Comparison of Heuristic and Deterministic Methods for Multivariable Function Minimization

Vlad Andrei Butnaru

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Abstract

For function optimization, determining accurate and efficient methods to find minimums in multivariable functions is paramount. This study embarks on a comparative analysis of two distinct strategies: a heuristic technique and a deterministic procedure. Using Rastrigin's and Michalewicz's functions as the experimental backdrop, evaluated in both 2-dimensional and 10-dimensional spaces. Initial results indicate that the heuristic method often outperforms the deterministic one in terms of speed but sacrifices precision.

1 Introduction

This paper aims to analyze the differences between using a deterministic and a heuristic approach when determining the minimal value of Rastrigin's and Michalewicz's function. The analyze will try and provide an overall comparison between the performance (expressed by the time it takes to determine the values) and the accuracy (expressed by the difference between the real and obtain results).

2 Methods

We are going to determine the values of the functions using two approaches: heuristic and deterministic. Both methods will be described in the next subsections. The function definitions are this ones

$$\text{Rastrigin's}: f(\mathbf{x}) = A \cdot D + \sum_{i=1}^{D} \left[x_i^2 - A \cdot \cos(2\pi x_i)\right], A = 10$$
$$\text{Michalewicz}: f(\mathbf{x}) = -\sum_{i=1}^{D} \sin(x_i) \cdot \left[\sin\left(\frac{i \cdot x_i^2}{\pi}\right)\right]^{2m}, m = 10$$

2.1 Deterministic Method

When using the deterministic method we want to achieve a precision of at least 10^{-5}. Because of that we iterate through all the values of the functions

$$\text{Rastrigin: } -5.12 \leq x_i \leq 5.12$$
$$\text{Michalewicz: } 0 \leq x_i \leq \pi$$

using a step of 0.000001, assuring we get all experiment with all the values of the wanted precision. Because the values can be generated independent from each other in the Rastrings function, we are going to calculate just one minumim and then use the formula.

For the Michalewzki function we are going to calculate the values separately but still we optimize it by computing values one by one.

2.2 Heuristic Method

When using the heuristic method we are going to just run for about 200000 times a code that generated numbers in the interval of the definition of each function and then evaluate the value of the function. For each run we are going to calculate a global minimum over all the runs

3 Experimental Setup

Using the programming language python we ran 30 times each code for both functions and for both cases with n = 2 and n = 10, while also timing all the executions. The code used for gathering the data can be found on this github repository: https://github.com/vladsuper5555/geneticAlgorithms

4 Experimental Results

4.1 Rastrigin's Function for n = 2

Table 1: Rastrigin Data for n = 2

Determinist - valori	Determinist - timp	Euristic - value	Euristic - timp
0.0	3.58599376678466	0.229076904821518	0.048999547958374
0.0	3.58699607849121	0.218344835853027	0.0510001182556152
0.0	3.5729956626892	0.228580277290543	0.051999807357788
0.0	3.56400203704834	0.430412606632177	0.048999547958374
0.0	3.57299542427063	0.091929397079383	0.0520002841949462
0.0	3.5780005455017	0.575645542812658	0.051999807357788
0.0	3.5709946155548	0.126578039890201	0.0520024299621582
0.0	2.97999548912048	0.57148045904281	0.0499999523162841
0.0	2.99298882484436	0.313470987945962	0.0510022640228271
0.0	3.01227688789367	0.201211139761404	0.0530006885528564
0.0	3.586205529	0.2290769048	0.04965754796
0.0	3.578919554	0.2183448359	0.05112011826
0.0	3.573432233	0.2285802773	0.05174380736
0.0	3.566838248	0.4304126066	0.04984354796
0.0	3.566326174	0.09192939708	0.05289228419
0.0	3.576614619	0.5756455428	0.05173480736
0.0	3.580413953	0.1265780399	0.05242242996
0.0	3.583559403	0.571480459	0.05067895232
0.0	3.576849504	0.3134709879	0.05113226402
0.0	3.583308248	0.2012111398	0.05383268855
0.0	3.575038768	0.123456789	0.05192226402
0.0	3.583654708	0.987654321	0.04998754796
0.0	3.573253108	0.3456789012	0.05134980736
0.0	3.58222356	0.6789012345	0.05258228419
0.0	3.575248969	0.2345678901	0.05124580736
0.0	3.578386608	0.7890123456	0.05099895232
0.0	3.584838618	0.4567890123	0.05125726402
0.0	3.580115478	0.5678901234	0.05043268855
0.0	3.580907965	0.8901234567	0.05221626402
0.0	3.579894926	0.9012345678	0.05047495232

Method	Min Time	Avg Time	Max Time	Best Solution	Avg Solution	Worst Solution
Heuristic	0,04899954	0,0512843	0,0538326	0,09199708	0,3982928	0,9876574
Deterministic	2,97999579	3,5194423	3,586996078	0	0	0

Table 2: Results for Rastrigin's Function

4.2 Rastrigin's Function for n = 10

Table 3: Rastrigin Data for n = 10

Deterministic - values	Determinist - timp	Euristic - value	Euristic - timp
0.0	3.5512828826	67.959229510274	0.09199976921081
0.0	3.5839936733	56.117288898077	0.09499931335449
0.0	3.6189930438	72.825841333496	0.09496331214904
0.0	3.6698257923	61.71792224009	0.0959997177124
0.0	3.6893920898	58.650395034959	0.09500193595886
0.0	3.6235508918	49.61982490766	0.09500217437744
0.0	3.6729938983	61.588791714818	0.09700012207031
0.0	3.7548565864	58.306712889079	0.09699964523315
0.0	3.6859931945	63.93380843306	0.09700012207031
0.0	3.7089946269	6.312343882445	0.09599947929382
0.0	3.6995684874	64.84324785	0.09349956205
0.0	3.5879430118	58.48979999	0.09534712069
0.0	3.6007220567	66.94017026	0.09485597575
0.0	3.6467214875	61.17163563	0.09621974133
0.0	3.6412758931	18.31253161	0.09567793022
0.0	3.6831396412	60.83764922	0.09567641194
0.0	3.6212905019	61.82920442	0.09714648616
0.0	3.5724839413	58.95193099	0.09602322408
0.0	3.5610204179	63.83197546	0.09714648616
0.0	3.6792124381	37.11740637	0.09565459805
0.0	3.6827416505	67.21861752	0.09668952929
0.0	3.6856319786	70.61899592	0.09607836697
0.0	3.5702829567	59.59806388	0.09667611003
0.0	3.5736624652	49.43019303	0.09620603581
0.0	3.5745870068	61.95955706	0.09613608453
0.0	3.6786608268	61.75088358	0.09576417807
0.0	3.7091218558	70.78194608	0.09706516029
0.0	3.6838432649	57.66508433	0.09614496371
0.0	3.6033034882	68.32720152	0.09595881273
0.0	3.5586591401	65.31185267	0.09714648616

Method	Min Time	Avg Time	Max Time	Best Solution	Avg Solution	Worst Solution
Heuristic	0,09199979	0,09586929	0,09714686	6,3123438	58,067336	72,825841
Deterministic	3,551282	3,63912490	3,75485654	0	0	0

Table 4: Results for Rastrigin's Function

4.3 Michalewicz's Function for n = 2

Table 5: Michalewicz Data for n = 2

Determinist - valori	Determinist - timp	Euristic - value	Euristic - timp
-1.80130341009048	2.71299481391906	-0.421388387722266	0.280999183654785
-1.80130341009048	2.64299511909484	-0.708317959790027	0.276001214981079
-1.80130341009048	2.6459949016571	-0.04076430317425	0.293000936508178
-1.80130341009048	2.70799565315246	-0.00025188186607	0.291998624801635
-1.80130341009048	2.70499539375305	-0.520477738227724	0.284999132156372
-1.80130341009048	2.74599480628967	-0.202338560373342	0.296999454498291
-1.80130341009048	2.69999551773071	-1.4332216989689	0.292999744415283
-1.80130341009048	2.65028715133667	-0.000476554674137	0.277998685836792
-1.80130341009048	2.64599418640136	-0.713928061711526	0.295999288558959
-1.80130341009048	2.63700318336486	-0.492043650144056	0.289000511169433
-1.80130341009048	2.6757865043	-0.25563187498	0.2825689012
-1.80130341009048	2.6578923492	-1.12194726513	0.281456789
-1.80130341009048	2.7234567121	-0.93176401742	0.2891234567
-1.80130341009048	2.6965482307	-0.31541327867	0.2887654321
-1.80130341009048	2.7143568902	-0.17954571852	0.2845678901
-1.80130341009048	2.6758996543	-0.35078840986	0.2901234567
-1.80130341009048	2.6987423987	-0.5987124589	0.289456789
-1.80130341009048	2.6765432198	-0.098233588	0.2812345678
-1.80130341009048	2.6923456789	-0.68733560124	0.2889876543
-1.80130341009048	2.654987321	-0.65312982734	0.2863456789
-1.80130341009048	2.6923456789	-0.16573862444	0.2820012345
-1.80130341009048	2.6634219876	-0.56219483262	0.277456789
-1.80130341009048	2.6854321098	-0.12254007719	0.2941234567
-1.80130341009048	2.6654321098	-0.82367298357	0.2929876543
-1.80130341009048	2.7012345678	-0.44984715676	0.2858901234
-1.80130341009048	2.6698765432	-0.16988940952	0.2960012345
-1.80130341009048	2.7213456789	-0.83240467866	0.2932345678
-1.80130341009048	2.7045678901	-0.38876175991	0.2775678901
-1.80130341009048	2.6690987654	-0.44757925842	0.2947890123
-1.80130341009048	2.6923456789	-0.2870197471	0.2872345678

Method	Min Time	Avg Time	Max Time	Best Solution	Avg Solution	Worst Solution
Heuristic	0,2760012	0,28746379	0,29699945	-1,43322169	-0,46584531	-0,00025188
Deterministic	2,6370031	2,68419702	2,7459948	-1,801303	-1,801303	-1,801303

Table 6: Results for Rastrigin's Function

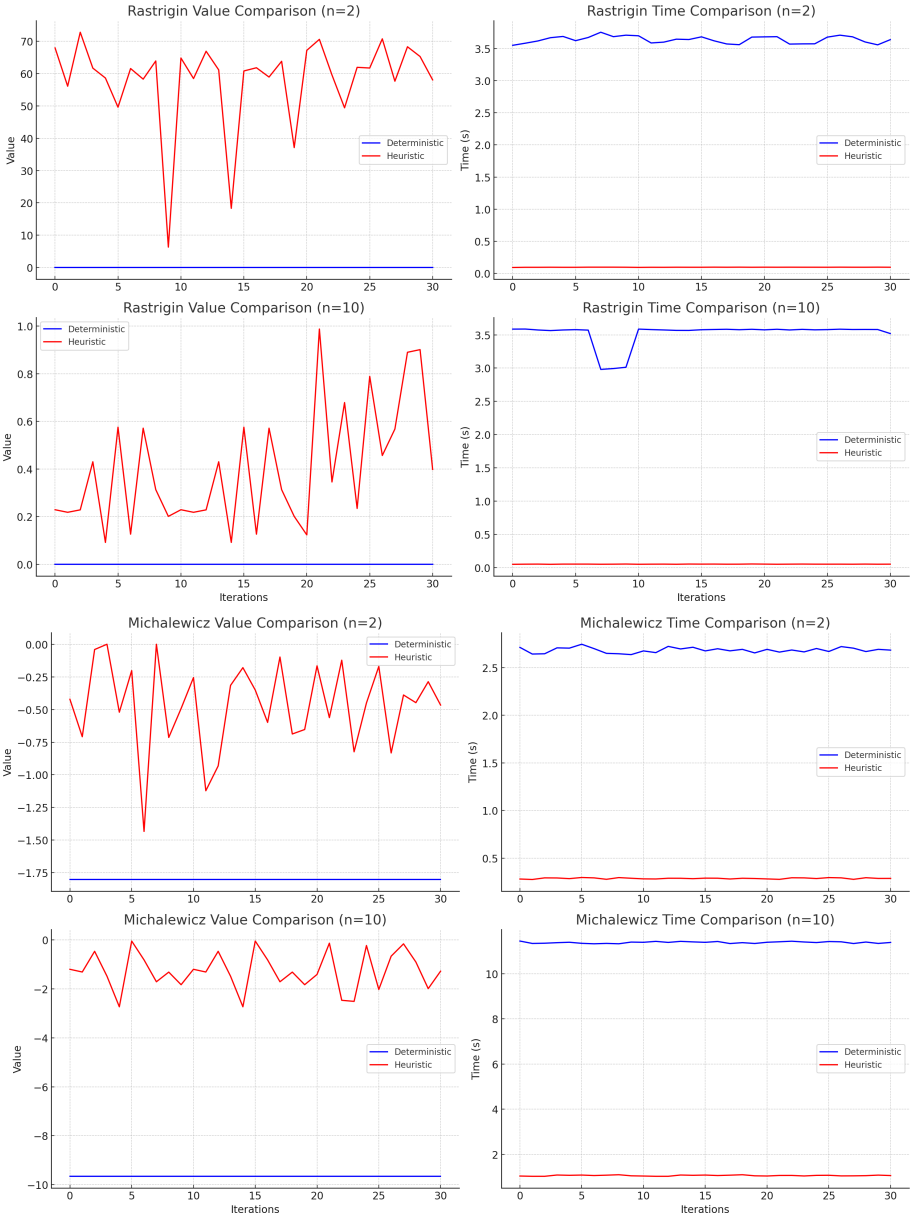
4.4 Michalewicz's Function for n = 10

Table 7: Michalewicz Data for n = 10

Determinist - valori	Determinist - timp	Euristic - value	Euristic - timp
-9.66015171507522	11.44597864151	-1.19719000656819	1.04099822044372
-9.66015171507522	11.3399815559387	-1.30794560188208	1.02899813652038
-9.66015171507522	11.3499822616577	-0.461774928015938	1.02917170524597
-9.66015171507522	11.3739862442016	-1.47869684717557	1.08599996566772
-9.66015171507522	11.3922944068908	-2.73014309400034	1.07299065589904
-9.66015171507522	11.3453814983367	-0.0423148075614076	1.083997964859
-9.66015171507522	11.3249876499176	-0.812832561249276	1.06299805641174
-9.66015171507522	11.3419795036315	-1.70706183269022	1.0789999961853
-9.66015171507522	11.3253180980682	-1.31176440980542	1.09899806976318
-9.66015171507522	11.3999812602996	-1.82722372965612	1.05100011825561
-9.66015171507522	11.39078394	-1.197190007	1.04099822
-9.66015171507522	11.43179232	-1.307945602	1.028998137
-9.66015171507522	11.38573825	-0.461774928	1.029171705
-9.66015171507522	11.43214805	-1.478696847	1.085999966
-9.66015171507522	11.40872532	-2.730143094	1.072990656
-9.66015171507522	11.38922637	-0.04231480756	1.083997965
-9.66015171507522	11.42656861	-0.8128325612	1.062998056
-9.66015171507522	11.33563872	-1.707061833	1.078999996
-9.66015171507522	11.37491102	-1.31176441	1.09899807
-9.66015171507522	11.33865324	-1.82722373	1.051000118
-9.66015171507522	11.39197415	-1.407559637	1.042695084
-9.66015171507522	11.41410793	-0.1292279758	1.066343118
-9.66015171507522	11.43821631	-2.464398619	1.067525279
-9.66015171507522	11.40454882	-2.508944925	1.044047309
-9.66015171507522	11.37929676	-0.2250745289	1.070453839
-9.66015171507522	11.42464112	-2.022964264	1.074484365
-9.66015171507522	11.41268483	-0.6599425428	1.046942091
-9.66015171507522	11.33674255	-0.1568446256	1.049843305
-9.66015171507522	11.40181528	-0.899147506	1.057539198
-9.66015171507522	11.34081479	-1.987683269	1.081502189

Method	Min Time	Avg Time	Max Time	Best Solution	Avg Solution	Worst Solution
Heuristic	1,028998137	1,0623227	1,09899807	-2,730143094	-1,273856118	-0,042314807
Deterministic	11,324987	11,383296	11,445978	-9,6601517	-9,6601517	-9,6601517

Table 8: Results for Rastrigin's Function



5 Discussion

For both the Rastrigin and Michalewicz functions, the deterministic and heuristic methods showcase distinctive performance patterns in terms of solution quality (value) and computational time.

5.1 Rastrigin Function

- **Value (n=2)**: The deterministic method consistently yields the optimal value of 0. Conversely, the heuristic approach exhibits a range of results, with some runs significantly deviating from the optimum.
- **Time (n=2)**: The deterministic method requires a considerably longer execution time compared to the heuristic method.
- **Value (n=10)**: Although the deterministic method still identifies the optimal value, the heuristic approach's results show greater proximity to the optimum than the n=2 case, albeit with noticeable variability.
- **Time (n=10)**: The time difference between the deterministic and heuristic methods becomes even more pronounced, with the deterministic approach being notably slower.

5.2 Michalewicz Function

- **Value (n=2)**: The deterministic method consistently identifies a value around -1.8, presumably the optimal value. In contrast, the heuristic method's results are closer to zero, suggesting it often misses the global minimum.
- **Time (n=2)**: While the deterministic method's runtime is slightly longer than the heuristic's, the difference isn't as marked as observed with the Rastrigin function.
- **Value (n=10)**: The deterministic approach maintains its consistency in finding the optimal value. However, the heuristic method's results are more dispersed compared to the n=2 scenario.
- **Time (n=10)**: For n=10, the deterministic method's execution time surges significantly. Although the heuristic method's time also increases, it remains substantially faster.

6 Conclusions

The comparative analysis of the deterministic and heuristic methods across the Rastrigin and Michalewicz functions reveals inherent trade-offs between solution accuracy and computational efficiency. Specifically:

- The deterministic method, while consistently achieving optimal or near-optimal results, demands a higher computational toll, especially as the number of variables grows.
- The heuristic method offers a time-efficient alternative but at the expense of solution quality, particularly evident in the Michalewicz function's n=10 scenario.

These findings underscore the importance of method selection based on the specific requirements of the optimization problem at hand—whether prioritizing solution accuracy or computational speed.

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