Post Quantum Cryptography and the future of the Internet

Willy Malvault

Snowcamp 2023

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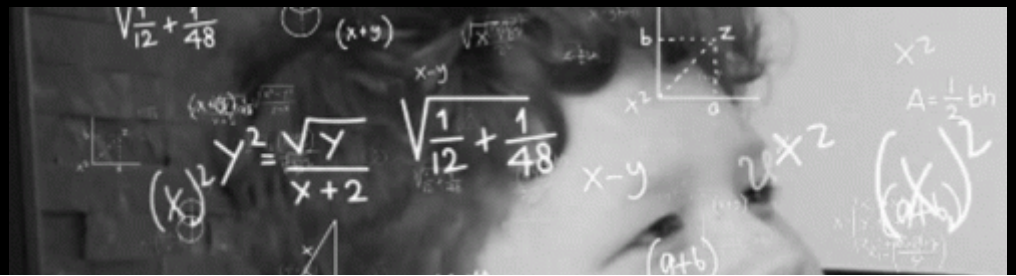
MOODY'S

It all begins with

My Approach

- Willy Malvault
- Principal Consultant at Sogilis
  - Cloud Native related stuff
  - Cybersecurity
- I started my carreer because of the need to understand how Internet works
- I needed to understand Post Quantum Cryptography



What is a public key cryptography scheme ?

$$Pk, Sk = \text{Keygen}()$$

Pk = Public Key

Sk = Secret Key

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Encryption

c = Enc(Pk,m)

m = Dec(Sk,c)

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Confidentiality

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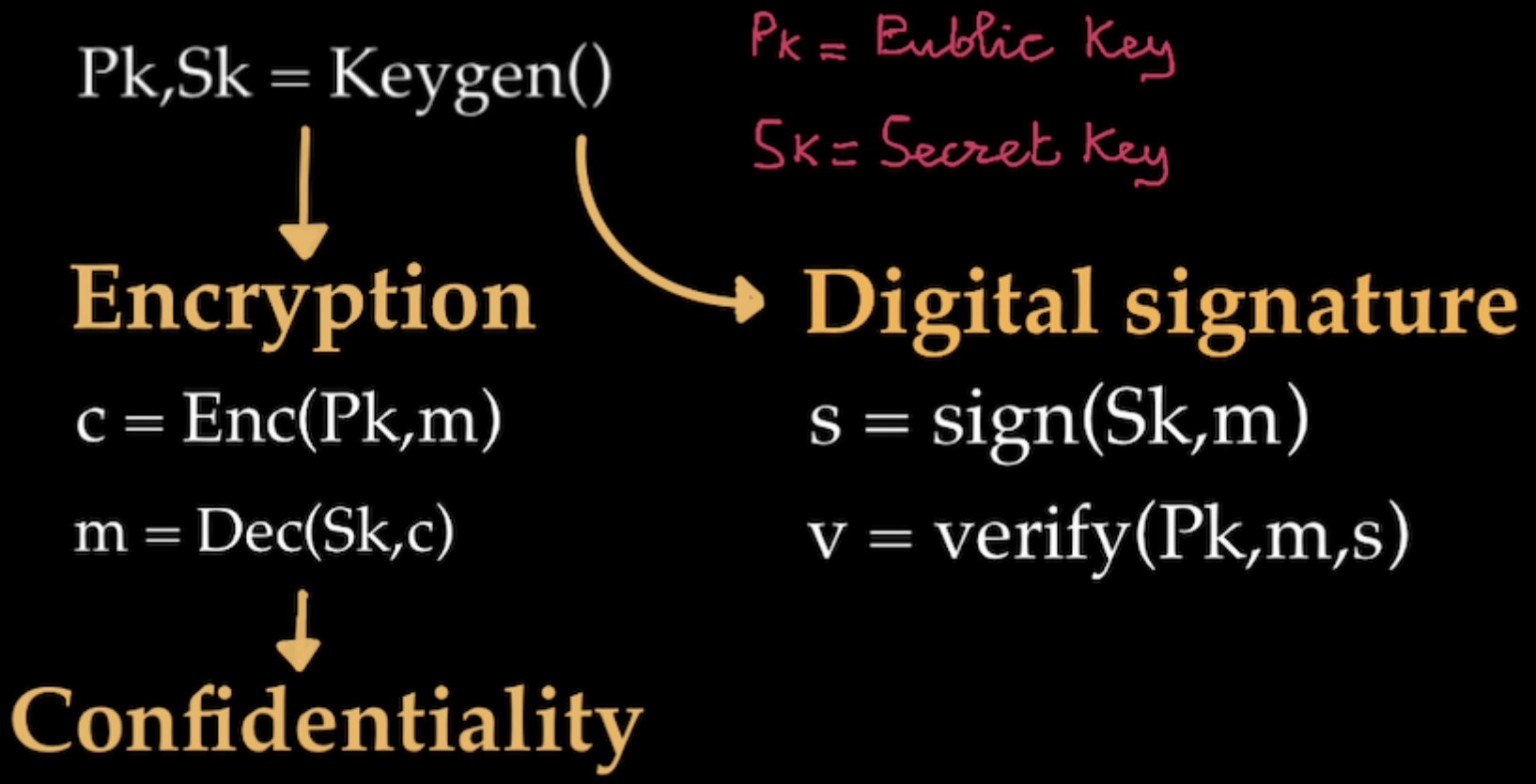

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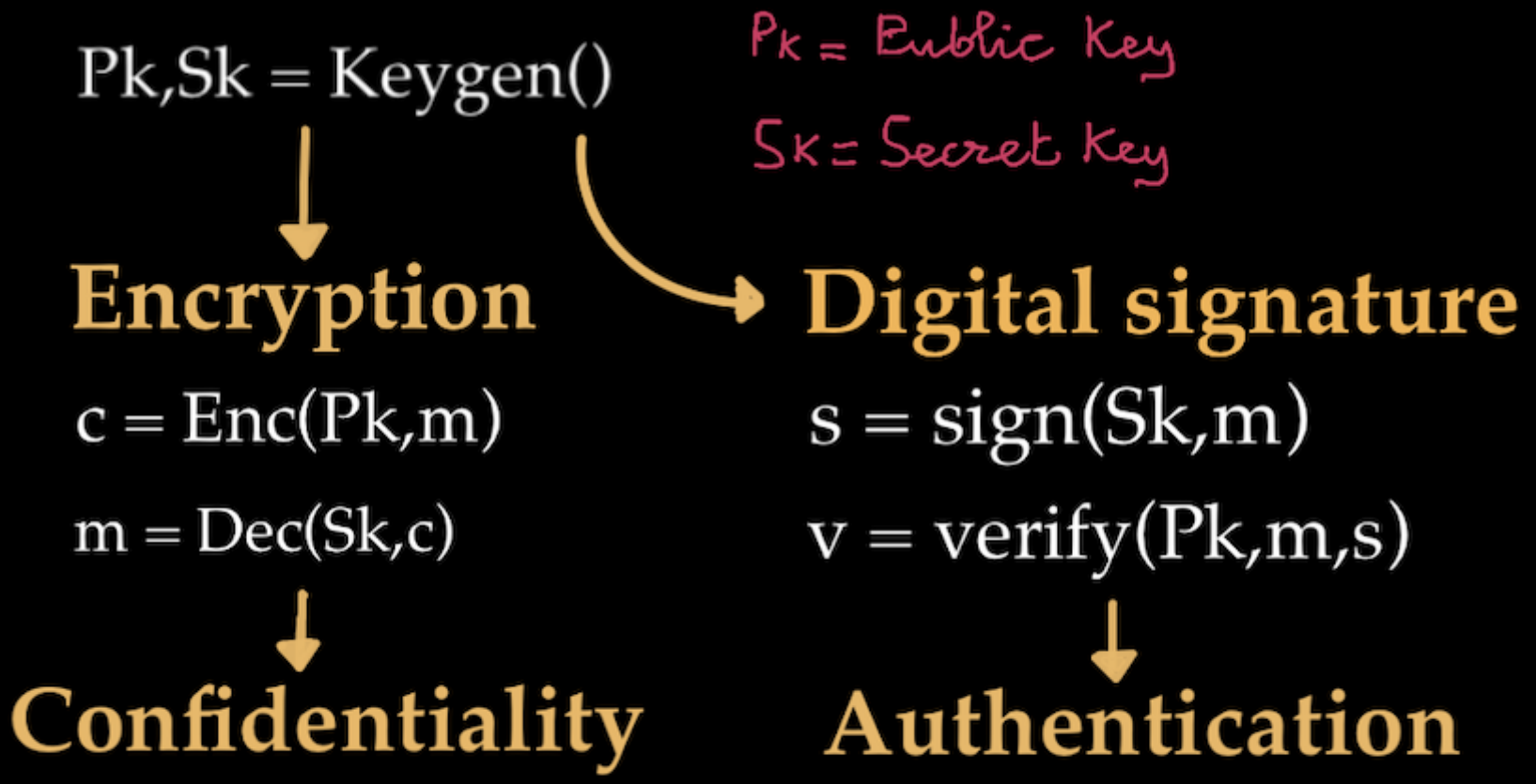

What's the point ?

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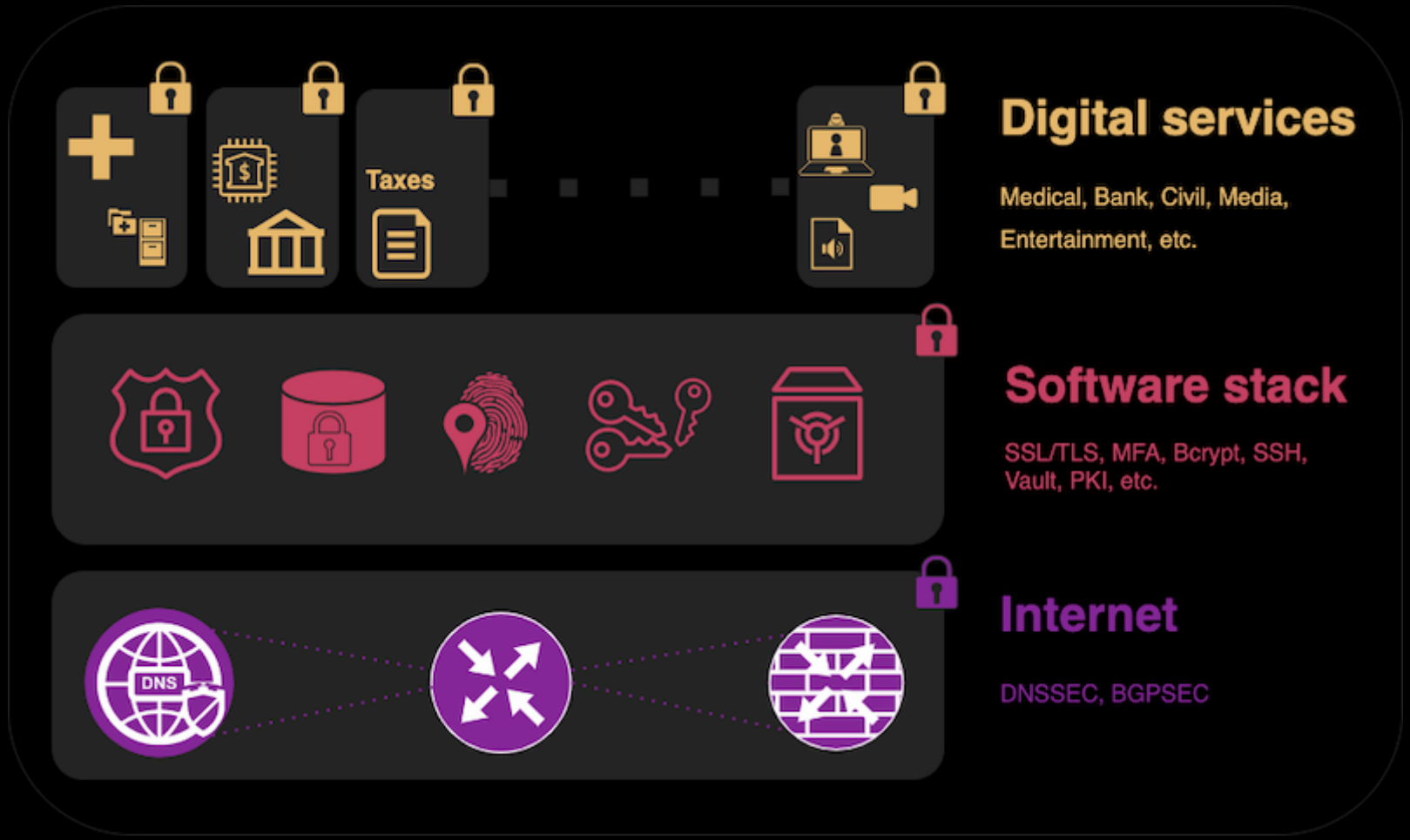

How to implement public key cryptography ?

> Ronald L. Rivest, Adi Shamir, Leonard M. Adleman: A Method for Obtaining Digital Signatures and Public-Key Cryptosystems. Commun. ACM 21(2): 120-126 (1978)

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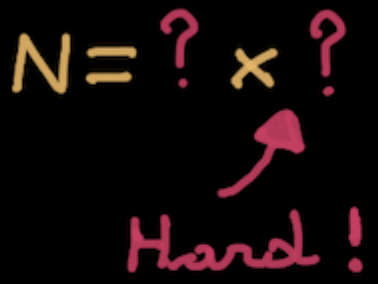
Prime number factorization



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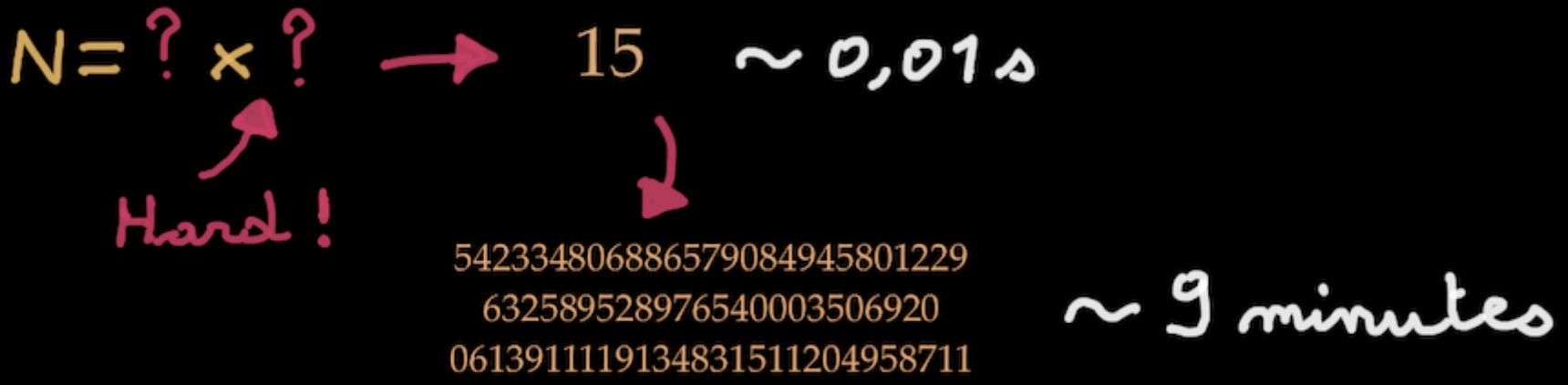

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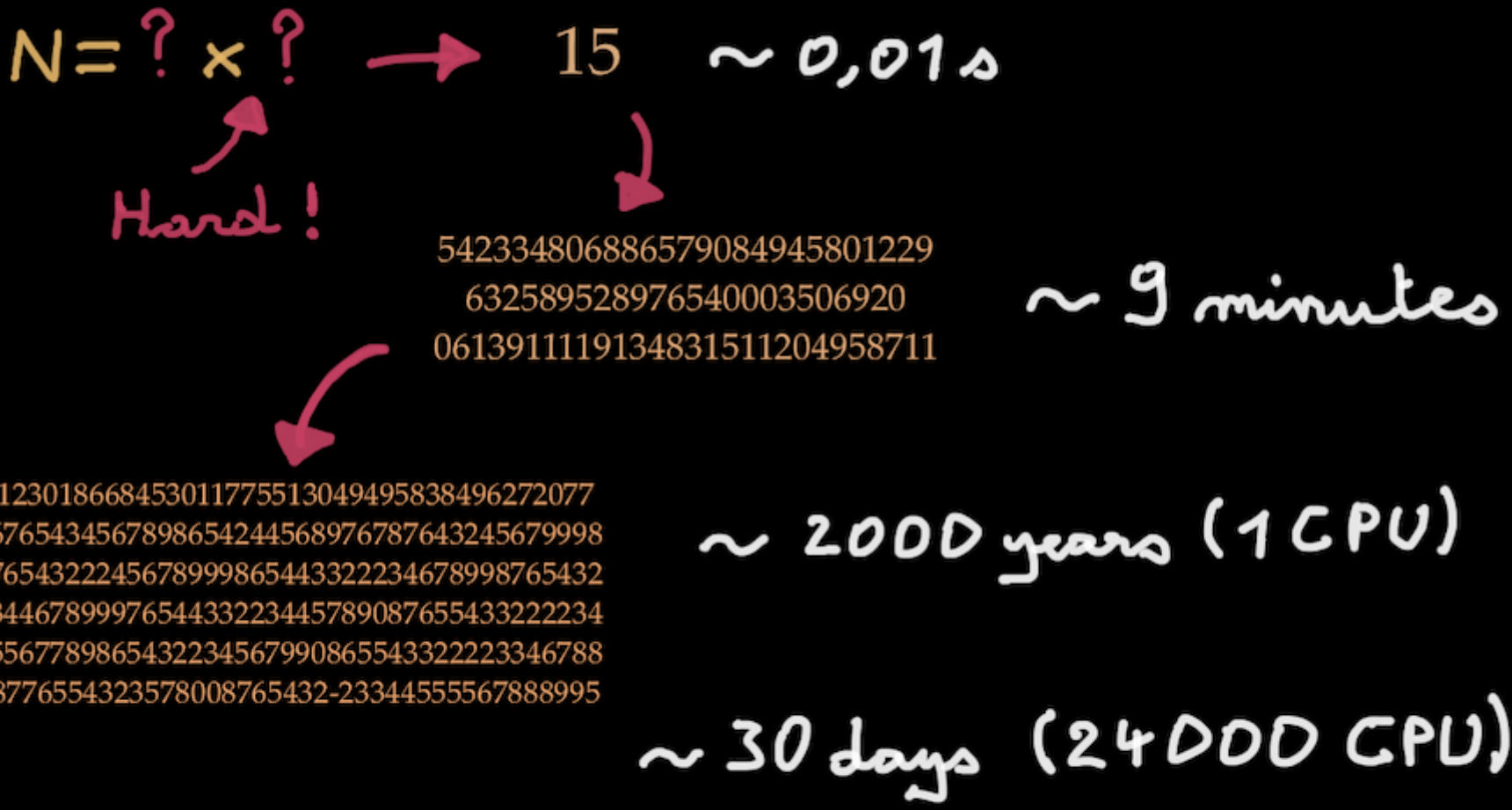

RSA cryptosystem (simplified)

KeyGen()

```
n = p * q

Pk = (c(p,q), n)

Sk = (mmi(p, q), n)
```

Enc(pk, m)

```
\(c = m^{Pk}\ mod\ n\)
```

Dec(sk, c)

```
\(m = c^{sk}\ mod\ n\)
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$$s = hash(m)^{sk} \mod n$$

Verify(pk, m)

$$h = s^{Pk} \mod n$$

OK if $hash(m)=h$

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RSA is working great !

The RSA problem/challenge

If an attacker can retrieve `p` and `q` from `n`, then she can retrieve `Sk`

It takes around 1000 core-years to factorize RSA-768 (record), with a classical computer.

> Fabrice Boudot, Pierrick Gaudry, Aurore Guillevic, Nadia Heninger, Emmanuel Thomé, Paul Zimmermann: Comparing the difficulty of factorization and discrete logarithm: a 240-digit experiment. IACR Cryptol. ePrint Arch. 2020: 697 (2020)

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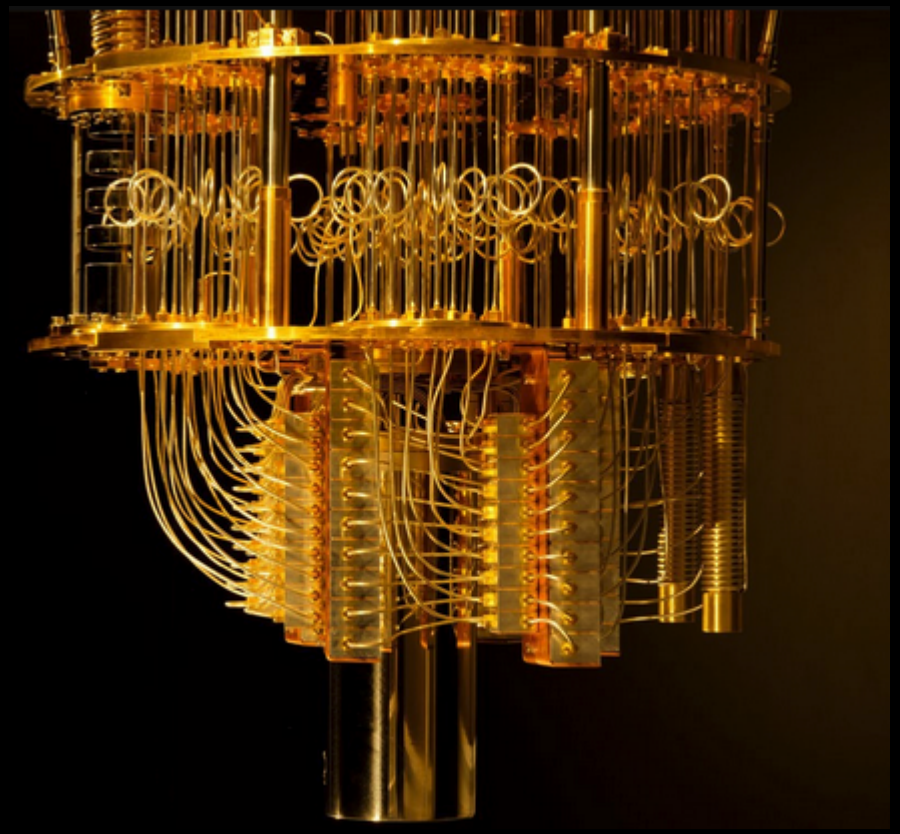
NIST standardization

But... this stands only for classical computers

Quantum computers is threatening !



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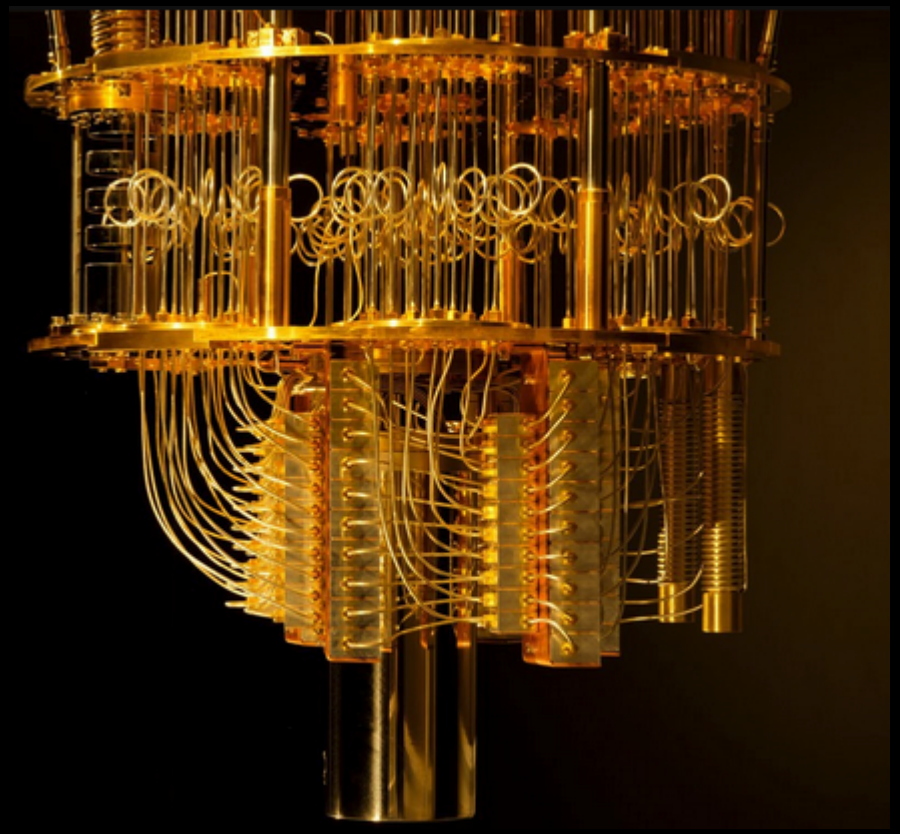

Qubit

Why is the qubit so powerfull ?

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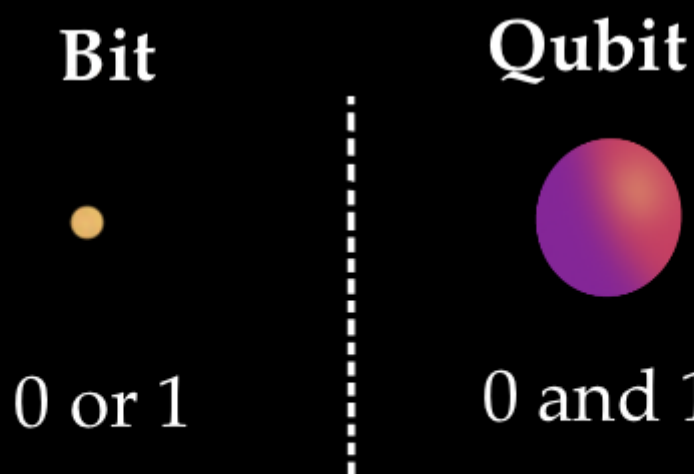


Quantum supremacy

Is the ability of quantum computing to outperform classical computing.

Typically resolving exponential time problems in polynomial time.

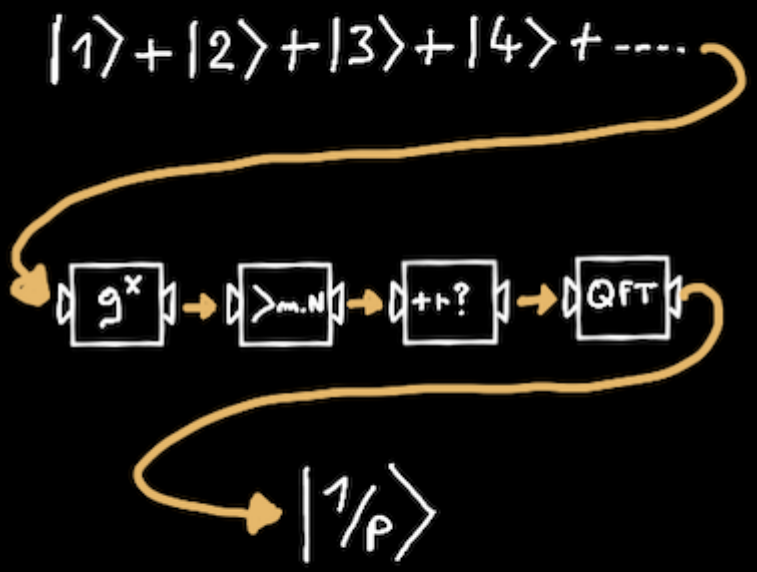
Strong at resolving high combinatory Problems:

Pharma, Nuclear, Meteo

Articficial intelligence

Breaking RSA !

The Famous Shor algorithm



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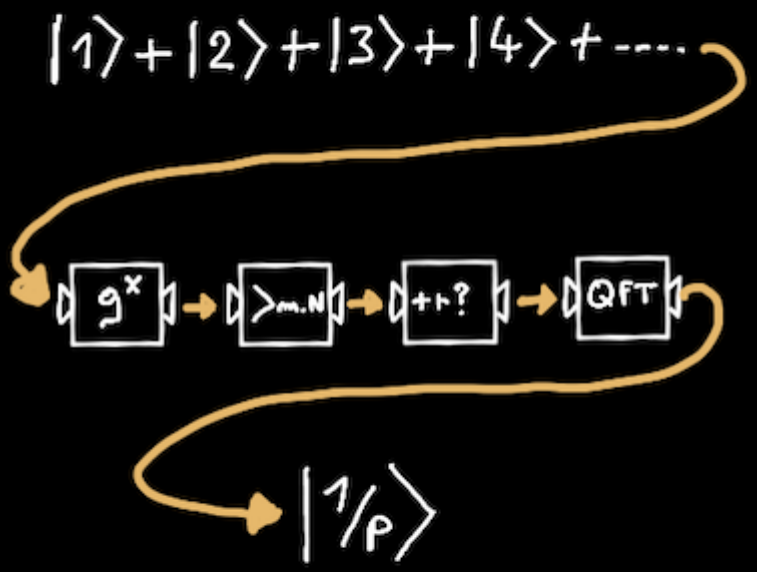

Should we be afraid of the monster ?

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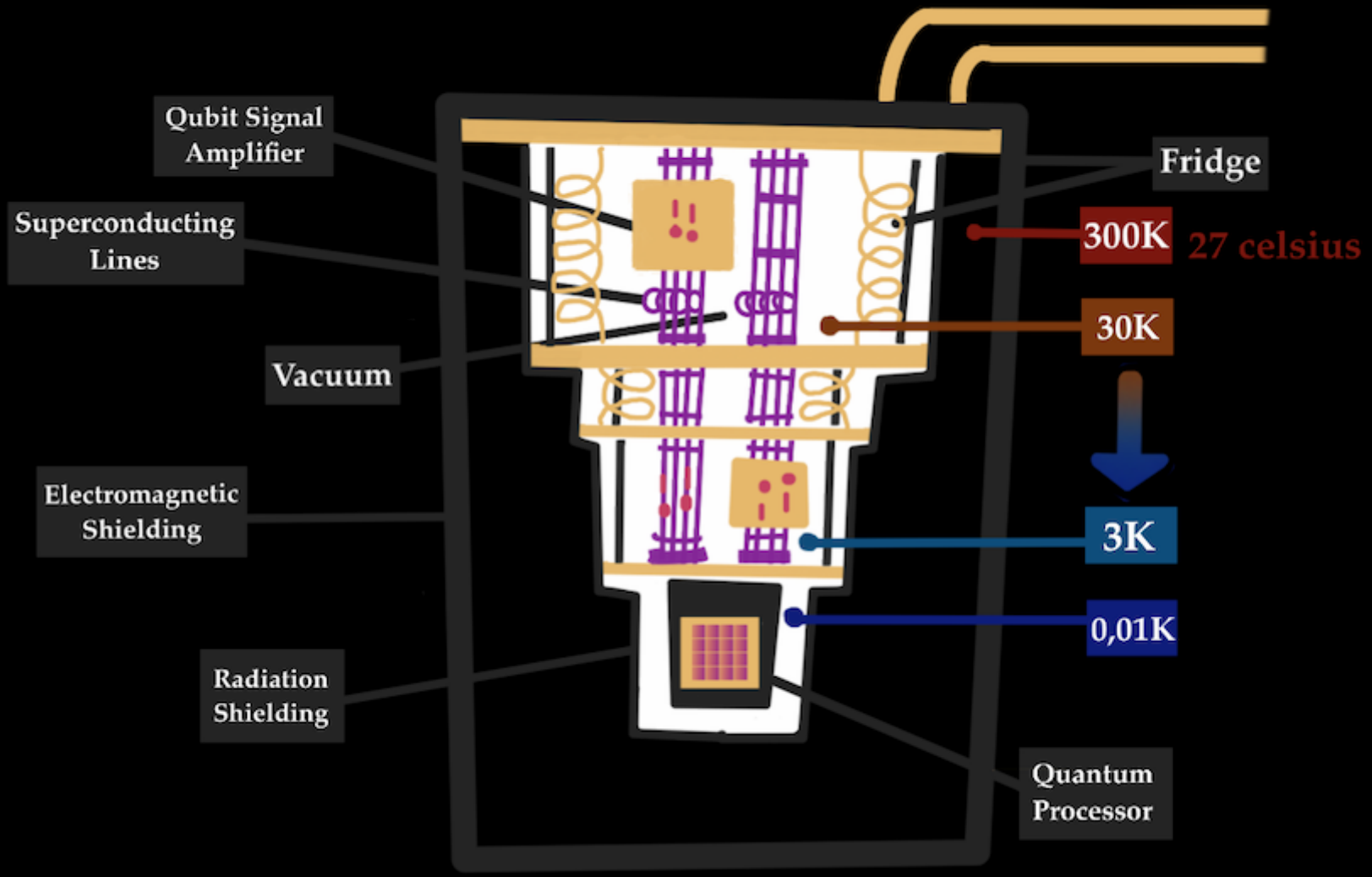
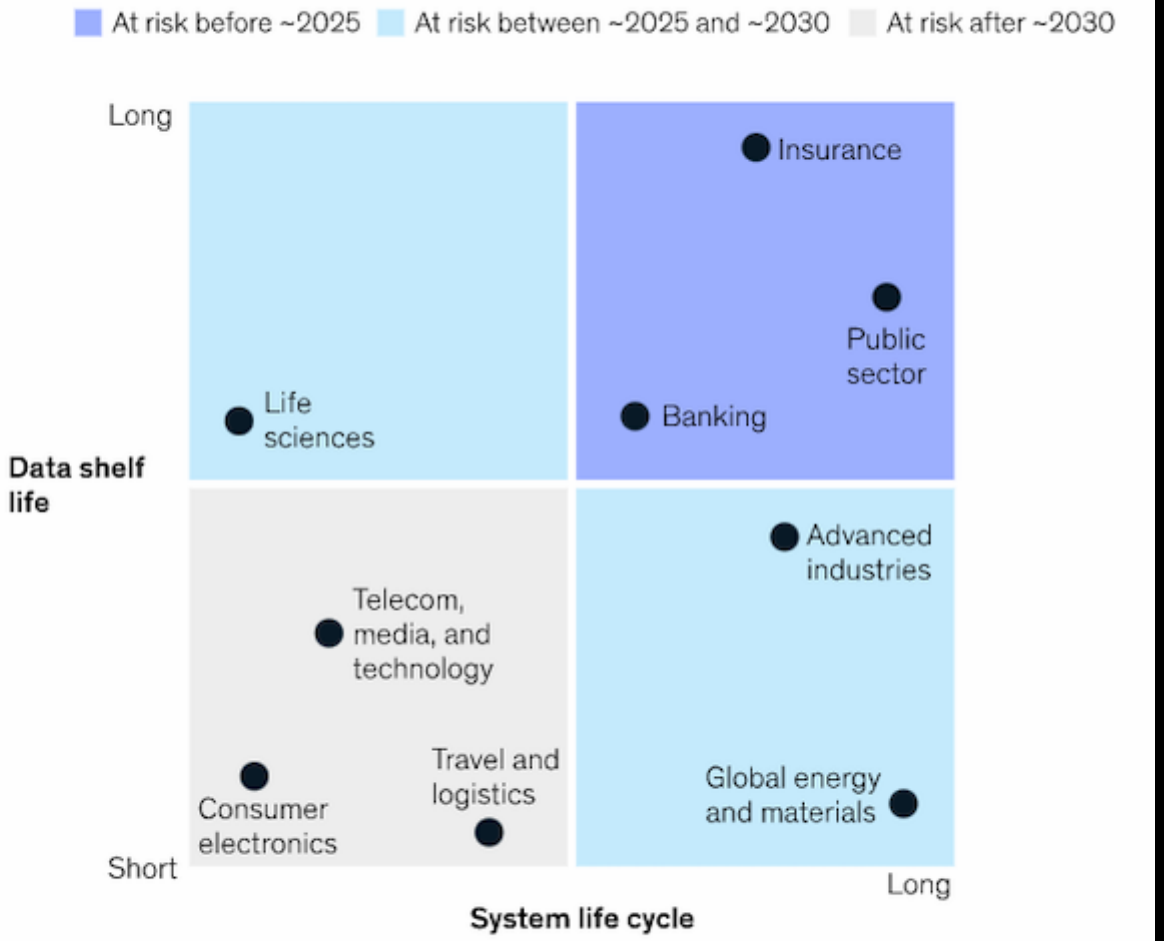

Qubit Signal Amplifier
Superconducting Lines
Vacuum
Electromagnetic Shielding
Radiation Shielding
Fridge
300K
27 celsius
30K
3K
0,01K
Quantum Processor

Risk of quantum-powered attack by industry



McKinsey & Company

Quantum Cryptography

Using quantum channels to exchange private keys.

> C. H. Bennett and G. Brassard. "Quantum cryptography: Public key distribution and coin tossing". In Proceedings of IEEE International Conference on Computers, Systems and Signal Processing, volume 175, page 8. New York, 1984

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Post-quantum Cryptography

Use traditionnal computers to use cryptography schemes robust to quantum attacks.

NIST PQC challenge

Selected protocols

Public Key encryption and key-establishment

Kyber: an IND-CCA2-secure key-encapsulation mechanism (KEM)

Digital signature

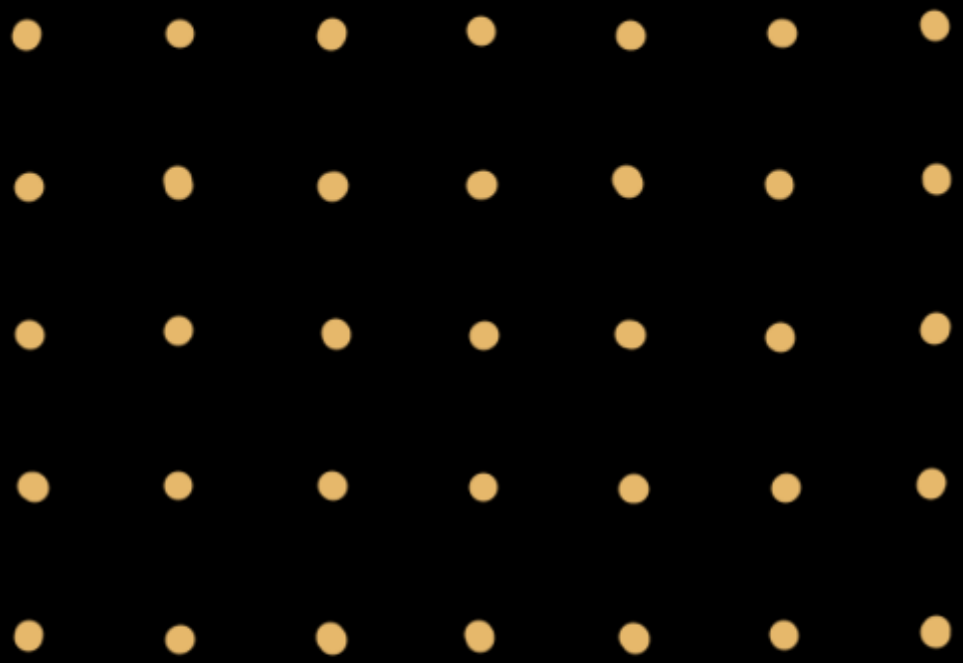
Dilithium: a strongly EUF-CMA-secure digital signature algorithm

Falcon: Fast-Fourier lattice-based compact signatures over NTRU

SPHINCS+: is a stateless hash-based signature scheme.

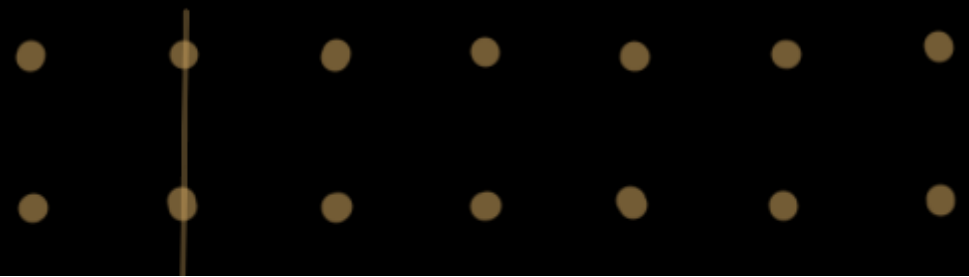
Introduction to lattice

- Infinite set of points in some N dimensional space



Introduction to lattice

- Infinite set of points in some N dimensional space
- Linear combination of vectors from a basis \({b_1, b_2, ...b_N}\) of \(\mathbb{R}^N\)
- \(L= \left\{ \sum a_{i} {b}_{i}\ :\ a_{i}\in \mathbb {Z} \right\} \)



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Fundamental hard problem

- Closest Vector Problem (CVP)
- Shortest Vector Problem (SVP)

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Fundamental hard problem

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- Shortest Vector Problem (SVP)

Learning With Error (LWE)

> Oded Regev, On lattices, learning with errors, random linear

Encrypt with Lattice

- Good basis (b_1, b_2) is the private key
- Bad basis (b_1', b_2') is the public key
- Encode bit with coordinates of a lattice point (according to public key, by instance)

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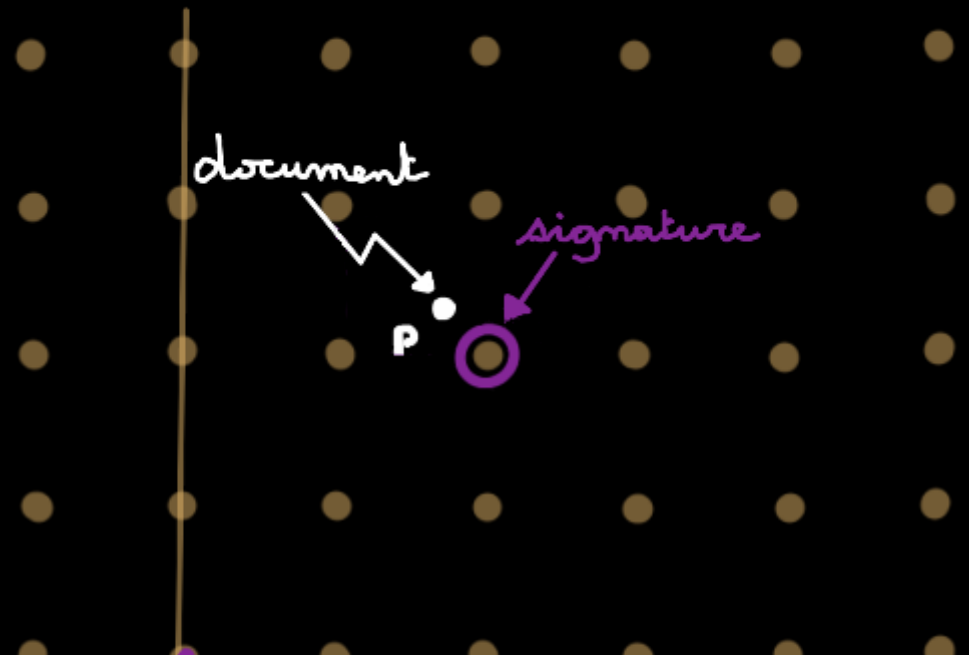
Decrypt with Lattice

- Try to surround the point
- Easy to find with good basis
- Hard to find with bad basis (or no basis)

Lattice digital signature

- Encode document with point in space
- Closest Vector is the signature

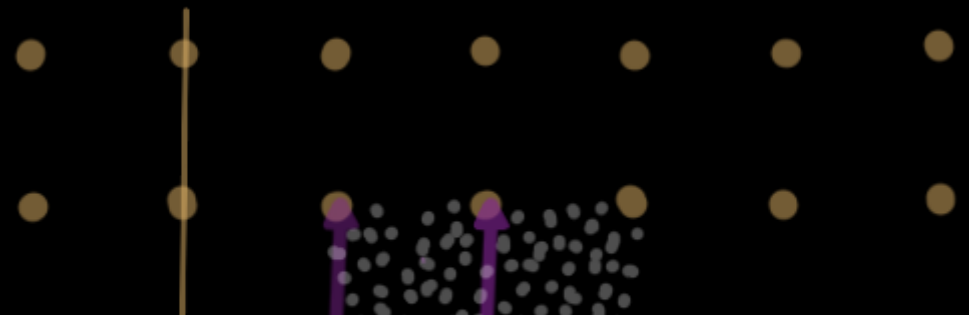


Lattice digital signature

- Encode document with point in space
- Closest Vector is the signature

Disadvantages

- Informations about private key can leak with a great number of signatures.



From lattice encryption to Kyber

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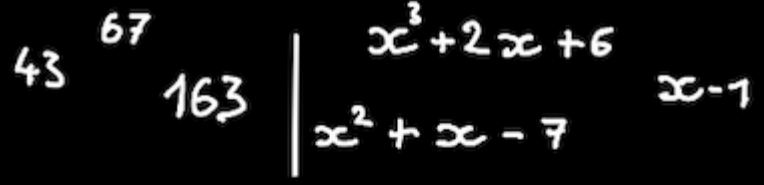
- Use polynimial rings rather than integers. Ring-LWE, then even Module-LWE (NTRU)



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$43 \quad 67 \quad 163 \;\Big|\; x^3+2x+6 \quad x-1$

x^2+x-7

We can add and multiple

$43 \times 67 = 2881 \;\Big|\; (x^2+x-7)(x-1) = x^3-8x+7$

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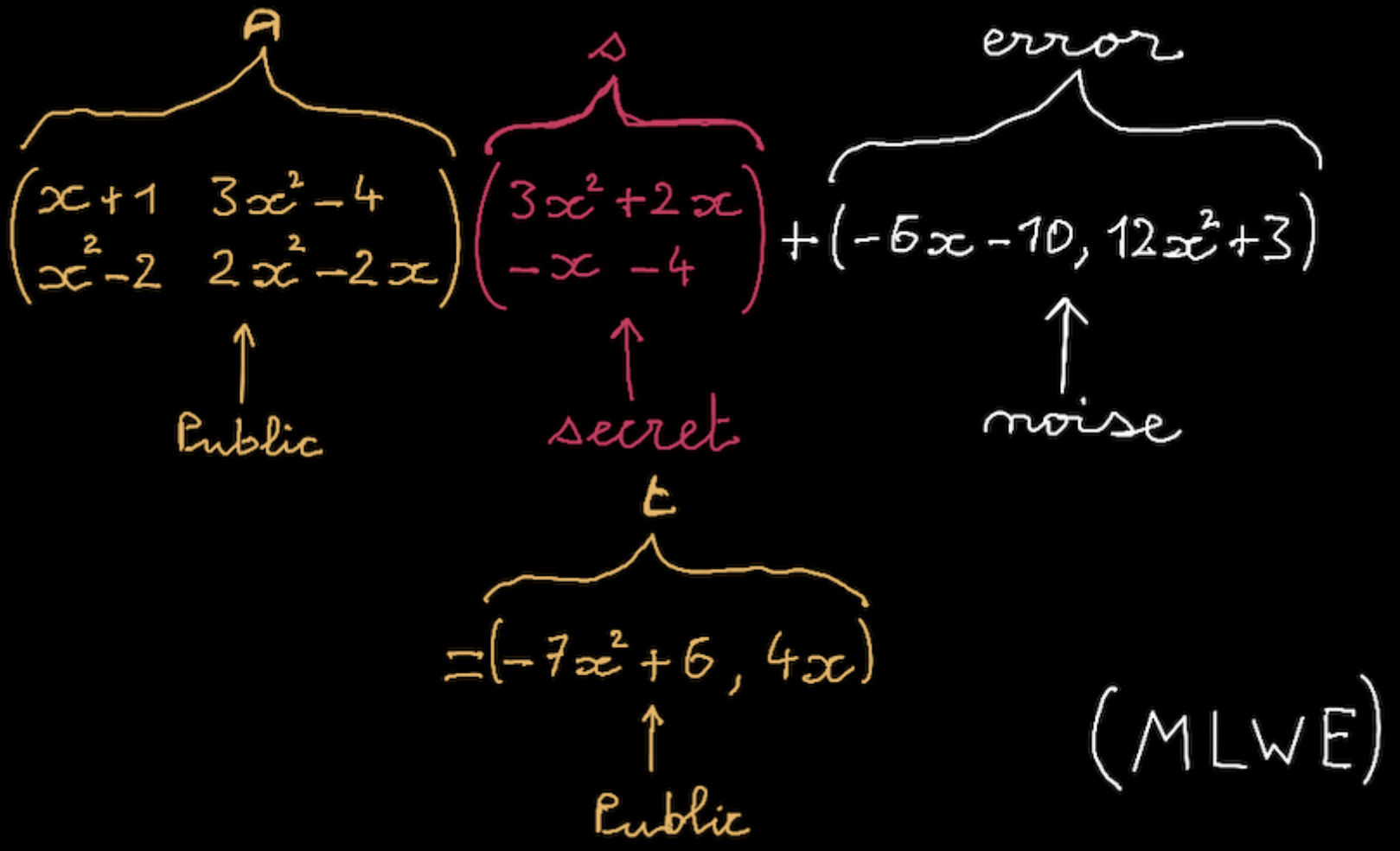
From lattice encryption to Kyber

- Use polynimial rings rather than integers. Ring-LWE, then even Module-LWE (NTRU)
- Public Key can be built using a square Matrix

$$\begin{pmatrix} 5 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} + (2,7) = (9,11)$$

Kyber matrix



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- Public Key can be built using a square Matrix
- **Secret and noise can come from the same distribution**

Kyber encryption

C

↓ ASCii to Binary

1000011

↓ Binary to Polynomial

$$m = 1x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 1x^1 + 1x^0$$

$$\underbrace{m = x^6 + x + 1}_{\text{Plaintext}}$$

Kyber encryption

$P_K = (A, t)$

$m = x^6 + x + 1$ } Plaintext

↓ Scale with large factor

$m = 1337\,m = 1337x^2 + 1337x + 1337$

m: Scaled Plaintext

↓ Add small errors *

$v = t * + * + m$

$u = A * + *$ } Ciphertext

Kyber decryption

$pk = (A, t)$

$sk = s$

$v = t * r + e_2 + m$

$u = A * r + e_1$

} Ciphertext

Kyber decryption

$pk = (A, t)$

$sk = s$

$v = t * + * + m$

$u = A * + *$

} Ciphertext

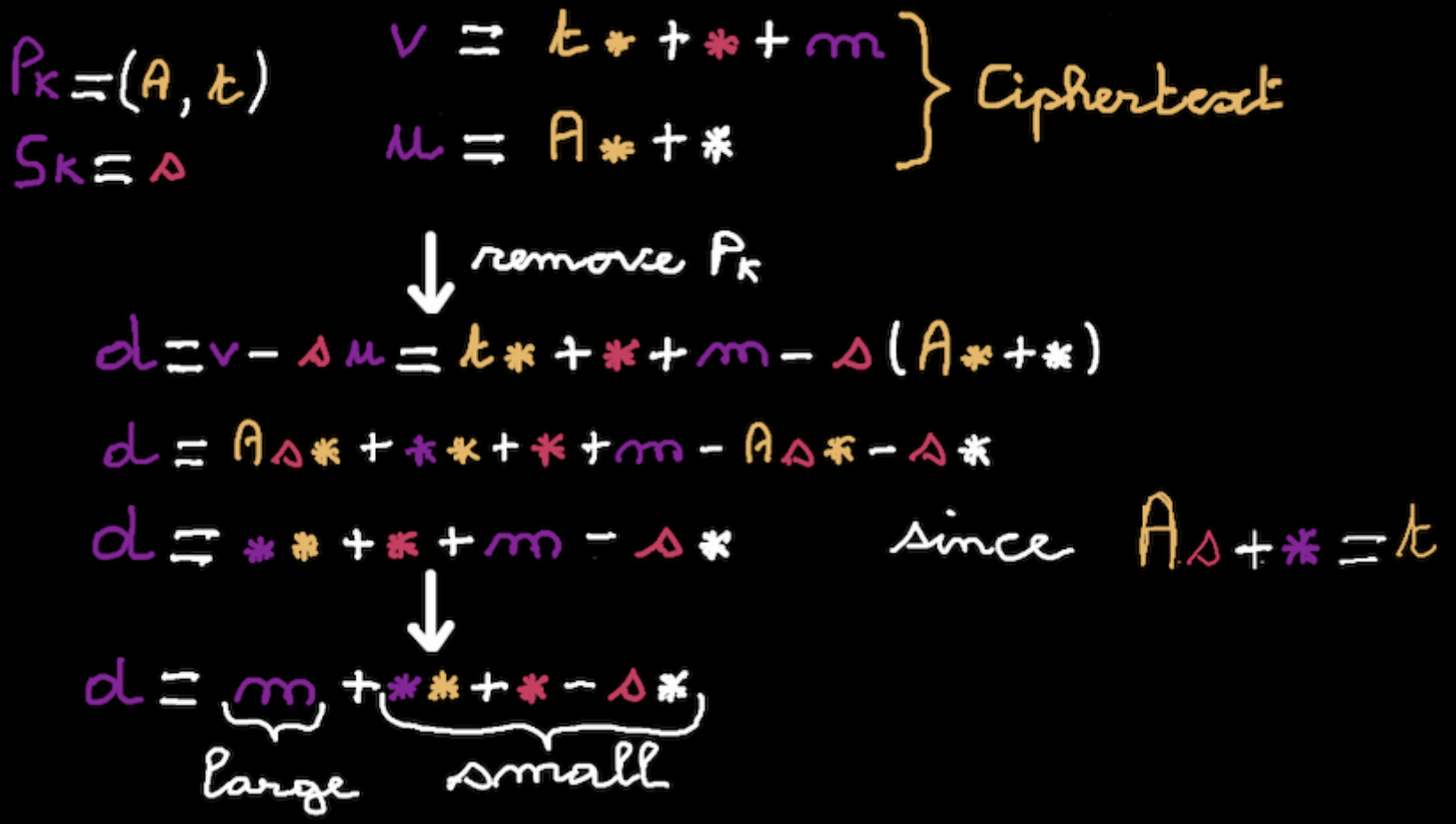
↓ remove pk

$d = v - s u = t * + * + m - s(A * + *)$

$d = A s * + * * + * + m - A s * - s *$

$d = * * + * + m - s *$ since $A s + * = t$

Kyber decryption



Kyber decryption

$pk = (A, t)$

$sk = s$

$v = t*r + *e + m$

$u = A*r + *e$

} Ciphertext

↓ remove pk

$d = v - su = t*r + *e + m - s(A*r + *e)$

$d = A*s*r + *e*r + *e + m - A*s*r - s*e$

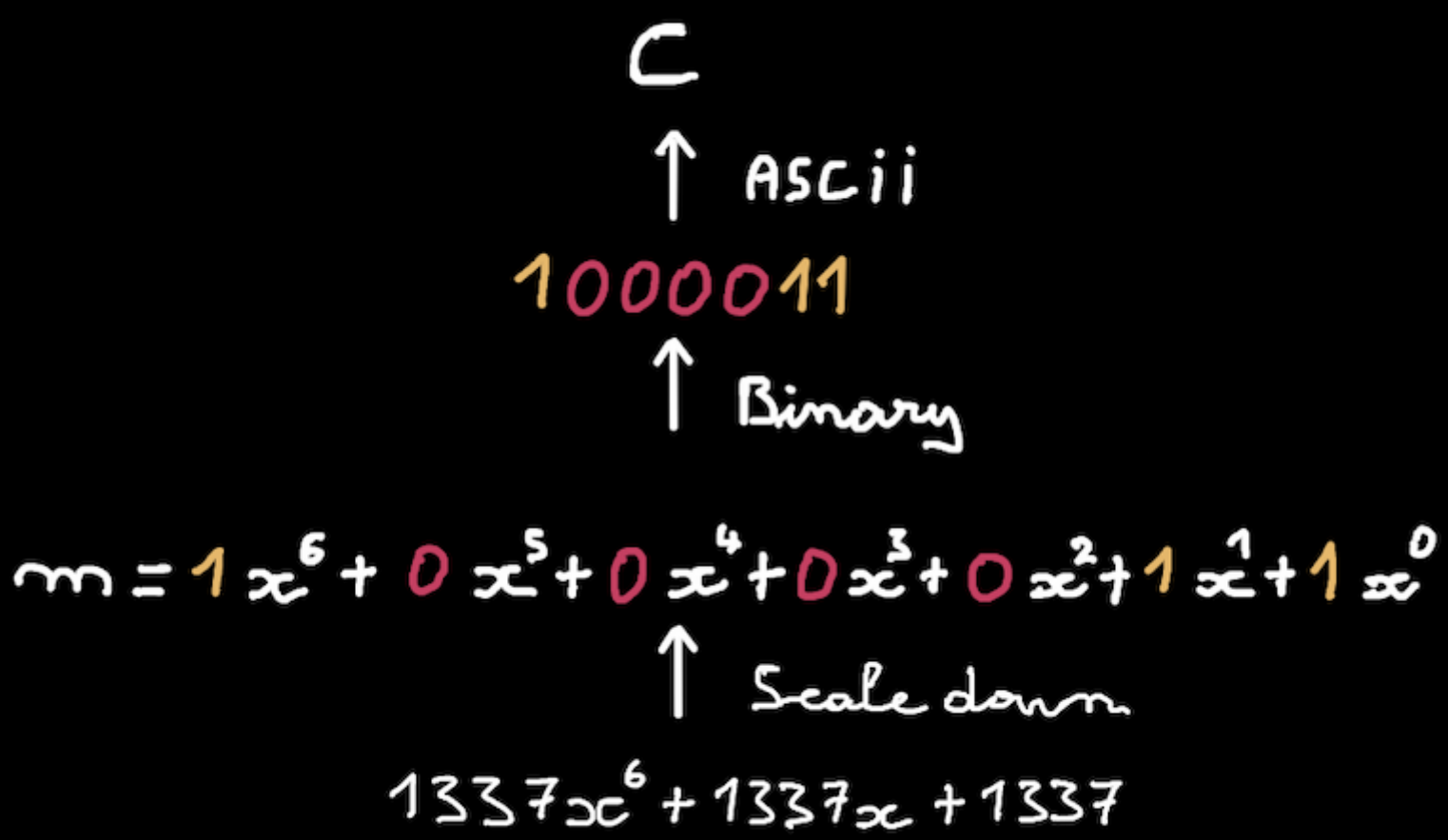
$d = *e*r + *e + m - s*e$ since $A*s + *e = t$

↓

$d = \underbrace{m}_{\text{large}} + \underbrace{*e*r + *e - s*e}_{\text{small}} = 1337x^6 + 1337x + 1337$

↑ rounding

Kyber decryption



Kyber performances

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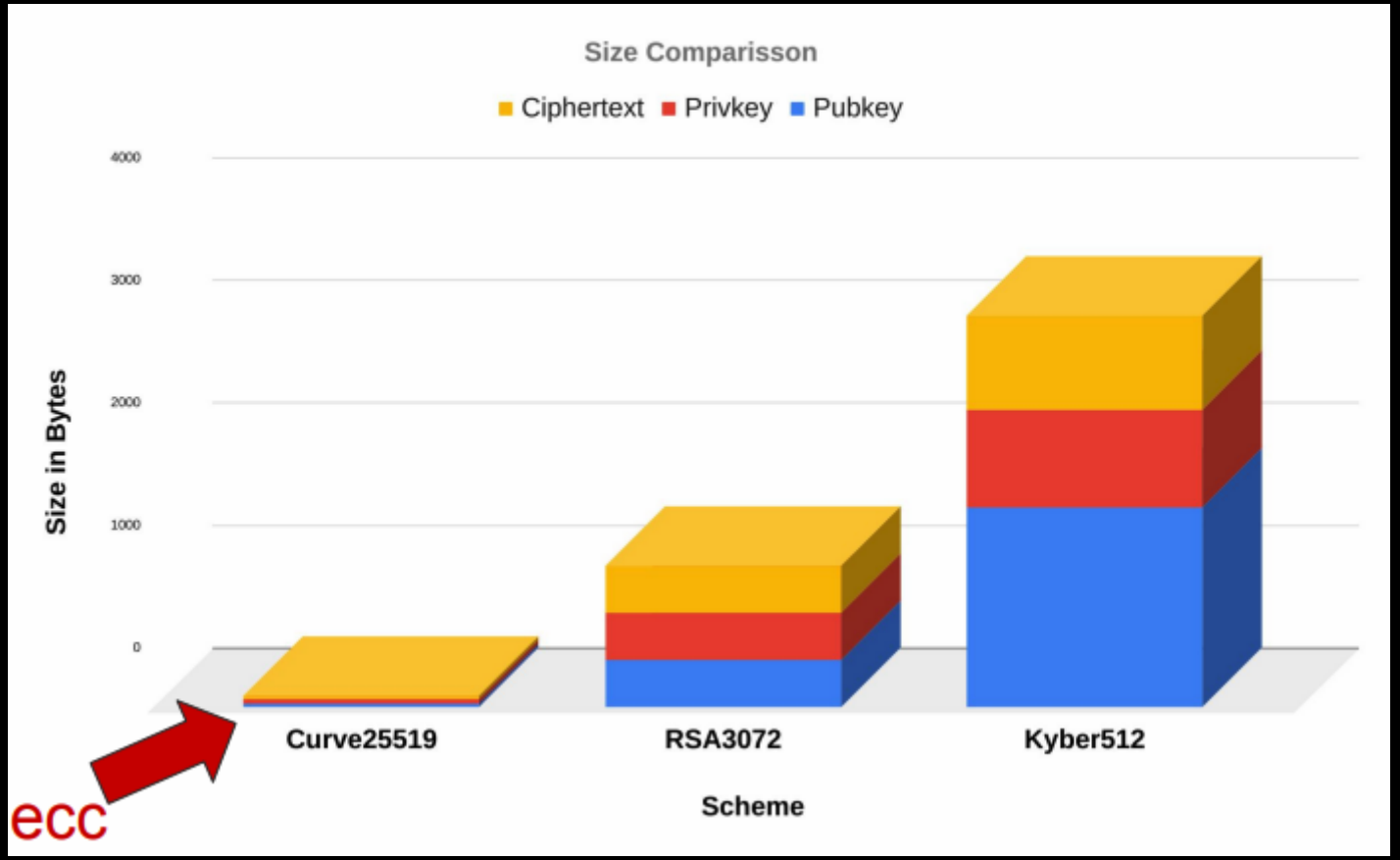

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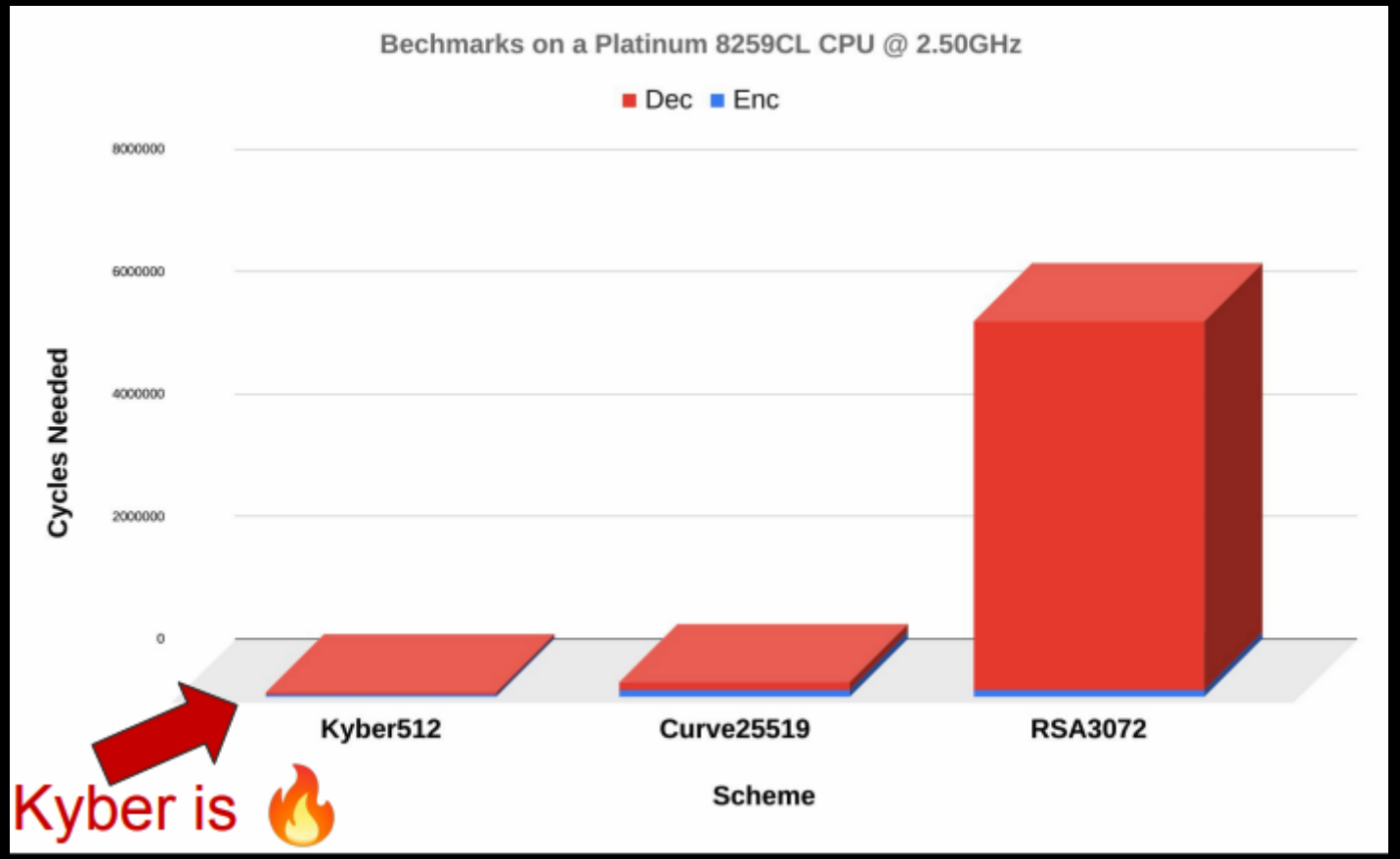

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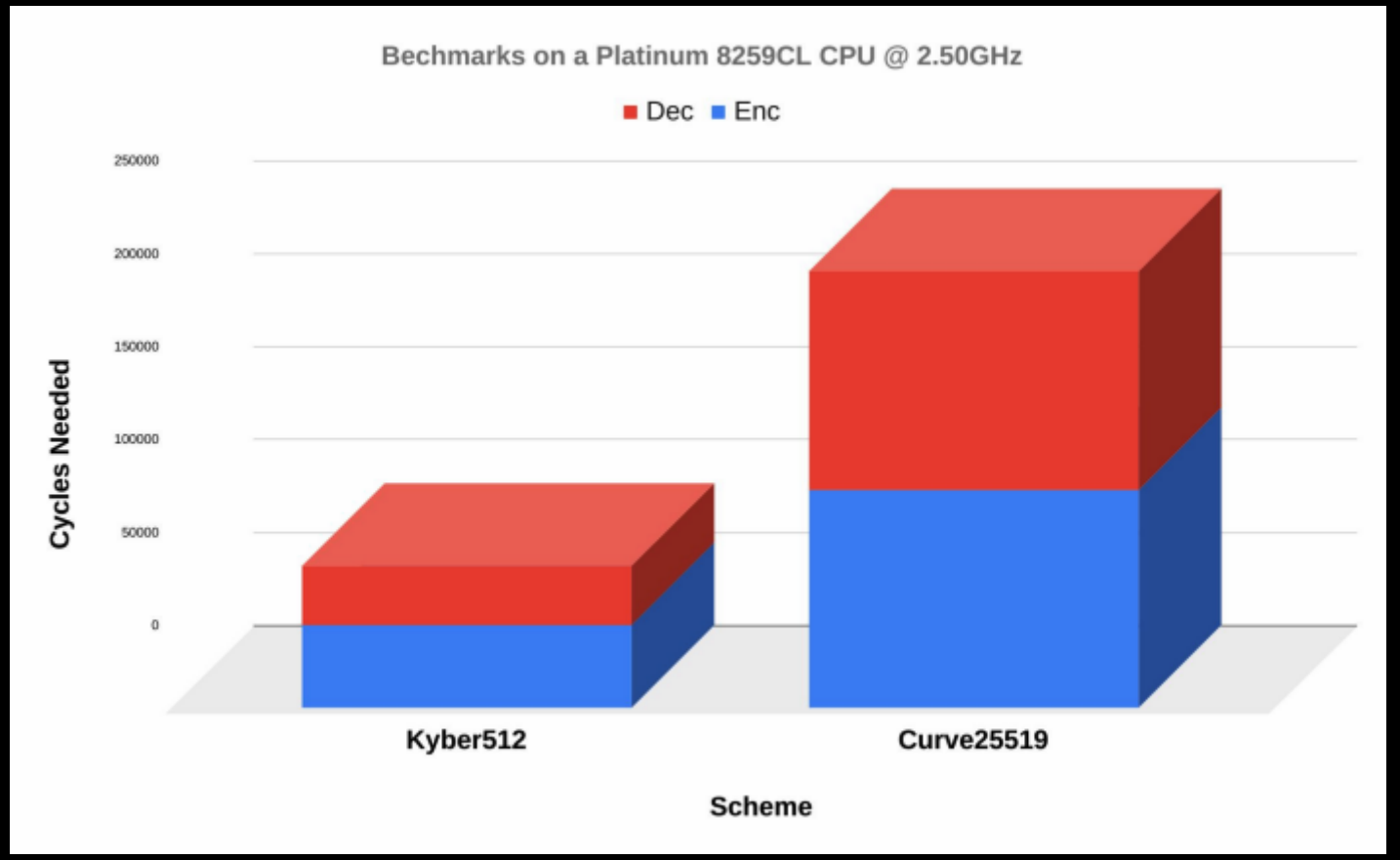

Conclusion

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Quantum computing

- Is a real threat for Cryptography and digital industry
- Is complex
  - Error management
  - Cold environment

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- Is complex
  - Error management
  - Cold environment

Lattice based protocol

- Are secure according to CVP and SVP
- Are just polynomial manipulation
- Provide efficient cryptography schemes
- Are being standardized anyway

Resources

RSA

RSA nist standardization

Quantum Computing

- IBM Composer Guide
- Blog (IBM) on factorizing 15 with Shor in 2021
- Are we doomed with Shor ?

Resources

Post quantum Cryptography

- Article de Stéphane Bortzmeyer sur l'annonce su NIST
- NIST PQC standardization page
- McKinsey - when-and-how-to-prepare-for-post-quantum-cryptography

Resources

Lattice based encryption explained

- Video on lattice based crypto
- Best video on Kyber
- Learning with Error explanation video
- Dilithium
- Kyber source code
- Réseaux euclidiens
- Course on SIS and LWE
- Course on ring LWE
- Kyber inspiration video

Learning with errors (LWE)

> Oded Regev. On lattices, learning with errors, random linear codes, and cryptography. pages 84–93, 2005.

Thank you !

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