Nuclear Structure Contribution to the Lamb Shift of Muonic Deuterium in Pionless EFT

Vadim Lensky

Institutsseminar Kern- und Hadronenphysik

Mainz December 02, 2019

Nuclear Structure Contribution to the Lamb Shift of Muonic Deuterium in Pionless EFT

Vadim Lensky

with A. Hiller Blin, V. Pascalutsa, B. Acharya, S. Bacca
M. Gorshteyn, M. Vanderhaeghen, C. Carlson

Institutsseminar Kern- und Hadronenphysik

Mainz December 02, 2019

Outline

- Introduction:
  → Nuclear radii and two-photon exchange (TPE) corrections
- Framework:
  → Pionless effective field theory ($\not{\pi}$EFT)
- In Practice:
  → Deuteron doubly virtual Compton scattering in $\not{\pi}$EFT
  → TPE corrections to the Lamb shift of μD in $\not{\pi}$EFT
- Detour:
  → TPE corrections via sum rules supplemented by EFTs
- Summary&Outlook

Nuclear Radii from Muonic Atoms

- Lamb shift is sensitive to the finite size of the atomic nucleus

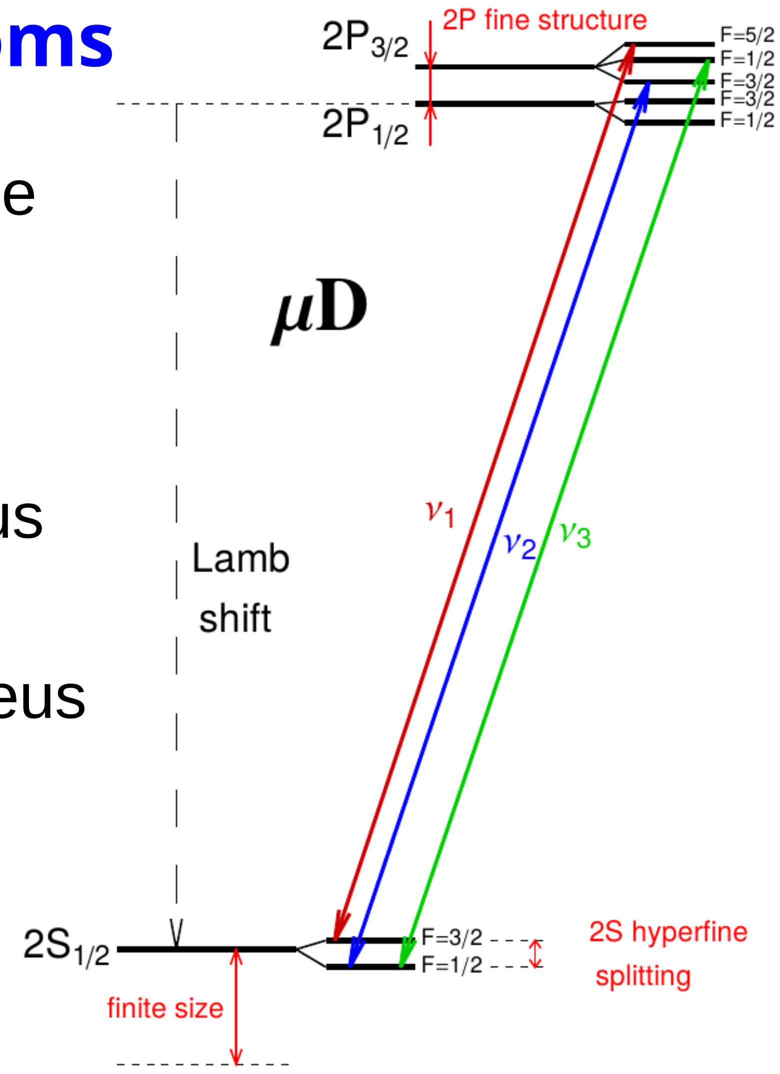
$$\Delta E^{\mathrm{f.s.}}_{2S} = m_r^3 \alpha^4 r^2 / 12$$

- Extraction of the nuclear charge radius from atomic spectra is possible
- Muon is heavier → closer to the nucleus → much larger effect $\sim 10^7 \times$electron
- Muonic atoms: better sensitivity to the radius, better precision
- Deuteron radius puzzle

$r_d(\mu\mathrm{D}) = 2.12562(78)\ \mathrm{fm}$ Pohl et al (2016)

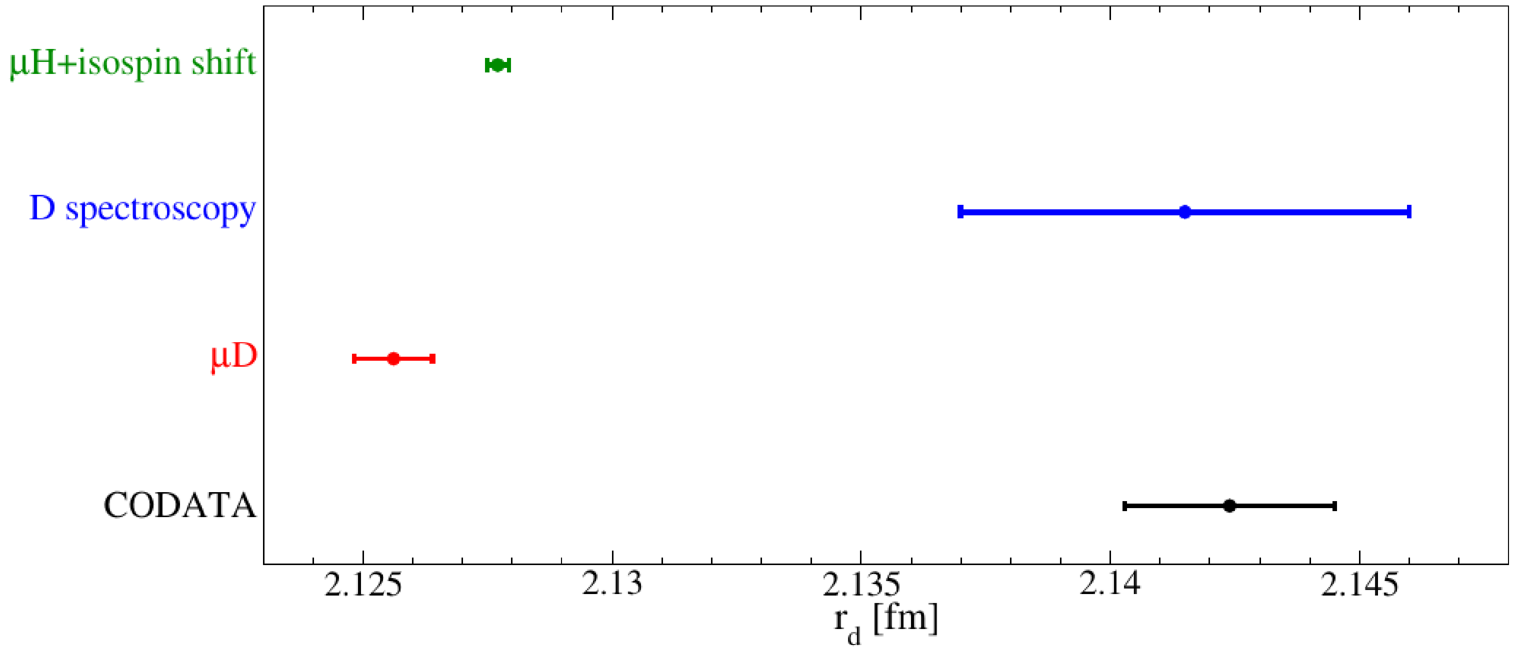
$r_d = 2.1424(21)\ \mathrm{fm}$ CODATA 2016

– 6..7σ discrepancy



Deuteron Radius Puzzle

- Deuteron radius puzzle



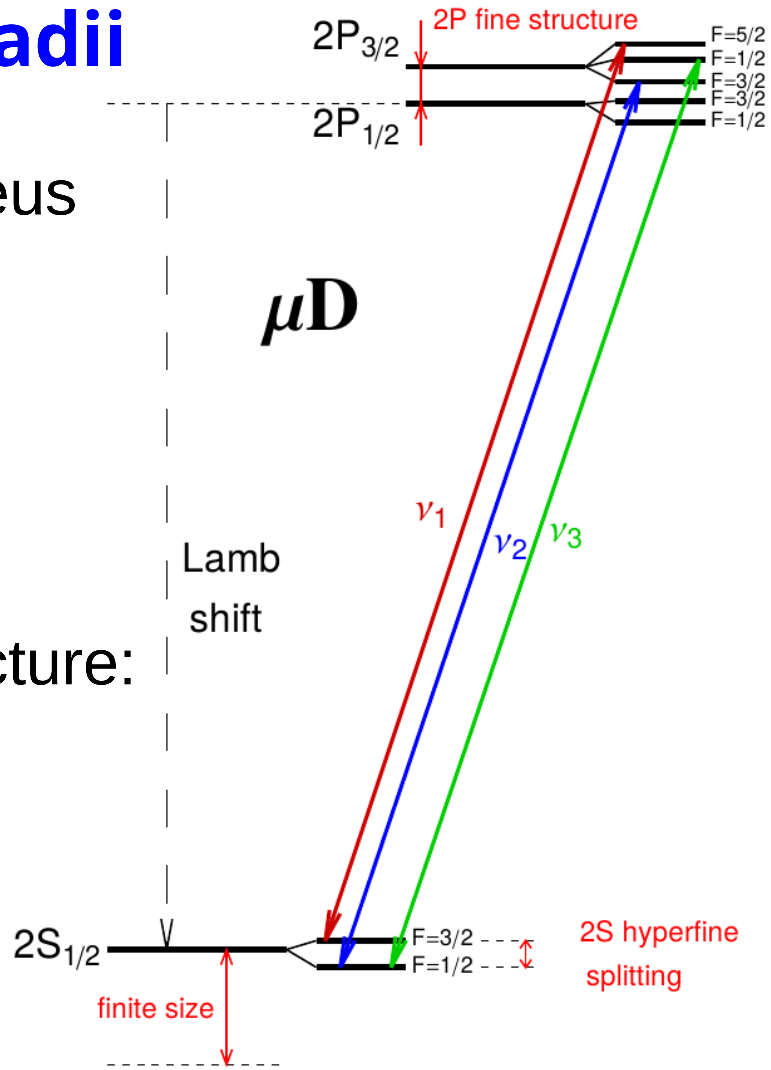
- CODATA uses the isotopic shift to evaluate r_d → dependent on the proton radius puzzle
- One can extract r_d from (electronic) deuterium spectroscopy alone (with a bigger error) → 3.5σ discrepancy on its own

Nuclear Structure in Nuclear Radii

- Muon is heavier → closer to the nucleus
- Better sensitivity to the radius!
- Side effect: better sensitivity to the detailed structure of the nucleus
- The leading effect of the nuclear structure:

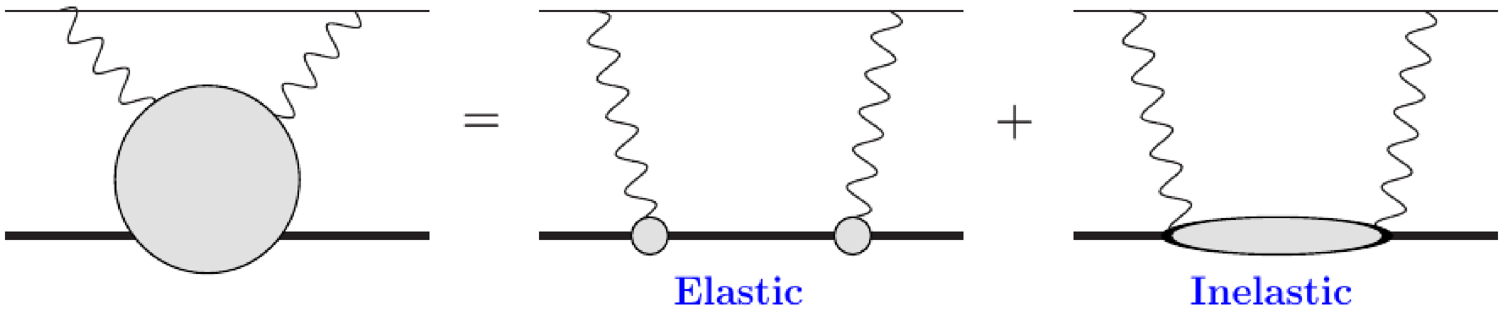
  two-photon exchange corrections

  - affect the extraction of nuclear radii
  - limit the theoretical accuracy



$$\Delta E_{LS}^{\text{theory}}(\mu \text{D}) = 228.7766(10) - 6.1103(3) r_d^2 + \Delta E_{\text{TPE}} \text{ meV}$$

Two-photon Exchange Corrections



- Elastic can be calculated in terms of e.m. deuteron FFs
- Ways to calculate inelastic
  - Hamiltonian approach (2nd order perturbation)
    Friar; Pachucki, Wienczek; Bacca, Barnea, Hernandez, Nevo Dinur, Ji
  - Sum rules (structure/response functions)
    Carlson, Gorshteyn, Vanderhaeghen
  - Nuclear VVCS amplitude
- We want to calculate the VVCS amplitude from pionless effective field theory – a systematically controlled approach
- I will also briefly discuss EFT input for sum rules

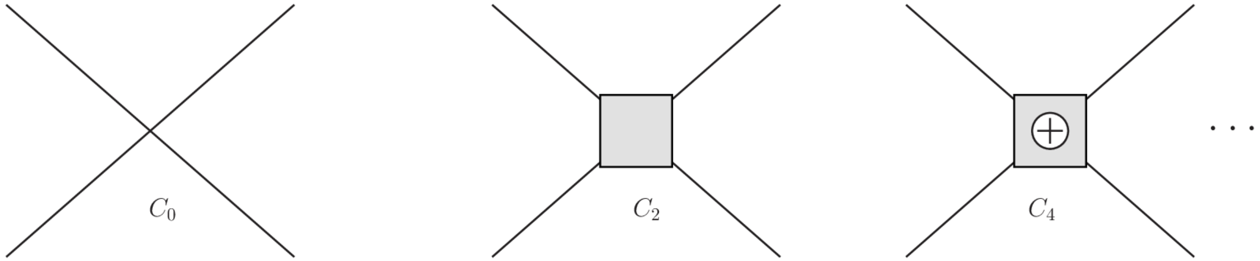
Pionless EFT

- Effective field theory of nucleon-nucleon interactions
- Works at very low energies/momenta $p \ll m_\pi$
- EFT of non-relativistic nucleons with contact interactions
  - Symmetries (isospin, Galilean, e.m. gauge, …)
  - Lagrangians are ordered according to the counting

$$\mathcal{L} = N^\dagger \left[iD_0 + \frac{\vec{D}^2}{2M} \right] N + \hat{\mu} N^\dagger \left(\vec{\sigma} \cdot \vec{B} \right) N$$

Kaplan, Savage, Wise;
Chen, Rupak, Savage; ...

$$+ C_0 \left(N^\dagger P^i N_c \right) \left(N_c^\dagger P^i N \right) + \frac{1}{8} C_2 \left\{ \left(N^\dagger P^i N_c \right) \left(N_c^\dagger P^i \left[\overleftarrow{D}^2 + \overrightarrow{D}^2 - 2 \overleftarrow{D} \cdot \overrightarrow{D} \right] N \right) + \mathrm{H.c.} \right\}$$
$$+ \ldots$$



Pionless EFT: Counting

- Nucleons are non-relativistic → $E \simeq p^2/M$: energies are $O(p^2)$
- Loop integrals are $dE\,d^3p = O(p^5)$
- Nucleon propagators are $(E - p^2/2M)^{-1} = O(p^{-2})$
- Typical momenta $p \sim \gamma = \sqrt{ME_d} \simeq 45\ \mathrm{MeV}$
- Expansion parameter $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$

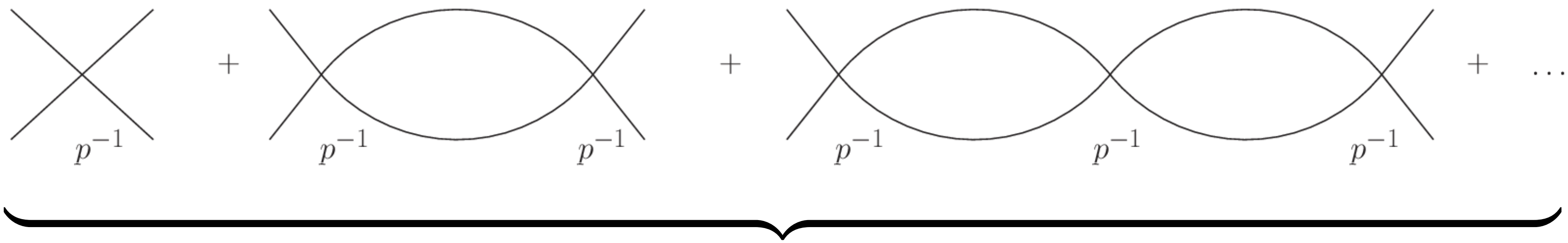
$$\mathcal{L} = N^\dagger \left[iD_0 + \frac{\vec{D}^2}{2M} \right] N + \hat{\mu} N^\dagger \left(\vec{\sigma} \cdot \vec{B} \right) N$$
$$+ C_0 \left(N^\dagger P^i N_c \right) \left(N_c^\dagger P^i N \right) + \frac{1}{8} C_2 \left\{ \left(N^\dagger P^i N_c \right) \left(N_c^\dagger P^i \left[\overleftarrow{D}^2 + \overrightarrow{D}^2 - 2 \overleftarrow{D} \cdot \overrightarrow{D} \right] N \right) + \mathrm{H.c.} \right\}$$
$$+ \ldots$$

- What is the counting for the coupling constants?
  - NN system has a low-lying bound/virtual state
  - Need to enhance constants: $C_{2n} = O(p^{-n-1})$

Pionless EFT: Resumming the LO

- $C_0 = O(p^{-1})$: leading-order loops need to be resummed

$\mathcal{M}_{-1} =$



$O(p^{-1})$

- Result: LO scattering amplitude reproduces the deuteron pole – the resummation is in fact a necessity!

$$\mathcal{M}_{-1} = \frac{4\pi}{M} \frac{1}{-\gamma - \mathrm{i}k}$$

- Higher-order terms (effective range, …) can be taken into account perturbatively, in the expansion in powers of p

Pionless EFT: Different Expansions Beyond LO

- Scattering amplitude in the deuteron channel

$$f(k) = \frac{1}{-\gamma - \mathrm{i}k + \frac{1}{2}\rho_d(k^2+\gamma^2) + w_2(k^2+\gamma^2)^2 + \ldots}$$

- Around the pole

$$f(k) = \frac{Z}{-\gamma - \mathrm{i}k} + R(k)$$ where $R(k)$ is regular at the pole and $Z = \dfrac{1}{1-\gamma\rho_d}$

- Reproduce the effective range at NLO (ρ-expansion)

$$f(k) = \frac{1}{(-\gamma - \mathrm{i}k)[1 + \frac{1}{2}\rho_d(\mathrm{i}k - \gamma) + \ldots]} = \frac{1}{-\gamma - \mathrm{i}k}\left[1 + \rho_d\gamma - \frac{1}{2}\rho_d(\mathrm{i}k+\gamma) + \ldots\right]$$

- The residue is reproduced perturbatively order-by-order

$$Z = 1 + (\gamma\rho_d) + (\gamma\rho_d)^2 + (\gamma\rho_d)^3 + \ldots$$

Pionless EFT: Different Expansions Beyond LO

- Scattering amplitude in the deuteron channel

$$f(k) = \frac{1}{-\gamma - \mathrm{i}k + \frac{1}{2}\rho_d(k^2+\gamma^2) + w_2(k^2+\gamma^2)^2 + \ldots}$$

- Around the pole

$$f(k) = \frac{Z}{-\gamma - \mathrm{i}k} + R(k)$$ where $R(k)$ is regular at the pole and $Z = \frac{1}{1-\gamma\rho_d}$

- … or reproduce the residue at NLO (Z-expansion)

$$f(k) = \frac{1}{(-\gamma - \mathrm{i}k)[\underbrace{1-\gamma\rho_d}_{1/Z} + \frac{1}{2}\rho_d(\mathrm{i}k+\gamma) + \ldots]} = \frac{Z}{-\gamma - \mathrm{i}k}\left[1 - \frac{Z}{2}\rho_d(\mathrm{i}k+\gamma) + \ldots\right]$$

- Z expansion yields better convergence for low-energy observables (→ asymptotic normalisation of the deuteron wave function is correct already at NLO)

Phillips, Rupak, Savage

In Practice: Counting

- We want to perform a NNLO calculation of the Lamb shift

$$\Delta E_{2S} = \frac{\alpha}{2\pi^2 m_\mu} \phi_{2S}(0)^2 \int \mathrm{d}^4 q \frac{f_L(-iq_0, Q^2) - 2(q_0^2/Q^2) f_T(-iq_0, Q^2)}{Q^2 \left(\frac{Q^4}{4m_\mu^2} + q_0^2\right)}$$

$$T_{\mathrm{VVCS}}^{\mathrm{scalar}} = \left[\epsilon_0\, \epsilon_0'^* f_L(\nu, Q^2) + \vec{\epsilon} \cdot \vec{\epsilon}'^* f_T(\nu, Q^2)\right] \vec{\epsilon}_d \cdot \vec{\epsilon}_d'^*$$

- Subtract the pole (elastic) pieces – calculated via emp. FFs
- The counting gives

$$f_L(\nu, Q^2) = p^{-2} + p^{-1} + \ldots$$

$$f_T(\nu, Q^2) = p^0 + p^1 + \ldots$$

- Together with the weighting in the integral, this means the transverse amplitude starts to contribute to LS only at N4LO

In Practice: Counting

- Relativistic corrections to one-nucleon vertex are counted at N4LO as well because of the $1/M^2$ suppression
- SD-mixing also starts only at N4LO
- The relevant Lagrangian up to NNLO [*]

$$\mathcal{L} = N^\dagger \left[iD_0 + \frac{\vec{D}^2}{2M} \right] N + \hat{\mu} N^\dagger \left(\vec{\sigma} \cdot \vec{B} \right) N$$

$$+ C_0 \left(N^\dagger P^i N_c \right) \left(N_c^\dagger P^i N \right) + \frac{1}{8} C_2 \left\{ \left(N^\dagger P^i N_c \right) \left(N_c^\dagger P^i \left[\overleftarrow{D}^2 + \overrightarrow{D}^2 - 2 \overleftarrow{D} \cdot \overrightarrow{D} \right] N \right) + \mathrm{H.c.} \right\}$$

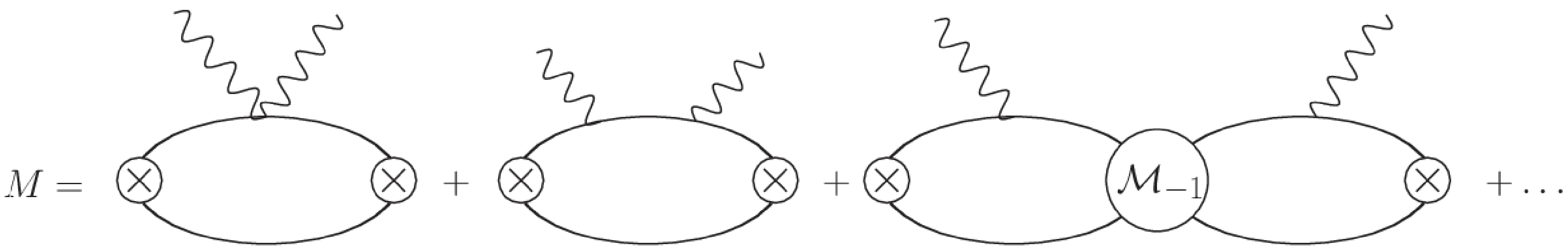
$$- \frac{C_4}{16} \left(N^\dagger \left[\overleftarrow{D}^2 + \overrightarrow{D}^2 - 2 \overleftarrow{D} \cdot \overrightarrow{D} \right] P^i N_c \right) \left(N_c^\dagger P^i \left[\overleftarrow{D}^2 + \overrightarrow{D}^2 - 2 \overleftarrow{D} \cdot \overrightarrow{D} \right] N \right) + \ldots$$

- In practice we calculate $f_T(\nu, Q^2)$ up to NLO completely
  - Add singlet NN interactions and two $NN\gamma$ contact terms
  - Cross check of the smallness of the $f_T(\nu, Q^2)$ contribution
  - Gauge invariance-conserving terms calculated up to NNLO

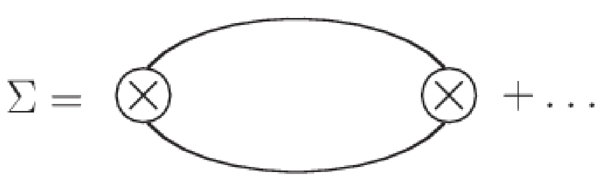
In Practice: Amplitude with Deuterons

- The reaction amplitude is given by the LSZ reduction

$$T = M \left[\left. \frac{\mathrm{d}\Sigma(E)}{\mathrm{d}E} \right|_{E=E_d} \right]^{-1}$$



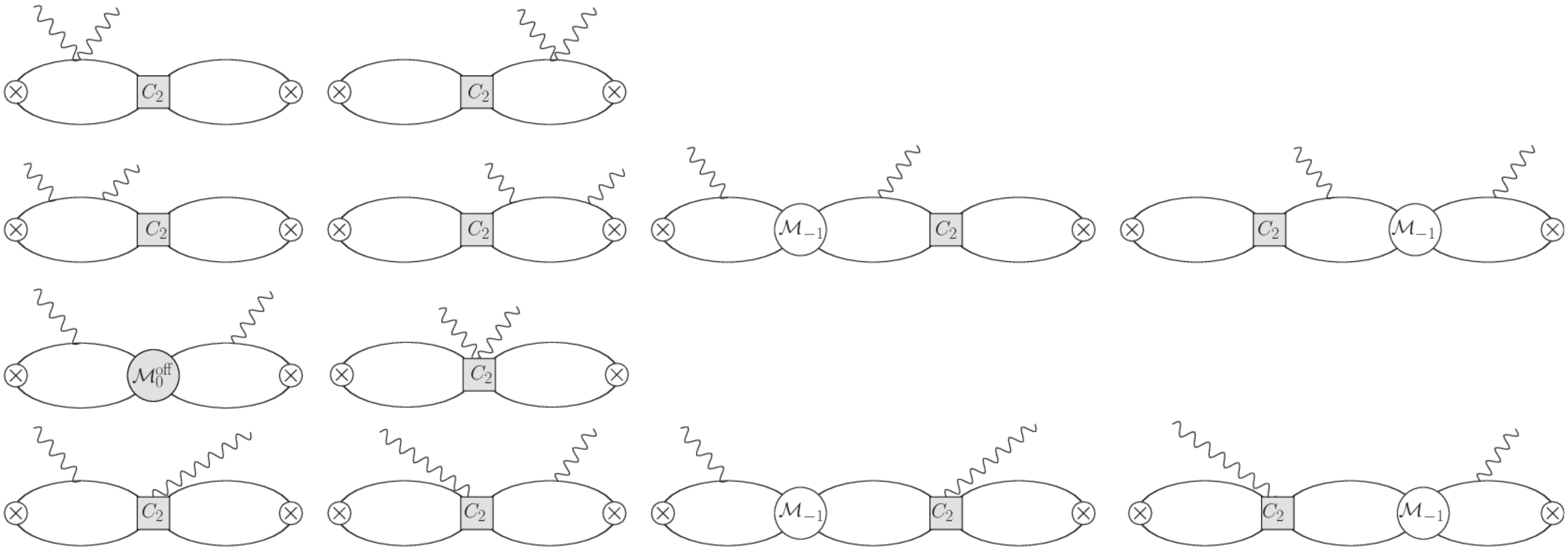
– irreducible VVCS graphs (here full LO in f_L; crossed not shown)



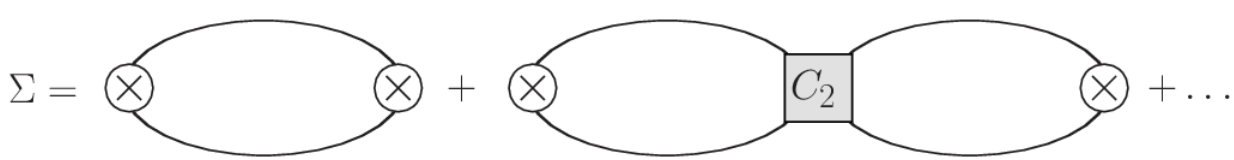
– deuteron self-energy at LO

In Practice: NLO + NNLO

- NLO irreducible VVCS graphs (f_L; crossed not shown)



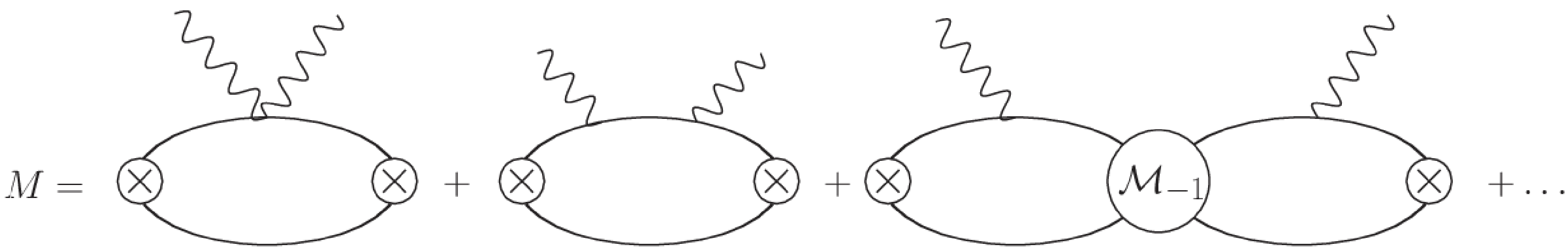
- LO+NLO deuteron self-energy



- NNLO: ~30 irreducible VVCS graphs, 2 self-energy graphs

In Practice: Evaluation and Checks

- The diagrams are evaluated analytically (pionless EFT!)
- We use the PDS scheme to regularise the divergences
- The regularisation scale dependence has to vanish in the total amplitude
  - If it does not: look for an error (or a missing NN contact term)
- Current conservation/gauge invariance is checked explicitly



$$M = \epsilon_\mu \epsilon'^*_\nu M^{\mu\nu}$$

$q_\mu M^{\mu\nu} = q_\nu M^{\mu\nu} = 0$ individually at each order

– all pieces that do not fulfil this have to cancel

In Practice: Contributions

- The expression for the residue is very simple:

$$\left.\frac{\mathrm{d}\Sigma(E)}{\mathrm{d}E}\right|_{E=E_d} = \frac{M^2}{8\pi\gamma}\left[1-(Z-1)+(Z-1)^2-(Z-1)^3+\dots\right]$$

$$\left[\left.\frac{\mathrm{d}\Sigma(E)}{\mathrm{d}E}\right|_{E=E_d}\right]^{-1} = \frac{8\pi\gamma}{M^2}\left[1+(Z-1)+0+0+\dots\right]$$

- The bulk of the contribution comes from the LO VVCS graphs + NLO residue correction
- NLO (+NNLO) VVCS graphs are small ~1%
- Transverse and (some of) the relativistic terms are ~1%

In Practice: the LS Integral

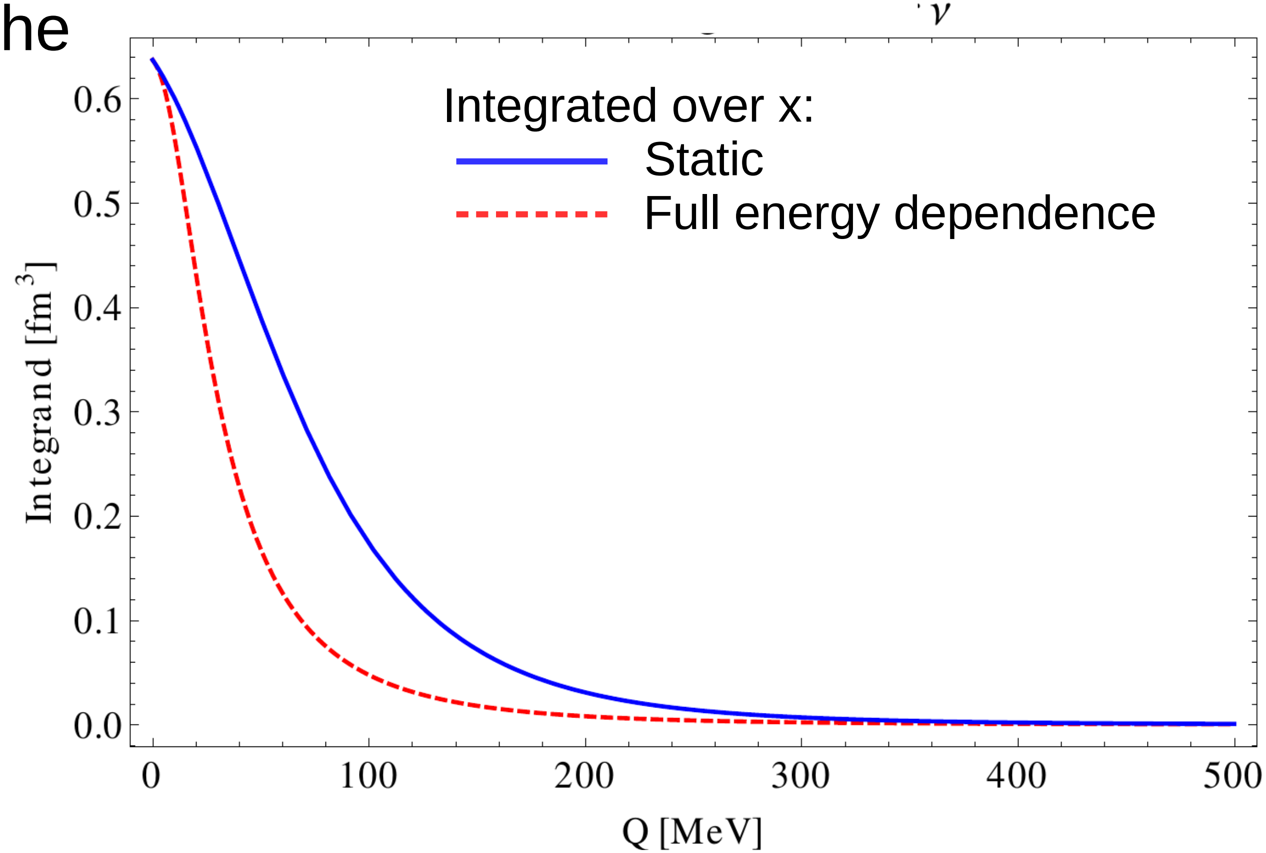
- The energy shift (non-pole f_L , Wick-rotated, hyperspherical)

$$\Delta E_{2S}^{\text{inel}} = \frac{2\alpha}{\pi m_\mu}[\phi_{2S}(0)]^2 \int_0^\infty \frac{\mathrm{d}Q}{Q} \int_{-1}^{1} \mathrm{d}x\sqrt{1-x^2}\, \frac{f_L(-iQx,Q^2)}{\frac{Q^2}{4m_\mu^2}+x^2}$$

- Static limit $f_L(-iQx,Q^2) \to f_L(0,Q^2)$ works well for muonic hydrogen but not here – keep the energy dependence

- Better convergence

- High-momenta contribution is (hopefully) under control



Results (Preliminary)

- Results for the µD Lamb shift contribution

$$\Delta E_{2S}^{\mathrm{inel}} = -1.58(0.04)\ \mathrm{meV}$$

- Other contributions:
  - Elastic: from deuteron FFs, well under control $= -0.415(2)\ \mathrm{meV}$
  - Coulomb: calculated in Hamiltonian approach $= \ \ 0.262\ \mathrm{meV}$
- Adding them up

$$\Delta E_{2S}^{\mathrm{TPE}} = \Delta E_{2S}^{\mathrm{inel}} + \Delta E_{2S}^{\mathrm{elastic}} + \Delta E_{2S}^{\mathrm{Coulomb}} = -1.73(0.05)\ \mathrm{meV}$$

- Compares well with other calculations
  - $-1.675(0.04)\ \mathrm{meV}$ Hernandez et al. 2019
  - $-1.680(0.016)\ \mathrm{meV}$ Pachucki 2011
  - $-1.7096(0.02)\ \mathrm{meV}$ Krauth et al. 2016 (theory summary)
- Differences are most likely due to nucleon FFs (relativistic corrections), the present calculation is for pointlike nucleons

Sum Rules

- The sum rules connect the TPE correction to the target structure functions F_1 and F_2 [inclusive electron scattering]

$$\Delta E_{2S}^{\mathrm{inel}} = -\frac{2\alpha^2}{M_d m_\mu}[\phi_{2S}(0)]^2 \int_0^\infty \frac{\mathrm{d}Q^2}{Q^2} \int_{\nu_{\mathrm{thr}}}^\infty \frac{\mathrm{d}\nu}{\nu} \left[\tilde{\gamma}_1(\nu, Q^2) F_1(\nu, Q^2) + \frac{M_d \nu}{Q^2} \tilde{\gamma}_2(\nu, Q^2) F_2(\nu, Q^2)\right]$$

- There is also a subtraction contribution expressed as a weighted integral of $F_1(\nu, Q^2) - F_1(\nu, 0)$ Gorshteyn
- Use a parameterisation of the structure function, fit to the available data – a data-driven approach

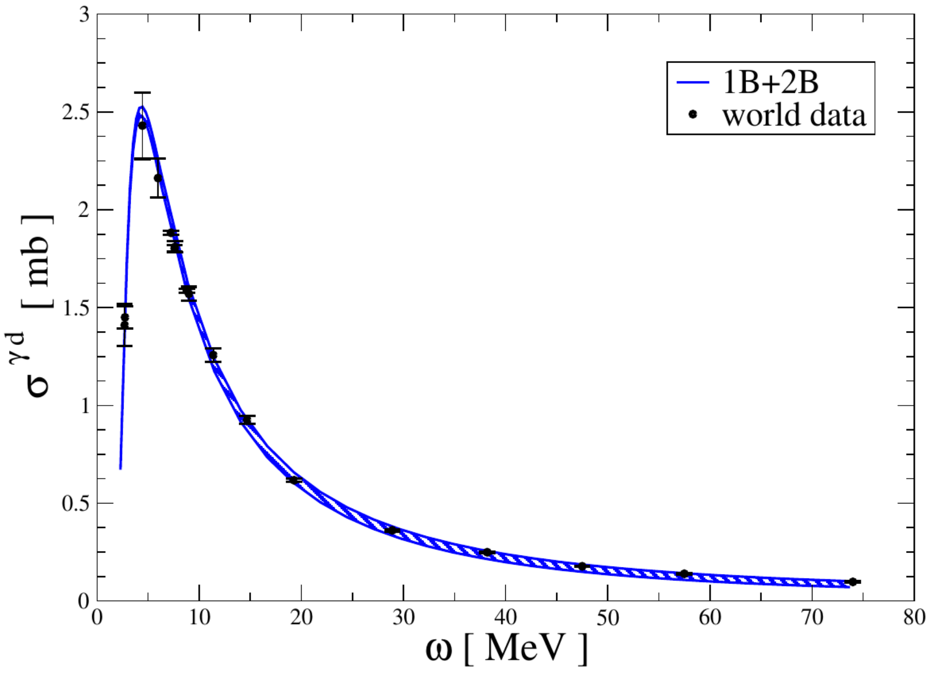
$$\Delta E_{2S}^{\mathrm{inel}} = -1.749 \pm 0.740 \ \mathrm{meV}$$ Carlson, Gorshteyn, Vanderhaeghen

- Problem: too few quality low-Q^2 data → big uncertainty
- Solution: use input from a theory (Chiral/pionless EFT)
- Verify theory using the available data, supplement the SR

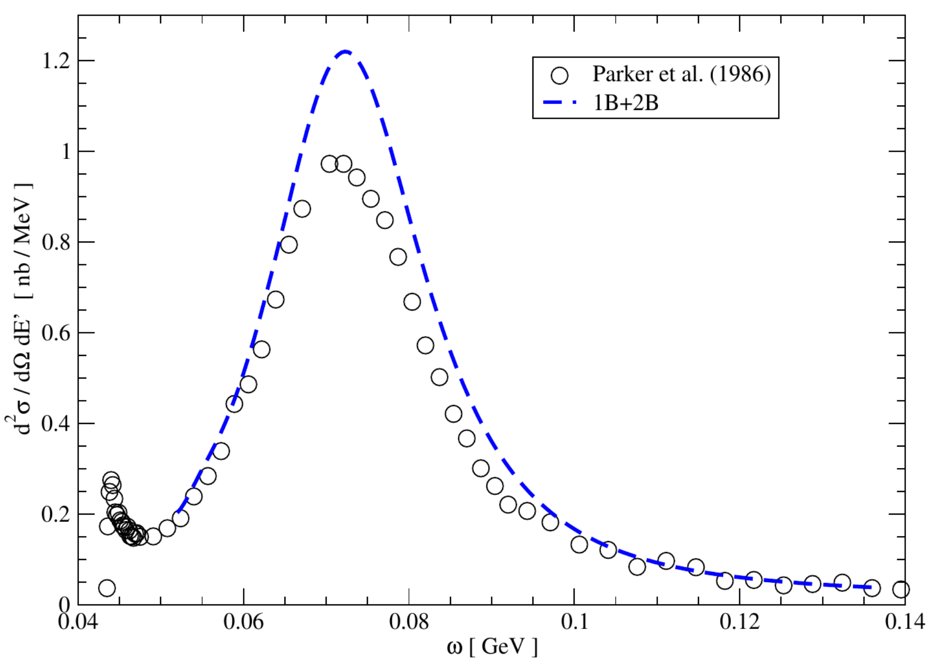
Sum Rules Aided by (Chiral) EFT

- NNLO Chiral EFT evaluation of the deuteron photo/electrodissociation cross section

Deuteron photodissociation



Electrodissociation E=220 MeV, θ=180°



- Data description is reasonably good so far
- Work in progress!
- Better data in low-Q^2 and low-energy region desirable!

Summary and Outlook

- Preliminary results for the TPE correction consistent with Hamiltonian approach and sum rules
- Full NNLO calculation, including error analysis (relativistic corrections, transverse contributions, ...) ready soon
- The most important relativistic corrections – due to the nucleon FFs – might consider to include them
- Coulomb [non-forward] distortions in EFT?
- The approach could be extended to the hyperfine splitting
- Sum rules with structure functions input from EFTs: verification of the theory and decreased error in the TPE correction evaluation