Alg drum cost min: grafuri orientate și ponderate de la un nod sursă

$G = (V, E)$, fct pondere $w : E \to \mathbb{R}$

$p = \langle v_1, v_2, \dots, v_k \rangle$

$s \rightsquigarrow v$ succesiune de mai multe arce

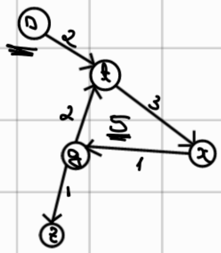
$w(p) = \sum_{i=1}^{k} w(v_{i-1}, v_i)$

$\delta(s, v) = \begin{cases} \min\{w(p) \mid s \overset{p}{\rightsquigarrow} v\}, & \text{dacă există } s \rightsquigarrow v \\ \infty, & \text{altfel} \end{cases}$

Bellman_Ford

$G = (V, E)$
- $w : E \to \mathbb{R}$
- s - nod sursă
- d - estimare cost drum
- π - predecesor

s.d = 0
t.d = ~~∞~~ 1
x.d = ~~∞~~ 4
y.d = 6
z.d = 7

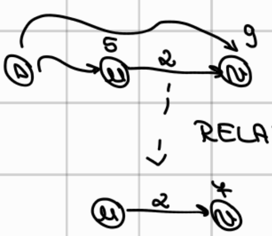


Bellman-Ford (G, s)   complex: O(VE)

```
1. INIȚIALIZARE (G)
2. for i = 1 to |V|-1 do
3.       for fiecare (u,v) ∈ E do
4.             RELAX (u,v,w)
5. for fiecare (u,v) ∈ E do
6.       if v.d > u.d + w(u,v)
7.             return FALSE
8. return TRUE
```

```
INIȚIALIZARE (G)
    for v ∈ V
        v.d = ∞
        v.π = NIL
    s.d = 0
```

```
RELAX (u,v,w)
    if v.d > u.d + w(u,v)
        v.d = u.d + w(u,v)
        v.π = u
```



$\delta(s, u) = 5$

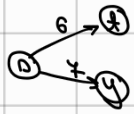
v.d = 9

v∈V	s	t	x	y	z
v.d	0	~~∞~~ ~~6~~ 2	~~∞~~ ~~7~~ 4	~~∞~~ 7	~~∞~~ ~~2~~ -2
v.π	–	~~t~~ ~~s~~ x	~~t~~ ~~t~~ y	~~t~~ s	~~t~~ t

(t,x), (t,y), (t,z), (x,t), (y,x), (y,z), (z,x), (z,s), (s,t), (s,y)

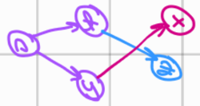
(∞ > 0+6 ?)
DA => optimizare

i = 1



i = 2

(y,x)

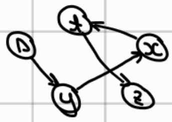


i = 3

t x minimic
t y minimic
t z minimic

(x,t)

y x
y z minimix
z x
z,s
s t
s y

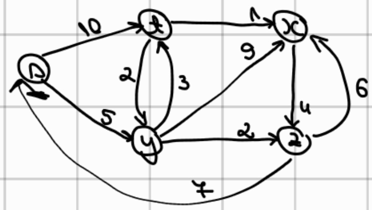


i = 4

TRUE

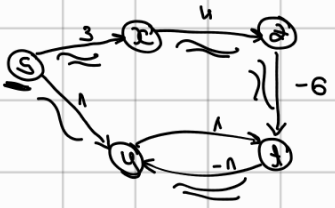
Dijkstra (G, s)

```
INITIALIZARE (G)                u ∈ V\S
S = Ø
Q = V
while Q ≠ Ø do                  sau |Q| > 1
      u = EXTRACT_MIN(Q)
      S = S ∪ {u}
      for v ∈ G.Adj[u]    (vecinii lui s)
            RELAX (u, v, w)
```



```
RELAX (u, v, w)
   if v.d > u.d + w(u, v)
      v.d = u.d + w(u, v)
      v.π = u
```

$v \in V$	a	b	c	d	e
$d.d$	0	∞	∞	∞	∞
$d.\pi$ (părinte)	—	—	—	—	—



$\sigma_{\Delta}(s,y) = 1$

$\sigma_{BF}(s,y) = 0$

Bellman Kalaba

$G = (V, E)$

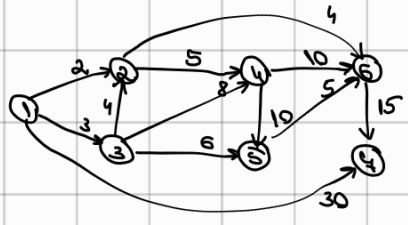
$A = (a_{ij})_{i,j=\overline{1,m}}$

$V^{(1)} = [V_i^{(1)}]_{i=\overline{1,m}}$

$V^{(k)} = V^{(k+1)}$

$$a_{ij} = \begin{cases} w(v_i, v_j), (v_i, v_j) \in E \\ 0, i = j \\ \infty, (v_i, v_j) \notin E \end{cases}$$

$$V_i^{(k)} = \min_{j=\overline{1,m}} \{ a_{ij} + V_j^{(k-1)} \} \quad i = \overline{1,m}$$



$$A = \begin{pmatrix} 0 & 2 & 3 & \infty & \infty & \infty & 3 \\ \infty & 0 & \infty & 5 & \infty & 5 & \infty \\ \infty & 4 & 0 & \infty & 6 & \infty & \infty \\ \infty & \infty & \infty & 0 & 10 & 10 & \infty \\ & & & & 0 & 5 & \infty \\ & & & & & 0 & 15 \\ & & & & & & 0 \end{pmatrix}$$

$$V^{(1)} = \begin{bmatrix} 30 \\ 8 \\ 8 \\ 8 \\ 8 \\ 5 \end{bmatrix}$$

$$V^{(2)} = \begin{bmatrix} 30 \\ 19 \\ 8 \\ 25 \\ 20 \\ 15 \\ 0 \end{bmatrix}$$

$8 + 30$
$0 + 0$
$8 + 8$
$5 + 8$
$8 + 8$
$7 + 15$
$8 + 0$

Reim (4+1)