# General Exam Instructions

If you are preparing for the Probability section of your general exam and plan to have me as your examiner, it is essential to review my notes, which cover both Probability I and II. The most recent version can be found at this link: http://merlot.stat.uconn.edu/~boba/stat6894/probabilityI.pdf

I anticipate that you will memorize all the significant definitions and theorem statements. While I won't require you to replicate complex proofs, you should be proficient in executing simple, standard techniques. For instance, you should be able to:

- Prove Chebyshev's Inequality,
- Derive formulas for the characteristic function of standard distributions,
- Verify that a sequence of random variables forms a martingale,
- Provide an example of a sequence of random variables that converges in probability but not with probability 1.

Below is a list of definitions, concepts, and theorem statements that are absolutely crucial to know. Please note, failing to answer any of these questions will result in automatic failure of the exam.

1. Definitions of $\sigma$-field.
2. Probability space.
3. Continuity and $\sigma$-additiveness of additive measure.
4. Random variable.
5. Definition of independence.
6. Borel-Cantelli lemmas.
7. Expectation of simple random variables.
8. Expectation of random variables.
9. Monotone convergence theorem.
10. Fatou's lemma. Dominated convergence theorem.
11. Chebyshev's inequality.
12. Cauchy-Schwarz's inequality.
13. Jensen's inequality.
14. Radon-Nikodym theorem.
15. Product spaces and Fubini's theorem.
16. Characteristic Functions.
17. Different Types of Convergence: with probability 1, in $L_p$, in probability, in distribution.
18. Continuity Theorem
19. Week Laws of Large Numbers for iid random variables.
20. Central Limit Theorem for iid random variable.
21. Conditional Expectation.
22. Martingale: Definition.
23. Doob's Optional Stopping Theorem.
24. Doob's Submartingale Inequality.

For instance, you might be asked to explain how we define the expectation of a non-negative random variable $X$. Here is the answer.

Any random variable is a measurable mapping of $(\Omega, \mathcal{F})$ to $(\mathbb{R}, \mathcal{B})$. Therefore, we can construct a sequence of non-negative simple random variables $\{X_n\}$ such that $X_n(\omega) \uparrow X(\omega)$ for any $\omega \in \Omega$. The expectation of a simple random variable is given by an explicit formula. Then $E(X) = \lim_{n\to\infty} E(X_n)$. This limit exists and does not depend on the approximation of $X$ by $X_n$.

In addition, you may be asked to define measurability, prove the existence of the limit, or define a simple random variable. However, you will not be asked to demonstrate that the limit is the same for any two approximations.