VI.—DISCUSSION.

THE EXISTENTIAL IMPORT OF PROPOSITIONS.

MR. MACCOLL's interesting paper in the January number of MIND, together with his note in the April number, raises certain points which call for an answer from those who (like myself) adhere to the usual standpoint of symbolic logicians on the subject of the existential import of propositions.

The first point in regard to which clearness is essential concerns the meaning of the word "existence". There are two meanings of this word, as distinct as stocks in a flower-garden and stocks on the Stock Exchange, which yet are continually being confused, or at least supposed somehow connected. Of these meanings, only one occurs in philosophy or in common parlance, and only the other occurs in mathematics or in symbolic logic. Until it is realised that they have absolutely nothing to do with each other, it is quite impossible to have clear ideas on our present topic.

(*a*) The meaning of *existence* which occurs in philosophy and in daily life is the meaning which can be predicated of an individual, the meaning in which we inquire whether God exists, in which we affirm that Socrates existed, and deny that Hamlet existed. The entities dealt with in mathematics do not exist in this sense: the number 2, or the principle of the syllogism, or multiplication, are objects which mathematics considers, but which certainly form no part of the world of existent things. This sense of existence lies wholly outside Symbolic Logic, which does not care a pin whether its entities exist in this sense or not.

(*b*) The sense in which existence is used in symbolic logic is a definable and purely technical sense, namely this: To say that A exists means that A is a class which has at least one member. Thus whatever is not a class (*e.g.*, Socrates) does not exist in this sense; and among classes there is just one which does not exist, namely, the class having no members, which is called the null-class. In this sense, the class of numbers (*e.g.*) exists, because 1, 2, 3, etc., are members of it; but in sense (*a*) the class and its members alike do not exist: they do not stand out in a part of space and time, nor do they have that kind of super-sensible existence which is attributed to the Deity.

It may be asked: How come two such diverse notions to be confounded? It is easy to see how the confusion arises, by considering classes which, if they have members at all, must have members that exist in sense (*a*). Suppose we say: "No chimeras exist". We may mean that the class of chimeras has no members, *i.e.*, does not exist in sense (*b*), or that nothing that exists in sense (*a*) is a chimera. These two are equivalent in the present instance, because if there were chimeras, they would be entities of the kind that exist in sense (*a*). But if we say "no numbers exist," our statement is true in sense (*a*) and false in sense (*b*). It is true that nothing that exists in sense (*a*) is a number; it is false that the class of numbers has no members. Thus the confusion arises from undue preoccupation with the things that exist in sense (*a*), which is a bad habit engendered by practical interests.

Mr. MacColl assumes (p. 74) two universes, the one composed of existences, the other of non-existences. It will be seen that, if the above discrimination is accepted, these two universes are not to be distinguished in symbolic logic. All entities, whether they exist or whether they do not (in sense (*a*)), are alike real to symbolic logic and mathematics. In sense (*b*), which is alone relevant, there is among classes not a multitude of non-existences, but just one, namely, the null-class. All the members of every class are among realities,[1] in the only sense in which symbolic logic is concerned with realities.

But it is natural to inquire what we are going to say about Mr. MacColl's classes of unrealities, centaurs, round squares, etc. Concerning all these we shall say simply that they are classes which have no members, so that each of them is identical with the null-class. There are no Centaurs; 'x is a Centaur' is false whatever value we give to x, even when we include values which do not exist in sense (*a*), such as numbers, propositions, etc. Similarly, there are no round squares. The case of nectar and ambrosia is more difficult, since these seem to be individuals, not classes. But here we must presuppose definitions of nectar and ambrosia: they are substances having such and such properties, which, as a matter of fact, no substances do have. We have thus merely a defining concept for each, without any entity to which the concept applies. In this case, the concept is an entity, but it does not denote anything. To take a simpler case: "The present King of England" is a complex concept denoting an individual; "the present King of France" is a similar complex concept denoting nothing. The phrase intends to point out an individual, but fails to do so: it does not point out an unreal individual, but no individual at all. The same explanation applies to mythical personages, Apollo, Priam, etc. These words have a *meaning*, which can be found by looking them up in a classical dictionary; but they have not a *denotation*: there is no entity, real or imaginary, which they point out.

[1] This holds even of the null-class. Of all the members of the null-class, *every* statement holds, since the null-class has no members of which it does not hold. See below, on the interpretation of the universal affirmative A.

It will now be plain, I hope, that the ordinary view of symbolic logicians as to existential import does not require Mr. MacColl's modifications. This view is, that A and E do not imply the existence, in sense (*b*), of their subjects, but that I and O do imply the existence, in sense (*b*), of their subjects. No one of the four implies the existence, in sense (*a*), either of its subject or of any of the members of its subject. We have, adopting Peano's interpretation:—

A. All S is P = For all values of x, 'x is an S' implies 'x is a P'.

E. No S is P = For all values of x, 'x is an S' implies 'x is not a P'.

I. Some S is P = For at least one value of x, 'x is an S' and 'x is a P' are both true.

O. Some S is not P = For at least one value of x, 'x is an S' and 'x is not a P' are both true.

Thus I and O require that there should be at least one value of x for which x is an S, *i.e.*, that S should exist in sense (*b*). I also requires that P should exist, and O requires that not-P should exist. But A and E do not require the existence of either S or P; for a hypothetical is true whenever its hypothesis is false,[1] so that if 'x is an S' is always false, 'All S is P' and 'No S is P' will both be true whatever P may be.

The above remarks serve to answer the objection raised by Mr. MacColl in the April number of MIND (p. 295) to the equation 0A = 0. To begin with, 0 does not represent the class of non-existences, but the non-existent class, *i.e.*, the class which has no members. Thus, if "XA = X" means "every X is an A,"[2] then "0A = 0" means "every member of the class which has no members is an A," or "for every value of x, 'x is a member of the class which has no members' implies 'x is an A'". This hypothetical is true for all values of x, because its hypothesis is false for all values of x, and a hypothetical with a false hypothesis is true. Thus Mr. MacColl's objection rests upon his taking 0 to be the class of non existences, presumably in sense (*a*), since only so would 0 be a class with many members, all of them unreal, as he supposes it to be. The true interpretation of 0, as the non-existent class, in sense (*b*), at once disposes of the difficulty.

The same principles solve Lewis Carroll's paradox, noticed by "W" in the April number of MIND (p. 293). I cannot agree with "W" in regarding the paradox as merely verbal; on the contrary, I consider it a good illustration of the principle that a false proposition implies every proposition. Putting p for 'Carr is out,' q for 'Allen is out,' and r for 'Brown is out,' Lewis Carroll's two hypotheticals are:—

(1) q implies r.
(2) p implies that q implies not-r.

[1] See my *Principles of Mathematics*, vol. i., p. 18.
[2] Not "every X is A," as Mr. MacColl says, and as most logicians say.

Lewis Carroll supposes that 'q implies r' and 'q implies not-r' are inconsistent, and hence infers that p must be false. But as a matter of fact, 'q implies r' and 'q implies not-r' must both be true if q is false, and are by no means inconsistent. The contradictory of 'q implies r' is 'q does not imply r,' which is not a consequence of 'q implies not-r'. Thus the only inference from Lewis Carroll's premisses (1) and (2) is that if p is true, q is false, *i.e.*, if Carr is out, Allen is in. This is the complete solution of the paradox.

B. RUSSELL.

MR. RUSSELL has very kindly and courteously sent me a proof of the above, so that logicians might, in the same number of MIND, have an opportunity of reading his criticism side by side with any comments I might desire to make. My comments shall be brief. With much of what Mr. Russell says in his able and interesting dissection of the question at issue I agree; but not with all. That the word *existence*, like many others, has various meanings is quite true; but I cannot admit that any of these "lies wholly outside Symbolic Logic". Symbolic Logic has a right to occupy itself with any question whatever on which it can throw any light. As regards Existential Import, the one important point on which I appear to differ from all other symbolists is the following. The null class o, which they define as containing no members, and which I, for convenience of symbolic operations, define as consisting of the null or unreal members o_1, o_2, o_3, etc., is understood by them to be *contained in every class*, real or unreal; whereas I consider it to be *excluded from every real class*. Their convention of universal inclusion leads to awkward and, I think, needless paradoxes, as, for example, that "Every round square is a triangle," because round squares form a *null* class, which, by them, is understood to be *contained in every class*. My convention leads, in this case, to the directly opposite conclusion, namely, that "No round square is a triangle," because I hold that every purely *unreal* class, such as the class of round squares, is necessarily excluded from every purely *real* class, such as the class of figures called triangles.

I may mention, as a fact not wholly irrelevant, that it was in the actual application of my symbolic system to concrete problems that I found it absolutely necessary to label realities and unrealities by special symbols e and o, and to break up the latter class into separate individuals, o_1, o_2, o_3, etc., just as I break up the former into separate individuals, e_1, e_2, e_3, etc. It is a vital principle in the evolution of any effective symbolic system that we should modify, and, whenever possible, simplify our notation, in order to adapt it to the varying needs of different classes of problems. It is this elastic adaptability to circumstances—this readiness to change the meaning of any symbol (not excepting *zero*), and even of any conventional arrangement of symbols, whenever it suits the purpose of the investigation—that enables my symbolic system to solve certain classes of problems (especially in mathematics) which lie entirely beyond the reach of any other symbolic system within my knowledge. In saying this I do not mean in any way to suggest that other symbolic systems may not have the advantage over mine in regard to other classes of problems which I have never studied. Mr. Russell's system in particular seems to be specially constructed to deal with problems which lie altogether out of the line of my researches. Different kinds of work require different kinds of instruments.

The following arrived too late to be added to the article on "Symbolic Reasoning":—

[My statement in § 3 (of "Symbolic Reasoning"), that "if any two classes X and Y are mutually exclusive, the complementary classes 'X and 'Y overlap," requires qualification. I should have said "if any two *non-complementary* classes, etc.". This I discovered symbolically as follows; though, of course, it may be proved more briefly. Let ϕ denote the *unqualified* statement. We get

$$\phi = (xy)^{\eta} : (x'y')^{-\eta} = (xy)^{\eta}\,(x'y')^{\eta} : \eta$$
$$= (x : y')\,(y' : x) : \eta = (x = y') : \eta = (x = y')^{\eta}.$$

Thus, ϕ is equivalent to the statement that *the class* X *cannot be the complement of the class* Y, a statement which only holds good when X and Y are understood throughout to be *non-complementary*.]

HUGH MACCOLL.