# **Genetic Algorithm Summary**

The research paper addresses fundamental limitations in traditional portfolio optimization approaches. The authors developed an innovative ensemble investment strategy that combines momentum-based analysis with Capital Asset Pricing Model (CAPM) valuations, optimized through genetic algorithms. Their methodology, termed "CAPM+", demonstrated superior performance by achieving over 400% cumulative returns during 2008-2018 testing periods, significantly outperforming both market indexes and momentum-only strategies across Korean (KOSPI200) and US (S&P500) markets. The research contributes computational efficiency improvements through Fund Standardization, reducing portfolio optimization complexity from O(n²) to O(1), while maintaining practical applicability through realistic transaction cost modeling.

I read the paper, implemented in python the described solution, and tested both CAPM and Momentum only versions. Here is the summary of the research paper in my understanding:

### The Portfolio Optimization Challenge

Traditional investment approaches face significant challenges in dynamic market environments such as **unpredictability of markets** (future market movements are inherently uncertain due to numerous socio-economic factors), **computational complexity** (Markowitz's Modern Portfolio Theory requires O(n²) calculations for variance-covariance matrices), **risk management** (need to balance between systematic (market-wide) and unsystematic (stock-specific) risks), **asset valuation** (Identifying undervalued stocks with growth potential requires sophisticated analysis)

#### **Theoretical Foundations**

#### **Modern Portfolio Theory (Markowitz)**

Markowitz's diversification theorem shows that combining assets with low correlation can reduce portfolio risk, but as the number of assets grows, covariance computation becomes prohibitive. The Sharpe ratio—in essence, excess return per unit of risk—provides a practical measure for evaluating risk-adjusted performance during optimization. Meanwhile, CAPM offers a framework to estimate a stock's fair expected return based on its systematic risk, using the classic equation:

$$E(Ri) = Rf + Bi(E(Rm) - Rf$$

Where Rf is the risk-free rate, E[Rm] is the expected return of the market portfolio and  $\beta$ i is stock i's systematic risk, estimated via a rolling 36-month regression of its returns on the market's returns (with at least 12 months of data).

## **CAPM+ Strategy**

### The ensemble Approach

The paper's primary innovation lies in combining two complementary investment strategies that address different market dynamics. The key insight is that momentum captures short-term market trends while CAPM valuation identifies fundamental mispricings. Together, they create a more robust investment approach.

**Momentum Strategy** - Assumes price trends continue (Newton's first law applied to finance)

Relies on the empirical persistence of asset returns, assuming that stocks exhibiting strong performance over a recent look-back period (e.g., 1 month) often continue their trend in the near term, so it captures market psychology and herding effects, enabling the strategy to exploit fast-moving bullish or bearish shifts. In practice, the algorithm ranks all universe stocks by their trailing returns and selects the top percentile as momentum candidates.

**CAPM Valuation** - Identifies undervalued stocks trading below their Security Market Line

Assigns each stock a theoretical fair return via CAPM formula, calculates rolling  $\beta$  over a 36-month window (requiring at least 12 months of data) and determines mispricing by comparing realized returns against CAPM expectations; stocks with positive deviations are deemed undervalued and likely candidates for mean-reversion.

## **Algorithm framework**

## Portfolio Representation and Optimization

Portfolios are encoded as binary chromosomes, where each gene indicates inclusion (1) or exclusion (0) of a stock. An initial population of 50 random portfolios evolves through iterations using tournament selection (size 3), two-point crossover applied at a 100% rate, and 1% bit-flip mutation (ensuring at least one stock remains). Elitism preserves the top two portfolios unchanged in each generation. The GA runs for up to 300 generations (150 in some smaller universes), or until no improvement in best fitness occurs for 20 consecutive generations.

Each stock's  $\beta$  is re-estimated monthly using a rolling 36-month window (with at least 12 months of returns) to reflect changing systematic risk

Also important note is that the realistic trading costs are built in and net returns in the fitness function subtract these before computing both the Sharpe component and the average CAPM mispricing.

#### **Fund Standardization**

So the idea is to rescale each stock's weight by its individual standard deviation over the look-back window. Because all assets are variance-normalized, portfolio variance reduces to the sum of pairwise correlations (a constant-time operation), avoiding the full covariance matrix.

To collapse the O(n²) covariance computation into O(1) time, each stock's raw return series is first **standardized** by its own volatility over the look-back window. After this transformation, the covariance matrix of the universe becomes the **correlation** matrix of the original returns. This approach lets the GA evaluate thousands of candidate portfolios per generation extremely rapidly, even in large universes like the S&P 500

#### **Fitness Function**

For each candidate portfolio, net returns are calculated by subtracting transaction fees (0.015%) and taxes (0.3%) from raw returns. The Sharpe component is then computed as the difference between average net return and the annualized risk-free rate, divided by the standard deviation of net returns. The CAPM component is the average mispricing across the selected stocks. The GA seeks to maximize the sum of these two components, thereby selecting portfolios with attractive risk-adjusted returns and positive fundamental signals. So the fitness function combines a Sharpe-ratio term (mean net return minus risk-free rate) / net return plus the average CAPM mispricing across the portfolio, ensuring that selected portfolios have both strong risk-adjusted momentum and positive fundamental signals

Fitness = (ROI -Riske-free-rate) / Risk + Portfolio CAPM

## **Backtesting and Results**

The original research conducted comprehensive backtesting across two major markets over an 11-year period (2008-2018) Korean KOSPI200 (~200 stocks) and US S&P500 (~500 stocks) The chosen timeframe deliberately included both the global financial crisis and subsequent bull market recovery, creating diverse market environments for strategy validation.

Three analysis periods tested (1-month, 3-month, and 6-month) to determine optimal portfolio refresh rates at realistic trading costs (0.015% fees + 0.3% taxes) incorporated to ensure practical applicability rather than theoretical performance.

Strategy performance was measured against market index returns and momentum-only approaches to isolate the CAMP component's contribution

## **Key Findings from Original Research**

**CAPM+** outperformed market indexes

CAPM+ strategy achieved over 400% cumulative returns during 1-month rebalancing periods, substantially outperforming both KOSPI200 and S&P500 index returns over the same 11-year period

### Superior to momentum-only strategies

The ensemble approach consistently outperformed momentum-only strategies across all testing periods, with CAPM valuation providing meaningful performance enhancement beyond pure trend-following

#### Shorter analysis periods work better

Monthly portfolio optimization (1-month analysis) delivered superior results compared to quarterly (3-month) or semi-annual (6-month) approaches, suggesting that asset mispricings correct relatively quickly in efficient markets

#### **Cross-market validation**

The strategy demonstrated robust performance across both Korean and US markets despite different economic conditions, regulatory environments, and market characteristics, indicating broad applicability

#### Crisis resilience

During the 2008 financial crisis when market indexes declined over 20%, the CAPM+ strategy showed superior downside protection, validating its risk management capabilities during extreme market stress