SAIR model for COVID-19 spread analysis

Victoria Landaberry y Juan Manuel Sancho

March 2020

1 Introduction

COVID-19 has spread all over the world and different measures have been taken by countries to control its propagation. We have used the SIR (Susceptible, Infected and Recovered) and SEIR (Susceptible, Exposed, Infected and Recovered) models to analyze the dynamics of the propagation, estimate the number of infected population, the duration of the spread and the capacity of the healthcare system to provide attention to patients with severe and critical symptoms.

One of the main issues of using these models to analyze COVID-19 is that the dynamics of the virus is different from what the classical models assume, and countries have taken measures such as quarantines, affecting the final results of the spread of the virus.

In particular, the SIR model supposes that a susceptible person that is infected by entering in contact with an infected person remains infected and, thus, infecting the rest of the population during all the period that it takes to recover. The SIR model, on the other hand, does not take into account the possibility that measures such as quarantine affect the spread once that one person has been detected as infected.

On the other hand, the SEIR model considers that there is a latent period in which the infected person does not transmit the disease (during this period, it's counted as Exposed). This assumption is not aligned with the evidence we have so far about the nature of contagion. Indeed, COVID-19 is highly contagious, and according to Laurel SA et al. (March, 2020) people that get the virus are contagious since then. Also, we can have some asymptomatic cases, about 30% of the population, that are infected and transmitting the virus, but they are not aware of this situation and they are not identified as infected by statistics.

We present a SAIR model (Susceptible, Asymptomatic infected, Infected tested and Recovered or dead model) that introduces some particular aspects of COVID-19. Our goal is to adjust the estimations that we obtained using the classical models by introducing some changes in the dynamics of the virus.

2 The model

In our model we have four classes of person:

- **Susceptibles (S)**: Person without immunity to the infectious agent, susceptible to be infected.
- **Asymptomatic infected(A)**: Asymptomatic infected group considers the person that got the disease and does not have any symptoms, the person that got the disease during the period of time where it doesn't present any symptoms (latent period) , and those that, even though they have symptoms, have not been identified because they haven't been tested. As long as a person is in this group, they propagate the disease.
- **Infected tested (I)**: A person that has the disease and has been tested is identified as infected. Once an Infected person is detected, they are put in quarantine and they stop spreading the virus.
- **Recovered or dead(R)**:Person recovered, that either has immunity or has died. They come from the infected group once they are recovered or from the asymptomatic group that has no symptoms or has not been identify as infected after the time it takes to get recovered. This group also assumes that a part of the infected population will die from the disease.

Total population is equal to N (the sum of all these groups).

$$N = S + A + I + R \tag{1}$$

β in our model is the mean human contact per day that a person has times the probability of contagion. This parameter can be affected by measures taken by countries, such as social distancing or isolation. Susceptible may become Asymptomatic infected if they enter in contact with an Asymptomatic agent.

$$\frac{dS}{dt} = -\beta \times \frac{S(t) \times A(t)}{N} \tag{2}$$

Asymptomatic group will grow each period by the number of persons that were susceptible and got the disease. They will decrease at a rate $\frac{1}{tau_I}$ if they recover without having been identified (this means that they were asymptomatic during all the period or that they weren't tested). tau_I is the mean time of recovery of the disease. The group will also decrease once they present symptoms if they are tested and identified as Infected tested. This will happen at a rate $\frac{1}{tau_I - \tau_E}$, where τ_E is the mean days of latent period, when a patient does not hay any symptoms if they are going to present some after. We also introduce ω, which is the probability of being asymptomatic during all the period or having symptoms but not being identified as infected because they're not tested. Asymptomatic infected agents are the ones that spread the disease.

$$\frac{dA}{dt} = \beta \times \frac{S(t) \times A(t)}{N} - \frac{\omega}{\tau_I} \times A(t) - \frac{(1-\omega)}{\tau_I - \tau_E} \times A(t) \tag{3}$$

Finally, there is a proportion of asymptomatic cases that present symptoms and are tested. Once they are identified as infectious tested, they are in quarantine and do not transmit the disease as a result of isolation. They will be in this group until they recover or die.

$$\frac{dI}{dt} = \frac{1-\omega}{\tau_I - \tau_E} \times A(t) - \frac{1}{\tau_I - \tau_E} I(t) \tag{4}$$

Once an infected or asymptomatic recovers (or dies), they are classified in the group of Recovered and they have immunity.

$$\frac{dR}{dt} = \frac{\omega}{\tau_I} \times A(t) + \frac{1}{\tau_I - \tau_E} I(t) \tag{5}$$