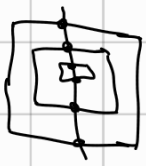
Grafuri Euler și Hamilton

- Euler → ciclu elementar care conține toate muchiile

  ↳ nu repetă muchii



T. caracterizare
1. G e eulerian
2. toate vf au grad par
3. G poate fi partiționat în cicluri disjuncte

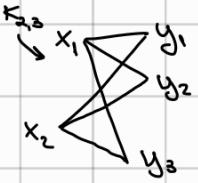
K_m $m = ?$ a.î. G eulerian

m-par? ✗ nu

m-impar $\quad \forall K_m = m-1$

$K_{m,m}$

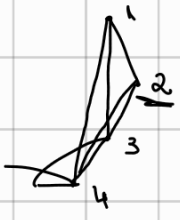
$m,m = ?$ a.î. $K_{m,m}$ eulerian $\quad m,m$ - par



- $G = (V, E)$, $n = |V|$, $V = \{1, 2, \dots, n\}$

  $(u,v) \in E \iff |u-v| \leq 2$

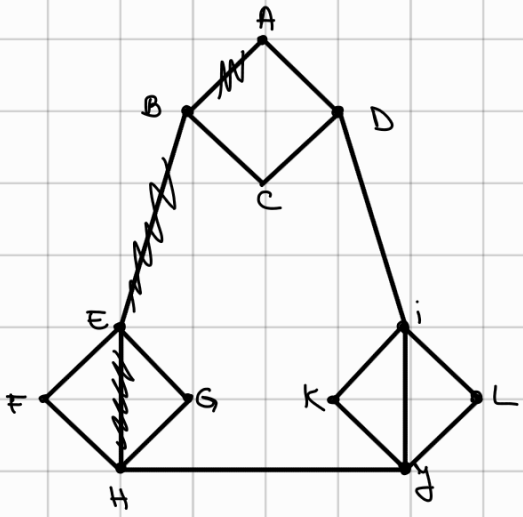
  e G eulerian? conține lanț Euler?



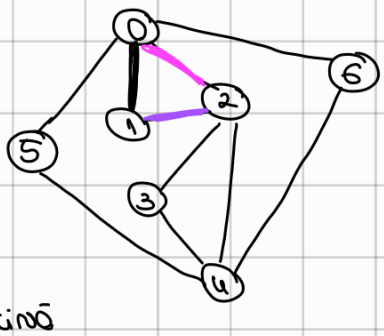
T: G are cel mult 2 vf de grad impar $\Rightarrow$ lanț Euler

Fleury (G) $\quad$ G - are Euler

1. $x = u \in V$
2. $A = E$
3. $L = \emptyset$
4. while $A \neq \emptyset$ do
5. alege e incidentă în x ($x \in g(e)$) și dacă se poate e să nu fie muchie punte
6. $L = L \cup \{e\}$
7. if $|g(e)| = 2$
8. $x = v \in g(e) \setminus \{x\}$
9. $A = A \setminus \{e\}$
10. return L



$L = \{ (A,B), (B,E), (E,H) \}$



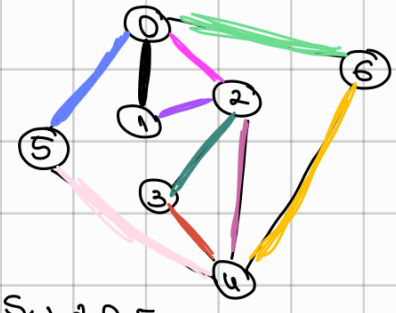
0: ~~5~~, ~~1~~, ~~2~~, 6
1: ~~0~~, ~~2~~
2: ~~0~~, ~~1~~, 3, 4
3: 2, 4
4: 5, 3, 2, 6
5: 0, 4
6: 0, 4

S - stinō

S: 1

S: 1, 2

S: 1, 2, 0



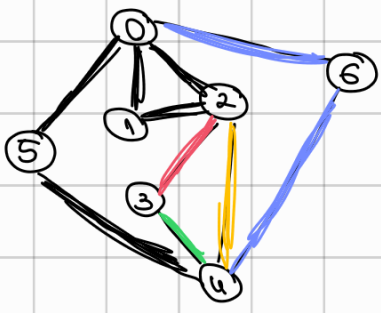
0: ~~5~~, 6
1: –
2: ~~3~~, 4
3: ~~2~~, 4
4: ~~5~~, ~~2~~, ~~3~~, 6
5: ~~0~~, 4
6: ~~0~~, 4

S: 1, 2, 0, 5

S: 1, 2, 0, 5, 4, 3, 2, 4

S: 1, 2, 0, 5, 4, 3, 2, 4, 6, 0

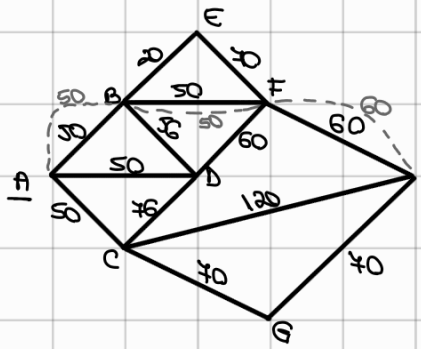
0 - 6 - 4 - 2 - 3 - 4 - 5 - 0 - 2 - 1 - 0



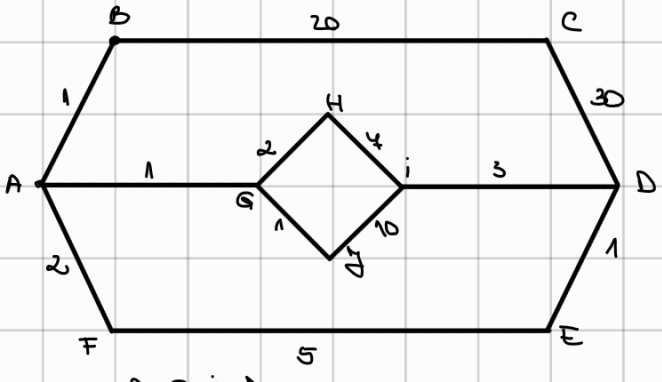
S: 1, 2, 0, 5, 4, 6, 0

0 - 6 - 4 - 2 - 3 - 4

S: 1, 2, 0, 5, ~~4~~, ~~3~~, ~~2~~, ~~4~~



H - grad impar
=> nu e ciclu Eulerian



A, G, i, D

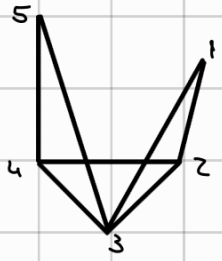
$\begin{cases} AG - 1 \\ iD - 3 \end{cases}$	$\begin{cases} Ai - 10 \\ GD - 9 \end{cases}$	$\begin{cases} AD - 8 \\ Gi - 9 \end{cases}$
4	19	17

Graf Hamiltonian → ciclu elementar care conține toate nf.

$n = 100$, $V = \{1 \dots 100\}$

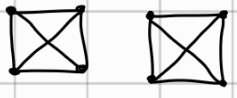
$(u, v) \in E \iff |u - v| \leq 2$

este G Hamiltonian? conține un lanț H?

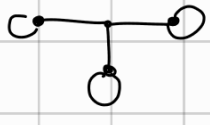


- desenați graf 3-regulat care nu e hamiltonian

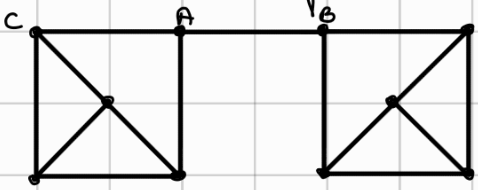
→ toate muchiile au grad 3



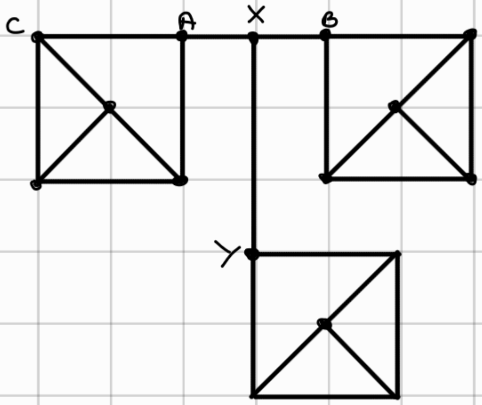
- desenați graf 3-regulat, convex, care nu e hamiltonian



• desenați graf 3-regular simplu, conex, care nu e hamilton
sol: să aibă muchie punte



• desenați graf 3-regular simplu, conex, care nu conț lanț hamilton



T Dirac

$G=(V,E)$ $n \geq 3$, $\delta(G) \geq \frac{n}{2}$ => G e hamiltonian

• 100 mărgele roșii
150 –"– verzi
200 –"– albastre

$\delta(v_{roșu}) = 350$ că un nod roșu leg toate nod verzi și albastre
grad minim

$\delta(v_{verde}) = 300$

$\delta(v_{albastru}) = 250$

T Dirac => $\frac{n}{2} = 225 < \delta(G)$
G este H

• $n = 100$

$i=1$ K_{100} $\delta(G) = 99$

$i=2$ $= 97$

⋮

i $= 99 - 2(i-1) \geq \frac{n}{2}$

• 3 × 3 × 3

→, ←, ↑, ↓,