EQUATIONS OF MOTION

Lagrange Method (Energy Method)

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = F_i$$

where:

- $T =$ ***Kinetic Energy***
- $U =$ ***Potential Energy***
- $D =$ ***Dissipative Energy***
- $F_i =$ ***Externally applied force/moment***
- $q_i =$ ***Generalized coordinate***

EQUATIONS OF MOTION

Lagrange Method (Energy Method)

Generalized Coordinates

$$q = x,\ y,\ z,\ \theta_x,\ \theta_y,\ \theta_z$$

Kinetic Energy

$$T = \sum_{i=1}^{N_m} \frac{1}{2}\ M_i\ \dot{x}^2 + \sum_{i=1}^{N_j} \frac{1}{2}\ J_i\ \dot{\theta}^2$$

Potential Energy

$$U = \sum_{i=1}^{N_k} \frac{1}{2}\ K_i\ x^2 + \sum_{i=1}^{N_m} \pm\ (\ M_i\ g\ )\ x$$

Dissipative Energy

$$D = \sum_{i=1}^{N_c} \frac{1}{2}\ C_i\ \dot{x}^2$$

EQUATIONS OF MOTION

Lagrange Method (Energy Method)

Typical problems with Lagrange formulation:

- ***Be sure to establish number of degree of freedom first and formulate all energy terms in only those variables. Clearly identify which degrees of freedom are relative coordinates versus absolute coordinates. Watch out for rotational/translational problems.***
- ***For kinetic energy terms, be sure to formulate*** *absolute* ***velocities before taking derivatives. Watch out for 2-D and 3-D vector motions.***
- ***For potential energy terms, be sure that the actual deflection, described by relative and/or absolute coordinates, in spring elements is described. Watch out for 2-D and 3-D vector motions.***
- ***There should be only one total kinetic energy equation and one total potential energy equation for the system. The kinetic and potential energy equations should involve only the N generalized coordinates and the constants (mass, damping, stiffness) of the system.***
- ***Apply the Lagrange Equation once for each generalized coordinate. For N degrees of freedom, N generalized coordinates will yield N equations of motion.***
- ***If necessary, linearize equations of motion by neglecting nonlinear terms in the equations of motion. Note that the linear equations of motion may not adequately describe the original equations of motion.***