WEEK 4 SUMMARY

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Joint posterior:

$$f(u,\tau,\alpha,M) \propto \exp(-\Phi(u)) \times \frac{1}{\sqrt{(2\pi)^d \det C(\tau,\alpha,M)}} \exp\left(-\frac{1}{2}\langle u, C(\tau,\alpha,M)^{-1}u\rangle\right) \times \pi_0(\tau,\alpha,M)$$

where $C(\tau,\alpha,M)$

Algorithm 1 Hierarchical on τ, α, M

Initialize $u^{(0)} = q_1$, the Fiedler vector expressed in the standard basis.
Initialize $\tau^{(0)}, \alpha^{(0)}, M^{(0)}$.
Pick $\beta \in [0,1]$. Pick ϵ_1, ϵ_2, the jump sizes for τ, α respectively. Pick K, the max absolute jump for M, with jumps Q distributed as $\mathbb{P}(Q=j) = \frac{1}{1+|j|}$.
for $k = 0$ to S **do**
  Sample v from the prior distribution and expressed in the eigenbasis ▷ $u|y,\tau,\alpha$.
  Expressing u in the eigenbasis, set a proposal $\hat{u}^{(k)} = (1-\beta^2)^{1/2}u^{(k)} + \beta v$
  Set $u^{(k+1)} = \hat{u}^{(k)}$ with probability

$$A(u^{(k)} \to \hat{u}^{(k)}) = \min\left\{1, \exp(\Phi(u^{(k)}) - \Phi(\hat{u}^{(k)}))\right\}$$

  Set a proposal $\hat{\tau}^{(k)} = \tau^{(k)} + \epsilon_1 t$ for $t \sim N(0,1)$ ▷ $\tau|y,u,\alpha$
  Set $\tau^{(k+1)} = \hat{\tau}^{(k)}$ with probability

$$A(\tau^{(k)} \to \hat{\tau}^{(k)}) = \min\left\{1, \frac{f(u^{(k+1)}, \hat{\tau}^{(k)}, \alpha^{(k)})}{f(u^{(k+1)}, \tau^{(k)}, \alpha^{(k)})}\right\}$$

(Using the eigenbasis representation of u for computation) ▷ f is the joint posterior
  Set a proposal $\hat{\alpha}^{(k)} = \alpha^{(k)} + \epsilon_2 a$ for $a \sim N(0,1)$ ▷ $\alpha|y,u,\tau$
  Set $\alpha^{(k+1)} = \hat{\alpha}^{(k)}$ with probability

$$A(\alpha^{(k)} \to \hat{\alpha}^{(k)}) = \min\left\{1, \frac{f(u^{(k+1)}, \tau^{(k+1)}, \hat{\alpha}^{(k)})}{f(u^{(k+1)}, \tau^{(k+1)}, \alpha^{(k)})}\right\}$$

end for
return $\{u^{(k)}, \tau^{(k)}, \alpha^{(k)}\}$
