Erasmus School of Economics

MOOC Econometrics

Training Exercise P.1

Questions

1. Let e be a random variable with mean μ_e and variance σ_e^2. The random variable y is defined by $y = a + b'x + e$, with a a constant scalar, x a non-random $(k \times 1)$ vector, and b a $(k \times 1)$ non-random coefficient vector. What are the mean and variance of y?

2. Let x_1 and x_2 be random variables, with means and variances μ_i and σ_i^2, $i = 1, 2$ and covariance σ_{12}. What are the mean and variance of $y = a_1 x_1 - a_2 x_2$, with a_1 and a_2 constant.

3. Let x_1 and x_2 be random variables, with means and variances $\mu_i = \mu$ and $\sigma_i^2 = \sigma^2$, $i = 1, 2$, and correlation ρ. What are the mean and variance of $y = a_1 x_1 + (1 - a_1) x_2$ with a_1 constant?

4. Let x_i, $i = 1, 2, \ldots, n$ be n random variables with mean $\mu_i = \mu$ and variances $\sigma_i^2 = \sigma^2$. The covariances are all zero. Consider the random variable y that is constructed as the average of the variables x_i, $y = \frac{1}{n} \sum_{i=1}^{n} x_i$.

   (a) Find the mean and variance of y.

   (b) Find the correlation of y with x_i.

5. Let x be an $(n \times 1)$ vector of random variables with mean vector $\mu\iota_n$ and variance matrix $\sigma^2 I_n$, with μ and σ^2 constant scalars, ι_n the $(n \times 1)$ unit vector and I_n the $(n \times n)$ identity matrix. Define the $(n \times 1)$ vector $h = \frac{1}{n}\iota$ and the $((n+1) \times n)$ matrix H whose first n rows are formed by the identity matrix, and whose last row contains the vector h',

$$H = \begin{pmatrix} I \\ h' \end{pmatrix}.$$

   (a) Derive the mean and variance of $y = h'x$.

   (b) Derive the mean and variance of $z = Hx$.

   (c) Compare your answers to the previous question.

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