1 Generate Matrix

general method to generate a square matrix of square game

[1]:
```
import numpy as np
def genenerate_neighbord_matrix(n) -> np.array:
    mat = np.zeros((n**2, n**2), dtype= np.int8)

    # the general case
    for j in range(0, n**2):
        if j-n > -1 :
            mat[j-n,j] = 1

        if j % n != 0 :
            mat[j-1,j] = 1

        mat[j,j] = 1

        if (j+1) % n != 0 :
            mat[j+1,j] = 1

        if j+n < n**2 :
            mat[j+n,j] = 1

    return mat
print(genenerate_neighbord_matrix(3))
```

```
[[1 1 0 1 0 0 0 0 0]
 [1 1 1 0 1 0 0 0 0]
 [0 1 1 0 0 1 0 0 0]
 [1 0 0 1 1 0 1 0 0]
 [0 1 0 1 1 1 0 1 0]
 [0 0 1 0 1 1 0 0 1]
 [0 0 0 1 0 0 1 1 0]
 [0 0 0 0 1 0 1 1 1]
 [0 0 0 0 0 1 0 1 1]]
```

2 Solving game

general method to how solve the game, by solving the matrix.

[2]:
```
from sage.all import *
n = 3
A = Matrix(Integers(2),genenerate_neighbord_matrix(n))
Y = vector([1 for x in range(n**2)])
Z = vector([0 for x in range(n**2)])
X = A.solve_right(Y)
print(X)
```

```
(1, 0, 1, 0, 1, 0, 1, 0, 1)
```

3 Spanish method

[3]:
```python
def gaussian_elimination_spanish_alg(mat : np.array, sol_vec :np.array):
    n = int(sqrt(mat.shape[0]))
    #all rows but the last one
    for i in range(0, n**2-n):
        # the lamp that is affected
        affected_lamp = i + n
        row_i = mat[i][:affected_lamp+1]
        # check rows below
        # for j in range(i+1, n**2):
        for j in [i-1 + n, i+n, i+n+1, i+ 2*n]:
            if j> -1 and j < n**2 and mat[j][affected_lamp] == 1:
                row_j = mat[j][:affected_lamp+1]
                row_j = row_j + row_i
                row_j = row_j % 2
                mat[j][:affected_lamp+1] = row_j
                sol_vec[j] = ( sol_vec[j] + sol_vec[i] ) % 2


def mul_mat_sol_based_on_res(mat : np.array, end_state : list, res : list):
    n = int(sqrt(mat.shape[0]))
    for i in range(0,n**2-n):
        res_i_plus_n = int(end_state[i])
        for j in range(0,i+n):
            res_i_plus_n = (res_i_plus_n + mat[i][j] * res[j]) % 2
        res.append(res_i_plus_n)

def generate_mat_spanish_alg(mat : np.array):
    n = int(sqrt(mat.shape[0]))
    end_state = np.ones(n**2)
    gaussian_elimination_spanish_alg(mat, end_state)
    # the matrix we need to solve
    new_mat = np.array(mat[n**2-n:n**2, 0:n], copy=True)
    new_sol = np.array(end_state[n**2-n:n**2],  copy=True)

    #find solution for n variables
    A = Matrix(Integers(2),new_mat)
    Y = vector(Integers(2),new_sol)
    X = A.solve_right(Y)
    res = [x for x in X]
    mul_mat_sol_based_on_res(mat, end_state, res)
    return res
```

```
mat = genenerate_neighbord_matrix(4)
A = Matrix(Integers(2),mat)
res = generate_mat_spanish_alg(mat)
print(mat)
print(res)

print('check solution:')
X = vector(Integers(2),res)
Y = A*X
print(Y)
```

```
[[1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
 [1 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0]
 [0 1 1 1 0 0 1 0 0 0 0 0 0 0 0 0]
 [0 0 1 1 0 0 0 1 0 0 0 0 0 0 0 0]
 [1 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0]
 [0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0]
 [1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]
 [0 1 0 1 0 0 0 0 0 0 0 1 0 0 0 0]
 [0 1 1 1 0 0 0 0 0 0 0 0 1 0 0 0]
 [1 1 0 1 0 0 0 0 0 0 0 0 0 1 0 0]
 [1 0 1 1 0 0 0 0 0 0 0 0 0 0 1 0]
 [1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]]
[0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1]
check solution:
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)
```

4 Minimal case

generate matrix for rectengle game.
searching for integer solution.

[4]:
```
import numpy as np

# to prove the minimal case on not square we need to build matrix for not
↪rectangler board
def genenerate_neighbord_matrix_m_n(m,n) -> np.array:
    mat = np.zeros((m*n, m*n), dtype= np.int8)

    # the general case
    for j in range(0, m*n):
        if j-n > -1 :
            mat[j-n,j] = 1

        if j % n != 0 :
            mat[j-1,j] = 1

        mat[j,j] = 1

        if (j+1) % n != 0 :
            mat[j+1,j] = 1

        if j+n < n**2 :
            mat[j+n,j] = 1

    return mat
print(genenerate_neighbord_matrix_m_n(3,2))
```

```
[[1 1 1 0 0 0]
 [1 1 0 1 0 0]
 [1 0 1 1 1 0]
 [0 1 1 1 0 1]
 [0 0 0 0 1 1]
 [0 0 0 0 1 1]]
```

[5]:
```
from sage.all import *
n = m = 4
a = genenerate_neighbord_matrix_m_n(m,n)
print(a)
A = Matrix(Integers(),a)
Y = vector([1 for x in range(m*n)])
Z = vector([0 for x in range(m*n)])
X = A.solve_right(Y)
print(X)
```

```
[[1 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
```

```
 [1 1 1 0 0 1 0 0 0 0 0 0 0 0 0 0]
 [0 1 1 1 0 0 1 0 0 0 0 0 0 0 0 0]
 [0 0 1 1 0 0 0 1 0 0 0 0 0 0 0 0]
 [1 0 0 0 1 1 0 0 1 0 0 0 0 0 0 0]
 [0 1 0 0 1 1 1 0 0 1 0 0 0 0 0 0]
 [0 0 1 0 0 1 1 1 0 0 1 0 0 0 0 0]
 [0 0 0 1 0 0 1 1 0 0 0 1 0 0 0 0]
 [0 0 0 0 1 0 0 0 1 1 0 0 1 0 0 0]
 [0 0 0 0 0 1 0 0 1 1 1 0 0 1 0 0]
 [0 0 0 0 0 0 1 0 0 1 1 1 0 0 1 0]
 [0 0 0 0 0 0 0 1 0 0 1 1 0 0 0 1]
 [0 0 0 0 0 0 0 0 1 0 0 0 1 1 0 0]
 [0 0 0 0 0 0 0 0 0 1 0 0 1 1 1 0]
 [0 0 0 0 0 0 0 0 0 0 1 0 0 1 1 1]
 [0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 1]]
(0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0)
```

5 Solution Amount

[6]:
```
n = 9
a = genenerate_neighbord_matrix(n)
A = Matrix(Integers(2),a)
print(2**A.kernel().dimension())
```

```
256
```

6 Benchmark

[7]:
```
import datetime
import numpy as np

def matrix_solve(mat):
    A = Matrix(Integers(2),mat)
    Y = vector([1 for x in range(n**2)])
    Z = vector([0 for x in range(n**2)])
    X = A.solve_right(Y)
    return X

val = []
# run on range(10 ,61,5)
for i,n in enumerate(range(10 ,15)):
    # print(i)
    mat = genenerate_neighbord_matrix(n)

    a0 = datetime.datetime.now()
    matrix_solve(mat)
```

```python
    b0 = datetime.datetime.now()
    c0 = b0 - a0
    t0 = c0.total_seconds()
    # print(t0)

    a1 = datetime.datetime.now()
    generate_mat_spanish_alg(mat)
    b1 = datetime.datetime.now()
    c1 = b1 - a1
    t1 = c1.total_seconds()
    # print(t1)

    val.append((n, t0, t1))

res = np.array(val)
# np.savetxt("benchmark.csv", res, delimiter = ',')
print(res)
```

```
[[10.        0.020791  0.184697]
 [11.        0.029358  0.261447]
 [12.        0.0316    0.366729]
 [13.        0.045727  0.51665 ]
 [14.        0.068553  0.670478]]
```