Bare-Bones LaTeX Template for AIAA Technical Conference Papers

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Nomenclature

J	Jacobian Matrix
f	Residual value vector
x	Variable value vector
F	Force, N
m	Mass, kg
Δx	Variable displacement vector
α	Acceleration, m/s^2

Subscript

i	Variable number

I. Feasible Path Generation

This section introduces an approach that supports mission planning and task execution steps by a fleet of multiple cooperative UAVs. As soon as the mission objectives are defined by the high level cognitive components, the set of specific tasks targeting an application of each of the UAVs in the fleet, needs to be designed. The task of each UAV in a cooperative mission should specify the trajectory (both the path and the velocity profile) such that the mission objectives are achieved, each of the UAVs and their payloads do not exceed the flight dynamics or operational constraints, and that the deconfliction and collision avoidance conditions are met. Thus, this section outlines a path generation approach that is suitable for near real-time computation of feasible trajectories for multiple UAVs that are de-conflicted in space and that can be followed by resorting to the cooperative path following algorithm briefly described later in the paper.

Consider a fleet of n UAVs that are tasked to start from different locations (initial conditions) and arrive at the mission required final conditions. The exact time of arrival is not specified, but it may be restricted to lie within certain bounds.Suppose the objective of the task for each UAV is to execute this multi-vehicle mission while avoiding inter-vehicle collisions, meeting dynamical constraints (e.g. bounds on maximum accelerations), and minimizing a weighted combination of vehicle energy expenditures.

At first inspection, a possible solution to this problem would be to solve a constrained optimization problem that would yield (if at all possible) feasible trajectories $p_{c_i}(t), t \in [0, t_f]; i = 1, 2, ..., n$ for the

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vehicles, where t_f denotes the final time. Trajectory tracking systems on-board the UAVs would then ensure precise tracking of the trajectories generated, thus meeting the mission objectives.

This seemingly straightforward solution suffers from a major drawback: it does not allow for any "deviations from the plan". Absolute timing becomes crucial because the strategy described does not lend itself to on-line modification in the event that one or more of the vehicles cannot execute trajectory tracking accurately (e.g. due to adverse winds or lack of sufficient propulsion power). For this reason, it is far more practical to adopt a different solution where absolute time is not crucial and enough room is given to each vehicle to adjust its motion along the path in response to the motions of the other vehicles.

Dispensing with absolute time is key to the solution proposed. In this set-up, the optimization process should be viewed as a method to produce paths $p_{c_i}(\tau_i)$ without explicit time constraints, but with timing laws for $\tau_i(t)$ that effectively dictate how the nominal speed of each vehicle should evolve along the path. Using this set-up, spatial and temporal constraints are decoupled and captured in the descriptions of $p_{c_i}(\tau_i)$ and $\eta_i(\tau_i) = d\tau_i/dt$, respectively, as will be seen later. Furthermore, adopting polynomial approximations for $p_{c_i}(\tau_i)$ and $\eta_i(\tau_i) = d\tau_i/dt$ keeps the number of optimization parameters reduced and makes near real-time computational requirements easy to achieve. Intuitively, by making the path of a generic vehicle a polynomial function of $\tau \in [0, \tau_f]$, the shape of the path in space can be changed by increasing or decreasing τ - a single optimization parameter. This, coupled with a polynomial approximation for $\eta(\tau) = d\tau/dt$, makes it easy to shape the speed and acceleration profile of the vehicle along the path so as to meet desired dynamical constraints. The paths thus generated are the "templates" used for path following, as explained later in the paper.

The above circle of ideas was first explored in Refs.[?,?,?,?] for a single aircraft. This paper extends these results to the case of multiple UAVs following earlier work by the authors reported in Ref.[?] As will be seen, the approach to path generation exploits decoupling of spatial and temporal specifications. Let $p_c(\tau) = [x(\tau), y(\tau), z(\tau)]^\top$ denote a desired path to be followed by a single UAV, parameterized by $\tau \in [0, \tau_f]$. For computational efficiency, assume each coordinate $x(\tau), y(\tau), z(\tau)$ is represented by an algebraic polynomial of degree N of the form

$$x_i(\tau) = \sum_{k=0}^{N} a_{ik}\tau^k, \qquad i = 1, 2, 3, \tag{1}$$

where we set $x_1 = x, x_2 = y, x_3 = z$ for notational convenience. The degree N of polynomials $x_i(\tau)$ is determined by the number of boundary conditions that must be satisfied. Notice that these conditions (that involve spatial derivatives) are computed with respect to the parameter τ. There is an obvious need to relate them to actual temporal derivatives, but this issue will only be addressed later. Let d_0 and d_f be the highest-order of the spatial derivatives of $x_i(\tau)$ that must meet specified boundary constraints at the initial and final points of the path, respectively. Then, the minimum degree N^* of each polynomial in (1) is $N^* = d_0 + d_f + 1$. For example, if the desired path includes constraints on initial and final positions, velocities, and accelerations (second-order derivatives), then the degree of each polynomial is $N^* = 2 + 2 + 1 = 5$. Explicit formulae for computing boundary conditions $p_c'(0), p_c''(0)$ and $p_c'(\tau_f), p_c''(\tau_f)$ are given later in this section. Additional degrees of freedom may be included by making $N > N^*$. As an illustrative example, Table 1 shows how to compute the polynomial coefficients in (1) for polynomial trajectories of 5^{th} and 6^{th} degree. For 6^{th} degree polynomial trajectories, an additional constraint on the fictitious initial jerk (third-order derivatives) is included, which increases the order of the resulting polynomial and affords extra (design) parameters $x_i'''(0); i = 1, 2, 3$. Fig. 1 shows examples of admissible 5^{th} and 6^{th} order polynomial paths when only τ_f or τ_f and $x_i'''(0); i = 1, 2, 3$, viewed as optimization parameters, vary. Fig. 1 (right) shows how an increase in the number of optimization parameters leads to a larger class of admissible paths (in this particular case, parameters corresponding to initial jerk are added as free variables).

It is important to clarify how temporal constraints may be included in the feasible path computation process. A trivial solution would be to make $\tau = t$. In this case, solving the polynomial fitting problem that is at the root of Fig. 1 yields the speed profiles of Fig. 2. Little control exists over the resulting speeds even with fifth and sixth order polynomials, because once $x_1(t), x_2(t), x_3(t)$ have been computed to satisfy the boundary constraints imposed, speed v is inevitably given by

$$v(t) = \sqrt{\dot{x}_1^2(t) + \dot{x}_2^2(t) + \dot{x}_3^2(t)}. \tag{2}$$

We therefore turn our attention to a different procedure that will afford us the possibility of meeting strict boundary conditions and constraints without increasing the complexity of the path generation process.

To this effect, let $v_{\min}, v_{\max}$ and $a_{\max}$ denote predefined bounds on the vehicle's speed and acceleration, respectively. Let $\eta(\tau) = d\tau/dt$, yet to be determined, dictate how parameter τ evolves in time. A path $p_c(\tau)$ (with an underlying assignment $\eta(\tau)$) is said to constitute a *feasible* path if the resulting trajectory can be tracked by an UAV without exceeding pre-specified bounds on its velocity and total acceleration along that trajectory. With an obvious use of notation, we will later refer to a spatial path only, without the associated $\eta(\tau)$, as a feasible path.

From (2), and for a given choice of $\eta(\tau)$, the temporal speed $v_p(\tau(t))$ and acceleration $a_p(\tau(t))$ of the vehicle along the path (abbv. $v_p(\tau)$ and $a_p(\tau)$, respectively) are given by

$$\begin{aligned} v_p(\tau) &= \eta(\tau)\sqrt{x_1'^2(\tau) + x_2'^2(\tau) + x_3'^2(\tau)} = \eta(\tau)||p_c'(\tau)||, \\ a_p(\tau) &= = ||\ddot{p}_c(t)|| = ||p_c''(\tau)\eta^2(\tau) + p_c'(\tau)\eta'(\tau)\eta(\tau)||. \end{aligned} \tag{3}$$

Table 1. Examples of computation of the coefficients of 5^{th} and 6^{th} order polynomial paths.

5^{th} order	
Boundary conditions	$x_i(0), x_i'(0), x_i''(0), x_i(\tau_f), x_i'(\tau_f), x_i''(\tau_f)$
d_0/d_f	2/2
N^*/N	5/5
Linear algebraic matrix equation to solve for the coefficients a_{ik}	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & \tau_f & \tau_f^2 & \tau_f^3 & \tau_f^4 & \tau_f^5 \\ 0 & 1 & 2\tau_f & 3\tau_f^2 & 4\tau_f^3 & 5\tau_f^4 \\ 0 & 0 & 2 & 6\tau_f & 12\tau_f^2 & 20\tau_f^3 \end{bmatrix} \begin{bmatrix} a_{i0} \\ a_{i1} \\ a_{i2} \\ a_{i3} \\ a_{i4} \\ a_{i5} \end{bmatrix} = \begin{bmatrix} x_i(0) \\ x_i'(0) \\ x_i''(0) \\ x_i(\tau_f) \\ x_i'(\tau_f) \\ x_i''(\tau_f) \end{bmatrix}$
6^{th} order	
Boundary conditions	$x_i(0), x_i'(0), x_i''(0), x_i'''(0), x_i(\tau_f), x_i'(\tau_f), x_i''(\tau_f)$
d_0/d_f	3/2
N^*/N	5/6
Linear algebraic matrix equation to solve for the coefficients a_{ik}	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 1 & \tau_f & \tau_f^2 & \tau_f^3 & \tau_f^4 & \tau_f^5 & \tau_f^6 \\ 0 & 1 & 2\tau_f & 3\tau_f^2 & 4\tau_f^3 & 5\tau_f^4 & 6\tau_f^5 \\ 0 & 0 & 2 & 6\tau_f & 12\tau_f^2 & 20\tau_f^3 & 30\tau_f^4 \end{bmatrix} \begin{bmatrix} a_{i0} \\ a_{i1} \\ a_{i2} \\ a_{i3} \\ a_{i4} \\ a_{i5} \\ a_{i6} \end{bmatrix} = \begin{bmatrix} x_i(0) \\ x_i'(0) \\ x_i''(0) \\ x_i'''(0) \\ x_i(\tau_f) \\ x_i'(\tau_f) \\ x_i''(\tau_f) \end{bmatrix}$

In this paper we are interested in small UAVs that operate essentially at constant speeds. Clearly, in this case speed constraints can be easily satisfied for any constant $v_p \in [v_{\min}, v_{\max}]$. This in turn defines

$$\eta(\tau) = \frac{v_p}{||p_c'(\tau)||} \tag{4}$$

$$\dot{p}_c(t) = v_p \frac{p_c'(\tau)}{||p_c'(\tau)||} \tag{5}$$

Now using (3, 4) we obtain

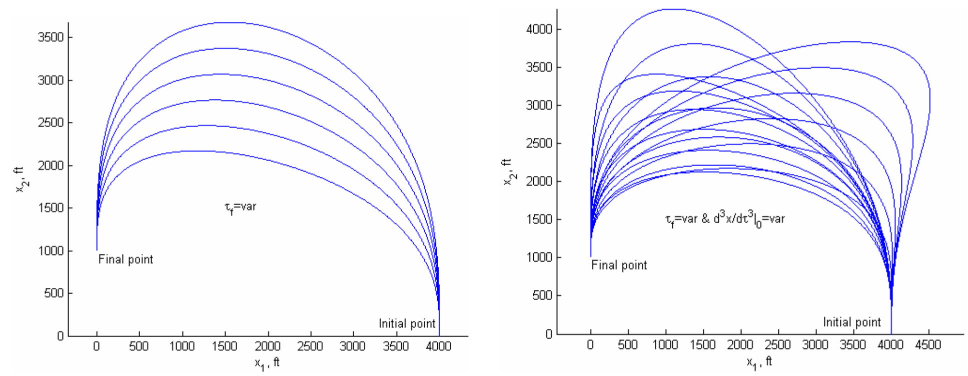

Figure 1. Admissible trajectories for 5^{th} and 6^{th} order polynomials.

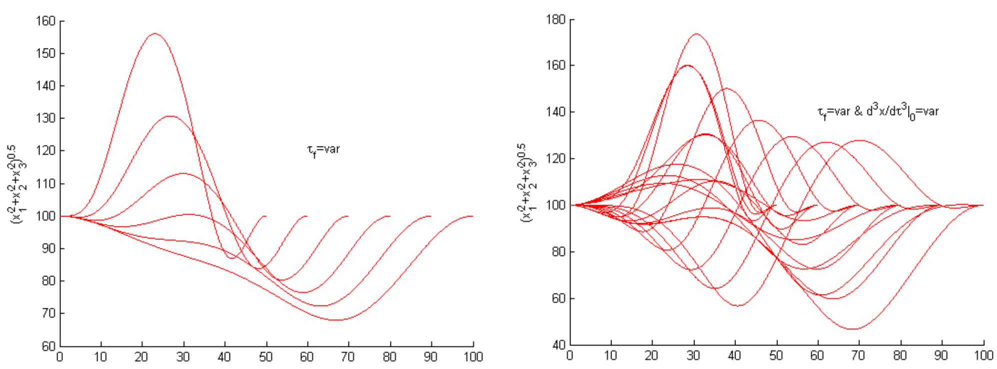

Figure 2. Speed profile corresponding to the paths shown in Figure 1 when $\tau = t$. Left: varying τ_f. Right: varying τ_f and the initial jerk.

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$$\ddot{p}_c(t) = \frac{v_p^2}{||p_c'(\tau)||^2}(I - \frac{p_c'(\tau)(p_c'(\tau))^T}{||p_c'(\tau)||^2})p''(\tau). \quad (6)$$

Therefore, we can choose

$$p_c'(0) = \frac{\dot{p}_c(0)}{||\dot{p}_c(0)||} \quad (7)$$

$$p_c'(\tau_f) = \frac{\dot{p}_c(t_f)}{||\dot{p}_c(t_f)||}. \quad (8)$$

to satisfy boundary conditions on $\dot{p}_c(t)$. Similarly, setting

$$\begin{aligned} p_c''(0) &= \ddot{p}_c(0) \\ p_c''(\tau_f) &= \ddot{p}_c(t_f), \end{aligned}$$

satisfies equation (6) at the boundaries.

On the other hand, the total acceleration a_p of a vehicle flying along the path $p_c(\tau)$ at a constant speed is the product of the curvature of the path with its velocity along the path squared. The curvature of the path $p_c(\tau)$ is given by

$$\kappa(\tau) = \frac{1}{||p'(\tau)||}||\frac{d}{d\tau}\frac{p_c'(\tau)}{||p_c'(\tau)||}||.$$

Thus, using simple algebra it can be shown that

$$\begin{aligned} a_p(\tau) &= v_p^2\kappa(\tau) \\ &= \frac{v_p^2}{||p_c'(\tau)||^2}||(I - \frac{p_c'(\tau)(p_c'(\tau))^T}{||p_c'(\tau)||^2})p''(\tau)||, \end{aligned}$$

which as expected is the norm of $\ddot{p}_c(t)$ (see equation (6). Therefore, for the case of constant velocities v_p a feasible path must satisfy the following set of constraints

$$v_{\min} \leq v_p \leq v_{\max}, \quad a_p(\tau) \leq a_{\max}, \ \forall\tau \in [0, \tau_f]. \quad (9)$$

for a pre-specified acceleration bound $a_{\max}$.

In this paper, we use the freedom afforded by a simple definition of a feasible path above to address the problem of time-coordinated control of multiple UAVs whereby all UAV's must arrive at their respective final destinations at the same time. The approach proposed here seeks to find a set of feasible paths that makes the coordinated arrival time problem easily solvable by a team of UAVs flying at constant speeds along these paths while coordinating their motion using the underlying communication network. In particular, these paths should be designed in such a way as to guarantee the simultaneous arrival by all UAVs at their respective destinations. Next, we make these ideas more precise.

Let l_{fi} denote the total path length of the ith path and v_{p_i} denote its velocity along this path. Then

$$l_{fi} = \int_0^{\tau_{fi}} ||p_{c_i}'(\tau_i)||\ d\tau_i.$$

It follows immediately that the time of flight t_{fi} of UAV i is given by

$$t_{fi_{\min}} = \int_0^{\tau_{fi}} \frac{||p_{c_1}'(\tau_i)||}{v_{\mathrm{p}_i}}\ d\tau_i.$$

Define a cost function $J = (\max_i t_{fi} - \min_i t_{fi})^2$. Then, making J arbitrarily small over the set of feasible paths, feasible velocities and accelerations will result in the desired solution to the simultaneous arrival problem discussed above. Therefore, we propose to solve the following path generation problem

$$
F1 : \begin{cases} \min\limits_{\tau_{fi}, v_{p_i},\, i=1,\ldots,n} \{J\} \\ \\ \quad \textit{subject to} \\ \\ \textit{boundary conditions and } (9) \textit{ for any } i \in [1, n] \\ \\ \min\limits_{j,k=1,\ldots,n, j\neq k} ||p_{c_j}(\tau_j) - p_{c_k}(\tau_k)||^2 \geq E^2 \textit{ for any } \tau_j, \tau_k \in [0, \tau_{fj}] \times [0, \tau_{fk}], \end{cases} \tag{10}
$$

Solution to the optimization problem $F1$ includes a set of n optimal paths and constant speed profiles that together minimize J subject to an additional constraint that these paths must be spatially separated by at least E meters: $\min\limits_{j,k=1,\ldots,n, j\neq k} ||p_{c_j}(\tau_j) - p_{c_k}(\tau_k)||^2 \geq E^2$ *for any* $\tau_j, \tau_k \in [0, \tau_{fj}] \times [0, \tau_{fk}]$ Clearly, this constraint guarantees collision avoidance.

The optimization problem $F1$ can be effectively solved in real-time by adding a penalty function G as discussed in Ref.[?] and by using any zero-order optimization technique. As an example, Fig. **??** illustrates the flexibility afforded by the reference polynomials to compute a coordinated target reconnaissance mission by three UAVs. In this case, the vector of optimization parameters is $\Xi = [\tau_{f1} \;\; \tau_{f2} \;\; \tau_{f3} \;\; v_{p_1} \;\; v_{p_2} \;\; v_{p_3}]$. The final value of the cost function $J = 1.6968e - 006$ corresponds to $|\max_i t_{fi} - \min_i t_{fi}| \leq \;\; 0.0013 \;\; sec$. The value of the optimization parameter vector $\Xi_{final} = [4010.0 \;\; 4999.7 \;\; 7487.6 \;\; 15.1380 \;\; 21.2238 \;\; 29.8054]$ resulted in spatially deconflicted paths where the minimum distance between any two paths did not fall below 350 m (the minimum required distance was 100 m). The optimal speed profiles [15.1380 m/s 21.2238 m/s 29.8054 m/s] are well within predefined limits of $v_{\min}, v_{\max}$ were $15 m/sec$ and $30 m/sec$, respectively. The maximum acceleration corresponding to each path did not exceed $0.89 m/sec^2$, well below the limit of $0.5g$. Finally, the resulting total path lengths for each path were $[l_{f1} \;\; l_{f2} \;\; l_{f3}] = [4535.4 \;\; 6358.7 \;\; 8929.7]$.

References

[1] Rebek, A., *Fickle Rocks*, Fink Publishing, Chesapeake, 1982.