CSE2315 — Assignment 1

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Exercise 1

Consider a language $L = \{ok, a, bad, dab, abba, hi, \varepsilon, acc, duck\}$

(a) Give a possible Σ such at $L \subseteq \Sigma^*$

$$\Sigma = \{a, b, c, d, h, i, k, o, u\}$$

(b) Why is this not only possible Σ

An alphabet can contain letters that do not appear in any word in the language, so $\Sigma' = \Sigma \cup \{x\}$ is also a valid alphabet.

(c) Give all words in L in shortlex order

$$\varepsilon, a, hi, okacc, bad, dab, abba, duck$$

Exercise 2

Consider the following claims (a) and (b). For each claim, verify whether it is true for arbitrary languages $L_1 \subseteq \Sigma_1^*$,$L_2 \subseteq \Sigma_2^*$,$L_3 \subseteq \Sigma_3^*$,$L_4 \subseteq \Sigma_4^*$. If a claim is true, give a proof; if it is not true, give a counterexample with an explanation how the counterexample shows the claim is false.

(a) If $L_1 \cup L_2 = L_3 \cup L_4$, then $\Sigma_1 \cup \Sigma_2 = \Sigma_3 \cup \Sigma_4$

FALSE.
Counterexample:

$$\begin{aligned} \Sigma_1 = \{a, x\}, \Sigma_2 = \{b\}, \Sigma_3 = \{a, y\}, \Sigma_4 = \{b\} \\ L_1 = \{a\}, L_2 = \{b\}, L_3 = \{a\}, L_4 = \{b\} \\ \\ L_1 \cup L_2 = L_3 \cup L_4 = \{a, b\} \\ \Sigma_1 \cup \Sigma_2 = \{a, b, x\} \neq \Sigma_3 \cup \Sigma_4 = \{a, b, y\} \end{aligned}$$

(b) If $L_1 \cup L_2 \subseteq L_3 \cap L_4$, then $L_1 L_2 \subseteq L_3 L_4$

TRUE.

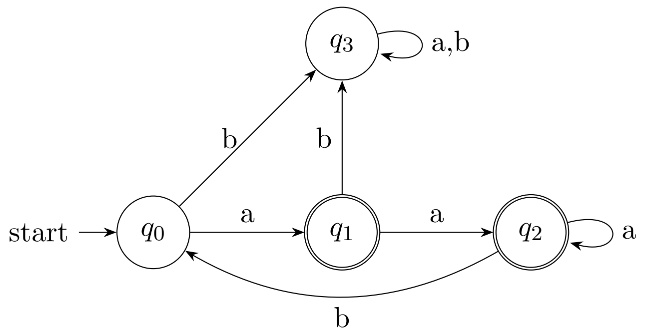
Proof. From the first part of the implication, it follows that every word in L_1 and every word is L_2 is in both L_3 and L_4, so $L_1 \subseteq L_3$ and $L_2 \subseteq L_4$
Take an arbitrary word $w \in L_1L_2$. By definition of concatenation, we have $w = xy$ such that $x \in L_1$ and $y \in L_2$. From earlier, $x \in L_3$ and $y \in L_4$. Therefore, $w = xy \in L_3L_4$. Since w is arbitrary, it holds that $\forall w \in L_1L_2 : w \in L_3L_4$. So $L_1L_2 \subseteq L_3L_4$. $\square$

Exercise 3

Suppose we have an alphabet $\Sigma = \{a, b\}$. Construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes the following language $L \subseteq \Sigma^*$:

$$L = \{w|\text{each } b \text{ in } w \text{ is immediately preceeded by at least two } a\text{s and w ends with an } a\}$$

(a) Give the transition graph of M. Use no more than 6 states.



(b) Describe briefly how M works.

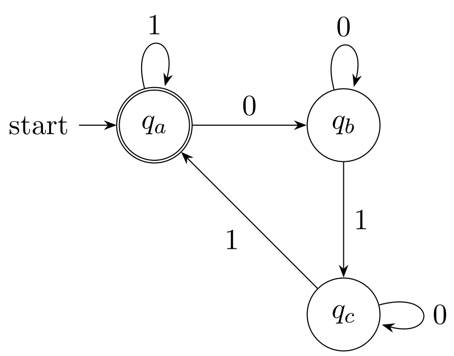
Basically it counts if there s been at least 2 as before a b. If not, the word is invalid so we go to a non-accepting state of q_3, which gets stuck there. If we get a b in a valid position, we go back to the beginning thing. Also, to check that it ends in a, only q_2 and q_3 are valid.

Exercise 4

Consider the DFA $D = (\{q_a, q_b, q_c\}, \{0, 1\}, \delta, q_a, \{q_a\})$, where δ is represented using the following transition table:

	0	1
q_a	q_b	q_a
q_b	q_b	q_c
q_c	q_c	q_a

(a) Give a transition diagram that corresponds to D.



(b) Give a word w of length 3 that is not in D but is in $D' = (Q, \Sigma, \delta, q_a, F \cup \{q_c\})$

$$010$$

(c) Now imagine that we create a DFA $D'' = (Q \cup Q_2, \Sigma, \delta_1, q_a, F)$, what do we know about δ_2?

$\delta_1(q,s) = \begin{cases} \delta(q,s) & \text{if } q \in Q \\ \delta_2(q,s) & \text{else} \end{cases}$ for some function $\delta_2 : Q_2 \times \Sigma \to Q \cup Q_2$. $Q \cap Q_2 = \emptyset$ Every state in Q_2 must have a transition defined for all inputs, so in our case, they must have a transition for 0 and for 1. So the number of extra transitions is $2|Q_2|$

(d) Describe as accurately as possible the relation between $L(D)$ and $L(D'')$ in terms of $\subset$,$\subseteq$,$=$ relations. Explain your answer.

$$L(D) = L(D'')$$

The starting state of D'' is in Q, and all transitions in δ, which cover all states in Q always remain in Q. So therefore, it is impossible for an input to leave Q and go into Q_2. Therefore, D and D'' are basically equivalent, as they have the same set of reachable states and stuff.

Exercise 5

Consider a modification of the DFA model, called NEFA. The NEFA model is identical to the DFA with the extra requirement that $q_o \in F$

(a) Is the NEFA model equally expressive as the DFA model?

It is not as expressive. Consider the DFA $Q = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q1\})$ with $\delta(\_,\_) = q_1$ The language of D does not contain ε, whereas any language recognized by a NEFA must contain ε, thus they can never be equivalent.

(b) Take 2 DFAs D_1 and D_2. Now take the language $L = (L(D_1) \cap L(D_2))^*$. Is L regular?

We have seen in the book that intersection and the $*$ operation are closed under the regural languages. So the answer is L is regular.

Bonus Exercise