CM12004: Problem Sheet

1. (1 is $True$)

   (i) $P = X \lor$ Y,Q= ¬(X ∧ Y)
   P→Q= (X ∨ Y) → ¬(X ∧ Y)
   P→Q= (¬X ∧ ¬Y) ∨ (¬X ∨ ¬Y)
   P→Q= (¬X ∨¬X ∨ ¬Y) ∧ (¬X ∨¬Y ∨ ¬Y)
   P→Q= ¬X ∨ ¬Y
   $Answer$: $neither$

(ii) $P = X \lor$ Y,Q= ¬X ∧ ¬Y
P→Q= (X ∨ Y) → (¬X ∧ ¬Y)
P→Q= (¬X ∧ ¬Y) ∨ (¬X ∧ ¬Y)
P→Q= ¬X ∧ ¬Y
$Answer$: $neither$

(iii) $P = X \to$ Y,Q= (¬X ∨ Y) ∧ (¬X ∨ X)
P→Q= (X→ Y) → (¬X ∨ Y) ∧ 1
P→Q= (X ∧ ¬Y) ∨ (¬X ∨ Y)
P→Q= (X ∨ ¬X ∨ Y) ∧ (X ∨ ¬Y ∨ Y)
P→Q= (1 ∨ Y) ∧ (1 ∨ ¬X) =1∧1=1
$Answer$: $tautology$

(iv) $P = X \to$ ¬Y,Q=Y → ¬X
P→Q= (¬X ∨ ¬Y) → (¬Y ∨ ¬X)
P→Q= (X ∧ Y) ∨ (¬Y ∨ ¬X)
P→Q= (X ∨ ¬X ∨ ¬Y) ∧ (¬X ∨ ¬Y ∨ Y)
P→Q= (1∨ ¬Y) ∧ (1∨ X) =1∧1=1
$Answer$: $tautology$

(v) $P = X \land$ (Y ∨ Z),Q= (X ∨ Y) ∧ (X ∨ Z)
P→Q=X ∧ (Y ∨ Z) → (X ∨ Y) ∧ (X ∨ Z)
P→Q= ((¬X ∨ ¬Y) ∧ (¬Z ∨ ¬X)) ∨ ((X ∨ Y) ∧ (Z ∨ X))
P→Q= (¬Y ∧ ¬Y) ∨ (¬Y ∧ ¬X) ∨ (¬X ∧ ¬Z) ∨ (¬X ∧ ¬X) ∨ (X ∧ X) ∨ (X ∧ Z) ∨ (X ∧ Y) ∨ (Z ∧ Y)
P→Q=X ∨ ¬X ∨ (¬Y ∧ ¬Y) ∨ (¬Y ∧ ¬X) ∨ (¬X ∧ ¬Z) ∨ (X ∧ Z) ∨ (X ∧ Y) ∨ (Z ∧ Y)
P→Q=1 ∨ (¬Y ∧ ¬Y) ∨ (¬Y ∧ ¬X) ∨ (¬X ∧ ¬Z) ∨ (X ∧ Z) ∨ (X ∧ Y) ∨ (Z ∧ Y)
P→Q=1
$Answer$: $tautology$

(vi) $P = X \to$ Y,Q= ¬X → ¬Y
P→Q= (X → Y) → (¬X → ¬Y)
P→Q= (X ∧ ¬Y) ∨ (X ∨ ¬Y)
P→Q= (X ∨ X ∨ ¬Y) ∧ (X ∨ ¬Y ∨ ¬Y)
P→Q= (X ∨ ¬Y) ∧ (X ∨ ¬Y)
P→Q=X ∨ ¬Y
$Answer$: $neither$

(vii) $P = X \to$ Y,Q= ¬(Y → ¬X)
P→Q= (X → Y) → ¬(Y → ¬X)
P→Q= (¬X ∨ Y) → (Y ∧ ¬X)

P→Q= (X ∧ ¬Y) ∨ (Y ∧ ¬X)

X	Y	$\neg X$	$\neg Y$	$X \wedge \neg Y$	$\neg X \wedge Y$	$(X \wedge \neg Y) \vee (\neg X \wedge Y)$
T	T	F	F	F	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

Answer: neither

(viii) $P = (X \ \rightarrow$ Y) ∧ (Y → Z),Q= (X → Z)
P→Q= ((X → Y) ∧ (Y → Z)) → (X → Z)
P→Q= ((¬X ∨ Y) ∧ (¬Y ∨ Z)) → (¬X ∨ Z)
P→Q= ((X ∧ ¬Y) ∨ (Y ∧ ¬Z)) ∨ (¬X ∨ Z)
P→Q= (¬X ∨ (X ∧ ¬Y)) ∨ (Z ∨ (Y ∧ ¬Z))
P→Q= ((¬X ∨ X) ∧ (X ∨ ¬Y)) ∨ ((Z ∨ ¬Z) ∧ (Y ∨ Z))
P→Q= ¬X ∨ (Y ∨ ¬Y) ∨ Z
P→Q=1 ∨ ¬X ∨ Z
P→Q=1
$Answer : tautology$

2. (a) There exist 16 binary connectives as there are 16 binary truth tables.

(b) (i) Note: A XOR $B = (A \vee$ B) ∧ (¬A ∨ ¬B)

A XOR B= (A∨ B) ∧ (¬A ∨ ¬B) = (A∨ B) ∧ ¬(A ∧ B) = ¬(¬A ∧ ¬B) ∧ ¬(A ∧ B)

(ii) A XOR $B = (A \vee$ B) ∧ (¬A ∨ ¬B) = ¬(¬(A ∨ B)) ∨ ¬(¬A ∨ ¬B))

(iii) A XOR $B = (A \vee$ B) ∧ (¬A ∨ ¬B) = (¬A→ B) ∧ (A → ¬B) = ¬((¬A→B) → ¬(A→ ¬B))

(c) (i) We can notice that:

X	X NAND X	$\neg X$
T	F	F
F	T	T

X NAND X = ¬X

X ∧ Y= (X ∨ Y) ∧ (X ∨ Y) = ¬(X ∨ Y) $NAND$ ¬(X ∨ Y) = (X $NAND$ Y) $NAND$ (X $NAND$ Y)

(ii) X ∨ Y= ¬X $NAND$ ¬Y= (X $NAND$ X) $NAND$ (Y $NAND$ Y)

(iii) X →Y= ¬X ∨ Y=X $NAND$ (Y $NAND$ Y)

(iv) X NAND X = ¬X

(d) (i) We can notice that:

X	X NOR X	$\neg X$
T	F	F
F	T	T

X NOR X = ¬X

X∧ Y= ¬X NOR ¬Y= (X NOR X) NOR (Y NOR Y)

(ii) X ∨ Y= (X ∨ Y) ∧ (X ∨ Y) = ¬(X ∨ Y) NOR ¬(X ∨ Y) = (X NOR Y) NOR (X NOR Y)

(iii) $X \rightarrow$Y= ¬X ∨ Y= ¬((X *NOR* X) *NOR* Y) = ((X *NOR* X) *NOR* Y) *NOR* ((X *NOR* X) *NOR* Y)

(iv) X NOR X = ¬X

3. (a) (i) $\forall x \in Z \exists y \in Z (x^2 < y+1)$

   For every integer x we can find an integer y such that $(x^2 < y+1)$.
   If we take $y = x^2$, $x^2 < x^2 + 1$. It means that this statement is True.

   (ii) $\exists x \in Z \forall y \in Z (x^2 < y+1)$

   We can find an integer x such that for every integer y, $(x^2 < y+1)$.
   We know that $x^2 \geq 0$, however integer y can be less than 0.
   If we take $y = -2$, $x^2 < -1$. It means that this statement is False.

   (iii) $\exists y \in Z \forall x \in Z (x^2 < y+1)$

   We can find an integer y such that for every integer x, $(x^2 < y+1)$.
   Assume that $x = y+5$, $((y+5)^2 - y - 1 < 0) = (y^2 + 9y + 24 < 0)$. It means that this statement is False.

   (iv) $\forall x \in Z \exists y \in Z ((x < y) \rightarrow (x^2 < y^2))$

   For every integers x we can find an integer y such that $((x < y) \rightarrow (x^2 < y^2))$.
   If we take $y = x^2 + 1$, $x < x^2 + 1$ and this is True for all integers, as well as $x^2 < (x^2+1)^2$. True implies True, it means that this statement is True.

   (b) (i) $\forall x \in Z \forall y \in Z (x^2 < y+1)$

   For every two integers x and y, $(x^2 < y+1)$
   If $x = -1$ and $y = -1$, then $(-1)^2 < -1 + 1 = 1 < 0$. That is why this statement is False.

   (ii) $\exists x \in Z \exists y \in Z (x^2 < y+1)$

   There exist two integers x and y such that $(x^2 < y+1)$
   If $x = 1$ and $y = 1$, then $1^2 < 1 + 1 = 1 < 2$. That is why this statement is True.

   (iii) $\forall y \in Z \forall x \in Z (x^2 < y+1)$

   For every integer y we can find an integer x such that $(x^2 < y+1)$
   We know that $x^2 \geq 0$. If $y = -2$, then $x^2 < -2 + 1 = x^2 < -1$. That is why this statement is False.

   (iv) $\exists x \in Z \forall y \in Z ((x < y) \rightarrow (x^2 < y^2))$

   We can find an integer x such that for every integer y, $((x < y) \rightarrow (x^2 < y^2))$
   If we take $x = 1$ knowing that, if $1 < y$ then $1 < y^2$. That is why this statement is True.

4. (a)
   - $2^{\emptyset}$
     $|\{\emptyset\}| = 1$
   - $2^{\{0\}}$
     $|\{\emptyset, \{0\}\}| = 2$
   - $2^{\{0\} \cup \{1\}}$
     $|\{\emptyset, \{0\}, \{1\}, \{0,1\}\}| = 4$
   - $2^{\{0\} \cap \{1\}}$
     $|\{\emptyset\}| = 1$

- $2^{\{\emptyset,0,1\}}$
  $|\{\emptyset, \{\emptyset\}, \{0\}, \{1\}, \{0,1\}, \{0,\emptyset\}, \{\emptyset,1\}, \{0,1,\emptyset\}\}| = 8$

- $2^{2^{2^{\{0,1\}}}} = 2^{2^{\{\emptyset,\{0\},\{1\},\{0,1\}\}}} = 2^{\{\emptyset,\{0\},\{1\},\{0,1\}...\{\emptyset,\{0\},\{1\},\{0,1\}\}\}} = 2^{16}$
  $|\ 2^{2^{2^{\{0,1\}}}}\ | = 65536$

(b) (i) $\{(x,S)|x \in S, S \in 2^A\}, A = \{1,2,...,n\}$

Assume that $n = 3$, then $A = \{1,2,3\}$. As 2^A is the set of all possible subsets of A, then S is one of those subsets. $2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$. If $x \in S$, then x can be one of the elements of the not empty S, so x can potentially be 1, 2 or 3. For example: if x is 1, then it appears in 4 possible subsets of A. Pairs with x can be (1, {1}), (1, {1,2}), (1, {1,3}), (1, {1,2,3}). We can do the same for $x = 2$ and $x = 3$.
So there are 3 possible x and since x must be in S, there are combinations for S that include x with the remaining two elements 2 and 3 $(n-1)$ that can be either included or excluded: $3 * 2^{3-1}$.

Answer: $n * 2^{n-1}$

(ii) $\{(S,T)|S \in 2^A, T \in 2^A, S \cap T = \emptyset\}, A = \{1,2,...,n\}$

Suppose $n = 2$, then $A = \{1,2\}$. Since 2^A is the set of all possible subsets of A, then S is one of those subsets. $2^A = \{\emptyset, \{1\}, \{2\}, \{1,2\},\}$. If S can be chosen from 2^n and as $S \cap T = \emptyset$, then T cannot contain any elements of S (complement of S in A), so $2^{n-|S|}$ (the cardinality of S shows how many different elements have been taken).

Let us apply this for $n = 2$ and $S = \emptyset$. $2^{2-|0|} = 4$, in fact we have four pairs, because the conjunction of two emptysets equals emptyset. $(\emptyset, \emptyset)$, $(\emptyset, \{1\})$, $(\emptyset, \{2\})$, $(\emptyset, \{1,2\})$.

For $n = 2$ and $S = \{1,2\}$. $2^{2-|2|} = 1$, we actually have a pair $(\{1,2\}, \emptyset)$

For $n = 3$ and $S = \{2\}$, $2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$. $2^{3-|1|} = 4$, in fact we have four pairs $(\{2\}, \emptyset)$, $(\{2\}, \{1\})$, $(\{2\}, \{3\})$, $(\{2\}, \{1,3\})$.

We can see that for each element n we first check if it is in S, then if it is not in S we check if it is in T. Then, for example, for $n = 3$ the pair $S = \{1\}$, $T = \{2\}$, we have element 3 which is not in sets T and S. So if we consider all possible disjoint subsets S and T of A, each element n, $n \in A$, can be in S, can be in S or neither.

Answer: 3^n

Let us prove it, for $n = 2$, by rewriting all possible pairs:
$A = \{1,2\}$
All possible subsets of A: $2^A = \{\emptyset, \{1\}, \{2\}, \{1,2\},\}$
If $S = \emptyset$: $(\emptyset, \emptyset), (\emptyset, \{1\}), (\emptyset, \{2\}), (\emptyset, \{1,2\})$
If $S = \{1\}$: $(\{1\}, \emptyset), (\{1\}, \{2\})$
If $S = \{2\}$: $(\{2\}, \emptyset), (\{2\}, \{1\})$
If $S = \{1,2\}$: $(\{1,2\}, \emptyset)$
The sum of all possible pairs: $4 + 2 + 2 + 1 = 9$ or $3^2 = 9$

5. (i) Number of maps from {1, 2} to {1, 2, 3}:
For domain we have 2 elements that can be mapped to 3 elements from codomain. For the first element from domain we have 3 choices and for the second element we have 3 choices.
Answer: 3 * 3 = 9

(ii) Number of injective maps from {1, 2} to {1, 2, 3}:
As with injective maps, two different arguments must have different argument images, for the first element we have 3 choices, for the second we only have 3 - 1 = 2 (because we used an image).
Answer: 3 * 2 = 6

(iii) Number of bijective maps from {1, 2} to {1, 2, 3}:
The bijective map must be one-to-one. Since the number of elements from the first set is smaller than from the second set, not all images would have an argument. This means that there are 0 bijective maps
Answer: 0

(iv) Number of bijective maps from {1, 2, 3} to {1, 2}:
Domain has 3 elements, codomain has 2, so for each argument there are two images to choose from. The first preimage is 2, the second and third are also 2.
Answer: $2^3 = 8$

(v) Number of surjective maps from {1, 2, 3} to {1, 2}:
A surjective map is a map where all elements of the codomain have at least one pre-image. Since the first set is larger than the second by one element, one of the images must have 2 arguments. This means that there are 3 ways of choosing which two domain elements would have the same image. There are also 2 choices of which image would have 2 arguments, the remaining argument must be mapped to the remaining image.
Answer: 3 (ways to choose arguments) * 2 (two possible images) * 1 (remaining) = 6

6. (a) Base case: $\sum_{i=1}^{n}(2i-1) = n^2$

For n = 1, $\sum_{i=1}^{1}(2i-1) = 1^2$

$(2-1) = 1^2$

$1 = 1$

Inductive step: assume the statement is true for some integer k, $\sum_{i=1}^{k}(2i-1) = k^2$

Lets prove for k+1: $\sum_{i=1}^{k+1}(2i-1) = (k+1)^2$

We can rewrite left side as ($\sum_{i=1}^{k}(2i{-}1)$ + (2(k + 1) - 1)) (the last element) = $k^2{+}(2(k{+}1){-}1) = k^2{+}2k{+}1$

$k^2 + 2k + 1 = (k+1)^2$

(b) Base case: $\sum_{i=1}^{n} i^2 = \frac{n}{6}(n+1)(2n+1)$

For n = 1, $\sum_{i=1}^{1} i^2 = \frac{1}{6}(1+1)(2*1+1)$

$1^2 = \frac{1}{6} * 2 * 3$

$1 = 1$

Inductive step: assume the statement is true for some integer k, $\sum_{i=1}^{k} i^2 = \frac{k}{6}(k+1)(2k+1)$

Lets prove for k+1: $\sum_{i=1}^{k+1} i^2 = \frac{1}{6}(k+1)(k+2)(2k+3)$

We can rewrite left side as $\sum_{i=1}^{k+1} i^2 = \frac{k}{6}(k+1)(2k+1) + (k+1)^2$ (the last element)

$\frac{k}{6}(k+1)(2k+1) + (k+1)^2 = \frac{1}{6}(k+1)(k+2)(2k+3)$

$\frac{k}{6}(2k+1) + (k+1) = \frac{1}{6}(k+2)(2k+3)$

$\frac{(2k^2+k)+6k+6}{6} = \frac{2k^2+7k+6}{6}$

$\frac{2k^2+7k+6}{6} = \frac{2k^2+7k+6}{6}$

(c) $\sum_{i=3}^{n-2}(i^2)$

As we know formula from the first example, we can rewrite it as:
$\sum_{i=3}^{n-2}(i^2) = \sum_{i=1}^{n}(i^2) - \sum_{i=1}^{2}(i^2)$ (two first elements of the series) $-((n-1)^2 + n^2)$ (last two elements of the series).

$\frac{n}{6}(n+1)(2n+1) - 5 - 2n^2 + 2n - 1 = \frac{n}{6}(2n^2 + 3n + 1) - 5 - 2n^2 + 2n - 1$

$\frac{2n^3+3n^2+n-30-12n^2+12n-6}{6} = \frac{2n^3-9n^2+13n-36}{6} = \frac{n^3}{3} - \frac{3n^2}{2} + \frac{13n}{6} - 6$

(d) $7^n - 1$ is divisible by 6

Base case: for n $= 1$, $7^1 - 1 = 6$ is divisible by 6.

Inductive step: Assume the statement is true for some integer k: $7^k - 1 = 6m$; $m, n \geq 0$
Lets prove it for k+1:
$7^k - 1 = 6m$, let's multiply both sides by 7
$7^{k+1} - 7 = 42m$, let's add 6 to both sides
$7^{k+1} - 1 = 42m + 6$, as the right side is divisible by 6, the left side is divisible by 6 too.

(e) $2n + 1 \leq 2^n$ for $n \geq 3$

Note that since $n \geq 3$, the left side is never equal to the right side. If we start with $n = 3$, then $6+1 \leq 8$. For $n = 4$, $8+1 \leq 16$. For $n = 5$, $10+1 \leq 32$. So, as we can see, the left side is incremented by 2 each time we take the previous n plus 1, while the right side is always doubled. The smallest difference between the left and right sides is for $n = 3$ and is equal to 1. So we can rewrite $2n+1 \leq 2^n$ as $2n+1 < 2^n$

Base case: for $n = 3$, $7 < 8$

Inductive step: assume the statement is true for some integer k: $2k + 1 < 2^k$ for $k \geq 3$
Lets prove it for k+1:
Lets add 2 to both sides $(2k + 1) + 2 < 2^k + 2$
We know that $2^k > 2$ for $k \geq 3$, if $(2k + 1) + 2 < 2^k + 2 < 2^k + 2^k$
Then it means that $2k + 3 < 2^{k+1}$

7. (i) x + y is an odd integer ($*$ is relation)

Relation is reflexive on a set if every element is related to itself. For every $x \in Z$, $x + x$ should be odd.
As $x + x = 2x$ and $2x$ is always even, then it is not a reflexive relation.

Relation is symmetric for every $x, y \in Z$, if $x * y$ then $y * x$.
$x + y$ and $y + x$ are always equal as $x + y = y + x$. It is a symmetric relation.

Relation is transitive for every $x, y, z \in Z$, if $x * y$ and $y * z$, then $x * z$.
If $x + y$ is odd, $y + z$ is odd, $x + z$ is not necessarily odd. For example: $x = 1, y = 4, z = 3$, then $x + y = 5$, $y + z = 7$, but $x + z = 4$. It is not a transitive relation.

Answer: symmetric

(ii) $x + y$ is an even integer

Relation is reflexive on a set if every element is related to itself.
For every $x \in Z$, $x + x$ should be even. As $x + x = 2x$ and $2x$ is always even, then it is a reflexive relation.

Relation is symmetric for every $x, y \in Z$, if $x * y$ then $y * x$.
$x + y$ and $y + x$ are always equal as $x + y = y + x$. It is a symmetric relation.

Relation is transitive for every $x, y, z \in Z$, if $x * y$ and $y * z$, then $x * z$.
For even sum both elements must be even or odd. If x is even, then y should be even. If y is even, then z is even too, so $x + z$ is always even if $x + y$ and $y + z$ are even. (we can do the same for x is odd)

Answer: equivalence relation

(iii) xy is an odd integer

Relation is reflexive on a set if every element is related to itself.
For every $x \in Z$, x^2 should be odd. For $x = 2$, $2^2 = 4$, so it is not a reflexive relation.

Relation is symmetric for every $x, y \in Z$, if $x * y$ then $y * x$.
xy and yx are always equal as $xy = yx$. It is a symmetric relation.

Relation is transitive for every $x, y, z \in Z$, if $x * y$ and $y * z$, then $x * z$.
If xy and yz are odd, then xz should be odd. Result of multiplication can be odd if only two of the multiples are odd. If x and y are both odd, then xy is odd. If y is odd, then z must be odd. If z and x are odd, then xz is odd. It is a transitive relation.

Answer: symmetric, transitive

(iv) $x + xy$ is an even integer.

Relation is reflexive on a set if every element is related to itself.
For every $x \in Z$, $x + x^2$ should be even. Whatever x is odd or even, $x(x + 1)$ is always even, because one element of multiplication is even while another is odd. It is a reflexive relation.

Relation is symmetric for every $x, y \in Z$, if $x * y$ then $y * x$.
If $x + xy$ is even, then $y + yx$ should be also even. If x is even, y is odd, then $x + xy$ is even, but $y + yx$ is odd, because the sum of even and odd is always odd. It is not a symmetric relation.

Relation is transitive for every $x, y, z \in Z$, if $x * y$ and $y * z$, then $x * z$.
If $x + xy$ is even, $y + yz$ is even, then $x + xz$ should be even.

(a) Assume that x is even, y is even, then $x(z + 1)$ is always even, because the multiplication has even element.

(b) Assume that x is even, y is odd, then z must be odd, because $y(z + 1)$ has to be even. $x(z + 1)$ is even,because the multiplication has even element.

(c) If x is odd, then y should be odd, because $x(y + 1)$ has to be even. If y is odd, then z is odd, because $y(z + 1)$ has to be even. If x and z are odd, then $x(z + 1)$ is even.

Answer: reflexive, transitive

8. Subset relation $\subset$ on the set $A = \{\{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ (R is relation)

(i) Partial order is reflexive, antisymmetric and transitive, whereas strict partial order is asymmetric (irreflexive and antisymmetric) and transitive.

Relation is reflexive on a set if every element is related to itself, for every $x \in A$, $x \subset x$.
If we consider that this subset sign is not strict, then every set is a subset of itself, it means that $x \subset x$. It is a reflexive relation.

Relation is antisymmetric on a set if $x, y \in A$, xRy and yRx, then $x = y$.
If we take two A set elements, x is a subset of y, y is a subset of x, then indeed x and y are equal,

according to the set theory that every set is a subset of itself (if the subset sign is not strict). For example: $\{a,b\}R\{a,b\}$, $\{a,b\}R\{a,b\}$, then $\{a,b\} = \{a,b\}$. It is an antisymmetric relation.

Relation is transitive for every $x, y, z \in A$, if xRy and yRz, then xRz.
If $x \subset y$, $y \subset z$, then $x \subset z$. If all elements of a set x are in a set y and all elements of y are in a set z, then all elements of x are in z. This means that $x \subset z$. It is a transitive relation.

Answer: it is a partial order.

(ii) A partial order with an additional condition, such as if $x, y \in A$, then either xRy or yRx is true, is called a total order. Take two elements $\{b\}$ and $\{a,c\}$. Both are in the set A, but neither is a subset of the other $\{b\} \not\subset \{a,c\}$, $\{a,c\} \not\subset \{b\}$.

Answer: it is not a total order.

(iii) Maximal element x, $x \in A$ is the element such that there is no y, $y \in A$ that xRy:
Set $\{a,b,c\}$,$\{a,b,c\} \in A$, is not a subset of any other elements in A.

Minimal element x, $x \in A$ is the element such that there is no y, $y \in A$ that yRx:
Sets $\{b\}$ and $\{c\}$, $\{b\}, \{c\} \in A$, do not have any subsets from A.

Answer: {a,b,c} and {b}, {c}