Modelling, measuring and incorporating uncertainty

Introducing Type-2 intuitionistic fuzzy-set QCA (T2iQCA)

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Set-theoretic methods are blooming in social sciences.But why?

- They address a **new class of questions**.
- **Flexibility** in terms of data requirements.
- Ability to capture **conjunctural causation** and **equifinality**.
- Formal elegance that ensures **replicability**.

In particular, the number of papers employing Qualitative Comparative Analysis (QCA) has grown exponentially:

- From 60 papers between 1981-1999, to over 130 just in 2020.
- Used in business studies, political science, sustainability, criminology, health sciences.

What caused this ascent?

One driver of growth was the introduction of fuzzy-set QCA by Charles Ragin (2000).

Why was Ragin's new framework so important?

- Recognized how messy social reality is.
- Acknowledged that mathematical theory had moved beyond Boolean logic.
- Allowed researchers to **incorporate uncertainty**.

However, are we getting complacent?

- Fuzzy-set theory dates from 1965 (Zadeh, 1965; Klaua, 1965)
- Other types of uncertainty (e.g., linguistic) are obscured by the lack of development.

What is to be done?

- Focus not only on designing new estimators, but on **introducing new classes of models**.
  - Apply new, more general mathematical theories.
  - Be more transparent about the linguistic origins of QCA.
- (Re)construct the most robust QCA estimators for the new class of models.

Solution?

Type 2 Intuitionistic Fuzzy Set QCA

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Crisp sets

- Membership of cases in a set is assessed in binary terms according to a bivalent membership function $\mu_a \in \{0, 1\}$
- Founded on the mathematical theory of Boolean logic.

Table 1: Example of crisp set

	High inequality	High development	High carbon emissions
Canada	0	1	1
US	1	1	1
Russia	1	1	1
France	0	1	0

Fuzzy sets

- Cases have continuous degrees of membership in a set, $\mu_a \in [0, 1]$
- Three fundamental membership scores: full membership (1), full non-membership (0), indifference point (0.5)
- Extends Boolean logic, but maintains most of its operational properties.

Table 2: Example of Type 1 fuzzy set

	High inequality	High development	High carbon emissions
Canada	0.4	1	0.6
US	0.7	0.9	0.7
Russia	0.75	0.55	0.9
France	0.3	1	0.4

Type 2 fuzzy sets

- Type-2 fuzzy sets operate with a membership function that is fluctuating; that is, membership scores are themselves *fuzzy*.
- This implies two *de facto* membership functions:
  - lower membership function($\underline{\mu_a}$)
  - upper membership function($\overline{\mu_a}$)
- The interval $[\underline{\mu_a}, \overline{\mu_a}]$ is called the footprint of uncertainty (FOU).
- Incorporate linguistic and conceptual uncertainty without loss of information.

Table 3: Example of Type 2 fuzzy set

	High inequality	High development	High carbon emissions
Canada	0.30 — 0.40	0.98 — 1	0.50 — 0.65
US	0.58 — 0.77	0.89 — 0.95	0.65 — 0.72
Russia	0.70 — 0.77	0.50 — 0.60	0.85 — 0.97
France	0.25 — 0.35	0.95 — 1	0.30 — 0.50

Intuitionistic fuzzy sets

- Characterized by two distinct functions: membership function (μ_a) and non-membership function (ν_a), with $\mu_a + \nu_a \leqslant 1$
- Allows the researcher to incorporate uncertainty without assuming anything about its distribution across the population of cases.

Table 4: Example of Intuitionistic fuzzy set

μ_a	High inequality	High development	High carbon emissions
Canada	0.4	1	0.6
US	0.7	0.9	0.7
Russia	0.75	0.55	0.9
France	0.3	1	0.4

ν_a	High inequality	High development	High carbon emissions
Canada	0.6	0	0.4
US	0.3	0	0.25
Russia	0.15	0.45	0
France	0.65	0.0	0.5

Type 2 Intuitionistic fuzzy set

- Membership function $\mu_a$defined by:
  - lower membership function($\underline{\mu_a}$), upper membership function($\overline{\mu_a}$)
  - $[\underline{\mu_a}, \overline{\mu_a}]$ — membership footprint of uncertainty (mFOU).
- Non-membership function ν_a defined by:
  - lower membership function($\underline{\nu_a}$), upper membership function($\overline{\nu_a}$)
  - $[\underline{\nu_a}, \overline{\nu_a}]$ — non-membership footprint of uncertainty (nFOU).

Table 5: Example of Type 2 Intuitionistic fuzzy set

ν_a	High inequality	High development	High carbon emissions
Canada	0.30 — 0.40	0.98 — 1	0.50 — 0.65
US	0.58 — 0.77	0.89 — 0.95	0.65 — 0.72
Russia	0.70 — 0.77	0.50 — 0.60	0.85 — 0.97
France	0.25 — 0.35	0.95 — 1	0.30 — 0.50
ν_a	High inequality	High development	High carbon emissions
Canada	0.52 — 0.60	0 — 0.02	0.30 — 0.45
US	0.20 — 0.30	0.05 — 0.10	0.20 — 0.30
Russia	0.20 — 0.30	0.35 — 0.45	0 — 0.10
France	0.60 — 0.70	0 — 0.03	0.45 — 0.65

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T2iQCA

- **A new model, but not a new estimator!**
- The necessary and sufficiency relationships can be derived using the same algorithm (extended) as fsQCA.
- Allows for more precise measurement of uncertanty, but can be reduced to fsQCA.

Table 6: Example of Type 2 Intuitionistic fuzzy set

μ_a	High inequality	High development	High carbon emissions
Canada	0.30 — 0.40	0.98 — 1	0.50 — 0.65
US	0.58 — 0.77	0.89 — 0.95	0.65 — 0.72
Russia	0.70 — 0.77	0.50 — 0.60	0.85 — 0.97
France	0.25 — 0.35	0.95 — 1	0.30 — 0.50
ν_a	High inequality	High development	High carbon emissions
Canada	0.52 — 0.60	0 — 0.02	0.30 — 0.45
US	0.20 — 0.30	0.05 — 0.10	0.20 — 0.30
Russia	0.20 — 0.30	0.35 — 0.45	0 — 0.10
France	0.60 — 0.70	0 — 0.03	0.45 — 0.65

What is the membership and non-membership score in the combination of conditions?

- Use the logical AND or any other valid t-norm.

Table 7: Example of Type 2 Intuitionistic fuzzy set

μ_a	HI $\wedge HD \wedge HE$	HI $\cdot HD \cdot HE$
Canada	0.30 — 0.40	0.147 — 0.26
US	0.58 — 0.72	0.335 — 0.52
Russia	0.50 — 0.60	0.29 — 0.45
France	0.25 — 0.35	0.057 — 0.175
ν_a	HI $\wedge HD \wedge HE$	HI $\cdot HD \cdot HE$
Canada	0 — 0.02	0 — 0.005
US	0.05 — 0.10	0. — 0.009
Russia	0 — 0.10	0 — 0.013
France	0 — 0.03	0 — 0.0.13

- This process enables the creation of truth tables for T2 Intuitionistic fuzzy sets.

What if the mFOU and nFOU pass through the point of indiference?

- Reconsult the data, check the coding.
- Re(consider) the reasons for which this happens.
- Make an informed choice!

Table 8: Example of Type 2 Intuitionistic fuzzy set

μ_a	High inequality	High development	High carbon emissions
Canada	0.30 — 0.40	0.98 — 1	0.45 — 0.65
ν_a	High inequality	High development	High carbon emissions
Canada	0.52 — 0.60	0 — 0.02	0.30 — 0.55

What comes next?

- A more precise characterization of estimators in relation to the model.
- A better visual characterization of sets.
- An R pakacge to deal with T2iQCA (and hopefully a STATA command).