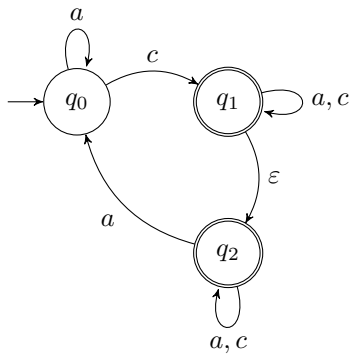
DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

CSE2315 Lab Course
Assignment 2

- This assignment consists of several exam-level questions and sometimes an optional extra exercise.
- See Brightspace, *Course Information* for instructions about the lab course, rules and grading procedure.
- Your solution has to be handed in on paper, typeset in LaTeX, using a word processor or readable handwriting. Handing in via email is not permitted.
- Cooperating with a colleague is permitted and *strongly encouraged!* In such cases, please hand in a single copy of the work.
- Total number of pages, without this cover page: 3.

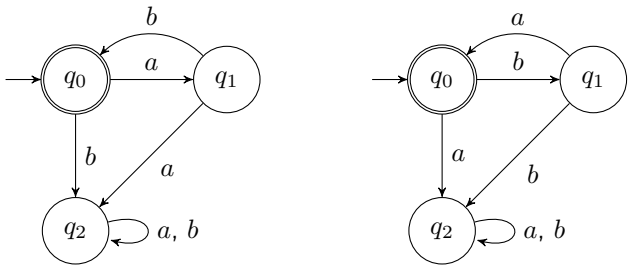
1. Consider the following language:
$$L = (a^*b)^*.$$
Construct a NFA N for which $L(N) = L$. To this end, please answer the following questions.
   (a) Consider a symbol x. Construct a NFA N_1 for which $L(N_1) = \{x\}$.
   (b) Construct the NFA N so that $L(N) = L$ in a systematic way. Use your construction in (a) as well as the constructions used in the proofs of Theorem 1.47 and 1.49 in Sipser. Please give the resulting NFA and do not simplify it.

2. *Taken from Resit 2021-2022:* Consider the NFA $N = \{\{q_0, q_1, q_2\}, \{a, b, c\}, \delta, q_0, \{q_1, q_2\}\}$ where δ is depicted using the tra nsition diagram below.



Construct a DFA D such that $L(D) = L(N)$. Use the procedure given in Sipser. Please give the resulting DFA and leave out unreachable states.

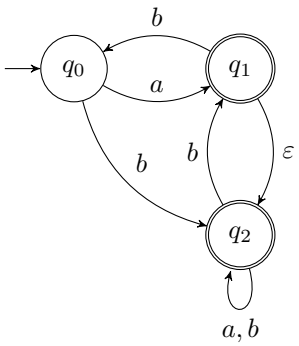
3. Consider the following two DFAs D_1, D_2:



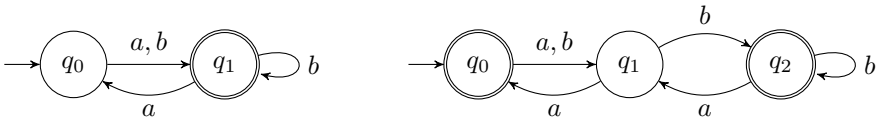
   (a) Describe the languages of D_1 and D_2.
   (b) Construct a DFA D such that $L(D) = L(D_1) \cap L(D_2)^c$, where $L(D_2)^c$ is the complement of the language of D_2. Adapt the procedure given in the proof of Theorem 1.25 in Sipser. Leave out unreachable states.

4. Suppose we have the following language L over the alphabet $\Sigma = \{a, b, c\}$:
$$L = \{w \in \Sigma^* \mid w \text{ has an } a \text{ and every } a \text{ after the first } a \text{ in } w \text{ is immediately followed by a } c\}.$$
For example $aacb$ is in the language, so is $acbac$ (note that it is okay for the first a to *also* be followed by a c). Words like $abca$ and $abaab$ are not in the language.
   (a) Give a regular expression R such that $L(R) = L$.
   (b) Explain how you arrived at your answer.

5. Convert the following NFA N to a regular expression R such that $L(R) = L(N)$. Please show all the intermediate steps and eliminate the states in the following order: q_0, q_1, q_2.



6. *Revision, old exam question midterm 18-19* Consider the following two DFAs, D on the left and D' on the right:



   (a) Construct a new DFA D'' of at most 2 states, such that $L(D'') = \overline{L(D)}$. A transition diagram and a short explanation suffice.

   (b) Use the procedure from Sipser to construct a DFA D''', such that $L(D''') = L(D) \cup L(D')$. You should leave out unreachable states.

See next page for an optional extra ~~exercise~~ puzzle.

Optional extra exercise This last exercise is not part of the learning material. However, if you like puzzles and have some time left, you might like to do this challenging puzzle.

Although your adventure of last week was a little more dangerous than expected, you decide it was worth the treasure and even want to find some more. However, this time you should be a bit better prepared. Searching for information, you find a bit about how the teleportation devices work. They deconstruct you to atoms at one place, send the information of how that was done to another place and construct you back to normal there, with completely new atoms. But since you are now consisting of different matter, are you still you? You decide to not ponder about that too much and search further. You find another mystical mansion somewhere but you read that it is a bit different than the previous one. While the teleportation devices from the other mansion always built you back up in exactly 1 place, the teleporation devices from this mansion might build you up in 0 or multiple places. Multiple places might not be so bad (or maybe it is, depending on how much you like yourself), but if you are build up in 0 places you just dissapear (how dare they still call that a teleportation device!). Because of this danger, you decide to build a remote-controlled robot. On the control are 2 buttons, one to let the robot take the left teleportation device, one to let it take the right one. Whenever the robot enters a room, you get a message on a screen telling you what colour the walls are.

You send in the robot using a teleportation device and let it take the treasure (with a different button I forgot to mention earlier). The walls of the first room are grey. Then, you press the button that lets the robot go left. This time, you get 2 messages: 1 tells you that the robot is in a grey room, the other that the robot is in a white room. You press a few more buttons (given below), but then the walls start to crumble. This mansion is falling apart as well. There is only time to use 6 teleportation devices. How can you be sure to get the robot with the treasure out?

Start	Left	Right	Left	Right	Left	Left	Left
{Grey}	{Grey,White}	{White}	{Grey}	{Grey,White}	{Grey,White}	{Grey,White}	{Grey,White}

Left	Right	Left	Left	Left	Right	Left
{Grey,Grey,White}	{Grey,White}	{Grey,White}	{Grey,White}	{Grey,White}	{White}	{Grey}

- Each room has 2 teleportation devices, a left one and a right one
- Teleportation devices are deterministic (the same device in the same room will always lead to the same room(s))
- If 2 (or more) teleportation devices teleport the same thing to the same room at the same time, only 1 of those will be build (the device cannot see that they are 2 signals)
- At least 1 teleportation device leads outside
- There are at most 4 rooms
- Rooms will not change colour (the same room will always have the same colour)
- After the robot has taken the teleportation devices as given in the table (and entered rooms with wall colours as given in the table), how can you get the robot outside to collect the treasure?

- Extra: How can you get 2 treasures?
- Extra: How can you get 3 treasures? (here, you have to think outside the box a little)
- Extra: Given that you have time left to use n teleporation devices (instead of 6), how many treasures would you be able to collect?

End of the assignment