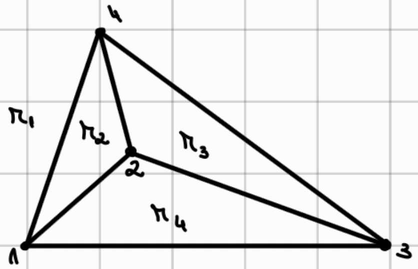
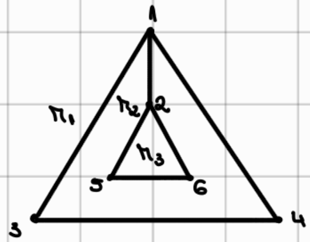
Grafuri planare, colorare

luni de la 18 - 20 - 2 ore



Formula lui Euler $\quad n - m + r = 2$

↳ $K_{3,3}$ și K_5 nu sunt planare

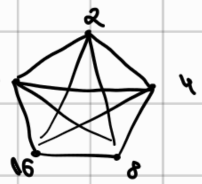


$G = (V, E)$ $\quad V = \{1 \dots n\}$

$(i, j) \in E \iff \frac{i}{j}$ sau $\frac{j}{i}$

G planar? $\quad$ X

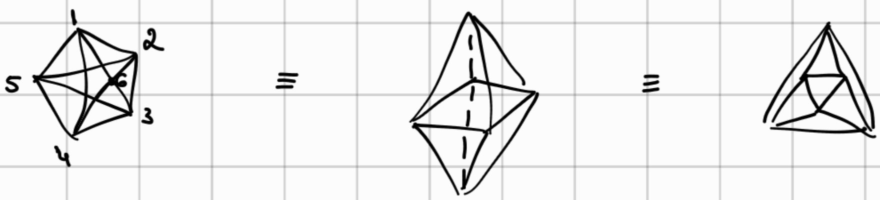
K_5



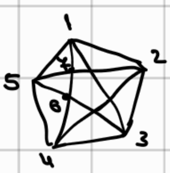
nr min ? a.î. G să fie 4-regular simplu și planar

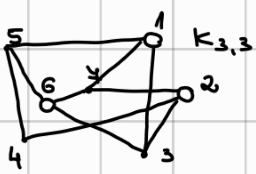
$n=5$: K_5 nu e bun că nu e planar

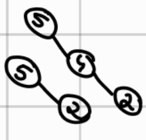
$n = 6$? $\quad$ da?



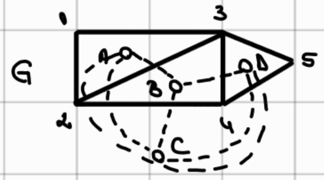
G







dualul unui graf planar



G	dual
$n=5$	$n=4$
$m=7$	$m=7$
$r=4$	$r=5$

Colorarea unui graf

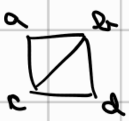
2 noduri vecine să nu aibă aceeași culoare (harta)

nr minim de culori : 4 (graf planar)

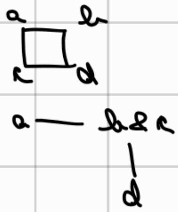
a) $c_G(k) = c_{G-e} - c_{G/e}(k)$ $\qquad E_n, T_n, K_n$

b) $c_G(k) = c_{\bar{G}}(k) + c_{G/e}(k)$

$\chi(G)$ - nr cromatic

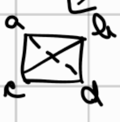


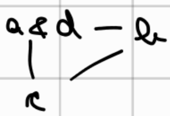
a) (c,b) $\xrightarrow{G-e}$ / $\xrightarrow{G/e}$



b) $(a,d) \notin E$

$\bar{G} = G+e$ $\qquad$ $G\backslash e$





E_n

$\dot{1} \;\; \dot{2} \; \ldots \; \dot{n}$

$c_{E_n}(k) = k^n$, $\chi(E_n) = 1$

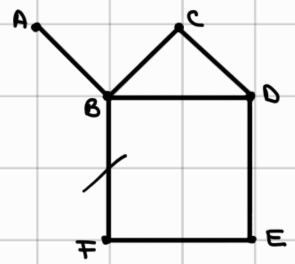
T_n

$c_{T_n}(k) = k(k-1)^{n-1}$ $\qquad$ $\chi(T_n) = 2$ (primul nr care nu i soluție)

polinom cromatic

K_n

$c_{K_n}(k) = k(k-1)(k-2)\cdot\ldots\cdot(k-n+1)$

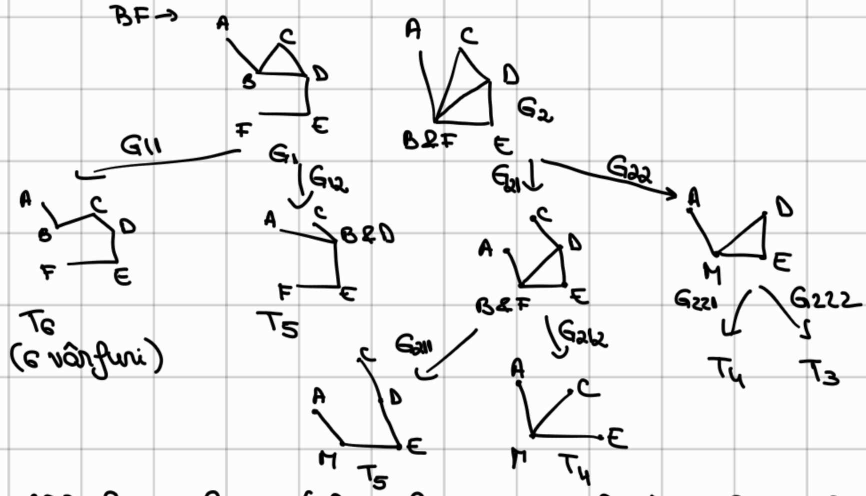


$\frac{n(n-1)}{2} = 15$ muchii - graf complet

$\Rightarrow$ mai repede șterg

$P_G(k) = P_{G_1}(k) - P_{G_2}(k)$

Val a)



a) $P_G(k) = C_{T_6} - C_{T_5} - (C_{T_5} - C_{T_4} - C_{T_4} + C_{T_3}) = C_{T_6} - 2C_{T_5} + 2C_{T_4} - C_{T_3}$

$P_G(k) = P_{G_1}(k) - P_{G_2}(k) \Leftrightarrow$

$P_{G_1}(k) = P_{G_{11}}(k) - P_{G_{12}}(k)$

$P_{G_2}(k) = P_{G_{21}}(k) - P_{G_{22}}(k) = P_{G_{211}}(k) - P_{G_{212}}(k) - P_{G_{221}}(k) + P_{G_{222}}(k)$

$\cancel{k(k-1)^2[(k-1)^3 - 2(k-1)^1 - 1]}$ $\qquad \chi(G) =$

$k(k-1)^2[(k-1)^3 - 2(k-1)^2 + 2(k-1) - 1]$

$= k^6 - 7k^5 + 20k^4 - 29k^3 + 2k^2 - 6k$

$\chi(G) = 3 \qquad P_G(3) = 36$

- $n = 128$

  $V = \{1, \dots, 128\}$

  $(i,j) \in E \Leftrightarrow \text{cmmdc}(i,j) \neq 1$

  $\chi(G) = ?$ 64

  $K_{64} \subset G$ (subgraf)

  $\downarrow$

  $\chi(K_{64}) = 64$

  $1 \to 2$

  $3 \to 4$

desenați un graf 3-regular pt nr cromatic de muchii ≥ 4