Programming Paradigms.

PS1.

1. a) $\lambda x.(\lambda y.xy)x = \lambda a.(\lambda b.ab)a$

b) $\lambda x.(\lambda x.x)x = \lambda x.(\lambda y.y)x$

c) $\lambda x.\lambda y.x\ y = \lambda a.\lambda b.ab$

d) $\lambda x.x(\lambda x.x) = \lambda y.y(\lambda x.x)$

e) $\lambda x.(\lambda x.x)x = \lambda a.(\lambda b.b)a$

f) $(\lambda x.\lambda y.y)\ z\ x = (\lambda a.\lambda b.b)\ z\ x$

2. a) $(\lambda x.\lambda y.x)\ y\ z \rightarrow$

This y is not bound with the one which is inside brackets as its in another scope $\rightarrow (\lambda x.\lambda y.x)\ y_0\ z \rightarrow$

$[x \rightarrow y_0]\ (\lambda y.x)z \rightarrow$

$(\lambda y.y_0)z \rightarrow$

y_0 which is y

b) $(\lambda x.\lambda y.x)(\lambda z.y)\ z\ w \rightarrow$

Firstly, let's change variable names to not confuse it in scopes

$(\lambda x.\lambda y.x)\ (\lambda z.y_0)\ z_0\ w \rightarrow$

$\rightarrow [x \rightarrow \lambda z.y_0]\ (\lambda y.x)z_0\ w \rightarrow$

$\rightarrow (\lambda y.\lambda z.y_0)z_0\ w$

$\rightarrow (\lambda y . \lambda z . y_0) z_0 \, w \rightarrow$

$([y \rightarrow z_0](\lambda z . y_0)) \, w \rightarrow$

$(\lambda z . y_0) \, w$

$\rightarrow [z \rightarrow w] \, y_0 \rightarrow y_0$

c) $(\lambda b . \lambda f . \lambda t . b \; t \; f) \underbrace{(\lambda f . \lambda t . t)}_{a} \rightarrow$

$\lambda f . \lambda t . \; a \; t f \overset{def}{=} \lambda f . \lambda t . (\lambda f . \lambda t . t) \, t \, f \rightarrow$

$\lambda f . \lambda t . \; (\lambda t . t) \, f \rightarrow \lambda f . \lambda t . \; f$

d) $(\lambda s . \lambda z . s \, (s \, z)) \underbrace{(\lambda b . \lambda f . \lambda t . b \; t \; f)}_{A} \underbrace{(\lambda f . \lambda t . t)}_{B} \rightarrow$

$\rightarrow (\lambda z . \; A \, (A \, z)) \, B \rightarrow A \, (A \, B)$

$\rightarrow (\lambda b . \lambda f . \lambda t . b \; t \; f) \, (\underbrace{(\lambda b . \lambda f . \lambda t . b \; t f)(\lambda f . \lambda t . t)}_{\text{same as in previous}}) \rightarrow$

$\rightarrow (\lambda b . \lambda f . \lambda t . b \; t \; f) \, (\lambda f . \lambda t . f) \rightarrow$

$\rightarrow \lambda f . \lambda t . (\lambda f . \lambda t . f) \, t \, f \rightarrow \lambda f . \lambda t . t$

e) $\underbrace{(\lambda s . \lambda z . s \, (s \, z)) (\lambda s . \lambda z . s \, (s \, z))}_{A} \underbrace{(\lambda b . \lambda f . \lambda t . b \; t f)}_{B}$

$(\lambda z . \; A \, (A \, z)) (B) \, (\lambda f . \lambda t . t) \rightarrow$

$(A \, (A \; B)) \; (\lambda f . \lambda t . t)$

$A \; B = (\lambda s . \lambda z . s \, (s \, z)) \, (\lambda b . \underbrace{\lambda f . \lambda t . \; b \; t f}_{B}) \Longrightarrow$

e) $\underbrace{(\lambda s.\lambda z.s(s z))}_{A} \underbrace{(\lambda s.\lambda z.s(s z))}_{A} \underbrace{(\lambda b.\lambda f.\lambda t.b\ t f)}_{B} \underbrace{(\lambda f.\lambda t.t)}_{C} \equiv$

$\overset{def}{\Rightarrow} (\lambda z.\ A(A z))\ B\ C \to A(A\ B)C$

$\overset{def}{\Rightarrow} (\lambda s.\lambda z.\ s(s z))\ (A\ B)C$

$\to (A\ B)((A\ B)\ C)$

$\to (\lambda z.\ B\ (B z))\ ((\lambda z.B(B z))\ C)$

$\to (\lambda z.\ B\ (B z))\ (\ B\ (B C))$

$\to B\ (B\ (B\ (B C)))$

$B\ C \overset{def}{=} (\lambda b.\lambda f.\lambda t.\ b\ t f)\ (\lambda f.\lambda t.\ t) \to \lambda f.\lambda t.f$
from c).

$\to B(B(\underbrace{B\ (\lambda f.\lambda t.f)}_{\substack{\text{def from d)} \\ \lambda f.\lambda t.t}})))$

$\to B(\ B\ C) \to B\ (\ \lambda f.\lambda t.f) \to \lambda f.\lambda t.t$

3. $tru = \lambda t.\lambda f.t$
$fls = \lambda t.\lambda f.f$

a) $implies = \lambda b.\lambda c.\ b\ c\ tru$

b) $implies\ fls\ tru \to$

$(\lambda b.\lambda c.\ b\ c\ tru)\ fls\ tru \to$

$(\lambda c.\ fls\ c\ tru)\ tru \to$

$fls\ tru\ tru =$

$(\lambda t.\lambda f.f)$ tru tru $\rightarrow$

$(\lambda f.f)$ tru $\rightarrow$

tru.

For F $\rightarrow$ T the result is T.

4. $c_0 = \lambda s.\lambda z.z$

$c_1 = \lambda s.\lambda z.s\,z$

$c_2 = \lambda s.\lambda z.s\,(s\,z)$

...

plus $= \lambda m.\lambda n.\lambda s.\lambda z.\ m\ s\ (n\ s\ z)$

times $= \lambda m.\lambda n.\ \lambda s.\lambda z.m\ (n\ s)\ z$

a) I. $n \mapsto 2n+1$

(plus c_1 (times c_2 n)).

II. $n \rightarrow n^2 + 1$

(plus (times n n) c_1)

III. $n \rightarrow 2^n + 1$.

power $= \lambda m.\lambda n.\ n\ m$

(plus c_1 (power c_2 n))

IV. $n \rightarrow 2^{n+1}$

(power c_2 (plus n c_1))

b) I. (plus c_1 (times c_2 c_2)) →

(plus $\lambda s.\lambda z.\ s\ z$ ($\lambda s.\lambda z.\ c_2$ (c_2 s) z)) →

(plus $\lambda s.\lambda z.\ s\ z$ ($\lambda s.\lambda z.$ (c_2 s) ((c_2 s) z)) →

(plus $\lambda s.\lambda z.\ s\ z$ ($\lambda s.\lambda z.$ ($\lambda z.\ s\ (s\ z)$) (s (s z)))) →

(plus $\lambda s.\lambda z.\ s\ z$ $\lambda s.\lambda z.\ s(s(s(s\ z))))$ →

$\lambda s.\lambda z.\ s(s(s(s(s\ z)))) = c_5$ V

II. (plus c_1 (times c_2 c_2)) – same as previous.

III. (plus c_1 (power c_2 c_2)) →

(plus c_1 (c_2 c_2)) →

(plus c_1 ($\lambda s.\lambda z.\ s(s\ z)$) $\lambda a.\lambda b.\ a(a\ b)$) →

(plus c_1 ($\lambda z.$ ($\lambda a.\lambda b.\ a(a\ b)$) ($\lambda a.\lambda b.\ a(a\ b)$ z))) →

(plus c_1 ($\lambda z.$ (. . .) ($\lambda c.\ z(z\ c)$))) →

(plus c_1 (λz $\lambda b.$ ($\lambda c.\ z(z\ c)$) ($\lambda c.\ z(z\ c)$ b)))) →

(plus c_1 ($\lambda z.\lambda b.$ ($\lambda c.\ z(z\ c)$) (z (z b))))) →

(plus c_1 ($\lambda z.\lambda b.\ z(z\ (z(z\ b))))$) →

$\lambda z.\lambda b.\ z(z(z(z(z\ b))))$ /5

IV. (power c_2 (plus c_2 c_1)) →

(power c_2 c_3) → c_3 c_2. It will require a lot of writings to compute this expression. Will work in the same way as in prev. example.