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PSYCHOLOGY AND PHILOSOPHY.

I.—IS POSITION IN TIME AND SPACE ABSOLUTE OR RELATIVE?

BY BERTRAND RUSSELL.

THE purpose of the present paper is to reopen a discussion which has been generally regarded as definitively closed. We have all been taught to believe that time and space consist wholly of relations, and that moments and points are mathematical fictions. It is this opinion that I wish to challenge. I shall endeavour to state, as precisely as possible, both the absolute and the relative theory in each case; to show that, in the case of time, provided we take any account of facts, it is difficult, if not impossible, to free the relational theory from contradiction; to prove that, in the case of space, the relational theory must be so modified, if it is to be logically permissible, as to lose all the advantages which it claims over the absolute theory; and finally, by an examination of Lotze's arguments against absolute space, to demonstrate the falsity of the logic from which these arguments are derived. The discussion has to be conducted differently in the two cases of space and time. The paper, therefore, will consist of three parts: the first will be concerned with time, the second with space, and the third with Lotze's arguments concerning space. I shall not be concerned directly with the existence of space and time, but only with their logical analysis: that is, I shall not raise the question whether what appears to exist in space and time is mere appearance, but only the question what space and time must be if what appears to occupy them does exist. It should be observed, however, that, if I succeed in constructing a logically permissible account of space and time, the chief reason for regarding their contents as mere appearance is thereby destroyed.

With regard to time, I assume that there is some definite series whose relations are temporal, *i.e.*, some series such that any two of its terms are either simultaneous, or are one before and the other after. This series is one-dimensional, and is also *compact*, *i.e.*, if A be before B, there is some term C which is before B and after A. With regard to such series in general, we may distinguish two theories, the absolute and the relative.[1] In the absolute theory, we have two classes of entities, (1) those which *are* positions, (2) those which *have* positions. Any two terms of the first class have an asymmetrical transitive relation: in the present case, either *before* or *after*. The terms which have positions are terms each of which has, to one or more of the terms which are positions, a certain specific relation, which may be expressed by saying that the new terms are *at* the positions, or that they occupy the positions. By compounding a term which has one or more positions with one of the positions which it has, we obtain a complex term, *i.e.*, the said term at the position; this new complex term contains the position as a constituent, but contains only one such position. In the case of time, two such terms which contain the same moment as constituent are said to be simultaneous; two which contain different moments are *said* to be one before and the other after, by correlation with the moments they contain, but this new relation of before and after is complex, containing as a constituent the before and after of moments. We may call *qualities* the terms which have positions in time; thus a quality may be at many moments, or even at all moments. The compound formed of a quality at a time may be called an *event;* thus an event is logically incapable of recurrence. Two events are simultaneous when both are qualities *at* one time; otherwise they are successive. We have thus three simple relations, *before*, *after* and *at*. Before and after are transitive and asymmetrical, *at* is intransitive and asymmetrical. The relation of an event to its moment, which is derivative, is also intransitive and asymmetrical; moreover it is, unlike *at*, a many-one relation: *i.e.*, though many events may be in a moment, there is only one

[1] See MIND, January, 1901, p. 35.

moment in which a given event can be. This is the absolute theory of time.

In the relational theory we do not require two different classes of entities, but we still require three simple relations. We have here a single class of entities, which may be called events, any two of which have one and only one of three unanalysable relations, simultaneity, priority and posteriority. All three are transitive; the first is symmetrical, while the other two are asymmetrical. Further, if A be simultaneous with B, and B be before C, A is before C; and if B be after C, A is after C. (These two properties are independent.) This is the relational theory.

The relational theory may seem, at first sight, simpler than the absolute theory, but in its application a great difficulty arises from the absence of any such class of entities as the events which it requires. Before developing this difficulty, we must observe that simultaneity must be strictly unanalysable. If two events were simultaneous because they had some common property not shared by events not simultaneous with them, the collection of all such common properties of different groups of simultaneous events would have exactly the characteristics by which we defined absolute time. Now for my part I consider it self-evident that all symmetrical transitive relations are analysable; and if this axiom were admitted, the relational theory would fall at once. But as my opponents will allow no such principle, I shall not appeal to it in what follows.

When we begin the search for events answering to the above definition, we are met by the following difficulty. Whatever can, in ordinary language, recur or persist, is not an event; but it is difficult to find anything logically incapable of recurrence or persistence, except by including temporal position in the definition. When we think of the things that occur in time—pleasure, toothache, sunshine, etc.—we find that all of them persist and recur. In order to find something which does not do so, we shall be forced to render our events more complex. The death of Cæsar or the birth of Christ, it may be said, were unique: they happened once, but will never happen again. Now it is no doubt probable that nothing exactly similar to these events will recur; but, unless the date is included in the event, it is impossible to maintain that there would be a logical contradiction in the occurrence, in the future, of a precisely similar event. Perhaps it may be said that the whole state of the universe has the required uniqueness: we may be told that it is logically impossible for the universe to be twice in the same state. But let us examine this opinion. In the first place, it receives no countenance from science, which, though it admits such recurrence to be improbable, regards it as by no means impossible. In the second place, the present state of the universe is a complex, of which it is admitted that every part may recur. But if every part may recur, it seems to follow that the whole may recur. In the third place, this theory, when developed so as to meet the second objection, becomes really indistinguishable from that of absolute position. There is no longer an unanalysable relation of simultaneity: there are a series of states of the universe, each of which, as a whole and only as a whole, has to each other a simple relation of before and after; an event is any part of a state of the universe, and is simultaneous with any other part of the same state, simultaneity meaning merely the being parts of some one state; before and after do not hold between events directly, but only by correlation. Thus the theory in question, except for the fact that *at* is no longer simple, is merely the absolute theory with states of the whole universe identified with moments. The reasons against such an identification are, first, that events seem to have an order which does not, in its very meaning, involve reference to the whole universe, and secondly, that immediate inspection seems to show that recurrence of the whole state of the universe is not logically absurd.

If the advocate of the relational theory abandons the search for events with a unique temporal position, and confines himself to what, in the absolute theory, we called qualities, he will find it impossible to obtain a time-series at all. In this view, we endeavour to obtain events by means of the mutual relations of qualities, but this endeavour is frustrated by the fact that simultaneity, priority and posteriority are no longer mutually incompatible. It is necessary now to hold that qualities as such have temporal relations. Thus if I feel first pain and then pleasure, we shall be forced to hold that pain as such precedes pleasure as such. We cannot say that this sequence occurs now, but not always, for *now* is constituted by its contents, of which the pain and the pleasure form part. And we have already seen that it is useless to take qualities more complex than pleasure. But we must also admit that pleasure precedes pain, and that pleasure and pain are simultaneous; for these cases also occur. But this destroys the whole time-series, which depends, as all series do, upon the mutual incompatibility of its constitutive relations. The *now* is wholly vague, for among the qualities simultaneous with a given quality, many, if not all, will also be prior or posterior to it; if the quality be one which persists or recurs, all qualities simultaneous with it must also be prior and posterior to it. Thus the whole time-series collapses: the passage from events to qualities cannot be made, and the edifice of temporal propositions crumbles.

The same argument may be put as follows. It must be admitted that there is some class of entities for which simultaneity is transitive. If A, B, C be such entities, and A and B, B and C are simultaneous, so are A and C. But among qualities this does not hold. For qualities may recur, and recurrence can only mean, on the theory in question, simultaneity with qualities not simultaneous with one another. This contradicts our principle, and shows that it is not between qualities as such that temporal relations hold. In short, if terms which are not positions are to be arranged serially by means of three relations, all transitive and two asymmetrical, while the third is symmetrical, then these three relations must be mutually incompatible. This condition is satisfied by events, but not by qualities. It is impossible, however, owing to the fact of recurrence, to pass from qualities to events without assuming moments. We may divest the argument of all temporal reference, and our result is then seen to be at once demonstrable and obvious. It is as follows: When a collection of terms are capable of serial arrangement, but some among them occur in several positions in the series, then the terms in question form a series which is not independent, but is obtained by some many-many or one-many relation which each of them has to one or more terms of some independent series.

In regard to space, no such simple argument is possible. For this there are three chief reasons. (1) There are, apparently, no existents in space which are not also in time, whereas mental qualities are in time, but are often held to be not in space. (2) It cannot be taken as a self-evident premiss that two things or qualities may have a symmetrical transitive relation such as would be called occupation of the same place, whereas it is evident that two events may be simultaneous. (3) Common sense recognises no entities having a fixed position in space, as events have in time: there is in space nothing corresponding to dates, but all things in space are held to be perpetually in motion. To these must be added, as a minor cause of difficulty, the greater complexity introduced by dimensions. For these reasons, a direct proof of absolute position, from premisses universally admitted, will not be found in what follows, though it is possible that such a proof may be discoverable. Nevertheless, I shall give grounds for thinking that the relational theory leads to paradoxical results, and that it needs emendations which rob it of the merits that have endeared it to philosophers. When this has been done, I shall endeavour to refute the current refutations of the absolute theory.

The first requisite—though it is one which is universally neglected—is to set forth precisely the two theories which are to be compared. And since it is in the detailed development that the relational theory mainly shows its weakness, it seems necessary to give a fairly full statement of what space is on the two theories. For this purpose, I shall assume that space is Euclidean or hyperbolic—an assumption which does not affect the present argument.

In the absolute theory, a space is a class of entities, called *points*, and having relations of a certain type. The type is somewhat complicated, but may be set forth as follows: Any two distinct points have one and only one asymmetrical transitive relation of a certain class; the converse relation also belongs to the class, and no relation of the class holds between anything except points. If a and b, c and d, be pairs of points having the same relation of the class in question, then a and c, b and d also have this relation or its converse. All the points forming what may be called the extension of a given relation of the class are a series which is endless and compact (*i.e.*, has a term between any two). Such a series is called a *straight line*. It follows from what has been said that the converse relation defines the same straight line, and that two lines cannot have more than one point in common. There are points not on a given straight line. If ab be a given straight line, c a point not on ab, e a point on ab, and d a point on ce such that e is between c and d, then the class of points x, such that either x lies on ab, or cx meets ab in a point between c and x, or dx meets ab in a point between d and x, is called the *plane abc*. If a, b, c be any three non-collinear points, and g be a point between b and c, e a point between a and g, then there is a point f which is between a and c, and between which and b the point e is. Also if f is given between a and c, there is a point e between a and g and between b and f, which is the converse of the former property. Hence it follows that a plane is determined by any three of its points, and contains the line joining any two of its points. But given any plane, there will be points outside the plane. Proceeding to define a three-dimensional figure as the plane was obtained from the straight line and the point, this new figure will contain all points.[1] It remains to add the metrical

[1] The above is obtained from Peano, *Sui Fondamenti della Geometria*, *Rivista di Matematica*, iv., p. 51 ff.

properties of our space. The stretch of points between two given points has a certain magnitude; from any given point, and in either direction along any given line through the point, there is one and only one stretch equal to any given stretch; if $a_0\, a_1,\, a_1\, a_2,\, \ldots\, a_{n-1}\, a_n$ be n equal stretches along the same straight line, and d be any given stretch, then, for some finite value of n, $a_0\, a_n$ will be greater than d; if a_0, a_n be any two points, and n any finite integer, there are n-1 distinct points $a_1, a_2, \ldots a_{n-1}$ on the line $a_0\, a_n$, such that the stretches $a_0\, a_1,\, a_1\, a_2,\, \ldots\, a_{n-1}\, a_n$ are all equal. In virtue of our previous assumptions, the defining relations of the lines in a plane through a point form a closed series; a stretch of such relations is called an angle; three assumptions similar to the three concerning stretches of points are required for angles. A few more axioms are required for triangles, areas and volumes; but these need not be here enumerated.[1]

The absolute theory of space asserts that there are entities (points) which answer to the above description, and that the spatial relations in the actual world hold primarily between points, and only by correlation between the contents of space. It may be observed that, complicated as the above account is, it introduces only two indefinables, namely the class *point* and the class of relations by which straight lines are defined. Magnitude of divisibility may be regarded as a third indefinable introduced by metrical geometry, but this is a general logical notion which has no special reference to space.

A detailed statement of the relational theory, comparable to the above statement of the absolute theory, has never been attempted. Let us first endeavour to characterise generally the points in which the relational theory differs from the above, and then, if possible, to state this theory with all the requisite detail.

There are two main differences of the two theories. First, the relational theory holds that there are no entities which, as such and timelessly, have certain spatial relations, but that all spatial relations involve reference to some time at which they hold between some given pair of entities. Even if there be a pair of entities whose spatial relations are unchanging, we can only say that they happen to have these relations at all times, not that they have them timelessly. Or, if we allow that two entities may have a given spatial relation eternally, we shall have to allow also that, as an eternal relation, the relation in question is not incompatible with another which becomes incompatible with it when the time

[1] See Pasch, *Neuere Geometrie*, Leipzig, 1882, § 13.

is taken into account. That is, if we allow that two material points may be eternally a yard apart, we must allow that they may also be eternally two yards apart, and that, from a given eternal relation, nothing can be inferred as to the relation at any given time. Secondly, the relational theory endeavours to dispense with all such entities and relations as are completely indistinguishable and impossible for us to identify. Thus distances, angles, areas, and volumes are more or less sensible, and are (within limits) capable of recognition. But individual points, straight lines, and planes are wholly impossible to recognise. It is not theoretically possible to discover, by immediate inspection, whether the part of space which I see now is the same as or different from the part which I saw at this time yesterday. The relational theory aims at dispensing with all this apparatus of indistinguishable and unrecognisable entities. This second point forms the motive of the theory, while the first seems to constitute its logical definition. I shall endeavour to show that the facts render it impossible wholly to satisfy the motive, and that a relational space according to the definition must still retain material points which, though capable of motion, are not assumed to exist, which will be as unrecognisable as the points of the absolute theory.

In the relational theory, space consists solely of relations; the terms of these relations are therefore non-spatial. Considered merely as terms of spatial relations, they may be called material points. The spatial relations of material points are to be, if possible, all comprised under distance, rectilinear angle, angle between planes, solid angle, area and volume, right and left. Advocates of the relational theory never (so far as I know) mention any relation except distance, but it is to be supposed that they would mention the others if they condescended to details. Any pair of material points has, at a given instant, one and only one distance; any three have three and only three angles, which are, like *between*, relations of one term to two others. Three particles also have one and only one area at a given time. Four particles have one and only one volume, four and only four solid angles, and six and only six angles between planes. All these relations are quantities, and all fulfil the three axioms (or their analogues) mentioned, as regards stretches, in the statement of the absolute theory. Finally, if four material points be such that the four solid angles which they determine are not all zero, then any two of the four have to the remaining two one or other of two symmetrical relations, which may be called right- and left-handedness: these are such that, if one of them holds between *ab* and *cd*, the other holds between *ab* and *dc*; further, in respect to these relations, motions are not continuous, *i.e.*, if *a*, *b*, *c*, be fixed, and *d* movable, *ab*, *cd* may suddenly cease to have either of the two relations, and may, by a further motion of *d*, however small, acquire the opposite relation to that which they had before.[1]

The above outline must now be filled in in detail. Let us begin with distances. In the first place, we may inquire whether distances are symmetrical or asymmetrical. In this inquiry, we must remember that we are not yet in possession of the straight line, so that a given distance cannot be specified as being in a given direction. Thus there may be any number[2] of particles whose distance from a given particle at a given instant is a given distance: this must, in fact, be set up as an axiom. But we cannot infer the distance of two particles from their distances from a third particle. A distance is neither a transitive nor an intransitive relation: if *a* and *b*, *b* and *c* both have a given distance, this may or may not be the distance of *a* and *c*. It seems then that, so long as the straight line has not been obtained, distances must be taken as symmetrical, *i.e.*, the distance of *a* from *b* is the same as that of *b* from *a*. Distances, moreover, must be a kind of magnitudes forming an endless compact series. If the zero of distance be included, distances are a series which has one end; but it must be set up as an axiom that the distance of two distinct material particles is never zero.

The next point to be considered is the nature of angles. Angles are not relations between distances, since they may change when distances remain constant, and remain constant when distances change. They might, it is true, be taken as relations of *three* distances, but it would still be necessary to specify the distances as forming a triangle, which imposes upon our distances limitations of an unknown nature. For until we have the straight line, we cannot add distances, and therefore we cannot say that, in a triangle, two distances together must be greater than the third. Thus it seems best to regard angle as a new fundamental relation of three particles. Between three particles *a*, *b*, *c*, there are to be always three angles, which may be regarded as relations of *a* to *b* and *c*, *b* to *c* and *a*, *c* to *a* and *b* respectively. These relations are to be symmetrical as regards the extremes, *i.e*, the angle which is a relation of *a* to *b* and *c* may be regarded

[1] The reader who is not interested in semi-mathematical details may omit the next three paragraphs with advantage.

[2] "Any number" means here any cardinal number not exceeding that of the continuum.

as a relation of *a* to *c* and *b*, and denoted by *bac* or *cab* indifferently. But the relation of the mean to the extreme is to be asymmetrical, *i.e.*, we must not interchange *a* and *b* or *a* and *c* in the above case. All angles are to be magnitudes, and the series of these magnitudes is to be compact, but unlike distances, it is to have two ends. One of these ends is the zero of angle, which, unlike the zero of distance, may actually hold between distinct particles. This case is specially important, and several axioms are required for dealing with it. If *a*, *b*, *c*, etc., be all mutually distinct, it may happen that the angles *bac* and *bca* are both zero. In this case the angle *abc* will be the maximum of angle, which will be called two right angles. If *d* and *e* be points such that *bdc* and *bea* are both zero, then *dbc* and *eba* are both two right angles; also *bda* or *bad* will be zero, and *bad* or *bda* will be two right angles; with similar remarks as regards *e*. Formally, we may set up the following definitions and axioms.[1] If the angle *abc* is two right angles, we say that *b* is between *a* and *c*. The relation of *between* has the following properties. Neither *a* nor *c* is between *a* and *c*, but so long as *a* and *c* are distinct, there always may be material points between them, and there always may be material points between which and *b* lies the material point *a*. The points *a* and *b*, the points (if any) between them, the points *x* (if any) such that *a* lies between *x* and *b*, and the points *y* (if any) such that *b* lies between *y* and *a*, are said to lie on the *straight line ab*, and no other points are said to lie on this straight line. If *a*, *d* be material points, *c* a material point between them, and *b* a material point between *a* and *c*, then *b* is between *a* and *d*. If *b* and *c* be between *a* and *d*, then either *b* is between *a* and *c*, or *b* is identical with *c*, or *b* is between *c* and *d*. If *b* lies between *a* and *c*, and also between *a* and *d*, then either *c* and *d* coincide, or *c* lies between *b* and *d*, or *d* lies between *b* and *c*. If *b* lies between *a* and *c*, and *c* between *b* and *d*, then *c* lies between *a* and *d*. These properties are axiomatic, and mutually independent. From all of them together, but not from any selection of them, it results that the material points on one straight line, however numerous they may be, provided there are more than two, form a series. In virtue of our axioms concerning the connexion of maximum and minimum angles, we may define the straight line, in place of the former definition, as follows: The point *c* is collinear with *a* and *b* when the angle *acb* is either zero or two right angles. A further axiom is needed,

[1] *Cf.* Peano, *Rivista di Matematica*, iv., p. 55.

namely: if c be between a and b, the stretches ac, cb are both less than ab.

The above properties are sufficient to characterise maximum and minimum angles. But new axioms are required for dealing with other angles. To begin with, if a, b, c be any three non-collinear points, and d a point between a and c, the angle dab is the same as the angle cab. Next, if b, c, e be collinear, and c be between b and e, and a be a point not on the line bc, then the angle bae is greater than either of the angles bac, cae. By means of these axioms, we are able to arrange serially all such lines through a as contain material points on some line not through a, and to show that, if the same lines can be arranged serially by reference to some other line not through a, the order will be unchanged. But we cannot show that *all* lines through a and in one plane form a series, for some triads of them may not have material points on any one given line.

The development of the theory of planes, angles between planes, and solid angles would be extremely complicated, and I shall not undertake it. I shall content myself with a few remarks on areas and volumes. Three material points have at any instant a certain relation called their *area*. This relation is a magnitude, which, in Euclidean space, has no maximum, but which always has a zero. The relation of the kind in question between three collinear points is a zero area. Area is symmetrical with respect to the three terms of the relation. If a, b, c be three non-collinear points, and d a point between b and c, the area abc is defined to be the sum of abd and acd. Similarly if a', b', c', d' be four points in one plane, no three of which are collinear, and there is a material point e' between a' and c' and also between b' and d'; if, further, $a'b'c'$ be equal to our former area abd, and $a'd'c$ to our former area acd, then it is said that a', b', c', d' determine an area equal to abc. But if it should happen that there is no such material point as e', and if neither the stretches $a'b'$, $c'd'$ nor the stretches $a'd'$, $b'c'$ have a common material point e'; or if, though e' exists, there are not, and never have been, and never will be, four material points a, b, c, d satisfying the above conditions; then we cannot say that a', b', c', d', determine an area at all. Moreover we shall need an axiom to assure us that the area $a'b'c'd'$ so defined is the same as the area obtained by summing $b'a'd'$ and $b'c'd'$. Similar remarks apply to polygons of more than four sides. As regards circles, or other curvilinear figures, they can have no area at all; for their area could only be the limit of a sum of a continually increasing number of continually diminishing triangles, and the conditions for the method of limits are here not fulfilled, since nothing can be *merely* a limit. As regards volumes, we must make precisely similar remarks.

I have now done my best to state precisely some of the more important parts of the relational theory. It will be admitted that its complexity is bewildering. But what is more important is, that it compels us either to abandon many of the usual propositions of geometry, or to allow the notion of *possible* material points. I have hitherto excluded this notion, and I shall continue to do so until the consequences of such exclusion have been examined. We have just seen that only triangles always have areas, and only tetrahedra always have volumes. This is one result which flagrantly contradicts both geometry and common sense. To maintain that a sphere has no volume and a circle no area is surely to condemn the theory from which such consequences follow. Another similar difficulty arises concerning the intersection of straight lines. If two lines *ab*, *cd* intersect, that must mean, on the relational theory, that there is now a material point *e* collinear both with *a* and *b* and with *c* and *d*. But geometrically, the question whether there is an intersection depends only upon *a*, *b*, *c*, and *d*, and not upon the empirical existence of some fifth point *e*. Thus it is an axiom which is essential to geometry that, if *a*, *b*, *c*, *d* be four points in a plane, either *ab*, *cd*, or *ac*, *bd*, or *ad*, *bc* intersect. Without this axiom, Euclid's seventh proposition cannot be proved. (Euclid's pretended proof, it may be observed, is worthless.) But when an intersection is taken to be a material point, it is plain that no such axiom can be set up. Hence we shall have to take the seventh proposition as an axiom. And the distinction, which must be recognised, between lines which intersect and lines which do not intersect, becomes impossible to state. The same thing appears in regard to points on a line. There is certainly some sense in which it is always true that there *may* be a material point between any two; but on our present theory it is hard to see what this sense can be. The serial arrangement of the lines in a plane through a point, or of the planes through a line, also fails to be general when we confine ourselves to actual material points. These and many analogous respects, in which the relational theory fails, could not be detected formerly, because the axioms of geometry had not been carefully scrutinised; for example, it is only very recently that the order of points, lines, or planes, has been seen to require special axioms. In some sense, it is plain, space is a plenum; that is, there are entities which have any spatial relation that is possible. But we cannot make these entities actual material points, since, even if there are actual material points everywhere, this is a purely empirical fact, which cannot be invoked as an escape from a logical difficulty.

It remains to examine the notion of possible material points. Since, in the relational theory, all spatial relations hold, not eternally, but at times, the possible material points, if they are to be geometrically useful, must be in time, and they must have changing spatial relations to actual material points. The actual points, in fact, must be merely those among the possible ones which happen to exist. It must be set up as an axiom that, if *a*, *b* be any two possible or actual material points, there is a possible material point between them; and so on. But now we must ask ourselves what reason we have for assuming such possible points at all. We have now a set of spatial relations, all of which always hold between entities, actual or possible, and whose change from time to time consists solely in the fact that it is between different entities that they hold. Given a collection of geometrical relations all belonging at one time to one particle, this collection will belong at every time to some particle. Moreover the collection of geometrical relations belonging to one particle at one time is exactly identical with the collection belonging to the same or any other particle at the same or any other time; for all possible distances, angles, etc., are at all times relations of any given particle to other particles. Motion consists merely in the fact that the same relations which *a* has at one time to (say) *b*, *c*, *d*, etc., it has at another ~~time~~ to (say) *c*, *d*, *b*, etc. That is, the same relations are, as it were, differently shuffled; and this constitutes all that motion can mean. In this notion there is, so far as I can see, nothing self-contradictory, provided it be admitted that a non-existent may be in time and have changing relations. But it must be admitted that the introduction of non-existent material points as an essential part of geometry destroys the plausibility of the relational theory, and renders it no longer able to content itself with data that are in some degree sensible. It seems, at this stage, merely a natural simplification to suppose that there are spatial points, between which primarily spatial relations hold, and *at* some or all of which there are at any given time material points. And then all spatial relations except right and left become definable in terms of *at* and straight lines.

We have now seen that the relational theory, if it confines itself to actual material points, is unable, since the question as to the geometrical distribution of such points must be empirical, to give an account, which will consist with the facts, of the intersection of geometrical figures, the order of lines and planes, and the nature of areas and volumes. If it invokes other material points, not assumed to exist, but regarded as entities related to time and to existent material points as these are related to time and to each other, it becomes logically admissible, but loses whatever advantage it may possess over the absolute theory. And however we develop it, the complications which it introduces are so great that the mere enumeration of axioms becomes a herculean task. When we come to motion, the additional complications become infinitely greater, since the motion of one particle can only be specified by the changes in its relations to *all* other particles; whereas, in the absolute theory, the motion is specified by the mention of the points at which the particle is at the two times. And there is a special argument, derived from dynamics, to show that the straight line must be an independent relation, not, as in the relational theory, derivative from the collinearity of three particles. This is the argument from what is called absolute rotation, which I have omitted because it demands a long discussion of the laws of motion. But this argument alone, in my opinion, suffices, in the absence of logical grounds on either side, to turn the scale in favour of absolute position.

I come now to the third section of my paper, in which I shall endeavour to answer the current arguments against absolute position. If it can be shown that there are no *a priori* arguments on either side, simplicity and the dynamical argument should decide in favour of the absolute theory. The arguments against this theory are, in my opinion, one and all fallacious. They are best collected in Lotze's *Metaphysic* (§ 108 ff). They are there confused with arguments for the subjectivity of space—an entirely distinct question, as should have been evident from the fact that Kant, in the *Critique*, advocated the theory of absolute position.[1] Omitting arguments only bearing on this latter point, we have the following summary of Lotze's arguments against absolute space.

(1) Relations are only either (α) as presentations in a relating consciousness, or (β) as internal states in the real elements which are said to stand in these relations (§ 109).

(2) The being of empty space is neither the being which works effects (which belongs to a thing), nor the mere validity of a truth, nor the fact of being presented by us. What kind of being is it then? (§ 109).

[1] *Cf.* Vaihinger, *Commentar*, pp. 189-190.

(3) All points are exactly alike, yet every pair have a relation peculiar to themselves; but being exactly like every other pair, the relation should be the same for all pairs (§ 111).

(4) The being of every point must consist in the fact that it distinguishes itself from every other, and takes up an invariable position relatively to every other. Hence the being of space consists in an active mutual conditioning of its various points, which is really an interaction (§ 110).

(5) If the relations of points were a mere fact, they could be altered, at least in thought; but this is impossible: we cannot move points or imagine holes in space. This impossibility is easily explained by a subjective theory (§ 110).

(6) If there are real points, either (α) one point creates others in appropriate relations to itself, or (β) it brings already existing points into appropriate relations, which are indifferent to their natures (§ 111).

(1) All these arguments depend, at bottom, upon the first, the dogma concerning relations. As it is of the essence of the absolute theory to deny this dogma, I shall begin by examining it at some length.[1] "All relations," Lotze tells us, "only *are* as presentations in a relating consciousness, or as internal states in the real elements which, as we are wont to say, stand in these relations." This dogma Lotze regards as self-evident, as indeed he well may; for there is not one anterior philosopher, unless it be Plato, who does not employ the dogma as an essential part of his system. To deny it, therefore, is a somewhat hardy undertaking. Let us, nevertheless, examine the consequences to which the dogma leads us.

It would seem that, if we accept the dogma, we must distinguish two kinds of relations, (α) those which are presentations in a relating consciousness, and (β) those which are internal states of the elements supposed to be related. These may be ultimately identical, but it will be safer in the mean time to treat them as different. Let us begin with those which are only presentations in a relating consciousness. These presentations, we must suppose, are beliefs in propositions asserting relations between the terms which appear related. For it must be allowed that there are beliefs in such propositions, and only such beliefs seem capable of being regarded as presentations in which relations have their being. But these beliefs, if the relations believed

[1] The logical opinions which follow are in the main due to Mr. G. E. Moore, to whom I owe also my first perception of the difficulties in the relational theory of space and time.

to hold have no being except in the beliefs themselves, are necessarily false. If I believe A to be B's father, when this is not the case, my belief is erroneous; and if I believe A to be west of B, when westerliness in fact exists only in my mind, I am again mistaken. Thus this first class of relations has no validity whatever, and consists merely in a collection of mistaken beliefs. The objects concerning which the beliefs are entertained are as a matter of fact wholly unrelated; indeed there cannot even be *objects*, for the plural implies diversity, and all beliefs in the relation of diversity must be erroneous. There cannot even be one object distinct from myself, since this would have to have the relation of diversity to me, which is impossible. Thus we are committed, so far as this class of relations goes, to a rigid monism.

But now, what shall we say of the second class of relations, those namely which are reducible to internal states of the apparently related objects? It must be observed that this class of relations presupposes a plurality of objects (two at least), and hence involves the relation of diversity. Now we have seen that, if there be diversity, it cannot be a relation of the first class; hence it must itself be of the second class. That is, the mere fact that A is different from B must be reducible to internal states of A and B. But is it not evident that, before we can distinguish the internal states of A from those of B, we must first distinguish A from B? *i.e.*, A and B must *be* different, before they can have different states. If it be said that A and B are precisely similar, and are yet two, it follows even more evidently that their diversity is not due to difference of internal states, but is prior to it. Thus the mere admission that there are internal states of different things destroys the theory that the essence of relations is to be found in these states. We are thus brought back to the notion that the apparent relations of two things consist in the internal states of one thing, which leads us again to the rigid monism implied in the first type of relation.

Thus the theory of relations propounded by Lotze is, in fact, a theory that there are no relations. This has been recognised by the most logical adherents of the dogma—*e.g.*, Spinoza and Mr. Bradley—who have asserted that there is only one thing, God or the Absolute, and only one type of proposition, namely that ascribing predicates to the Absolute. In order to meet this development of the above theory of relations, it will be necessary to examine the doctrine of subject and predicate.

Every proposition, true or false—so the present theory contends—ascribes a predicate to a subject, and—what is a corollary from the above—there is only one subject. The consequences of this doctrine are so strange, that I cannot believe they have been realised by those who maintain it. The theory is in fact self-contradictory. For if the Absolute has predicates, then there are predicates; but the proposition "there are predicates" is not one which the present theory can admit. We cannot escape by saying that the predicates merely qualify the Absolute; for the Absolute cannot be qualified by nothing, so that the proposition "there are predicates" is logically prior to the proposition "the Absolute has predicates". Thus the theory itself demands, as its logical *prius*, a proposition without a subject and a predicate; moreover this proposition involves diversity, for even if there be only one predicate, this must be different from the one subject. Again, since there is a predicate, the predicate is an entity, and its predicability of the Absolute is a relation between it and the Absolute. Thus the very proposition which was to be non-relational turns out to be, after all, relational, and to express a relation which current philosophical language would describe as purely external. For both subject and predicate are simply what they are—neither is modified by its relation to the other. To be modified by the relation could only be to have some other predicate, and hence we should be led into an endless regress. In short, no relation ever modifies either of its terms. For if it holds between A and B, then it is between A and B that it holds, and to say that it modifies A and B is to say that it really holds between different terms C and D. To say that two terms which are related would be different if they were not related, is to say something perfectly barren; for if they were different, they would be other, and it would not be the terms in question, but a different pair, that would be unrelated. The notion that a term can be modified arises from neglect to observe the eternal self-identity of all logical concepts or Platonic ideas, which alone form the constituents of propositions.[1] What is called modification consists merely in having at one time, but not at another, some specific relation to some other specific term; but the term which sometimes has and sometimes has not the relation in question must be unchanged, otherwise it would not be *that* term which had ceased to have the relation.

The general objection to Lotze's theory of relations may

[1] See Mr. G. E. Moore's paper on "The Nature of Judgment," MIND, N.S., vol. viii.

be thus summed up. The theory implies that all propositions consist in the ascription of a predicate to a subject, and that this ascription is not a relation. The objection is, that the predicate is either something or nothing. If nothing, it cannot be predicated, and the pretended proposition collapses. If something, predication expresses a relation, and a relation of the very kind which the theory was designed to avoid. Thus in either case the theory stands condemned, and there is no reason for regarding relations as all reducible to the subject-predicate form.

(2) I come now to the second of Lotze's objections to empty space. This is again of a somewhat abstract logical character, but it is far easier to dispose of, since it depends upon a view more or less peculiar to Lotze. There are, it says, three and only three kinds of being, no one of which belongs to space. These are (α) the being of things, which consists in activity or the power to produce effects; (β) the validity of a truth; (γ) the being which belongs to the contents of our presentations.

The answer to this is, that there is only one kind of being, namely being *simpliciter*, and only one kind of existence, namely existence *simpliciter*. Both being and existence, I believe, belong to empty space; but being alone is relevant to the refutation of the relational theory—existence belongs to the question which Lotze confounds with the above, namely as to the reality or subjectivity of space. It may be well first to explain the distinction of being and existence, and then to return to Lotze's three kinds of being.

Being is that which belongs to every conceivable term, to every possible object of thought—in short to everything that can possibly occur in any proposition, true or false, and to all such propositions themselves. Being belongs to whatever can be counted. If A be any term that can be counted as one, it is plain that A is something, and therefore that A is. "A is not" must always be either false or meaningless. For if A were nothing, it could not be said not to be; "A is not" implies that there is a term A whose being is denied, and hence that A is. Thus unless "A is not" be an empty sound, it must be false—whatever A may be, it certainly is. Numbers, the Homeric gods, relations, chimeras and four-dimensional spaces all have being, for if they were not entities of a kind, we could make no propositions about them. Thus being is a general attribute of everything, and to mention anything is to show that it is.

Existence, on the contrary, is the prerogative of some only amongst beings. To exist is to have a specific relation to existence—a relation, by the way, which existence itself does not have. This shows, incidentally, the weakness of the existential theory of judgment—the theory, that is, that every proposition is concerned with something that exists. For if this theory were true, it would still be true that existence itself is an entity, and it must be admitted that existence does not exist. Thus the consideration of existence itself leads to non-existential propositions, and so contradicts the theory. The theory seems, in fact, to have arisen from neglect of the distinction between existence and being. Yet this distinction is essential, if we are ever to deny the existence of anything. For what does not exist must be something, or it would be meaningless to deny its existence; and hence we need the concept of being, as that which belongs even to the non-existent.

Returning now to Lotze's three kinds of being, it is sufficiently evident that his views involve hopeless confusions.

(α) The being of things, Lotze thinks—following Leibniz here as elsewhere—consists in activity. Now activity is a highly complex notion, which Lotze falsely supposed unanalysable. But at any rate it is plain that, if there be activity, what is active must both be and exist, in the senses explained above. It will also be conceded, I imagine, that existence is conceptually distinguishable from activity. Activity may be a universal mark of what exists, but can hardly be synonymous with existence. Hence Lotze requires the highly disputable proposition that whatever exists must be active. The true answer to this proposition lies (1) in disproving the grounds alleged in its favour, (2) in proving that activity implies the existence of time, which cannot be itself active. For the moment, however, it may suffice to point out that, since existence and activity are logically separable, the supposition that something which is not active exists cannot be logically absurd.

(β) The validity of a truth—which is Lotze's second kind of being—is in reality no kind of being at all. The phrase, in the first place, is ill-chosen—what is meant is the truth of a truth, or rather the truth of a proposition. Now the truth of a proposition consists in a certain relation to truth, and presupposes the being of the proposition. And as regards being, false propositions are on exactly the same level, since to be false a proposition must already be. Thus validity is not a kind of being, but being belongs to valid and invalid propositions alike.

(γ) The being which belongs to the contents of our presentations is a subject upon which there exists everywhere the greatest confusion. This kind is described by Lotze as "*ein Vorgestelltwerden durch uns*". Lotze presumably holds that the mind is in some sense creative—that what it intuits acquires, in some sense, an existence which it would not have if it were not intuited. Some such theory is essential to every form of Kantianism—to the belief, that is, that propositions which are believed solely because the mind is so made that we cannot but believe them may yet be true in virtue of our belief. But the whole theory rests, if I am not mistaken, upon neglect of the fundamental distinction between an idea and its object. Misled by neglect of being, people have supposed that what does not exist is nothing. Seeing that numbers, relations, and many other objects of thought, do not exist outside the mind, they have supposed that the thoughts in which we think of these entities actually create their own objects. Every one except a philosopher can see the difference between a post and my idea of a post, but few see the difference between the number 2 and my idea of the number 2. Yet the distinction is as necessary in one case as in the other. The argument that 2 is mental requires that 2 should be essentially an existent. But in that case, it would be particular, and it would be impossible for 2 to be in two minds, or in one mind at two times. Thus 2 must be in any case an entity, which will have being even if it is in no mind.[1] But further, there are reasons for denying that 2 is created by the thought which thinks it. For, in this case, there could never be two thoughts until some one thought so; hence what the person so thinking supposed to be two thoughts would not have been two, and the opinion, when it did arise, would be erroneous. And applying the same doctrine to 1, there cannot be one thought until some one thinks so. Hence Adam's first thought must have been concerned with the number 1; for not a single thought could precede this thought. In short, all knowledge must be recognition, on pain of being mere delusion; Arithmetic must be discovered in just the same sense in which Columbus discovered the West Indies, and we no more create numbers than he created the Indians. The number 2 is not purely mental, but is an entity which may be thought *of*. Whatever can be thought of has being, and its being is a precondition, not a result, of its being thought of. As regards the existence of an object of thought, however, nothing can be inferred from the fact of its being thought of, since it certainly does not exist in the thought which

[1] *Cf.* Frege, *Grundgesetze der Arithmetik*, p. xviii.

thinks of it. Hence, finally, no special kind of being belongs to the objects of our presentations as such. With this conclusion, Lotze's second argument is disposed of.

(3) Lotze's third argument has been a great favourite, ever since Leibniz introduced it. All points, we are told, are exactly alike, and therefore any two must have the same mutual relation as any other two; yet their mutual distances must differ, and even, according to Lotze (though in this he is mistaken), the relation of every pair must be peculiar to that pair. This argument will be found to depend again upon the subject-predicate logic which we have already examined. To be exactly alike can only mean—as in Leibniz's Identity of Indiscernibles—not to have different predicates. But when once it is recognised that there is no essential distinction between subjects and predicates, it is seen that any two simple terms simply differ immediately—they are two, and this is the sum-total of their differences. Complex terms, it is true, have differences which can be revealed by analysis. The constituents of the one may be A, B, C, D, while those of the other are A, E, F, G. But the differences of B, C, D from E, F, G are still immediate differences, and immediate differences must be the source of all mediate differences. Indeed it is a sheer logical error to suppose that, if there were an ultimate distinction between subjects and predicates, subjects could be distinguished by differences of predicates. For before two subjects can differ as to predicates, they must already be two; and thus the immediate diversity is prior to that obtained from diversity of predicates. Again two terms cannot be distinguished in the first instance by difference of relation to other terms; for difference of relation presupposes two distinct terms, and cannot therefore be the ground of their distinctness. Thus if there is to be any diversity at all, there must be immediate diversity, and this kind belongs to points.

Again points have also the subsequent kind of diversity consisting in difference of relation. They differ not only, as Lotze urges, in their relations to each other, but also in their relations to the objects in them. Thus they seem to be in the same position as colours, sounds, or smells. Two colours, or two simple smells, have no intrinsic difference save immediate diversity, but have, like points, different relations to other terms.

Wherein, then, lies the plausibility of the notion that all points are exactly alike? This notion is, I believe, a psychological illusion, due to the fact that we cannot remember a point, so as to know it when we meet it again. Among simultaneously presented points it is easy to distinguish; but though we are perpetually moving, and thus being brought among new points, we are quite unable to detect this fact by our senses, and we recognise places only by the objects they contain. But this seems to be a mere blindness on our parts—there is no difficulty, so far as I can see, in supposing an immediate difference between points, as between colours, but a difference which our senses are not constructed to be aware of. Let us take an analogy: Suppose a man with a very bad memory for faces: he would be able to know, at any moment, whether he saw one face or many, but he would not be aware whether he had ever seen any of the faces before. Thus he might be led to define people by the rooms in which he saw them, and to suppose it self-contradictory that new people should come to his lectures, or old people cease to do so. In the latter point, at least, it will be admitted by lecturers that he would be mistaken. And as with faces, so with points—inability to recognise them must be attributed, not to the absence of individuality, but merely to our incapacity.

(4) Lotze's fourth argument is an endeavour to effect a *reductio ad absurdum*, by proving that, on the absolute theory, points must interact. The being of every point, Lotze contends, must consist in the fact that it distinguishes itself from every other, and takes up an invariable position relatively to every other. Many fallacies are contained in this argument. In the first place, there is what may be called the ratiocinator's fallacy, which consists in supposing that everything has to be explained by showing that it is something else. Thus the being of a point, for Lotze, must be found in its difference from other points, while, as a matter of fact, its being is simply its being. So far from being explained by something else, the being of a point is presupposed in all other propositions about it, as, *e.g.*, in the proposition that the point differs from other points. Again, the phrase that the point distinguishes *itself* from all other points seems to be designed to imply some kind of self-assertion, as though the point would not be different unless it chose to differ. This suggestion helps out the conclusion, that the relations between points are in reality a form of interaction. Lotze, believing as he does that activity is essential to existence, is unable to imagine any other relation between existents than that of interaction. How hopelessly inapplicable such a view is, will appear from an analysis of interaction. Interaction is an enormously complex notion, presupposing a host of other relations, and involving, in its usual form, the distinction of

a thing from its qualities—a distinction dependent on the subject-predicate logic already criticised. Interaction, to begin with, is either the simultaneous action of A on B and B on A, or the action of the present states of A and B conjointly on their states at the next instant. In either case it implies action. Action generally may be defined as a causal relation between one or more states of one or more things at the present instant and one or more states of the same or different things at the next instant. When there is only one thing in both cases, the action is immanent if the thing be the same in cause and effect, transient if the cause be in one thing and the effect in another. In order to speak of action, rather than causality simply, it is necessary to suppose things enduring for a certain time, and having changing states. Thus the notion of interaction presupposes the following relations: (1) diversity between things; (2) diversity between the states of things; (3) simultaneity; (4) succession; (5) causality; (6) the relation of a thing to its states. This notion, involving, as a moment's inspection shows, six simpler relations in its analysis, is supposed to be *the* fundamental relation! No wonder absurdities are produced by such a supposition. But the absurdities belong to Lotze, not to space. To reduce the relations of points to interactions, on the ground that interaction is the type of all relations, is to display a complete incapacity in the simplest problems of analysis. The relations of points are not interactions, any more than before and after, or diversity, or greater and less, are interactions. They are eternal relations of entities, like the relation of 1 to 2 or of interaction itself to causality. Points do not *assign* positions to each other, as though they were each other's pew-openers: they eternally have the relations which they have, just like all other entities. The whole argument, indeed, rests upon an absurd dogma, supported by a false and scholastic logic.

(5) The fifth argument seems to be designed to prove the Kantian apriority of space. There are, it says, necessary propositions concerning space, which show that the nature of space is not a "mere fact". We are intended to infer that space is an *a priori* intuition, and a psychological reason is given why we cannot imagine holes in space. The impossibility of holes is apparently what is called a necessity of thought. This argument again involves much purely logical discussion. Concerning necessities of thought, the Kantian theory seems to lead to the curious result that whatever we cannot help believing must be false. What we cannot help believing, in this case, is something as to

the nature of space, not as to the nature of our minds. The explanation offered is, that there is no space outside our minds; whence it is to be inferred that our unavoidable beliefs about space are all mistaken. Moreover we only push one stage farther back the region of "mere fact," for the constitution of our minds remains still a mere fact.

The theory of necessity urged by Kant, and adopted here by Lotze, appears radically vicious. Everything is in a sense a mere fact. A proposition is said to be proved when it is deduced from premisses; but the premisses, ultimately, and the rule of inference, have to be simply assumed. Thus any ultimate premiss is, in a certain sense, a mere fact. On the other hand, there seems to be no true proposition of which there is any sense in saying that it might have been false. One might as well say that redness might have been a taste and not a colour. What is true, is true; what is false, is false; and concerning fundamentals, there is nothing more to be said. The only logical meaning of necessity seems to be derived from implication. A proposition is more or less necessary according as the class of proposition implying it is greater or smaller.[1] In this sense the propositions of logic have the greatest necessity, and those of geometry have a high degree of necessity. But this sense of necessity yields no valid argument from our inability to imagine holes in space to the conclusion that there cannot really be any space at all except in our imaginations.

(6) The last argument may be shortly disposed of. If points be independent entities, Lotze argues—so I interpret him—that we can imagine a new point coming into existence. This point, then, must have the appropriate relations to other points. Either it creates the other points with the relations, or it merely creates the relations to already existing points. Now it must be allowed that, if there be real points, it is not self-contradictory to suppose some of them non-existent. But strictly speaking, no single proposition whatever is self-contradictory. The nearest approach would be "No proposition is true," since this implies its own truth. But even here, it is not strictly self-contradictory to deny the implication. Everywhere we come upon propositions accepted because they are self-evident, and for no other reason: the law of contradiction itself is such a proposition. The mutual implication of all the points of space seems to be another; the denial of some only among points is rejected

[1] *Cf.* G. E. Moore, "Necessity," MIND, N. S., No. 35.

for the same reason as the assertion that such and such a proposition is both true and false, namely, because both are obviously untrue. But if, *per impossibile*, a point previously missing were to come into existence, it would not create new points, but would have the appropriate relations to already existing points. The point, in fact, would have already had being, and as an entity would have eternally had to other points the same relations as it has when it comes into existence. Thus Lotze's argument on this, as on other points, depends upon a faulty logic, and is easily met by more correct views as to the nature of judgment.

I have not criticised in detail the current arguments against absolute time, as they are all included under those against space. In favour of absolute space, in addition to the paradoxes of the relational theory, I have to urge the very much greater simplicity of the absolute theory. This appears most plainly in the case of motion. On the relational theory, it is essential to a motion to specify all the changes of distance and angle with respect to all other particles in the universe, since all combinations are possible, and no reason exists for preferring one distance to another. But on the absolute theory, we have merely one change of relation: the particle, which was *at* one point, is now *at* another. The geometrical relations hold primarily between points, and only by correlation between particles. Thus geometrical propositions become timeless, and motion is infinitely simplified. I cannot persuade myself that there is anything to set against this except an antiquated logic, not re-examined in its fundamentals, and capable, if I am not mistaken, of an easy and simple refutation.