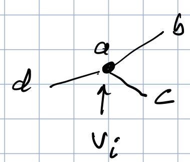
Let V be a set of vertices of a graph G

In the initial state each vertex is in a state 0.

Pressing vertex change its states from 0 to 1 or from 1 to 0 and also change the state of all neighborhoods vertecs.

We denote by variable $a, b, \dots$ the action that we do on vertex (press or not press it), $a, b, \dots \in \{0, 1\}$




Vertex $v_i$ will be in the state 1 if and only if $a + b + c + d = 1$

Our problem: from initial state $v_i = 0$, $i = 1, \dots n$ go to the state $v_i = 1$, $i = 1, \dots, n$

We have very simple alghoritm to solve this problem.

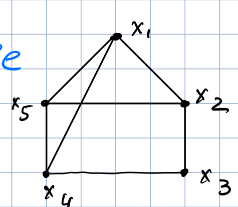
Assign to each vertex $v_i$ a variable $x_i$, $x_i \in \{0, 1\}$

For each vertex write down a linear equatuon:

Sum of its variable with variables of all its neighbors is equal 1.

Therefore, we get a system of n linear equcetions with n unknowns modulo 2.

Example




$$\begin{cases} x_1 + x_2 + x_4 + x_5 = 1 \\ x_2 + x_1 + x_3 + x_5 = 1 \\ x_3 + x_2 + x_4 = 1 \\ x_4 + x_3 + x_1 + x_5 = 1 \\ x_5 + x_1 + x_2 + x_4 = 1 \end{cases} \qquad \begin{cases} x_1 + x_2 + x_4 + x_5 = 1 \\ x_1 + x_2 + x_3 + x_5 = 1 \\ x_2 + x_3 + x_4 = 1 \\ x_1 + x_3 + x_4 + x_5 = 1 \\ x_1 + x_2 + x_4 + x_5 = 1 \end{cases}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & & 1 & 1 & 1 \\ 1 & 1 & 1 & & 1 & 1 \\ & 1 & 1 & 1 & & 1 \\ 1 & & 1 & 1 & 1 & 1 \\ 1 & 1 & & 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 1 & & 1 & 1 & 1 \\ & & 1 & 1 & & \\ & 1 & 1 & 1 & & 1 \\ & 1 & 1 & & & \\ & & & & & \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 1 & & 1 & 1 & 1 \\ & & 1 & 1 & & \\ & 1 & 1 & 1 & & 1 \\ & & & 1 & & 1 \\ & & & & & \end{array}\right]$$

$x_1 + 1 + 1 + x_5 = 1$

$x_3 + 1 = 0,\ x_3 = 1$

$x_2 + 1 + 1 = 1,\ x_2 = 1$

$x_4 = 1$

$x_1 + x_5 = 1$

$x_1 = 1 + t,\ x_2 = 1,\ x_3 = 1,\ x_4 = 1,\ x_5 = t$

$x_5 = t \in \{0, 1\},\ x_1 = 1 + t$

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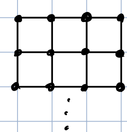
For grid graph  ... we can improve the above alghoritm as follows.

Denote values (pressed or not) of verteces of the first row as $x_1, x_2, x_3, x_4$. Then the verteces of the second row must be

$x_5$ such that $x_1 + x_2 + x_5 = 1 \Rightarrow x_5 = x_1 + x_2 + 1$

$x_6$ such that $x_2 + x_1 + x_3 + x_6 = 1 \Rightarrow x_6 = x_1 + x_2 + x_3 + 1$

$x_7$ such that $x_3 + x_2 + x_4 + x_7 = 1 \Rightarrow x_7 = x_2 + x_3 + x_4 + 1$

$x_8$ such that $x_4 + x_3 + x_8 = 1 \Rightarrow x_8 = x_3 + x_4 + 1$

In the same way the verteces of the third row must be

$x_9$ such that $x_5 + x_1 + x_6 + x_9 = 1 \Rightarrow x_9 = x_1 + x_5 + x_6 + 1 =$

$= x_1 + x_1 + x_2 + 1 + x_1 + x_2 + x_3 + 1 + 1 = x_1 + x_3 + 1$

$x_{10}$ such that $x_6 + x_5 + x_2 + x_7 + x_{10} = 1 \Rightarrow x_{10} = x_2 + x_5 + x_6 + x_7 + 1 =$

$= x_2 + x_1 + x_2 + 1 + x_1 + x_2 + x_3 + 1 + x_2 + x_3 + x_4 + 1 + 1 = x_4$

$x_{11}$ such that $x_7 + x_6 + x_8 + x_3 + x_{11} = 1 \Rightarrow x_{11} = x_3 + x_6 + x_7 + x_8 + 1 =$

$= x_3 + x_1 + x_2 + x_3 + 1 + x_2 + x_3 + x_4 + 1 + x_3 + x_4 + 1 + 1 = x_1$

$x_{12}$ such that

After we expressed the values of the verteces in the last row by the values of the verteces in the first row we write down the conditions for the values of the verteces in the last row.

$$\left.\begin{array}{ll} x_9 + x_5 + x_{10} = 1 & \text{for } x_9 \\ x_{10} + x_6 + x_9 + x_{11} = 1 & \text{for } x_{10} \\ x_{11} + x_7 + x_{10} + x_{12} = 1 & \text{for } x_{11} \\ x_{12} + x_8 + x_{11} = 1 & \text{for } x_{12} \end{array}\right\}$$

We get a system of $n$ equations with $n$ unknowns.