# CSE2315 — Assignment 1

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## 1 Exercise 1

Consider a language $L = \{ok, a, bad, dab, abba, hi, \varepsilon, acc, duck\}$

### (a) Give a possible $\Sigma$ such at $L \subseteq \Sigma^*$

$$\Sigma = \{a, b, c, d, h, i, k, o, u\}$$

### (b) Why is this only possible $\Sigma$

Supposing this question is asking why this is the only possible **minimal** alphabet, the answer is that it must contain exactly the set of symbols that appear in the words in $L$, and that set is uniquely determined.

### (c) Give all words in $L$ in shortlex order

$$\varepsilon, a, hi, okacc, bad, dab, abba, duck$$

## 2 Exercise 2

Consider the following claims (a) and (b). For each claim, verify whether it is true for arbitrary languages $L_1 \subseteq \Sigma_1^*$,$L_2 \subseteq \Sigma_2^*$,$L_3 \subseteq \Sigma_3^*$,$L_4 \subseteq \Sigma_4^*$. If a claim is true, give a proof; if it is not true, give a counterexample with an explanation how the counterexample shows the claim is false.

### (a) If $L_1 \cup L_2 = L_3 \cup L_4$, then $\Sigma_1 \cup \Sigma_2 = \Sigma_3 \cup \Sigma_4$

**FALSE.**
Counterexample:

$$\begin{aligned} \Sigma_1 = \{a, x\}, \Sigma_2 = \{b\}, \Sigma_3 = \{a, y\}, \Sigma_4 = \{b\} \\ L_1 = \{a\}, L_2 = \{b\}, L_3 = \{a\}, L_4 = \{b\} \\ \\ L_1 \cup L_2 = L_3 \cup L_4 = \{a, b\} \\ \Sigma_1 \cup \Sigma_2 = \{a, b, x\} \neq \Sigma_3 \cup \Sigma_4 = \{a, b, y\} \end{aligned}$$

### (b) If $L_1 \cup L_2 \subseteq L_3 \cap L_4$, then $L_1 L_2 \subseteq L_3 L_4$

**TRUE.**

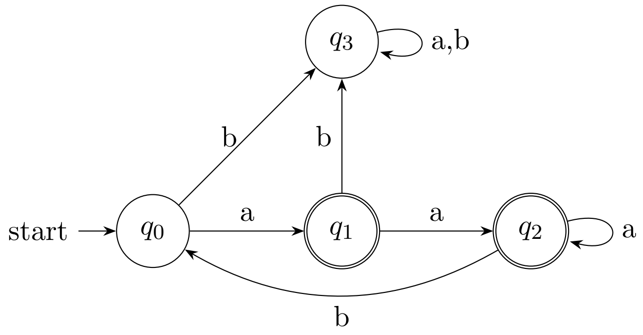
*Proof.* From the first part of the implication, it follows that every word in $L_1$ and every word is $L_2$ is in both $L_3$ and $L_4$, so Ł$_1 \subseteq L_3$ and $L_2 \subseteq L_4$
Take an arbitrary word $w \in L_1L_2$. By definition of concatenation, we have $w = xy$ such that $x \in L_1$ and $y \in L_2$. From earlier, $x \in L_3$ and $y \in L_4$. Therefore, $w = xy \in L_3L_4$. Since w is arbitrary, it holds that $\forall w \in L_1L_2 : w \in L_3L_4$. So $L_1L_2 \subseteq L_3L_4$. □

# 3 Exercise 3

Suppose we have an alphabet $\Sigma = \{a, b\}$. Construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes the following language $L \subseteq \Sigma^*$:

$$L = \{w | \text{each } b \text{ in } w \text{ is immediately preceeded by at least two } a\text{s and w ends with an } a\}$$

### (a) Give the transition graph of $M$. Use no more than 6 states.



### (b) Describe briefly how $M$ works.

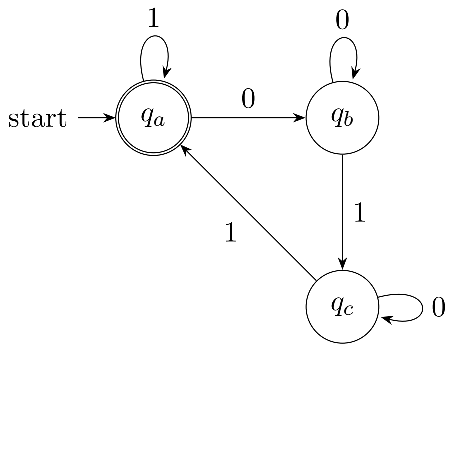
Basically it counts if there s been at least 2 $a$s before a $b$. If not, the word is invalid so we go to a non-accepting state of $q_3$, which gets stuck there. If we get a $b$ in a valid position, we go back to the beginning thing. Also, to check that it ends in a, only $q_2$ and $q_3$ are valid.

# 4 Exercise 4

Consider the DFA $D = (\{q_a, q_b, q_c\}, \{0, 1\}, \delta, q_a, \{q_a\})$, where $\delta$ is represented using the following transition table:

|  | 0 | 1 |
|---|---|---|
| $q_a$ | $q_b$ | $q_a$ |
| $q_b$ | $q_b$ | $q_c$ |
| $q_c$ | $q_c$ | $q_a$ |

### (a) Give a transition diagram that corresponds to $D$.



**(b) Give a word $w$ of length 3 that is not in $D$ but is in $D' = (Q, \Sigma, \delta, q_a, F \cup \{q_c\})$**

$$010$$

**(c) Now imagine that we create a DFA $D'' = (Q \cup Q_2, \Sigma, \delta_1, q_a, F)$, what do we know about $\delta_2$?**

$\delta_1(q, s) = \begin{cases} \delta(q, s) & \text{if } q \in Q \\ \delta_2(q, s) & \text{else} \end{cases}$ for some function $\delta_2 : Q_2 \times \Sigma \to Q \cup Q_2$. $Q \cap Q_2 = \emptyset$ Every state in $Q_2$ must have a transition defined for all inputs, so in our case, they must have a transition for 0 and for 1. So the number of extra transitions is $2|Q_2|$

**(d)Describe as accurately as possible the relation between $L(D)$ and $L(D'')$ in terms of $\subset$,$\subseteq$,$=$ relations. Explain your answer.**

$$L(D) = L(D'')$$

The starting state of $D''$ is in $Q$, and all transitions in $\delta$, which cover all states in $Q$ always remain in $Q$. So therefore, it is impossible for an input to leave $Q$ and go into $Q_2$. Therefore, $D$ and $D''$ are basically equivalent, as they have the same set of reachable states and stuff.

# 5 Exercise 5

# 6 Bonus Exercise