Edge metric dimension of some generalized Petersen graphs☆

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Abstract

The edge metric dimension problem was recently introduced, which initiated the study of its mathematical properties. The theoretical properties of the edge metric representations and the edge metric dimension of generalized Petersen graphs $GP(n,k)$ are studied in this paper. We prove the exact formulae for $GP(n,1)$ and $GP(n,2)$, while for the other values of k the lower bound is stated.

Keywords: Generalized Petersen graphs, edge metric dimension

1. Introduction

The concept of metric dimension of a graph G was introduced independently by Slater (1975) in [1] and Harary and Melter (1976) in [2]. This concept is based on the notion of resolving set R of vertices, which has the property that each vertex is uniquely identified by its metric representations with respect to R. The minimal cardinality of resolving sets is called the metric dimension of graph G.

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1.1. Literature review

Kelenc, Tratnik and Yero (2016) in [3] recently introduced a similar concept of edge metric dimension and initiated the study of its mathematical properties. They made a comparison between the edge metric dimension and the standard metric dimension of graphs while presenting realization results concerning the edge metric dimension and the standard metric dimension of graphs. They also prove that edge metric dimension problem is NP-hard in a general case, and provided approximation results. Additionally, for several classes of graphs, exact values for edge metric dimension were presented, while several others were given upper and lower bounds. In [4], the authors presented results of mixed metric dimension alongside the edge metric dimension for some classes of graphs.

Zubrilina, in paper [5], firstly proposed the classification of graphs of n vertices for which the edge metric dimension is equal to its upper bound $n-1$. The second result states that the ratio between edge metric dimension and metric dimension of an arbitrary graph is not bounded from above. Third result characterize change of the edge dimension of an arbitrary graph upon taking a Cartesian product with a path, and change of the edge dimension upon adding a vertex adjacent to all the original vertices.

The edge metric dimension of the Erdös-Rényi random graph $G(n, p)$ is given by Zubrilina in [6] and it is equal to $(1 + o(1)) \cdot \frac{4log(n)}{log(1/q)}$, where $q = 1 - 2p(1 - p)^2(2 - p)$.

1.2. Generalized Petersen graphs

Generalized Petersen graphs were first studied by Coxeter [7]. Each such graph, denoted as $GP(n, k)$, is defined for $n \geq 3$ and $1 \leq k < n/2$. It has $2n$ vertices and $3n$ edges, with vertex set $V(GP(n, k)) = \{u_i, v_i \mid 0 \leq i \leq n-1\}$ and edge set $E(GP(n, k)) = \{u_iu_{i+1},\ u_iv_i,\ v_iv_{i+k} \mid 0 \leq i \leq n-1\}$. It should be noted that vertex indices are taken modulo n.

Example 1. *Consider the Petersen graph, numbered as $GP(5, 2)$, shown in Figure 1. It is easily calculated, by using total enumeration technique, that its edge metric dimension is equal to 4 (it is also presented in Table 5). In Figure 2, $GP(6, 1)$ with edge metric dimension equal to 3, is presented. This can also be concluded by Theorem 2.*

The metric dimension of generalized Petersen graphs $GP(n, k)$ is studied for different values of k:

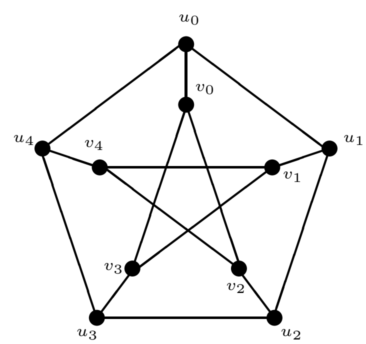

Figure 1: Petersen graph GP(5,2)

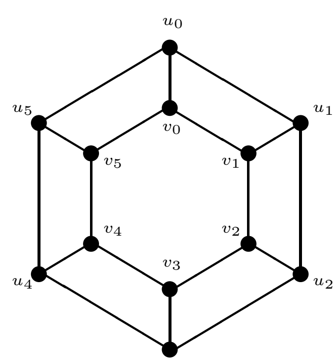

Figure 2: Graph GP(6,1)

- Case $k = 1$ is concluded from [8];
- Case $k = 2$ is proven in [9];
- Case $k = 3$ in [10].

Various other properties of generalized Petersen graphs have been recently theoretically investigated in the following areas: Hamiltonian property [11], the cop number [12], the total coloring [13], etc.

1.3. Definitions and previous work

Given a simple connected undirected graph $G = (V, E)$, for $u, v \in V$ $d(u, v)$ denotes the distance between u and v in G, i.e. the length of a shortest $u - v$ path. A vertex x of the graph G is said to resolve two vertices u and v of G if $d(x, u) \neq d(x, v)$. An ordered vertex set $R = \{x_1, x_2, ..., x_k\}$ of G is a resolving set of G if every two distinct vertices of G are resolved by some vertex of R. A metric basis of G is a resolving set of the minimum cardinality. The metric dimension of G, denoted by $\beta(G)$, is the cardinality of its metric basis.

Similarly, for a given connected graph G, a vertex $w \in V$ and an edge $uv \in E$, the distance between the vertex w and the edge uv is defined as $d(w, uv) = \min\{d(w, u), d(w, v)\}$. A vertex $w \in V$ resolves two edges e_1 and e_2 ($e_1, e_2 \in E$), if $d(w, e1) \neq d(w, e2)$. A set S of vertices in a connected graph G is an edge metric generator for G if every two edges of G are resolved by some vertex of S. The smallest cardinality of an edge metric generator of G is called the edge metric dimension and is denoted by $\beta_E(G)$. An edge metric basis for G is an edge metric generator of G with cardinality $\beta_E(G)$. Given an edge $e \in E$ and an ordered vertex set $S = \{x_1, x_2, ..., x_k\}$, the k-touple $r(e, S)$ = $(d(e, x_1), d(e, x_2), ..., d(e, x_k))$ is called the edge metric representation of e with respect to S.

Example 2. *Consider the generalized Petersen graph $GP(6, 1)$ given on Figure 2. The set $S_1 = \{u_0, u_1, u_3\}$ is an edge metric generator for G since the vectors of metric coordinates for edges of G with respect to S_1 are mutually different:* $r(u_0u_1, S_1)$=*(0,0,2);* $r(u_0u_5, S_1)$=*(0,1,2);* $r(u_0v_0, S_1)$=*(0,1,3);* $r(u_1u_2, S_1)$=*(1,0,1);* $r(u_1v_1, S_1)$=*(1,0,2);* $r(u_2u_3, S_1)$=*(2,1,0);* $r(u_2v_2, S_1)$=*(2,1,1);* $r(u_3u_4, S_1)$=*(2,2,0);* $r(u_3v_3, S_1)$=*(3,2,0);* $r(u_4u_5, S_1)$=*(1,2,1);* $r(u_4v_4, S_1)$=*(2,3,1);* $r(u_5v_5, S_1)$=*(1,2,2);* $r(v_0v_1, S_1)$=*(1,1,3);* $r(v_0v_5, S_1)$=*(1,2,3);* $r(v_1v_2, S_1)$=*(2,1,2);*

$r(v_2v_3, S_1)$=(3,2,1); $r(v_3v_4, S_1)$=(3,3,1); $r(v_4v_5, S_1)$=(2,3,2). From Corollary 4, it holds that for $GP(6,1)$, as for any other generalized Petersen graph, cardinality of edge metric generator must be at least 3, so S_1 is an edge metric basis for $GP(6,1)$. This implies that its edge metric dimension is equal to 3, i.e. $\beta_E(GP(6,1)) = 3$.

Two edges are called an incident, if both contain one common endpoint. For a given vertex $v \in V$, its degree deg_v is equal to number of its neighbors, i.e. number of edges in which it is endpoint. Maximum and minimum degree over all vertices of graph G is noted as $\Delta(G)$ and $\delta(G)$, respectively. Formally, $\Delta(G) = \max\limits_{v \in V} \deg_v$ and $\delta(G) = \min\limits_{v \in V} \deg_v$.

Proposition 1. ([3]) *For $n \geq 2$ it holds $\beta_E(P_n) = \beta(P_n) = 1$, $\beta_E(C_n) = \beta(C_n) = 2$, $\beta_E(K_n) = \beta(K_n) = n-1$. Moreover, $\beta_E(G) = 1$ if and only if G is a path P_n.*

Proposition 2. ([3]) *Let G be connected graph and let $\Delta(G)$ be the maximum degree of G. Then $\beta_E(G) \geq log_2\Delta(G)$*

Proposition 3. ([3]) *Let G be a connected graph and let S be an edge metric basis with $|S| = k$. Then S does not contain a vertex with the degree greater than 2^{k1}.*

From each of these propositions, given in [3], it follows the next two statements:

Corollary 1. *Edge metric dimension of the 3-regular graph is at least 2.*

Corollary 2. $\beta_E(GP(n,k)) \geq 2$.

2. Main results

2.1. Lower bound

Having in mind the fact that vertices from an edge metric basis are also endpoints for some (incident) edges, the bound presented in Proposition 2, could be improved in some cases.

Theorem 1. *Let G be a connected graph and let $\delta(G)$ be the minimum degree of G. Then, $\beta_E(G) \geq 1 + \lceil log_2\delta(G) \rceil$.*

Proof. Suppose the contrary, that there exists edge metric generator $S = \{w_1, w_2, ..., w_p\}$ with cardinality $p < 1 + \lceil log_2\delta(G)\rceil$. Vertex w_1 is incident to at least $\delta(G)$ edges. Name them as e_1, ..., $e_{\delta(G)}$. Since w_1 is incident with e_1, ..., $e_{\delta(G)}$, it is obvious that $d(e_1, w_1) = ... = d(e_{\delta(G)}, w_1) = 0$. By the definition of distances between vertices and edges, it is clear that for an arbitrary vertex $v \in V(G)$ there can be only two different distances to some set of incident edges. Then, for each i, $i = 2, ..., p$, distances $d(e_1, w_i)$, ..., $d(e_{\delta(G)}, w_i)$ have only two different values, so since $d(e_1, w_1) = ... = d(e_{\delta(G)}, w_1) = 0$ there exists at most 2^{p-1} different edge metric representations of edges e_1, ..., $e_{\delta(G)}$ with respect to S, so $\delta(G) \leq 2^{p-1}$. Next, because p has integer value, it follows that $\lceil log_2\delta(G)\rceil \leq p-1 \Rightarrow p \geq 1+\lceil log_2\delta(G)\rceil$, which is in contradiction to starting assumption! Therefore, $\beta_E(G) \geq 1 + \lceil log_2\delta(G)\rceil$. □

In case of regular graphs, the bound presented in Proposition 2 is improved by one.

Corollary 3. *Let G be an r-regular graph. Then, $\beta_E(G) \geq 1 + \lceil log_2 r\rceil$.*

Since $GP(n, k)$ is 3-regular graphs, and $\lceil log_2 3\rceil = 2$ then it holds next corollary.

Corollary 4. $\beta_E(GP(n, k)) \geq 3$.

2.2. Exact value for $GP(n, 1)$

In this section, we are studding generalized Petersen graphs $GP(n, 1)$. The following theorem gives the exact value of edge metric dimension of generalized Petersen graphs $GP(n, 1)$.

Theorem 2. $\beta_E(GP(n, 1)) = 3$

Proof. Let $S = \{u_0, u_1, v_0\}$.
<u>*Case 1.*</u> $n = 2t$
Edge metric representations with respect to S are:

$$r(u_iu_{i+1}, S) = \begin{cases} (0,0,1), \ i = 0 \\ (i, i-1, i+1), \ 1 \leq i \leq t-1 \\ (t-1, t-1, t), \ i = t \\ (2t-1-i, 2t-i, 2t-i), \ t+1 \leq i \leq 2t-1 \end{cases}.$$

$$r(u_iv_i,S)=\begin{cases}(0,1,0),\ i=0\\(i,i-1,i),\ 1\le i\le t\\(2t-i,2t+1-i,2t-i),\ t+1\le i\le 2t-1\end{cases}.$$

$$r(v_iv_{i+1},S)=\begin{cases}(1,1,0),\ i=0\\(i+1,i,i),\ 1\le i\le t-1\\(t,t,t-1),\ i=t\\(2t-i,2t+1-i,2t-1-i),\ t+1\le i\le 2t-2\\(1,2,0),\ i=2t-1\end{cases}.$$

Since all edge metric representations with respect to S are mutually different, then S is an edge metric generator. Since $|S|=3$ and from Corollary 4 it follows $\beta_E(GP(2t,1))=3$.

Case 2. $n=2t+1$

Edge metric representations with respect to S are:

$$r(u_iu_{i+1},S)=\begin{cases}(0,0,1),\ i=0\\(i,i-1,i+1),\ 1\le i\le t\\(2t-i,2t+1-i,2t+1-i),\ t+1\le i\le 2t\end{cases}.$$

$$r(u_iv_i,S)=\begin{cases}(0,1,0),\ i=0\\(i,i-1,i),\ 1\le i\le t\\(t,t,t),\ i=t+1\\(2t+1-i,2t+2-i,2t+1-i),\ t+2\le i\le 2t\end{cases}.$$

$$r(v_iv_{i+1},S)=\begin{cases}(1,1,0),\ i=0\\(i+1,i,i),\ 1\le i\le t\\(2t+1-i,2t+2-i,2t-i),\ t+1\le i\le 2t\end{cases}.$$

Similarly as in Case 1, all edge metric representations with respect to S are mutually different, so S is an edge metric generator. Having in mind that $|S|=3$ and from Corollary 4 it follows $\beta_E(GP(2t+1,1))=3$. □

In [3], the authors came to an interesting question of relation between metric dimension and edge metric dimension of some graphs (called *realization question*), with the conclusion that it is possible to find all three cases, i.e. graphs G such that $\beta_E(G) = \beta(G)$, $\beta_E(G) > \beta(G)$ or $\beta_E(G) < \beta(G)$. For $GP(n,1)$ there are only two cases, since, from [8], it follows $\beta(GP(n,1)) = \begin{cases} 2, n \text{ is odd} \\ 3, n \text{ is even} \end{cases}$, so for $n = 2t$ it holds $\beta_E(GP(n,1)) = \beta(GP(n,1)) = 3$, while for $n = 2t+1$ it holds $3 = \beta_E(GP(n,1)) > \beta(GP(n,1)) = 2$.

2.3. Exact value for $GP(n,2)$

Analyze in this section is focused on edge metric dimension of generalized Petersen graphs $GP(n,2)$. Next theorem determines the exact value of the edge metric dimension for such graphs.

Theorem 3. $\beta_E(GP(n,2)) = \begin{cases} 3, & n = 8 \vee n \geq 10 \\ 4, & n \in \{5,6,7,9\} \end{cases}$

Proof. In the case of $n = 4t$, $t \geq 4$, let $S = \{u_0, v_3, v_{2t+3}\}$. All edge metric representations with respect to S are given in Table 1. The first column is related to edge $e \in E(GP(4t,2))$, the second column presents its edge metric representation $r(e)$, while the last column gives the condition in which statement in the second column is true. As it can be observed from Table 1, all edge metric representations with respect to S are mutually different, so S is an edge metric generator for $GP(4t,2)$. Having in mind that $|S| = 3$ and from Corollary 4 it follows that for $t \geq 4$, $\beta_E(GP(4t,2)) = 3$ holds.

If $n = 4t+1$, $t \geq 4$, then let $S = \{u_0, v_{2t-5}, v_{2t-4}\}$. All edge metric representations with respect to S are given in Table 2. As it can be seen in Table 2 all edge metric representations with respect to S are mutually different, so S is a edge metric generator for $GP(4t+1,2)$. Again, having in mind that $|S| = 3$ and from Corollary 4 it follows that for $t \geq 4$, $\beta_E(GP(4t+1,2)) = 3$ holds.

For $t \geq 4$ in cases when $n = 4t+2$ or $n = 4t+3$ let us define $S = \{u_0, v_{2t-2}, v_{2t-1}\}$. All edge metric representations of $GP(4t+2,2)$ and $GP(4t+3,2)$, with respect to S, are given in Table 3 and Table 4, respectively. It can be seen in Table 3 that all edge metric representations of $GP(4t+2,2)$, with respect to S, are mutually different, so S is a edge metric generator for $GP(4t+2,2)$. Again, having in mind that $|S| = 3$ and from Corollary 4 it follows that for $t \geq 4$, $\beta_E(GP(4t+2,2)) = 3$ holds. The same conclusion can

Table 1: Edge metric representations for $GP(4t, 2)$

e	$r(e)$	$Condition$
$u_{2i}u_{2i+1}$	$(0, 2, t)$	$i = 0$
	$(2, 1, t+1)$	$i = 1$
	$(i+2, i, t+2-i)$	$2 \leq i \leq t$
	$(2t+2-i, 2t+2-i, i-t)$	$t+1 \leq i \leq 2t-3$
	$(3, 4, t-2)$	$i = 2t-2$
	$(1, 3, t-1)$	$i = 2t-1$
$u_{2i+1}u_{2i+2}$	$(1, 2, t)$	$i = 0$
	$(3, 1, t+1)$	$i = 1$
	$(i+3, i, t+2-i)$	$2 \leq i \leq t-1$
	$(t+1, t, 2)$	$i = t$
	$(2t+1-i, 2t+2-i, i-t)$	$t+1 \leq i \leq 2t-3$
	$(2, 4, t-2)$	$i = 2t-2$
	$(0, 3, t-1)$	$i = 2t-1$
$u_{2i}v_{2i}$	$(0, 3, t)$	$i = 0$
	$(2, 2, t+1)$	$i = 1$
	$(i+1, i, t+3-i)$	$2 \leq i \leq t$
	$(t, t+1, 2)$	$i = t+1$
	$(2t+1-i, 2t+3-i, i-t)$	$t+2 \leq i \leq 2t-1$
$u_{2i+1}v_{2i+1}$	$(1, 1, t-1)$	$i = 0$
	$(i+2, i-1, t+1-i)$	$1 \leq i \leq t-1$
	$(t+1, t-1, 1)$	$i = t$
	$(2t+1-i, 2t+1-i, i-t-1)$	$t+1 \leq i \leq 2t-2$
	$(1, 2, t-2)$	$i = 2t-1$
$v_{2i}v_{2i+2}$	$(1, 3, t+1)$	$i = 0$
	$(2, 3, t+2)$	$i = 1$
	$(i+1, i+1, t+3-i)$	$2 \leq i \leq t-1$
	$(t, t+1, 3)$	$i = t$
	$(t-1, t+2, 3)$	$i = t+1$
	$(2t-i, 2t+3-i, i+1-t)$	$t+2 \leq i \leq 2t-2$
	$(1, 4, t)$	$i = 2t-1$
$v_{2i+1}v_{2i+3}$	$(2, 0, t-1)$	$i = 0$
	$(i+2, i-1, t-i)$	$1 \leq i \leq t-1$
	$(t, t-1, 0)$	$i = t$
	$(2t-i, 2t-i, i-t-1)$	$t+1 \leq i \leq 2t-2$
	$(2, 1, t-2)$	$i = 2t-1$

be drawn for $GP(4t+3, 2)$, since all its edge metric representations presented in Table 4 are also mutually different, so for $t \geq 4$, $\beta_E(GP(4t+3, 2)) = 3$ holds.

For the remaining cases when $n \leq 15$, edge metric dimension of $GP(n, 2)$ is found by the total enumeration technique, and it is presented in Table 5, together with the corresponding edge metric bases. It should be stated that edge metric dimension is equal to 3, except in cases for $n \in \{5, 6, 7, 9\}$, when it is equal to 4. $\square$

For $GP(n, 2)$ there are only two cases for realization question. From [9] it follows that $\beta(GP(n, 2)) = 3$, so for $n \notin \{5, 6, 7, 9\}$ edge metric dimension of $GP(n, 2)$ is equal to its metric dimension. Only in cases when $n \in \{5, 6, 7, 9\}$, it holds $4 = \beta_E(GP(n, 2)) > \beta(GP(n, 2)) = 3$.

Table 2: Edge metric representations for $GP(4t+1,2)$

e	$r(e)$	*Condition*
$u_{2i}u_{2i+1}$	$(0,t-2,t-1)$	$i=0$
	$(2,t-3,t-2)$	$i=1$
	$(i+2,t-2-i,t-1-i)$	$2\le i\le t-3$
	$(i+2,i+4-t,i+3-t)$	$t-2\le i\le t$
	$(2t+2-i,i+4-t,i+3-t)$	$t+1\le i\le 2t-3$
	$(4t-2i,3t-1-i,3t-1-i)$	$2t-2\le i\le 2t$
$u_{2i+1}u_{2i+2}$	$(1,t-2,t-2)$	$i=0$
	$(3,t-3,t-3)$	$i=1$
	$(i+3,t-2-i,t-2-i)$	$2\le i\le t-3$
	$(t+1,2,2)$	$i=t-2$
	$(t+2,3,3)$	$i=t-1$
	$(2t+2-i,i+4-t,i+4-t)$	$t\le i\le 2t-3$
	$(3,t,t+1)$	$i=2t-2$
	$(1,t-1,t)$	$i=2t-1$
$u_{2i}v_{2i}$	$(0,t-1,t-2)$	$i=0$
	$(2,t-2,t-3)$	$i=1$
	$(i+1,t-1-i,t-2-i)$	$2\le i\le t-3$
	$(i+1,i+4-t,i+2-t)$	$t-2\le i\le t$
	$(2t+2-i,i+4-t,i+2-t)$	$t+1\le i\le 2t-3$
	$(4,t,t)$	$i=2t-2$
	$(3,t-1,t+1)$	$i=2t-1$
	$(1,t-2,t)$	$i=2t$
$u_{2i+1}v_{2i+1}$	$(1,t-3,t-1)$	$i=0$
	$(i+2,t-3-i,t-1-i)$	$1\le i\le t-3$
	$(t,1,2)$	$i=t-2$
	$(t+1,2,3)$	$i=t-1$
	$(2t+1-i,i+3-t,i+4-t)$	$t\le i\le 2t-3$
	$(3,t+1,t)$	$i=2t-2$
	$(2,t,t-1)$	$i=2t-1$
$v_{2i}v_{2i+2}$	$(i+1,t-1-i,t-3-i)$	$0\le i\le t-4$
	$(t-2,3,0)$	$i=t-3$
	$(t-1,3,0)$	$i=t-2$
	$(t,4,1)$	$i=t-1$
	$(2t+1-i,i+5-t,i+2-t)$	$t\le i\le 2t-4$
	$(2t+1-i,3t-3-i,i+2-t)$	$2t-3\le i\le 2t-1$
	$(2,t-3,t)$	$i=2t$
$v_{2i+1}v_{2i+3}$	$(2,t-4,t-1)$	$i=0$
	$(i+2,t-4-i,t-1-i)$	$1\le i\le t-4$
	$(t-1,0,3)$	$i=t-3$
	$(t,1,3)$	$i=t-2$
	$(2t-i,i+3-t,i+5-t)$	$t-1\le i\le 2t-4$
	$(3,t,t)$	$i=2t-3$
	$(2,t+1,t-1)$	$i=2t-2$
	$(1,t,t-2)$	$i=2t-1$

Table 3: Edge metric representations for $GP(4t+2,2)$

e	$r(e)$	*Condition*
$u_{2i}u_{2i+1}$	$(0,t,t+1)$	$i=0$
	$(2,t-1,t)$	$i=1$
	$(i+2,t-i,t+1-i)$	$2\le i\le t-1$
	$(t+2,2,1)$	$i=t$
	$(2t+3-i,i+2-t,i+1-t)$	$t+1\le i\le 2t-2$
	$(3,t+1,t)$	$i=2t-1$
	$(1,t+1,t+1)$	$i=2t$
$u_{2i+1}u_{2i+2}$	$(1,t,t)$	$i=0$
	$(3,t-1,t-1)$	$i=1$
	$(i+3,t-i,t-i)$	$2\le i\le t-1$
	$(2t+2-i,i+2-t,i+2-t)$	$t\le i\le 2t-2$
	$(2,t+1,t+1)$	$i=2t-1$
	$(0,t+1,t+1)$	$i=2t$
$u_{2i}v_{2i}$	$(0,t+1,t)$	$i=0$
	$(2,t,t-1)$	$i=1$
	$(i+1,t+1-i,t-i)$	$2\le i\le t-1$
	$(t+1,2,0)$	$i=t$
	$(2t+2-i,i+2-t,i-t)$	$t+1\le i\le 2t$
$u_{2i+1}v_{2i+1}$	$(1,t-1,t+1)$	$i=0$
	$(i+2,t-1-i,t+1-i)$	$1\le i\le t-1$
	$(2t+2-i,i+1-t,i+2-t)$	$t\le i\le 2t-1$
	$(1,t,t+2)$	$i=2t$
$v_{2i}v_{2i+2}$	$(i+1,t+1-i,t-1-i)$	$0\le i\le t-2$
	$(t,3,0)$	$i=t-1$
	$(2t+1-i,i+3-t,i-t)$	$t\le i\le 2t-1$
	$(1,t+2,t)$	$i=2t$
$v_{2i+1}v_{2i+3}$	$(i+2,t-2-i,t+1-i)$	$0\le i\le t-2$
	$(t+1,0,3)$	$i=t-1$
	$(2t+1-i,i+1-t,i+3-t)$	$t\le i\le 2t-1$
	$(2,t-1,t+2)$	$i=2t$

3. Conclusions

In this article, the recently introduced edge metric dimension problem is considered. Exact formulae for generalized Petersen graphs $GP(n,1)$ and $GP(n,2)$ are stated and proved. Moreover, the lower bound for 3-regular graphs, which holds for all generalized Petersen graphs, is given.

Possible future research could be finding the edge metric dimension of some other challenging classes of graphs. Another research direction is construction of some metaheuristic approach for solving an edge metric dimension problem.

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Table 4: Edge metric representations for $GP(4t+3,2)$

e	$r(e)$	*Condition*
$u_{2i}u_{2i+1}$	$(0,t,t+1)$	$i=0$
	$(2,t-1,t)$	$i=1$
	$(i+2,t-i,t+1-i)$	$2\le i\le t-1$
	$(t+2,2,1)$	$i=t$
	$(2t+3-i,i+2-t,i+1-t)$	$t+1\le i\le 2t-1$
	$(2,t+2,t+1)$	$i=2t$
	$(0,t+1,t+1)$	$i=2t+1$
$u_{2i+1}u_{2i+2}$	$(1,t,t)$	$i=0$
	$(3,t-1,t-1)$	$i=1$
	$(i+3,t-i,t-i)$	$2\le i\le t-1$
	$(2t+3-i,i+2-t,i+2-t)$	$t\le i\le 2t-2$
	$(3,t+1,t+1)$	$i=2t-1$
	$(1,t+1,t+2)$	$i=2t$
$u_{2i}v_{2i}$	$(0,t+1,t)$	$i=0$
	$(2,t,t-1)$	$i=1$
	$(i+1,t+1-i,t-i)$	$2\le i\le t-1$
	$(t+1,2,0)$	$i=t$
	$(2t+3-i,i+2-t,i-t)$	$t+1\le i\le 2t-1$
	$(3,t+1,t)$	$i=2t$
	$(1,t,t+1)$	$i=2t+1$
$u_{2i+1}v_{2i+1}$	$(1,t-1,t+1)$	$i=0$
	$(i+2,t-1-i,t+1-i)$	$1\le i\le t-1$
	$(2t+2-i,i+1-t,i+2-t)$	$t\le i\le 2t-1$
	$(2,t+1,t+1)$	$i=2t$
$v_{2i}v_{2i+2}$	$(i+1,t+1-i,t-1-i)$	$0\le i\le t-2$
	$(t,3,0)$	$i=t-1$
	$(t+1,3,0)$	$i=t$
	$(2t+2-i,i+3-t,i-t)$	$t+1\le i\le 2t-2$
	$(3,t+1,t-1)$	$i=2t-1$
	$(2,t,t)$	$i=2t$
	$(2,t-1,t+1)$	$i=2t+1$
$v_{2i+1}v_{2i+3}$	$(i+2,t-2-i,t+1-i)$	$0\le i\le t-2$
	$(t+1,0,3)$	$i=t-1$
	$(2t+1-i,i+1-t,i+3-t)$	$t\le i\le 2t-2$
	$(2,t,t+1)$	$i=2t-1$
	$(1,t+1,t)$	$i=2t$

Table 5: Edge resolving bases of $GP(n,2)$

n	*basis*	$\beta_E(GP(n,2))$
5	$\{u_0, u_1, u_3, v_3\}$	4
6	$\{u_0, u_1, u_2, u_3\}$	4
7	$\{u_0, u_1, u_4, v_2\}$	4
8	$\{u_0, u_2, v_4\}$	3
9	$\{u_0, u_1, u_2, v_5\}$	4
10	$\{u_0, u_3, v_6\}$	3
11	$\{u_0, u_3, v_4\}$	3
12	$\{u_0, u_3, v_4\}$	3
13	$\{u_0, v_3, v_4\}$	3
14	$\{u_0, u_4, v_1\}$	3
15	$\{u_0, u_5, v_1\}$	3
$n = 4t \wedge t \geq 4$	$\{u_0, v_3, v_{2t+3}\}$	3
$n = 4t+1 \wedge t \geq 4$	$\{u_0, v_{2t-5}, v_{2t-4}\}$	3
$(n = 4t+2 \vee n = 4t+3) \wedge t \geq 4$	$\{u_0, v_{2t-2}, v_{2t-1}\}$	3

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