# CSE2315 — Assignment 2

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## Exercise 1

Suppose we have the following language $L$ over the alphabet $\Sigma = \{a, 0, 1\}$:

$$L = \left\{ua^k v | u, v \in \{0,1\}^* \text{ and } c_0(u) + c_1(v) > k\right\}$$

Where $c_x(w)$ is defined as the number of occurrences of $x$ in $w$. Use the pumping lemma to show that $L$ is not regular.

*Proof.* By contradiction.
Assume to the contrary that $L$ is regular. Let $p$ be the pumping length given by the pumping lemma. Choose $s$ to be the string $a^p 1^p 1$. Because $s$ is a member of $L$ and $s$ has length more than $p$, the pumping lemma guarantees that $s$ can be split into three pieces, $s = xyz$, where for any $i \geq 0$ the string $xy^i z$ is in $L$. We show this outcome is impossible.
Condition 3 of the pumping lemma says that $|xy| \leq p$. Thus, in the case of our word, y must consist of only $a$s. Let $k_{xyz}$ and $k_{kyyz}$ represent the number of $a$s in $xyz$ and $xyyz$ respectively. $k_{xyz} = p$ and $k_{xyyz} > k_{xyz} = p$, but the number of 1s in $xyyz$ is still $p + 1$. Thus in $xyyz$ we have $c_0(u) + c_1(v) \leq k_{xyyz}$, so $xyyz$ cannot be in $L$. Therefore, $s$ cannot be pumped and we have reached our desired contradiction. $\square$

## Bonus Exercise