I.

1) $\sum_{i=1}^{n} 2i - 1 = n^2$

$Claim:$ *The sum of* $2i - 1$ *for* $i = 1$ *to* n *is equal to* n^2 *for all* $n \geq 1$

$Define:$ $P(n) =$ *The sum of* $2i - 1$ *for* $i = 1$ *to* n *is equal to* n^2

$Base\ Case:$ $P(1)$ *is true because the LHS is* $2(1) - 1 = 1$ *and the RHS is* $1^2 = 1$

$Induction\ Step:$

Suppose that $P(n)$ is true for a given $n \geq 1$. Let us show that $P(n+1)$ is true :

Because $P(n)$ is true, we have :

sum of $2i - 1$ for $i = 1$ to $n = n^2$

Therefore, the sum of of $2i - 1$ for $i = 1$ to $n + 1$ is

(sum of $(2i - 1)$ for $i = 1$ to n) $= n^2$, then by adding $2(n+1)$ to both sides

$= 2(n+1) - 1 + n^2$

$= n^2 + 2n + 1$

$= (n+1)^2$

Therefore $P(n+1)$ is true

$PMI:$ By principal of MI, $P(n)$ is true for all $n \geq 1$.

2) $\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$

$Claim:$ The sum of $\frac{1}{2^i}$ for $i = 1$ to n is equal to $1 - \frac{1}{2^n}$ for all $n \geq 1$

$Define:$ $P(n) =$ The sum of $\frac{1}{2^i}$ for $i = 1$ to n is equal to $1 - \frac{1}{2^n}$

$Base\ Case:$ $P(1)$ is true because the LHS is $\frac{1}{2^1} = \frac{1}{2}$ and the RHS is $1 - \frac{1}{2^1} = \frac{1}{2}$

$Induction\ Step:$

Suppose that $P(n)$ is true for a given $n \geq 1$. Let us show that $P(n+1)$ is true :

Because $P(n)$ is true, we have sum of $\frac{1}{2^i}$ for $i = 1$ to n equal to $1 - \frac{1}{2^n}$

Therefore, sum of $\frac{1}{2^i}$ for $i = 1$ to $n + 1$ is

(sum of $\frac{1}{2^i}$ for $i = 1$ to n) $= (1 - \frac{1}{2^n})$, then by adding $\frac{1}{2^{n+1}}$ to both sides

$= (\frac{1}{2^{n+1}} + 1 - \frac{1}{2^n})$

$= \frac{1}{2^{n+1}} + \frac{2^{n+1}}{2^{n+1}} - \frac{2}{2^{n+1}}$

$= \frac{2^{n+1} - 1}{2^{n+1}}$

$= \frac{2^{n+1}}{2^{n+1}} - \frac{1}{2^{n+1}}$

$= 1 - \frac{1}{2^{n+1}}$

Therefore, $P(n+1)$ is true.

$PMI:$ By principal of MI, $P(n)$ is true for all $n \geq 1$.

3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Claim: *The sum of* i^2 *for* $i = 1$ *to* n *is equal to* $\frac{n(n+1)(2n+1)}{6}$ *for all* $n \geq 1$

Define: $P(n)$ = *The sum of* i^2 *for* $i = 1$ *to* n *is equal to* $\frac{n(n+1)(2n+1)}{6}$

Base Case: $P(1)$ *is true because* $(1)^2 = \frac{(1)(1+1)(2(1)+1)}{6}$

Induction Step:

Suppose that $P(n)$ *is true for a given* $n \geq 1$. *Let us show that* $P(n+1)$ *is true*:

Because $P(n)$ *is true, we have sum of* i^2 *for* $i = 1$ *to* n *equal to* $\frac{n(n+1)(2n+1)}{6}$

Therefore, sum of i^2 *for* $i = 1$ *to* $n+1$ *is*

(*sum of* i^2 *for* $i = 1$ *to* n) $= (\frac{n(n+1)(2n+1)}{6})$, *then by adding* $(n+1)^2$ *to both sides*

$$= (n+1)^2 + \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{6(n+1)^2 + n(n+1)(2n+1)}{6}$$
$$= \frac{(6n^2+12n+6)+(n^2+n)(2n+1)}{6}$$
$$= \frac{6n^2+12n+6+2n^3+n^2+2n^2+n}{6}$$
$$= \frac{(2n^3+9n^2+13n+6)}{6}$$
$$= \frac{(n+1)(n+2)(2n+3)}{6}$$

Therefore $P(n+1)$ *is true*

PMI: *By principal of MI,* $P(n)$ *is true for all* $n \geq 1$.

II.

Claim : $minEven(A, n)$ *returns the smallest even number up to index n or* $+\infty$ *if no elements are even when* $n \geq 0$.

Define : $P(n) = minEven(A, n)$ *returns the smallest even number up to index n or* $+\infty$ *if no elements are even.*

Base Case : $P(0)$ *is true because* $minEven(A, n)$ *returns* $+\infty$ *immediately since* $n = 0$.

To prove $P(1)$, *we consider two cases* : $A[0]$ *is either odd or even*

When the element of $A[0]$ *is odd,* $P(1)$ *is true because* $P(1)$ *is not even, so* $+\infty$ *is returned.*

When the element of $A[0]$ *is even,* $P(1)$ *is true because* $P(1)$ *is even, so the element of* $P(1)$ *is returned* .

Induction Step :

Let a number $n \geq 1$ *be given*

We're trying to show that if $minEven(A,\ n)$ *returns the smallest even number up to index n or* $+\infty$, *then* $minEven(A,\ n+1)$ *returns the smallest even number up to index* $n+1$ *or* $+\infty$.

Assume $P(n)$ *is true, that means* $minEven(A,\ n)$ *returns the smallest even number up to index n or* $+\infty$.

Case 1 : *We're trying to show that* $P(n+1)$ *is true if* $A[n]$ *is the smallest even number of array A.*

When we call $minEven(A,\ n+1)$, $minEven(A,\ n-1)$ *is recursively called and* $P(n)$ *is returned as best up to index n.*

Because the conditions $A[n] <$ *best and* $A[n]$ *is an even number are true, best is now to set to* $A[n]$.

Hence, $minEven(A,\ n+1)$ *returns* $A[n]$ *up to index* $n+1$.

Therefore, $P(n+1)$ *is true.*

Case 2 : *We're trying to show that* $P(n+1)$ *is true if* $A[n]$ *is an odd number.*

When we call $minEven(A,\ n+1)$, $minEven(A,\ n-1)$ *is recursively called and* $P(n)$ *is returned as best up to index n.*

Because $A[n]$ *is not an even number, the if condition is not true.*

Therefore, $minEven(A,\ n+1)$ *returns* $P(n)$ *up to index* $n+1$.

Hence, $P(n+1)$ *is true.*

PMI : *By the generalized principal of MI,* $P(n)$ *is true for all* $n \geq 0$.