
V	Output voltage of the PV module (V).
V_m	voltage at the maximum power point (V).
$V_{m,STC}$	voltage at the maximum power point under standard test conditions (V).
V_{oc}	open-circuit voltage of the PV module (V).
$V_{oc,STC}$	open-circuit voltage of the PV module at standard test conditions (V).
V_{th}	thermal voltage of the PV module (V).
W	Lambert W-function.
w_{STC}	Modified Lambert W-function at standard test conditions.

2. Theoretical bases of the analytical methods

The single-diode model of a PV cell, Fig. 1, is the most commonly used model for modelling PV modules and arrays. It has five parameters to be extracted when modelling a PV system; the photocurrent source, the ideality factor and the saturation current of the diode, and a series and a shunt resistance of the PV cell [11]. The associated transcendental implicit equation of the model is

$$I = I_{ph} - I_{sat} \left[\exp \left(\frac{V + IR_s}{n N_s V_{th}} \right) - 1 \right] - \frac{V + IR_s}{R_{sh}}, \quad \text{where } V_{th} = \frac{kT}{q} \quad (1)$$

This can be simplified into an explicit equation using the Lambert W-function as [12]

$$I = -\frac{V}{R_s + R_{sh}} + \frac{R_{sh}(I_{ph} + I_{sat})}{R_s + R_{sh}} - nV_{th} W \left(\frac{R_s R_{sh} I_{sat} \left(\frac{R_{sh}(R_s I_{ph} - R_s I_{sat} + V)}{nV_{th}(R_s + R_{sh})} \right)}{nV_{th}(R_s + R_{sh})} \right) / R_s \quad (2)$$

This explicit form is attractive due to its applications in modelling large photovoltaic systems especially under the effects of mismatch losses.

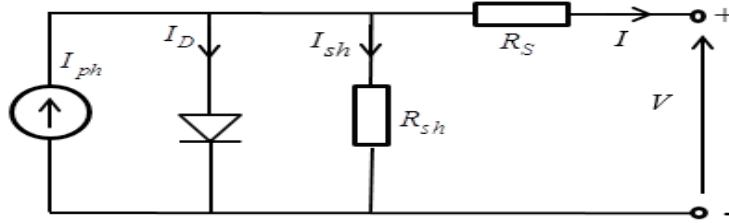


Fig. 1 Single-diode model of a PV module.

The I-V characteristics of a PV module are not always provided in the manufacturers' datasheets or given only at the standard test condition (STC) [6, 7, 13]. Analytical methods, usually, introduce some approximations in deriving the following three standard equations from (1) using the short-circuit (SC), the open-circuit (OC) and the maximum power points (MPP) on the I-V characteristics at STC as

$$\text{SC} : (0, I_{sc}) : I_{sc} = I_{ph} - I_{sat} \left[\exp \left(\frac{I_{sc} R_s}{n N_s V_{th}} \right) - 1 \right] - \frac{I_{sc} R_s}{R_{sh}} \quad (3)$$

$$\text{OC} : (V_{oc}, 0) : 0 = I_{ph} - I_{sat} \left[\exp \left(\frac{V_{oc}}{n N_s V_{th}} \right) - 1 \right] - \frac{V_{oc}}{R_{sh}} \quad (4)$$

$$\text{MPP} : (V_m, I_m) : I_m = I_{ph} - I_{sat} \left[\exp \left(\frac{V_m + I_m R_s}{n N_s V_{th}} \right) - 1 \right] - \frac{V_m + I_m R_s}{R_{sh}} \quad (5)$$

A fourth equation is obtained from the fact that derivative $dP / dV = 0$ at the MPP as

$$\left. \frac{dP}{dV} \right|_{MPP} = -\frac{I_m}{V_m} = \frac{-\frac{1}{R_{sh}} - \frac{I_{sat}}{n N_s V_{th}} \exp\left(\frac{V_m + I_m R_s}{n N_s V_T}\right)}{1 + \frac{R_s}{R_{sh}} + \frac{I_{sat} R_s}{n N_s V_{th}} \exp\left(\frac{V_m + I_m R_s}{n N_s V_T}\right)} \quad (6)$$

Another two equations are derived from the I-V characteristics as [6, 7, 13, 14]

$$\left. \frac{dI}{dV} \right|_{I_{sc}} = -\frac{1}{R_{sho}} \quad \text{and} \quad \left. \frac{dI}{dV} \right|_{V_{oc}} = -\frac{1}{R_{so}} \quad (7)$$

3. Analytical methods used for parameters extraction.

3.1. Method 1

This method uses the datasheet parameters I_{sc} , V_{oc} , I_m , and V_m and the slopes at the SC and OC points on the I-V curve [8]. It calculates a value of the ideality factor n as the main parameter and subsequently uses this value to compute the rest of the parameters. The method uses equations (3) to (7) to extract the model parameters of a cell after neglecting the terms N_s and using the following approximations

$$\exp\left(\frac{V_{oc}}{n V_{th}}\right) \gg \exp\left(\frac{I_{sc} R_s}{n V_{th}}\right), \quad R_s \ll R_{sh}, \quad \frac{1}{R_{sh}} \ll \frac{I_{sat}}{n V_{th}} \exp\left(\frac{V_{oc}}{n V_{th}}\right) \quad (8)$$

$$\frac{I_{sat}}{n V_{th}} \exp\left(\frac{I_{sc} R_s}{n V_{th}}\right) \ll \frac{1}{R_{sh}}, \quad \text{and} \quad \frac{I_{sat}}{n V_{th}} \exp\left(\frac{I_{sc} R_s}{n V_{th}}\right) \ll \frac{1}{R_{sho} - R_s}$$

Upon applying the above approximations, the ideality factor can be calculated from

$$n = \frac{V_m + I_m R_{so} - V_{oc}}{V_{th} \ln\left(I_{sc} + \frac{V_m}{R_{sho}} - I_m\right) - \ln\left(I_{sc} - \frac{V_{oc}}{R_{sho}}\right) + \frac{I_m}{(I_{sc} - V_{oc} / R_{sho})}} \quad (9)$$

Equation (9) represents the ideality factor of the solar cell in terms of measured parameters and in order to obtain the ideality factor for a module, its denominator is multiplied by N_s . The other four unknown parameters of single-diode model can be determined from

$$I_{sat} = \left(I_{sc} - \frac{V_{oc}}{R_{sho}}\right) \exp\left(\frac{-V_{oc}}{n V_{th}}\right) \quad (10)$$

$$R_s = R_{so} - \frac{n V_{th}}{I_{sat}} \exp\left(\frac{V_{oc}}{n V_{th}}\right) \quad (11)$$

$$R_{sh} = R_{sho} \quad (12)$$

$$I_{ph} = I_{sc} \left(1 + \frac{R_s}{R_{sh}}\right) + I_{sat} \exp\left[\left(\frac{I_{sc} R_s}{n V_{th}}\right) - 1\right] \quad (13)$$

3.2. Method 2

In this method, the series and shunt resistances are neglected, thus resulting in (1) becoming explicit [10], allowing calculation of the current, voltage, and power at the key operating points on the I-V curve. The method uses the following equations to describe the behaviour of the PV module

$$I_{sc} = I_{ph} \quad (14)$$

$$I_{sc} = I_{sat} \left[\exp\left(\frac{V_{oc}}{n N_s V_{th}}\right) - 1 \right] \quad (15)$$

$$V_{oc} = n N_s V_{th} \ln\left(1 + \frac{I_{sc}}{I_{sat}}\right) \quad (16)$$

$$I_{sat} = \frac{I_{sc}}{\exp\left[\left(\frac{V_{oc}}{n N_s V_{th}}\right) - 1\right]} \quad (17)$$

$$I_m = I_{sc} - I_{sat} \left[\exp\left(\frac{V_m}{n N_s V_{th}}\right) - 1 \right] \quad (18)$$

$$I_m = \frac{V_m I_{sat}}{n N_s V_{th}} \exp\left(\frac{V_m}{n N_s V_{th}}\right) \quad (19)$$

Substituting (15) and (18) into (19), yields

$$\exp\left(\frac{V_{oc}}{n N_s V_{th}}\right) = \left(1 + \frac{V_m}{n N_s V_{th}}\right) \exp\left(\frac{V_m}{n N_s V_{th}}\right) \quad (20)$$

$$V_m = n N_s V_T \ln\left(\frac{n N_s V_{th}}{I_{sat}} \frac{I_{sc}}{V_{oc}}\right) \quad (21)$$

$$I_m = I_{sc} + n N_s V_{th} \frac{I_{sc}}{V_{oc}} - I_{sat} \quad (22)$$

In order to calculate the ideality factor at standard test conditions using (17), and (18), yields

$$n = \frac{V_{m,STC} - V_{oc,STC}}{N_s V_{th,STC} \ln \left(1 - \frac{I_{m,STC}}{I_{sc,STC}} \right)} \quad (23)$$

The simplicity of the model is, however, achieved at the expense of reduced accuracy especially at low irradiance because of the effect of the PV resistances.

3.3. Method 3

In this method, the shunt resistance is omitted giving rise to what is known as the four parameter model. This model offers good accuracy that is comparable to that of the double-diode model as long as a negative value for the series resistance is acceptable, albeit with no physical meaning [15]. In general, the four parameters model provides a simple method of parameters extraction and is suitable for simulating PV systems under different operating conditions. This method depends on the manufacturer's datasheet parameters i.e. I_{sc} , V_{oc} , I_m , and V_m without the need for the full I-V curve [9, 15]. By eliminating the shunt resistance and reformulating (3), (4), and (5) and using this approximation

$$I_{ph} \gg I_{sat} \exp \left(\frac{I_{sc} R_s}{n N_s V_{th}} \right) \quad (24)$$

The following equations can be derived [9, 15]

$$I_{ph} = I_{sc} \quad (25)$$

$$I_{sat} = I_{sc} \exp(-V_{OC} / n N_s V_{th}) \quad (26)$$

$$R_s = [n V_{th} \ln(1 - I_m / I_{sc}) + V_{oc} - V_m] / I_m \quad (27)$$

$$n = (2V_m - V_{oc}) / N_s V_{th} \left[\frac{I_m}{I_{sc} - I_m} + \ln \left(1 - \frac{I_m}{I_{sc}} \right) \right] \quad (28)$$

3.4. Method 4

This method uses a piecewise linear curve fitting technique of the I-V characteristic combined with the four parameters model to extract the parameters of a PV module by producing an explicit system of five algebraic equations at the STC conditions [16]. The method uses the slopes at the SC and OC points and the derivative dP / dV at the maximum power point so as to obtain the solution for the five parameters of the single-diode model as follows

$$R_s = \frac{\left(\frac{dV}{dI} \Big|_{I=0} - \frac{dV}{dI} \Big|_{V=0} \right) \left[\frac{dV}{dI} \Big|_{V=0} (I_{sc} - I_m) + V_m \right] - \frac{dV}{dI} \Big|_{I=0} \left(\frac{dV}{dI} \Big|_{V=0} I_m + V_m \right) \left(\frac{dV}{dI} \Big|_{V=0} I_{sc} + V_{oc} \right)}{I_m \left(\frac{dV}{dI} \Big|_{I=0} - \frac{dV}{dI} \Big|_{V=0} \right) \left[\frac{dV}{dI} \Big|_{V=0} (I_{sc} - I_m) + V_m \right] + \left(\frac{dV}{dI} \Big|_{V=0} I_m + V_m \right) \left(\frac{dV}{dI} \Big|_{V=0} I_{sc} + V_{oc} \right)} \quad (29)$$

$$R_{sh} = -R_s - \frac{dV}{dI} \Big|_{V=0} \quad (30)$$

$$I_{ph} = I_{sc} (1 + R_s / R_{sh}) \quad (31)$$

$$a = \frac{\left(\frac{dV}{dI} \Big|_{I=0} + R_s \right) \left(\frac{dV}{dI} \Big|_{V=0} I_{sc} + V_{oc} \right)}{\left(\frac{dV}{dI} \Big|_{I=0} - \frac{dV}{dI} \Big|_{V=0} \right)} \quad (32)$$

$$I_{sat} = \left(I_{ph} - \frac{V_{oc}}{R_{sh}} \right) / \left(\exp \left(\frac{V_{oc}}{n N_s V_{th}} \right) - 1 \right) \quad (33)$$

3.5. Method 5

In order to improve the accuracy of parameters extraction using the single-diode model, two boundary conditions were considered; the first one is the dP/dV at the MPP, which enabled the derivation of an implicit form for the series resistance and an explicit form for the shunt resistance [8]. The second condition is the slope at the SC point which is used to determine the ideality factor. However in [17] the above conditions were used to derive an explicit equation for the series resistance from the manufacturer's datasheet and the slope at the SC point as

$$R_s = \frac{V_m \left[\left((V_m + (I_m - I_{sc}) R_{sho}) \ln \left(\frac{V_m + (I_m - I_{sc}) R_{sho}}{V_{oc} - I_{sc} R_{sho}} \right) \right) - (V_m - I_m R_{sho}) \right] + V_{oc} (V_m - I_m R_{sho})}{I_m \left[\left((V_m + (I_m - I_{sc}) R_{sho}) \ln \left(\frac{V_m + (I_m - I_{sc}) R_{sho}}{V_{oc} - I_{sc} R_{sho}} \right) \right) + (V_m - I_m R_{sho}) \right]} \quad (34)$$

Subsequently, this is used to derive the ideality factor and the shunt resistance as

$$nV_{th} = (V_m - I_m R_s) (V_m + (I_m - I_{sc}) R_{sho}) / (V_m - I_m R_{sho}) \quad (35)$$

$$R_{sh} = (V_m - I_m R_s) (V_m - R_s (I_{sc} - I_m) - nV_T) / [(V_m - I_m R_s) (I_{sc} - I_m) - nV_T I_m] \quad (36)$$

The slopes of the I-V curve at the SC and OC points can be obtained approximately as [14].

$$R_{sho} = C_{sh} V_{oc} / I_{sc}, \text{ and } R_s = C_s V_{oc} / I_{sc}, \text{ Where } C_{sh} = 34.49692 \text{ and } C_s = 0.11175 \text{ for silicon.} \quad (37)$$

3.6. Method 6

This method uses two approaches; the first one develops a new relationship between the modified ideality factor and the open-circuit voltage and relies upon the voltage and current temperature coefficients. The second approach relies on a simplified form of Lambert W-function of the single-diode model [12]. Using (4) after neglecting the shunt resistance and using the Townsend equation of the saturation current [6], the new equation connecting the modified ideality factor and the open-circuit voltage at STC becomes

$$\delta_{STC} = a_{STC} / V_{oc,STC} = 1 - T_{STC} \beta_{V_{oc}} / (50.1 - \alpha_{I_{sc}} T_{STC}) \quad (38)$$

The single-diode parameters using the Lambert W-function and properties of the ideal model to extract the five model parameters become

$$R_s = a_{STC} (w_{STC} - 1) - V_m / I_m \quad \text{and} \quad R_{sh} = a_{STC} (w_{STC} - 1) / [(1 - \frac{1}{w_{STC}}) I_{sc} - I_m] \quad (39)$$

$$I_{ph} = \left(1 + \frac{R_s}{R_{sh}} \right) I_{sc} \quad \text{and} \quad I_{sat} = I_{ph} e^{-\frac{1}{\delta_{STC}}} \quad (40)$$

$$\text{where : } w_{STC} = W(e^{\frac{1}{\delta_{STC}}}) = W(e^{\frac{1}{\delta_{STC}} + 1}) \quad (41)$$

4. Results and discussion

In order to validate and compare the above methods, the multi-crystalline Kyocera KC200GT PV module [4] was investigated in this work and results obtained using MATLAB simulations were compared against those obtained using well-documented numerical techniques, such as [3, 4, 5]. Table 1 shows that method 1 and method 5 are most accurate, whilst methods 2 and 3 resulted in higher values of the ideality factor. Method 4 produced lower than expected values for the ideality factor and the saturation current and consequently led to inaccurate values for the series and shunt resistance. Finally, the ideality factor of method 6 is always limited to the range 1 ± 0.05 [12]. Fig. 2(a) shows that methods 1 and 5 exhibit close similarity at the region where the current source is dominant because both methods use the slope at this region. At the region where the voltage source has strong affect, method 5 produces an underestimated I-V curve when compared with iterative methods, while methods 2 and 3 may be seen as the least accurate, however, as can be deduced from Fig. 2(b), they give good representation for the PV module characteristics at STC. This indicates that the resistances do not have significant effect on the both I-V and P-V characteristics curves at the STC, which is not the case under other operation conditions [10]. As shown in Fig. 2(c), method 4 shows underestimation of the current due to the fact that this method depends on a flat line when ignoring the shunt resistance. An overestimation of the power can be observed in this method, while all other methods have relatively the same maximum power. Finally, Fig. 2(d) shows that method 6 has good agreement with the iterative methods with slightly underestimation at the current region near the maximum power point.

Table.1 Parameters for the Multi-crystalline (Kyocera KC200GT).

Method	n	$R_s(\Omega)$	$R_{sh}(\Omega)$	$I_{sat}(A)$	$I_{ph}(A)$
Method 1	1.08317	0.27077	124	2.4885×10^{-9}	8.22793
Method 2	1.81764	0	∞	1.78074×10^{-5}	8.21
Method 3	1.40991	0.19455	∞	4.09919×10^{-7}	8.21
Method 4	0.65008	0.39999	82.55084	1.14541×10^{-15}	8.24978
method 5	0.88423	0.38033	123.61967	1.81544×10^{-11}	8.23526
Method 6	1.00258	0.30567	130.46626	4.43777×10^{-10}	8.22924
Iterative method [4].	1.3	0.2283	572.12367	9.89443×10^{-8}	8.21329
Numerical method Newton-Raphson [7].	1.3405	0.2172	951.3267	1.7097×10^{-7}	8.2119

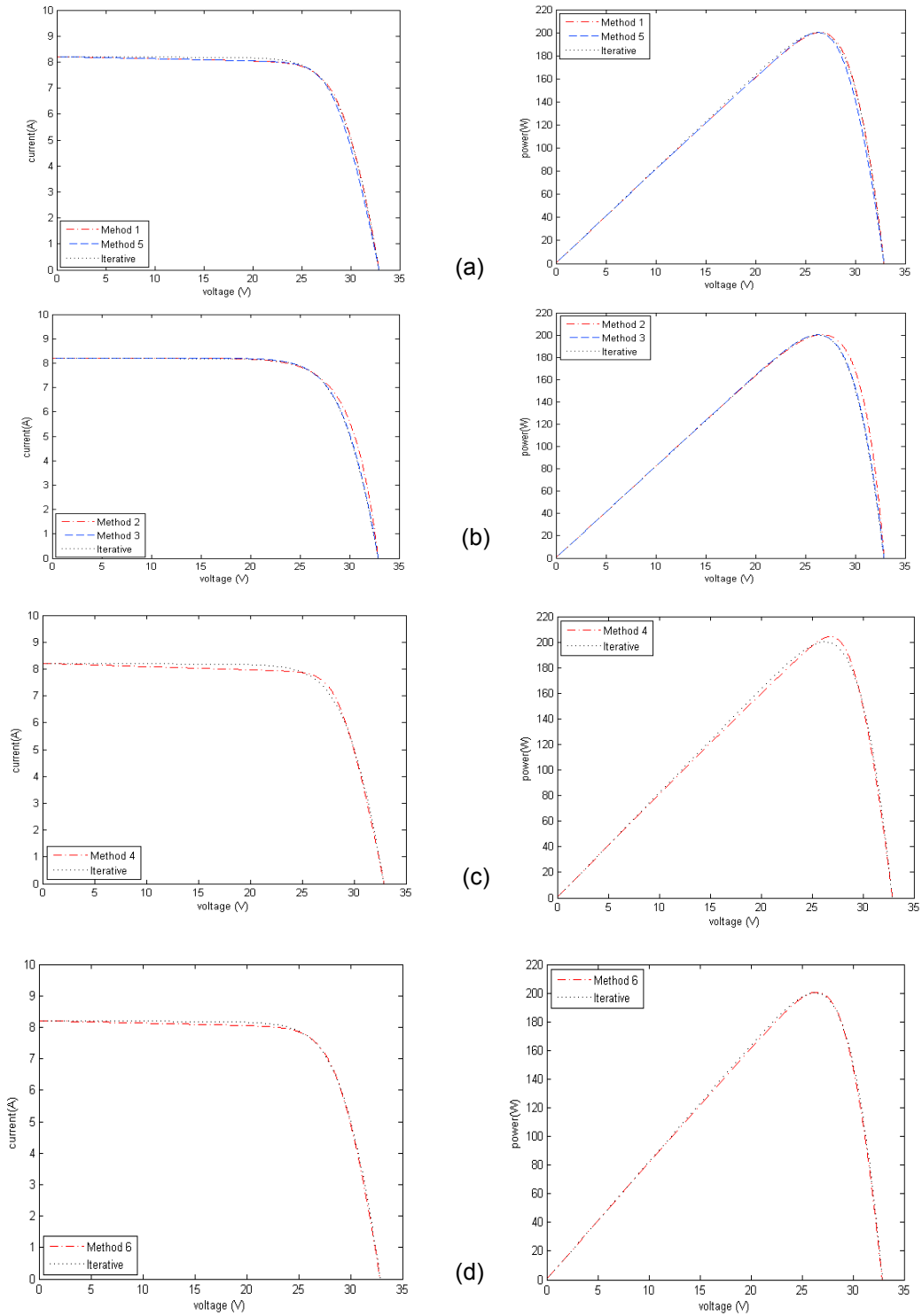


Fig.2. Represent the I-V and P-V characteristics of the KC200GT module,(a) comparison between methods 2,3 and iterative method ,(b) comparison between methods 1,5 and iterative method, (c) comparison between methods 4 and iterative method,(d) comparison between methods 6 and iterative method.

5. Conclusion

The paper presented an assessment of the complexity and accuracy of the prominent analytical methods presented in the literature for extracting lumped circuit parameters of PV modules. In addition to comparing the advantages and disadvantages of different analytical methods, the work demonstrated that simplified analytical methods with reduced computational complexity may substitute their complex numerical counterparts. Indeed, results indicate that analytical methods can produce results, which were comparable with those obtained using numerical techniques.

References

- [1] Anani N, Shahid M, Al-kharji O, Ponciano J. A CAD Package for Modeling and Simulation of PV Arrays. *Elsevier Energy Procedia* 2013; 42(1): 397-405.
- [2] Burns P, Anani N. Modelling and simulation of photovoltaic arrays under varying conditions. *Proceedings of the 9th IEEE Intern. Symp on Comm Sys, Networks and Dig Sig processing*. Manchester, UK: IEEE; 2014.
- [3] Cotfas D, Cotfas P, S. Kaplanis S. Methods to determine the dc parameters of solar cells. *Renewable and Sustainable Energy Reviews* 2013; 28(): 588-596.
- [4] Villalva MG, Gazoli JR, Filho ER. Comprehensive approach to modeling and simulation of photovoltaic arrays. *IEEE Trans.Power Electron* 2009; 94(5): 1198-1208.
- [5] Soto W, Klein S, Beckman W. Improvement and validation of a model for photovoltaic array performance. *Solar Energy* 2006; 80(1). 78-88.
- [6] Sera D, Teodorescu R, Rodriguez P. *Proc. of the IEEE International Symposium on Industrial Electronics*. Vigo: IEEE; 2007.
- [7] Can H, Ickilli D. Parameter estimation in modeling of photovoltaic panels based on datasheet values. *Solar Energy Engineering* 2013; 136(2).
- [8] Phang JCH, Chan DSH, Phillips JR. Accurate analytical method for the extraction of solar cell model parameters. *Electronics Letters* 1984; 20(10): 406-408.
- [9] Aldwane B. Modeling, simulation and parameters estimation for Photovoltaic module. (ed). *Proc. of the International Conference on Green Energy. tunisia: IEEE; 2014: 101-106.*
- [10] Saloux E, Teyssedou A., Sorin M. Explicit model of photovoltaic panels to determine voltages and currents at the maximum power point. *Electronics Letters* 2011; 85(5): 713-722.
- [11] Ma T, Yang H, Lu L. Development of a model to simulate the performance characteristics of crystalline silicon photovoltaic modules/strings/arrays. *Solar Energy* 2014; 100(): 31-41.
- [12] Batzelis EI, Papathanassiou SA. A method for the analytical extraction of the single-diode PV model parameters. *IEEE Transactions on Sustainable Energy* 2016; 7(2): 504-512.
- [13] Siddique HAB, Xu P, De Doncker RW. M. Parameter extraction algorithm for one-diode model of PV panels based on datasheet values. (ed). *Proc. of the international conference on clean electrical power. Alghero-Italy: IEEE; 2013.*
- [14] Orioli A, Gangi AD. A procedure to calculate the five-parameter model of crystalline silicon photovoltaic modules on the basis of the tabular performance data. *Applied Energy* 2013; 102(C): 1160–1177.
- [15] Sera D, Teodorescu R, Rodriguez P. Photovoltaic module diagnostics by series resistance monitoring and temperature and rated power estimation. (ed). *Proc. of the 34th Annual Conference of IEEE Industrial Electronics. Florida-USA: IEEE; 2008. pp. 2195-2199.*
- [16] Bai J, Liu S, Hao Y, Zhang Z, Jiang M, Zhang Y. Development of a new compound method to extract the five parameters of PV modules. *Energy Conversion and Management* 2014; 79(): 294-303.
- [17] Cubas J, Pindado S, Victoria M. On the analytical approach for modeling photovoltaic systems behavior. *Journal of Power Sources* 2014; 247(): 467-474.