VOL. LVII. No. 226.] [April, 1948

MIND

A QUARTERLY REVIEW

OF

PSYCHOLOGY AND PHILOSOPHY

I.—WHITEHEAD AND PRINCIPIA MATHEMATICA.

BY BERTRAND RUSSELL.

THERE is in some quarters a tendency to suppose that Whitehead's part in our joint work was less than in fact it was. As no one except myself now knows what were our respective shares, I will try to state the facts as nearly as I can remember them.

Whitehead was already a lecturer at Trinity when I was a freshman; in my first term I attended his lectures on Statics. He was one of the examiners when I was a candidate first for a scholarship and then for a fellowship. He was always quite exceptionally kind, and passed by gradual stages from a teacher to a friend.

In 1900 we went together to the International Congress of Philosophy in Paris, where I was impressed by Peano. I saw that methods analogous to his would clarify the logic of relations, and I was led to the definitions of cardinal, ordinal, rational, and real numbers which are given in *The Principles of Mathematics*. Very soon Whitehead became interested. The project of deducing mathematics from logic appealed to him, and to my great joy he agreed to collaborate. I knew that my mathematical capacity was not equal to accomplishing this task unaided. Moreover, in June 1901 I came upon the contradiction about classes which are not members of themselves, and from that time on a large part of my time was occupied with attempts to avoid contradictions.

In the early part of the *Principia*, Whitehead contributed the treatment of apparent variables and the notation $(x).\phi x$.

Chapters 10, 11, 13 of the *Principia* are in the main his work. He also invented the notations D'R, $\overrightarrow{R}$'x, R"α—in this last case, the concept as well as the notation. (My previous attempts at relational notation, which were clumsy, will be found in Peano's *Revue de Mathématiques*, Vols. VII and VIII.)

In the later parts, the primary responsibility was Whitehead's as regards cardinal arithmetic and mine as regards relation arithmetic. Whitehead alone was responsible for the section on Convergence and Limits of Functions, and for Part VI, on Quantity. Whitehead also contributed some portions which might have been thought to be more in my province, for instance the " Preparatory Statement of Symbolic Conventions " at the beginning of Vol. II, which is concerned with types and systematic ambiguity. He also wrote the bulk of the first chapter of the Introduction.

Whitehead was to have written a fourth volume, on geometry, which would have been entirely his work. A good deal of this was done, and I hope still exists. But his increasing interest in philosophy led him to think other work more important. He proposed to treat a space as the field of a single triadic, tetradic, or pentadic relation,[1] a treatment to which, he said, he had been led by reading Veblen.

When our work was sufficiently advanced, we parcelled out the topics, each produced a first draft of whatever was in his assignment, the other then went over it and probably made considerable changes, and then his revised draft was finally revised by the first author. In most parts of the book, there was, in the end, very little for which either had *sole* responsibility.

For the ten years from 1900 to 1910, the book took up practically all the available time of both of us. Our collaboration was always completely harmonious. Whitehead was more patient and accurate and careful than I was, and saved me often from a hasty and superficial treatment of difficulties that I found uninteresting. I, on the other hand, sometimes thought his treatment needlessly complicated, and found ways of simplifying his drafts. Neither of us alone could have written the book; even together, and with the alleviation brought by mutual discussion, the effort was so severe that at the end we both turned aside from mathematical logic with a kind of nausea. It was, I suppose, inevitable that we should turn aside in different directions, so that collaboration was no longer possible.

[1] And generally a space of n dimensions at the field of an $(n + 1)$-adic relation.