Artificial Neural Networks - Laboratory 5
Hopfield Networks

Tudor Berariu
tudor.berariu@gmail.com

AI-MAS Laboratory
Faculty of Automatic Control and Computers
University Politehnica of Bucharest

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1 Goals

The goal for today's laboratory is the implementation of a Hopfield network.

2 Hopfield Networks

Hopfield Networks are fully-connected recurrent asynchronuous networks with K neurons. The network is asynchronuous if each unit updates its state at random times, independently of other units update times.

In a Hopfield network, the activation function for a neuron is that from Formula 1.

$$x_i \longleftarrow sgn\Big(\sum_{j=1}^{K} w_{ij} x_j\Big) \tag{1}$$

Using Hebbian learning, a Hopfield network might be used as an associative memory in order to store a number of binary patterns (with values

from $\{-1, 1\}$). The weights of the network are computed from the original N patterns as in Formula 2.

$$\mathbf{W} = \sum_{n=1}^{N} \mathbf{x}^{(n)} \cdot (\mathbf{x}^{(n)})^T \; - \; N\mathbf{I} \tag{2}$$

Be careful: $w_{ii} = 0, \forall i \in \{1 \dots K\}$.

In order to use the network as a classifier or as a missing information reconstruction system, follow Algorithm 1

Algorithm 1 Hopfield

Require: the weights $\mathbf{W}$, given pattern $\mathbf{x}$
Ensure: recovered pattern $\mathbf{y}$

1: **repeat**
2: choose a random computing unit i
3: $x_i \longleftarrow sgn\Big(\sum_{j=1}^{K} w_{ij} x_j\Big)$
4: **until** no network unit changes its state
5: $\mathbf{y} \longleftarrow \mathbf{x}$

3 Tasks

Implement a Hopfield network and test it on the attached data set to recover digits.

```
__xxxxxxxx__
__xxxxxxxxx_
________xxx_
________xxx_
___xxxxxxx__
___xxxxxxx__
________xxx_
________xxx_
__xxxxxxxxx_
__xxxxxxxx__
```

1. Read the patterns from the file and store them as binary vectors.
2. Compute the weights for the Hopfield network, given M patterns out of the ten.
3. Produce noisy patterns from the original ones by changing Z bits.
4. Recover original pattern given a noisy one using the Hopfield network.
5. Test the network's limitations by increasing M and Z. Make plots, lots of plots.