DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

# CSE2315 Lab Course
# Assignment 1

- This assignment consists of several exam-level questions and sometimes an optional extra exercise.
- See Brightspace, *Course Information* for instructions about the lab course, rules and grading procedure.
- Your solution has to be handed in on paper, typeset in LaTeX, using a word processor or readable handwriting. Handing in via email is not permitted.
- Cooperating with a colleague is permitted and *strongly encouraged!* In such cases, please hand in a single copy of the work.
- Total number of pages, without this cover page: 2.

1. Consider a language $L = \{ok, a, bad, dab, abba, hi, \varepsilon, acc, duck\}$.
   (a) Give a possible $\Sigma$ such at $L \subseteq \Sigma^*$.
   (b) Why is this only possible $\Sigma$?
   (c) Give all words in $L$ in "modified lexographical order" (or shortlex order). (Note that we often refer to this just as a lexographical order.)

2. *Old lab question that was also used on the resit of 2021-2022 (modified):* Consider the following claims (a) and (b). For each claim, verify whether it is true for arbitrary languages $L_1 \subseteq \Sigma_1^*,\ \ L_2 \subseteq \Sigma_2^*,\ \ L_3 \subseteq \Sigma_3^*,\ \ L_4 \subseteq \Sigma_4^*$. If a claim is true, give a proof; if it is not true, give a counterexample with an explanation how the counterexample shows the claim is false.
   (a) If $L_1 \cup L_2 = L_3 \cup L_4$, then $\Sigma_1 \cup \Sigma_2 = \Sigma_3 \cup \Sigma_4$.
   (b) If $L_1 \cup L_2 \subseteq L_3 \cap L_4$, then $L_1L_2 \subseteq L_3L_4$.

3. Suppose we have an alphabet $\Sigma = \{a, b\}$. Construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes the following language $L \subseteq \Sigma^*$:

   $$L = \{w \mid \text{each } b \text{ in } w \text{ is immediately preceeded by at least two } a\text{s and } w \text{ ends with an } a\}.$$

   (a) Give the transition graph of $M$. Use no more than 6 states.
   (b) Describe briefly how $M$ works.

4. *Old exam question (modified), midterm 21-22* Consider the following DFA $D = (\{q_a, q_b, q_c\}, \{0, 1\}, \delta, q_a, \{q_a\})$

   where $\delta$ is represented using the following table:

   | | 0 | 1 |
   |---|---|---|
   | $q_a$ | $q_b$ | $q_a$ |
   | $q_b$ | $q_b$ | $q_c$ |
   | $q_c$ | $q_c$ | $q_a$ |

   (a) Give a transition diagram that corresponds to $D$.
   (b) Give a word $w$ of length 3 that is not in $D$ but is in $D' = (Q, \Sigma, \delta, q_0, F \cup \{q_c\})$. Explain your answer by giving the state transitions for $w$.
   (c) Now imagine that we create a DFA $D'' = (Q \cup Q_2, \Sigma, \delta_1, q_a, F)$, with
   $\delta_1(q, s) = \begin{cases} \delta(q, s) & \text{if } q \in Q \\ \delta_2(q, s) & \text{else} \end{cases}$ for some function $\delta_2 : Q_2 \times \Sigma \to Q \cup Q_2$
   *(Subquestion not included on exam for time reasons):* What do we know about $\delta_2$, specifically how many extra transitions will it have to contain? You may assume $Q \cap Q_2 = \emptyset$. Explain your answer.
   (d) Describe as accurately as possible the relation between $L(D)$ and $L(D'')$ in terms of $\subset, \subseteq, =$ relations. Explain your answer.

5. *Old exam question (modified), resit 18-19*
   (a) Consider a modification of the DFA model, called NEFA. The NEFA model is identical to the DFA with the extra requirement that $q_0 \in F$. Is the NEFA model equally expressive as the DFA model? If so, explain how we can construct an NEFA from a DFA. If not, give an example of a DFA that has no NEFA equivalent and argue why (no full proof is needed).
   (b) Take two DFAs $D_1$ and $D_2$. Now take the language $L = (L(D_1) \cap L(D_2))^*$. Is $L$ regular? Explain your answer in at most 5 lines.

See next page for an optional extra ~~exercise~~ puzzle.

**Optional extra exercise** This last exercise is not part of the learning material. However, if you like puzzles and have some time left, you might like to do this challenging puzzle.

In one of your adventures, you try to steal some treasure from an old mystical mansion. As this mansion has no door, you have build a teleportation device to teleport inside. This works as you expected and you find the treasure you were looking for. However, as the mansion still has no door, it will be a challenge to get out. The room you are in is mostly empty. The only things in there are 2 teleportation devices at the other side of the room. As you have no other choice, you decide to try one of them, and arbitrarily choose the left one. You teleport to an identical room, again with 2 teleportation devices. Now, you decide to use the right teleportation device. Again, you end up in an identical room. Since it's identical, you don't know whether this is a room you haven't been in before, the room you started in, or even the room you just left. You take the left teleportation device ending up in an identical room, and then the right teleportation device again, with the same result. Starting to lose hope, you enter the right teleportation device, ending up in a different room. This room has white walls, in contrast to the grey walls of before. This gives you hope. If there are different rooms, it might be easier to deduce where you are. Given the size of the mansion and the rooms, you know there are at most 4 rooms in the mansion. Also, you know that there is at least 1 teleportation device that leads outside. You use a few more teleportation devices (given below), but then the walls start to crumble. The mansion is falling apart. You have to get outside within 4 uses of a teleportation device. You can't be sure if it is possible, but which 4 teleportation devices should you use to have the only chance to get outside?

| Start | Left | Right | Left | Right | Right | Left | Left | Left | Left | Left | Left |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Grey | Grey | Grey | Grey | Grey | White | Grey | Grey | Grey | Grey | Grey | Grey |

| Left | Right | Right | Right | Left | Right | Left | ? | ? | ? | ? |
|---|---|---|---|---|---|---|---|---|---|---|
| Grey | Grey | White | White | Grey | White | Grey | | | | Outside |

- Each room has 2 teleportation devices, a left one and a right one
- Teleportation devices are deterministic (the same device in the same room will always lead to the same room)
- At least 1 teleportation device leads outside
- There are at most 4 rooms
- Rooms will not change colour (the same room will always have the same colour)
- After you have taken the teleportation devices as given in the table (and entered rooms with wall colours as given in the table), which 4 teleportation devices should you take to have a chance to get outside?

- Extra: The last sentence of the story says "You can't be sure if it is possible". Why?

End of the assignment