

# Unit 4: Projectiles

Fall 24



# Projectiles

Def: A projectile is any object that moves under the force of gravity only

## Type I: Drop an object

•  $V_0 = 0 \text{ m/s}$   
 $a = g \approx -10 \text{ m/s}^2$

Horizontal	Vertical
$\Delta x = 0 \text{ m}$	$\Delta y = \frac{1}{2} a_y t^2 + V_{oy} \cdot t$
$V_{ox} = 0 \text{ m/s}$	$V_{oy} = 0 \text{ m/s}$
$a_x = 0 \text{ m/s}^2$	$a_y = g \approx -10 \text{ m/s}^2$
$V_x = 0 \text{ m/s}$	$V_y = g t + V_0$

↑  $t$  ↑

## Example: Reaction time

→  $\Delta y \rightarrow \Delta y = \frac{1}{2} a_y t^2 + V_{oy} t$

## Type II: Throw down



Horizontal

$$\Delta x = 0 \text{ m}$$

$$V_{0x} = 0 \text{ m/s}$$

$$V_x = 0 \text{ m/s}$$

$$a_x = 0 \text{ m/s}^2$$

Vertical

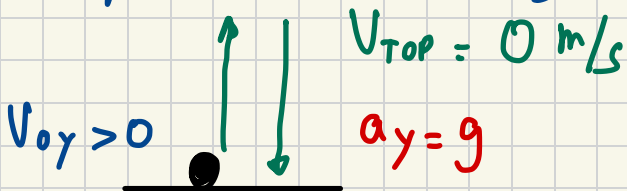
$$\Delta y = \frac{1}{2} a_y t^2 + V_{oy} t$$

$$V_{oy} < 0 \text{ (negative)}$$

$$a_y = g \approx -10 \text{ m/s}^2$$

$$V_y = a_y t + V_{oy}$$

## Type III: Throwing up



Horizontal

Vertical


$$\Delta y = \frac{1}{2} a_y t^2 + V_{oy} t$$

$$V_{oy} > 0 \text{ (positive)}$$

$$a_y = g \approx -10 \text{ m/s}^2$$

$$V_y = a_y t + V_{oy}$$

Example:

  
(same as above)

$V_{oy} = 2 \text{ m/s}$        $h_{\text{max}} = ?$   
 $t_{\text{up}} = ?$

1) First find  $t_{\text{up}}$

$$V_{\text{top}} = a_y t_{\text{up}} + V_{oy} \Rightarrow 0 = (-10) t_{\text{up}} + 2$$

$$t_{\text{up}} = \frac{-2}{-10} = 0.2 \text{ s}$$

2) Find  $\Delta y = ?$

$$\Delta y = \frac{1}{2} (-10) (0.2)^2 + (2)(0.2) = 0.2 \text{ m} \quad h_{\text{max}} = 0.2 \text{ m}$$

What about the motion down?

•  $V_{oy} = 0 \text{ m/s}$

•  $V_y(\text{at the bottom}) = -2 \text{ m/s}$

$$V_y(\text{bottom}) = g t_{\text{down}} + V_{oy} = -10 \cdot 0.2 = -2 \text{ m/s}$$

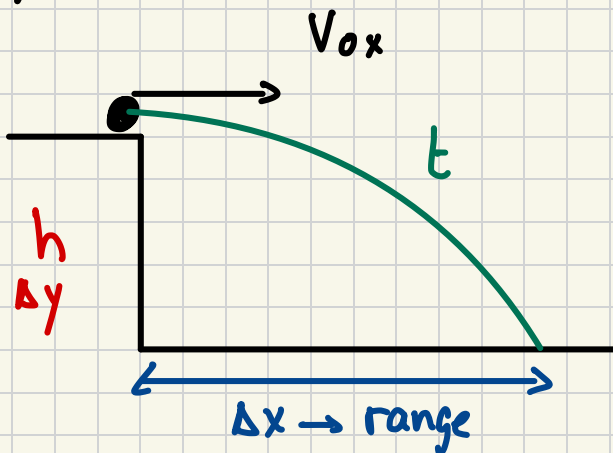
If the object lands at the same height where it started,  
 $t_{\text{up}} = t_{\text{down}}$

Trick:  $\Delta y_{\text{TOTAL}} = 0 \text{ m}$

$$t_T = t_{\text{up}} + t_{\text{down}} = 2 t_{\text{up}}$$

$t_T \rightarrow$  "hang time"

## Type IV: Horizontal Projectiles



The method is to treat each direction independently. The only thing in common is the TIME

Horizontal

$$a_x = 0 \text{ m/s}^2$$

$$V_{ox} \neq 0$$

$$V_x = V_{ox}$$

(constant)

$$\Delta x = \frac{1}{2} a_x t^2 + V_{ox} t$$

Vertical

$$a_y = g \approx -10 \text{ m/s}^2$$

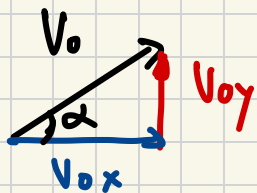
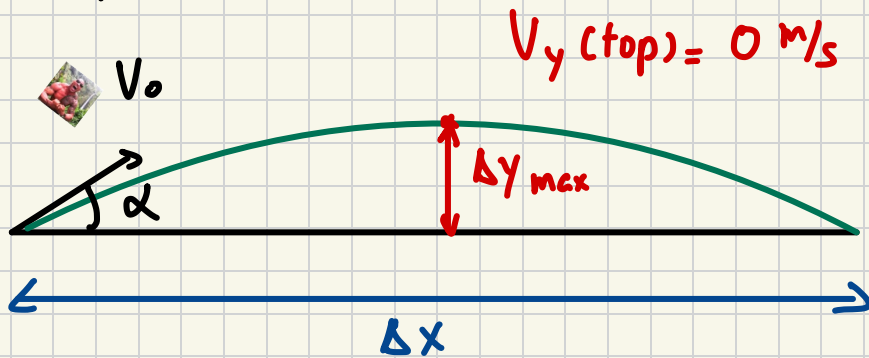
$$V_{oy} = 0 \text{ m/s}$$

$$V_y = g t + V_{oy}$$

$$\Delta y = \frac{1}{2} g t^2 + V_{oy} t$$

$t$

# Type V : Full Projectile



SOH CAHTOA  $a^2 + b^2 = c^2$

$$\sin \alpha = \frac{O}{H} \quad \tan \alpha = \frac{O}{A}$$

$$\cos \alpha = \frac{A}{H}$$

Now, the velocity has both **horizontal** and **vertical** components.

$V_{0y}$  : (opposite)

$$\sin \alpha = \frac{V_{0y}}{V_0} \Rightarrow V_{0y} = V_0 \sin \alpha$$

$V_{0x}$  : (adjacent)

$$\cos \alpha = \frac{V_{0x}}{V_0} \Rightarrow V_{0x} = V_0 \cos \alpha$$

**Horizontal**

**Vertical**

$$a_x = 0 \text{ m/s}^2$$

$$V_{0x} = V_0 \cos \alpha$$

$$V_x = V_0 \cos \alpha$$

$$\Delta X = V_{0x} \cdot t$$

$$a_y = g \approx -10 \text{ m/s}^2$$

$$V_{0y} = V_0 \sin \alpha$$

$$V_y = a_y t + V_{0y}$$

$$\Delta y = \frac{1}{2} a_y t^2 + V_{0y} \cdot t$$

\* NOTE:

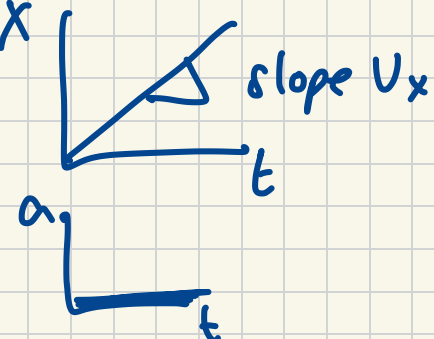
If the object lands at the same height it started:

$$\Delta y_t = 0 \text{ m}$$

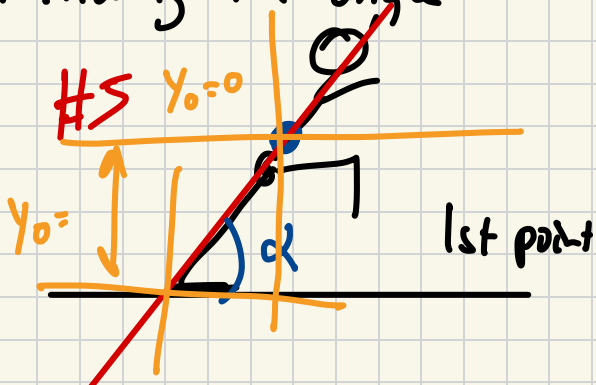
# Notes for Long jump

\* 2 position graphs, 2 velocity graphs, 2 accelerat.

horizontal: position graph  $x$



\* Finding the angle

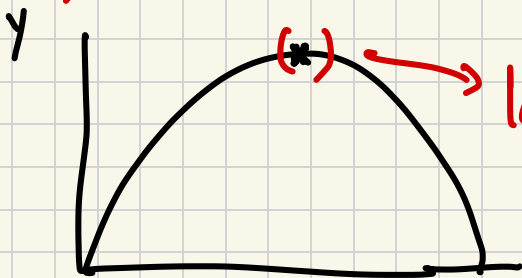


print a screenshot of that frame

$\Delta x$  (range) from video

$x_f, t_T \Rightarrow$  last point

$$x_0 = 0 \quad \Delta x = x_f - x_0$$



look at the table to find  $t$  &  $y$   
 $t_{up}, y_{max}$

$$V_{0x} = v_x = \frac{\Delta x}{t}$$

$\hookrightarrow$  from logger pro

$$\Delta y = \frac{1}{2} a t^2 + v_{0y} t \Rightarrow \Delta y_{total} = 0$$

$\downarrow$   
 $t_T \Rightarrow$  logger pro

$$a = -9.8 \text{ m/s}^2$$

angle  $\Rightarrow V_{0x}, V_{0y}$  (we calculated it)

$$\tan \alpha = \frac{V_{0y}}{V_{0x}} \Rightarrow \alpha = \tan^{-1} \frac{V_{0y}}{V_{0x}}$$

$$V_0^2 = V_{0x}^2 + V_{0y}^2$$

# Physics Unit 4 Projectiles

This packet will be due on Wednesday, December 4. To turn it in, scan your work and submit a single pdf on Canvas.

The extra practice problems on the last page are not part of your grade but they may be useful in helping you study for the test and midterm.

## Suggested Timeline

Date	Suggested problems
Nov. 19-20	1-5
Nov. 21-22	6-7
Dec. 2 in class	8-11
Dec. 2 homework	12-15

1. You are standing on top of a building and make the very poor decision to drop a 0.1-kg water balloon onto the head of Mr. Ramey, who is standing directly below you. You notice that it takes 0.85 seconds for the water balloon to strike Mr. Ramey.

a) Write down all the following “givens” and mark things you do not know with a question mark.

$$\Delta x =$$

$$\Delta y =$$

$$v_{0x} =$$

$$v_{0y} =$$

$$v_x =$$

$$v_y =$$

$$a_x =$$

$$a_y =$$

$$t =$$

b) What is the acceleration of the water balloon as it is falling?

c) How tall is the building you are standing on?

d) How fast is the water balloon moving when it strikes Mr. Ramey?



2. Max the Rottweiler likes to dig very deep holes to store bones in. He digs a hole that is 2.5 m deep, and he drops a bone into the hole. How long does it take the bone to hit the bottom of the hole?

First, write down all the following “givens” and mark things you do not know with a question mark.

$$\Delta x =$$

$$\Delta y =$$

$$v_{0x} =$$

$$v_{0y} =$$

$$v_x =$$

$$v_y =$$

$$a_x =$$

$$a_y =$$

$$t =$$

Then solve....

3. A ball is thrown down with an initial speed of 20 m/s.  
a) First, write down all the following “givens” and mark things you do not know with a question mark.

$$\Delta x =$$

$$\Delta y =$$

$$v_{0x} =$$

$$v_{0y} =$$

$$v_x =$$

$$v_y =$$

$$a_x =$$

$$a_y =$$

$$t =$$

- b) How far does the ball move in its first four seconds? Hint: be careful to make appropriate things negative!

- c) How much time is required for the ball to reach a speed of 50 m/s?

- d) What is the ball's velocity after it has traveled 100 m?

4. You decide that you are really sick of reading your U.S. History textbook and decide to throw it off the roof of your house. The roof is 7 meters high, and you throw the book straight down with an initial velocity of 12 m/s. How long does it take your book to hit the ground?

First, write down all the givens:

$$\Delta x =$$

$$\Delta y =$$

$$v_{0x} =$$

$$v_{0y} =$$

$$v_x =$$

$$v_y =$$

$$a_x =$$

$$a_y =$$

$$t =$$

Now solve....

5. A ball is thrown directly upward with a speed of 15 m/s.
- a) What is the acceleration of the rock?
  - b) What is the height of the rock after 1 s? Make sure to write all your givens before solving!
  - c) Calculate the time required to reach a velocity of 3 m/s upward.
  - d) Calculate the time required to reach a velocity of 5 m/s downward.
  - e) How long does it take the ball to reach its highest point?
  - f) How high is the highest point?

6. A ball punted vertically has a hang time of 3.8 s. What was its initial velocity? As part of your answer, include a force diagram of the situation and all relevant calculations and/or graphs.

First, write down all the givens:

$$\Delta x =$$

$$\Delta y =$$

$$v_{0x} =$$

$$v_{0y} =$$

$$v_x =$$

$$v_y =$$

$$a_x =$$

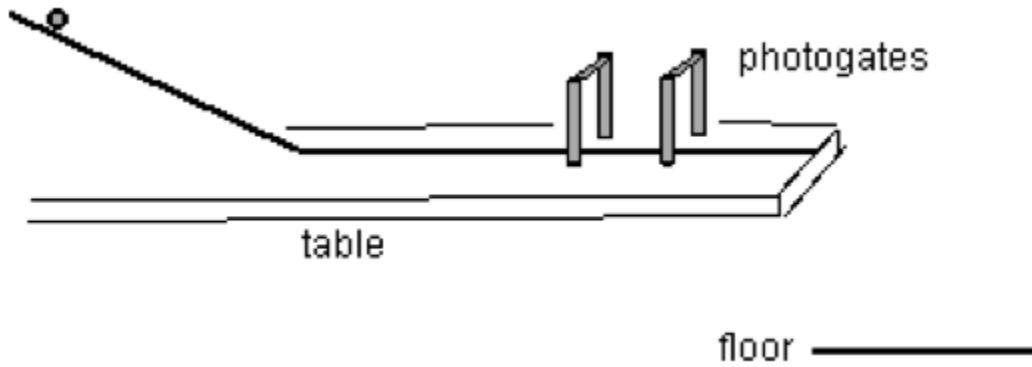
$$a_y =$$

$$t =$$

Now solve....

7. A rock is thrown straight up with an initial velocity of 22 m/s. How long is it in the air before it returns to the thrower? As part of your answer, include graphs of vertical position, velocity and acceleration.

8. A student finds that it takes 0.2 s for a ball to pass between two photogates placed 0.3 m apart on a level table. The end of the table is 0.92 m above the floor. Where could a coin be placed so that the ball would hit the coin exactly?



9. Suppose that in the same setup as #8 but with a different speed, the ball hits the ground exactly 0.25 m horizontally from the end of the table.

a) What was the speed of the ball as it left the table?

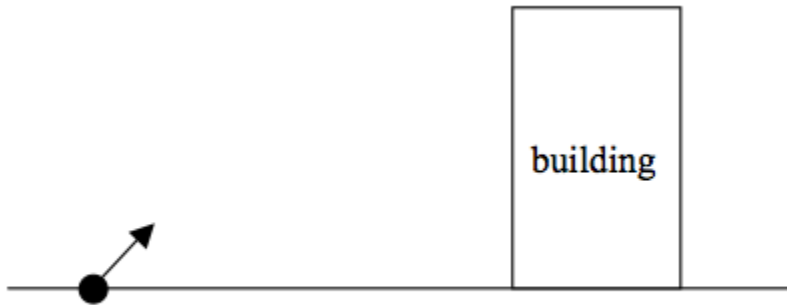
b) How long did it take the ball to pass between the photogates?

10. The movie "The Gods Must Be Crazy" begins with a pilot dropping a bottle out of an airplane. A surprised native below thinks it is a message from the gods. If the plane was flying at an altitude of 500 m, and the bottle lands 400 m horizontally from the initial dropping point, how fast was the plane going?
11. In many locations, old abandoned stone quarries have filled with water once excavating has been completed. While standing on a quarry wall a boy tosses a piece of granite into the water below. If he throws the rock horizontally with a speed of 3.0 m/s, and it strikes the water 4.5 m away from the wall, how high is the wall?



12. Tad drops a cherry pit out the car window 1.0 m above the ground while traveling at 18 m/s. How far horizontally from the initial dropping point will the pit land? If the car continues to go at the same speed, where will it be in relation to the pit when the pit hits the ground?

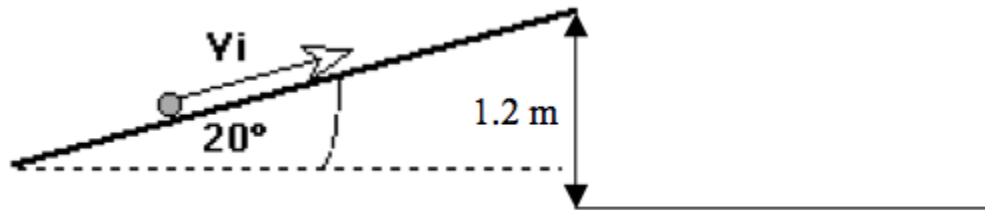
13. A water balloon is launched towards a building 24 m away with an initial velocity of 18 m/s at an angle of 50 degrees above the horizontal.



a) At what height will the balloon hit the building?

b) If the balloon misses or shoots over the building (assume the building is very short in this case) how far will the balloon land from its launch location? (Hint: 24 m is NOT  $\Delta x$ !)

14. A metal sphere is launched with an initial velocity of  $1.5 \text{ m/s}$  as it leaves the ramp. The end of the ramp is  $1.20 \text{ m}$  above the floor. Find the range of the ball. (The range is the horizontal distance the object travels before hitting the ground.)



15. A bullet is fired from a gun at a  $30.0^\circ$  angle with the horizontal and a muzzle velocity (speed when it comes out of the gun) of 600 m/s. What is the range of the gun for this angle of elevation?

# Projectiles: Extra Practice Problems

These problems are NOT part of your grade, but they may be useful for extra practice.

1. A girl tosses a softball vertically upward with an initial speed of 12 m/s.
  - a. What is the hang time of the ball if it returns to its starting height?
  - b. How high does the ball go?
  - c. How fast is the ball moving at a time of 1.5 seconds? Is it moving up or down?
  - d. At the high point, what is the value of the ball's acceleration?
2. A ball thrown horizontally from a 13-m-high building strikes the ground 5 m from the building.
  - a. What is the initial vertical velocity of the ball?
  - b. What are the horizontal and vertical accelerations of the ball?
  - c. With what horizontal speed was the ball thrown?
3.
  - a. A ship is 3110 m from an enemy ship. It shoots a projectile with an initial velocity of 250 m/s at an angle of 75 degrees. How close to the enemy ship does the projectile land? (Hint: 3110 m is not  $\Delta x$ !)
  - b. Suppose there is a volcanic mountain sticking out of the water at a distance of 2500 m from the original ship. How long does it take the projectile to reach the mountain?
  - c. If the mountain is 1800 m high, how far over the top of the mountain does the projectile go?
4. A ball is thrown down vertically with an initial speed of 20.5 m/s from a height of 58.8 m.
  - a. What values should you use for  $v_{0y}$  and  $\Delta y$  in this situation?
  - b. How long will it take for the ball to reach the ground?
  - c. What will be its speed just before it strikes the ground?

## Challenge Problem

The current world record for "punkin chunkin" was set on Sept. 9, 2010 in Moab, Utah.

According to the World Championship Punkin Chunkin Association (WCPCA), pumpkins may be chunked by any mechanical means (e.g., by slingshot, catapult, centrifugal device, trebuchet, or pneumatic cannon) as long as the pumpkin remains whole until it hits the ground.

The current world record was achieved using a pneumatic (air) cannon that was set to shoot the pumpkin at an angle of about  $40^\circ$  with respect to the horizontal, and the hang time of the pumpkin was 16.8 seconds.

- a) What was the record-setting range for the pumpkin?
- b) With what velocity did the record-setting pumpkin leave the pneumatic cannon?
- c) (fiction) In an attempt to repeat the record-setting chunk, a mishap occurred. The pumpkin hit Dumbo the Flying Elephant as he was flying south for the winter. (Yes, Dumbo is still alive and flying. And, yes, the event generated a firestorm in the media that resulted in the cancellation of the annual Punkin Chunkin Championships.) Here's the question... If the pumpkin hit Dumbo 10 seconds after the pumpkin was fired, how far below the maximum height of the pumpkin was Dumbo flying?

# In-class Problems

1. An object dropped from an airplane hits the ground 8 s later. How high is the airplane?

First, write down all the following "givens" and mark things you do not know with a question mark.

$$\Delta x = 0 \text{ m}$$

$$v_{0x} = 0 \text{ m/s}$$

$$v_x = 0 \text{ m/s}$$

$$a_x = 0 \text{ m/s}^2$$

$$t = 8 \text{ s}$$

$$\Delta y = \frac{1}{2} g t^2 + v_{0y} t$$

$$v_{0y} = 0 \text{ m/s}$$

$$v_y = g t + v_{0y}$$

$$a_y = g \approx -10 \text{ m/s}^2$$

Then solve....

$$\Delta y = \frac{1}{2} \times (-10)(8)^2 + 0 \cdot 8 = -5 \times 64 = -320 \text{ m}$$

so  $h_{\text{plane}} = 320 \text{ m}$

V of the object when it hits the ground?

$$V_y = g t + v_{0y} = -10 \text{ m/s}^2 \times 8 \text{ s} = -80 \text{ m/s}$$

2. With what speed does a freely falling object dropped from a height of 90 m hit the ground? How long does the object take to fall this distance? (Note that there are two parts to this problem!)

First, write down all the following "givens" and mark things you do not know with a question mark.

$$\Delta x = 0 \text{ m}$$

$$v_{0x} = 0 \text{ m/s}$$

$$v_x = 0 \text{ m/s}$$

$$a_x = 0 \text{ m/s}^2$$

$$t = ?$$

$$\Delta y = -90 \text{ m}$$

$$v_{0y} = 0 \text{ m/s}$$

$$v_y = gt + v_0$$

$$a_y = -10 \text{ m/s}^2$$

Then solve....

$$\Delta y = -90 = \frac{1}{2}gt^2 + v_0t$$

$$-90 = \frac{1}{2}(-10)t^2 + 0 \cdot t$$

$$-90 = -5t^2 \Rightarrow t^2 = \frac{-90}{-5} = 18 \quad t = \sqrt{18} = 4.24 \text{ s}$$

$$V = gt + v_0 = -10 \times 4.24 = -42.4 \text{ m/s}$$

3. An evil student throws his worthless lab partner off a bridge with an initial downward velocity of 10 m/s. It takes the lab partner 4 seconds to land in the water below the bridge.

a) Write down all the following "givens" and mark things you do not know with a question mark.

$\Delta x =$   
 $v_{0x} =$   
 $v_x =$   
 $a_x =$   
 $t = 4 \text{ s}$

$$\Delta y = \frac{1}{2} g t^2 + v_{0y} t$$

$$v_{0y} = -10 \text{ m/s}$$

$$v_y = g t + v_{0y}$$

$$a_y = g \approx -10 \text{ m/s}^2$$

b) How high is the bridge?  $h \rightarrow \Delta y$

$$\Delta y = \frac{1}{2} \times (-10) (4)^2 + (-10) (4) = -5 \times 16 - 40 = -120 \text{ m}$$

$$h = 120 \text{ m}$$

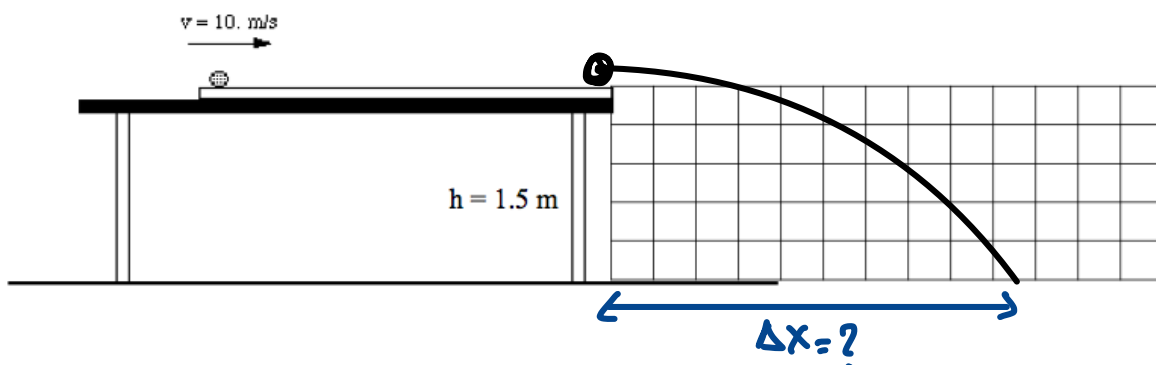
c) What is the velocity when the student reaches the water?

$$v_y = g t + v_{0y} = -10 \times 4 - 10 = -50 \text{ m/s}$$



4. When Tommy drops a rock off the edge of a cliff (initial velocity = 0), it takes 4 seconds to hit the ground. When he throws the same rock straight down, it takes 3 seconds to hit the ground. How fast did he throw it in the second case? Hint: treat this as two parts with separate givens. What is the same in both situations? Make sure to write all your givens for both situations.

5. A ball is rolling along a frictionless table with a speed of 10 m/s, as shown below.  
a) Sketch a diagram of the ball once it leaves the table.



- b) Write the givens...

$$\Delta x = v_{0x} t$$

$$v_{0x} = 10 \text{ m/s}$$

$$v_x = 10 \text{ m/s}$$

$$a_x = 0 \text{ m/s}^2$$

$$t = ?$$

$$\Delta y = -1.5 \text{ m} = y_f - y_0 = 0 - 1.5 \text{ m}$$

$$v_{0y} = 0 \text{ m/s}$$

$$\Delta y = \frac{1}{2} g t^2$$

$$v_y = g t$$

$$a_y = -10 \text{ m/s}^2$$

- c) How long does it take the ball to hit the floor?  $t = ?$

$$\Delta y = \frac{1}{2} g t^2 \Rightarrow -1.5 = \frac{1}{2} \times (10) \times t^2 \quad -1.5 = 5t^2$$

$$t^2 = \frac{1.5}{5} = 0.3 \quad t = 0.55 \text{ s}$$

- d) Determine how far forward the ball travels before it hits the floor.

$$\Delta x = v_{0x} t = 10 \times 0.55 = 5.5 \text{ m}$$

- e) Suppose the ball's speed were doubled to 20 m/s. Would that affect how long it takes to hit the ground (the time from part c)? Would it affect how far forward the ball goes?

The same  $t$  because the time to fall depends only on the height.

6. Suppose that a small plane, traveling at a height of 300 m at a speed of 60 m/s, dropped a bag of flour. How far horizontally from the point of release would the bag have traveled when it hit the ground? If the plane is moving with a constant velocity, how far forward will it have moved by the time the bag hits the ground? Where would the plane be in relation to the bag when the bag hits the ground?

7. A kickoff sends a football with an initial velocity of 25 m/s at an angle 50 degrees above the horizontal.

a) Find the x- and y-components of the initial velocity.

b) Write down all the givens:

$$\Delta x =$$

$$\Delta y =$$

$$v_{0x} =$$

$$v_{0y} =$$

$$v_x =$$

$$v_y =$$

$$a_x =$$

$$a_y =$$

$$t =$$

c) Find the time the ball is in the air.

d) Find the horizontal distance the ball travels before it hits the ground. (This is called the RANGE.)

e) Find the ball's maximum height.