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PANEL METHODS IN COMPUTATIONAL FLUID DYNAMICS

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INTRODUCTION

Apparently it is a truism in the field of computational fluid dynamics that the more generally the physics of the flow is modeled by a method, the less general is the boundary geometry to which the method is applicable. Part of the difficulty lies in the stringent requirements on the calculational grid in the field of flow. According to the above observation, the simplest meaningful flow physics—that of inviscid, incompressible potential flow—should be applicable to the most general geometries. Indeed, as is well known, this problem can be formulated as a linear integral equation over the boundary, thus eliminating the need for a field grid and allowing the possibility that a potential-flow method formulated in this way can be capable of obtaining flow solutions about completely arbitrary configurations. Happily, this generality substantially can be achieved in practice, although with more analytical and programming effort than most publications on the subject are prepared to admit. At the same time, the predictions of such methods have been found to agree well with experiment over a surprisingly large range of flow conditions. Even when their results fail to give the proper experimental values, they are frequently useful in predicting the incremental effect of a proposed design change or in ordering various designs in terms of effectiveness. This perhaps fortuitous agreement with real flow, combined with their geometric generality, has made numerical potential-flow methods indispensable design tools in many fields. For example, in the author's company a major design calculation (e.g. flow

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about a complete airplane) is performed by such a method approximately 10 times per day. Other facilities have similar frequencies of use.

Integral-equation methods for solving potential-flow problems became feasible with the advent of internally programmed digital computers. For the first half-dozen years or so, their effectiveness was proven by considering two-dimensional and axisymmetric problems (Smith & Pierce 1958). However, three-dimensional methods have been of principal concern for the past 25 years (Hess & Smith 1964). [A complete historical bibliography is contained in Hess (1986).] During most of this period, the main development effort has been carried out at aircraft companies, and the main problem of interest has been that of the steady flow about a three-dimensional lifting configuration in a uniform stream of what otherwise is an unbounded fluid. This is designated herein the three-dimensional lifting problem. For definiteness, attention is directed to this problem in order to compare and contrast the various theoretical formulations and numerical implementations that have been employed to solve the potential-flow problem. This discussion is the principal aim of the present article. A secondary purpose is to describe various modifications that have been made to the method in order to apply it to more general potential-flow problems.

Originally, investigators referred to these methods as surface singularity methods to contrast them with the older approximate internal singularity techniques. Soon, however, their discretization of the body surface into small quadrilaterals (Figures 5–7 below) led to their designation as panel methods, and this has become the commonly accepted name in the fluid-dynamics community. Later, it was realized that a similar approach could be used in other problems governed by linear partial differential equations. This later work, much of it on two-dimensional problems, has come to be called the boundary-element method, but the older fluid-dynamic designation is used here.

THREE-DIMENSIONAL LIFTING PROBLEM

In attempting to construct a potential-flow representation of the problem of steady three-dimensional lift, the investigator is obliged to make certain somewhat arbitrary decisions that render the result a potential-flow model of lifting flow rather than something more fundamental (Hess 1974). Figure 1 shows a relatively simple configuration, which contains all the essential features of the problem, and also displays the commonly accepted model of three-dimensional lift. The body consists of a lifting portion (e.g. a wing) having a sharp trailing edge from which issues a zero-thickness trailing vortex wake and along which a Kutta condition of finite flow is applied. A nonlifting portion (e.g. a fuselage) lacks a trailing edge, and no

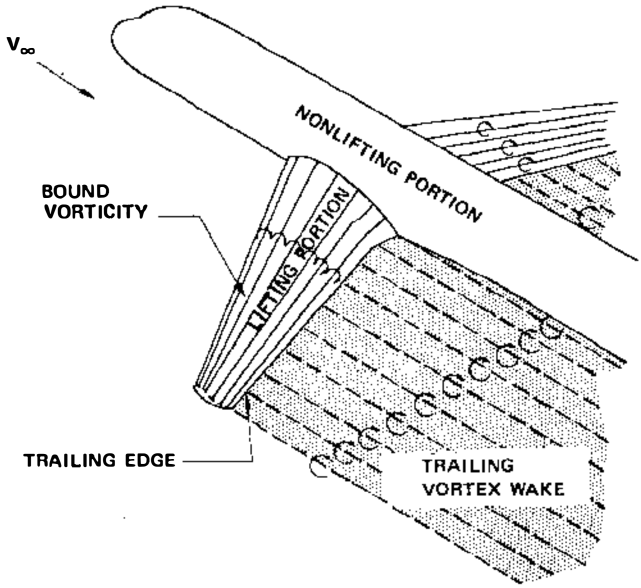

Figure 1 Potential-flow model of three-dimensional lifting flow.

Kutta condition is applied. Arbitrary decisions must be made: For example, where does the trailing edge end at the wing tip and at the wing-fuselage juncture? What happens at the rear of the fuselage in the absence of a Kutta condition? Experience has shown that the model illustrated in Figure 1, despite being somewhat arbitrary, does give useful results in a large number of flow situations.

The potential-flow problem for steady three-dimensional lift is defined by the following conditions: (*a*) The velocity potential satisfies Laplace's equation outside of the body and wake, (*b*) the disturbance due to the body vanishes at infinity, (*c*) the normal component of velocity is zero on the body, (*d*) the Kutta condition is satisfied along the trailing edge, and (*e*) the wake is a stream surface of the flow with equal pressure on both sides. The last condition indicates that the wake position is initially unknown and must be computed as part of the solution. In the vast majority of situations the flow quantities of interest are not sensitive to details of the wake shape, and an assumed shape is used.

It is well known (Lamb 1932) that any quantity satisfying Laplace's equation may be written as an integral over the boundary surface S of a source distribution σ per unit area and a normal dipole distribution μ per unit area. In this case, if $\mathbf{v}$ represents the disturbance velocity field due to the body (the difference between local velocity and freestream velocity $\mathbf{U}$), then $\mathbf{v} = \text{grad}\,\phi$, where ϕ is the disturbance potential and may be expressed as

$$\phi = \iint_{\text{body}} \left[\frac{1}{r}\sigma + \frac{\partial}{\partial n}\left(\frac{1}{r}\right)\mu\right] dS + \iint_{\text{wake}} \frac{\partial}{\partial n}\left(\frac{1}{r}\right)\mu\, dS. \tag{1}$$

Here r is the distance from the point where ϕ is being evaluated to an integration point on the surface of the body or wake, and n is the distance normal to S. Note that the wake is represented by a dipole distribution in Equation (1) rather than by the vorticity distribution of Figure 1 and the above discussion. The equivalence of vorticity and dipole distributions is derived in the report form of Hess (1974), where it is shown that the vorticity strength depends on certain derivatives of the dipole strength.

The potential ϕ as given by Equation (1) identically satisfies conditions (*a*) and (*b*) above for any functions σ and μ. Moreover, once conditions (*d*) and (*e*) have been satisfied, the function μ on the wake [second integral in Equation (1)] is uniquely determined by the values of the trailing edge. Thus the functions σ and μ on the body [first integral in Equation (1)] must be determined from the zero-normal-velocity boundary condition [condition (*c*)] and the Kutta condition [condition (*d*)]. Since the latter of these applies only on a single curve, the essence of the situation is that two functions σ and μ of position on the body are available for satisfying a single boundary condition over the body surface. Thus, there is a non-uniqueness that must be eliminated by arbitrarily imposing another condition over the body surface, and with certain restrictions, either σ or μ or a combination thereof may be specified and the remaining function or combination used to satisfy the boundary condition.

Application of the boundary condition results in an integral equation for the remaining unknown function. Applying the zero-normal-velocity boundary condition $\mathbf{n}\cdot\text{grad}\,\phi = -\mathbf{n}\cdot\mathbf{U}$ on the body involves differentiating Equation (1) and taking the limit as the point where ϕ is evaluated approaches the body. The limiting process usually produces a Fredholm equation of the second kind (Hess & Smith 1966). No matter how the singularities are chosen, the exterior flow field is theoretically unique independent of the individual values of source and dipole strength. The particular relation of σ and μ chosen to eliminate the above nonuniqueness may be called the singularity "mix." It is given principal emphasis in many theoretical presentations, the implication being that this choice is the most important determinant of the efficiency and accuracy of the resulting potential-flow method. In fact, it is less important than other theoretical considerations, such as the Kutta condition, and even certain details of the numerical implementation. Since all choices of formulation or implementation are ultimately dictated by the need to achieve favorable numerics, these choices are discussed in the following section.

NUMERICAL IMPLEMENTATION

Basic Discretization

To obtain a numerical solution of the governing integral equation, two separate discretizations are necessary: that of the body and wake geometries, and that of the singularity distribution. In fluid-dynamic applications the universal choice for the first of these is to represent the body and wake surfaces by a large number of small quadrilateral regions, which in most methods are planar. Figure 2 shows the discretization of the configuration shown in Figure 1. The resulting faceted or paneled appearance of the body has led this type of method to be called a panel method and the individual quadrilaterals to be denoted panels.

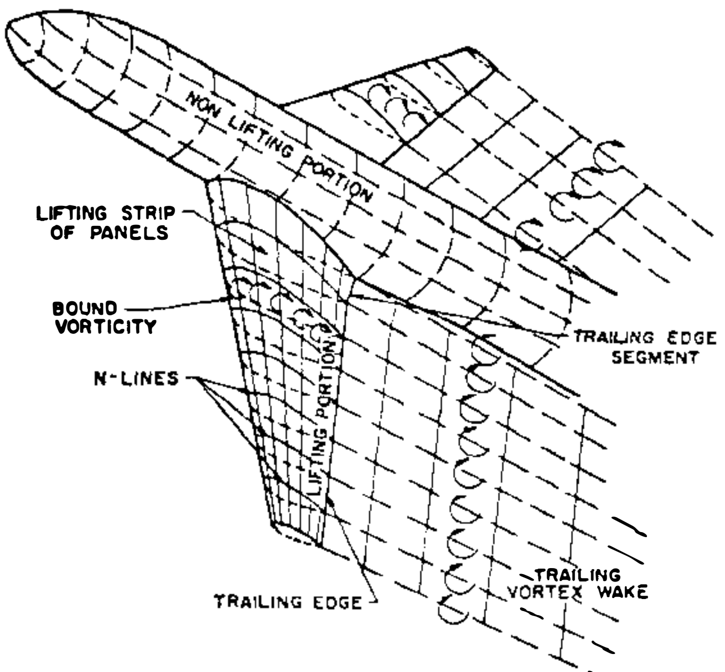

Figure 2 Numerical discretization of the three-dimensional lifting problem.

The simplest approximation for the singularities is that they be constant over each panel, but even if the representation is more elaborate, the logic of the procedure ensures that the singularity distribution can be expressed in terms of one unknown value per panel plus one unknown value for each segment into which the trailing-edge is divided (Figure 2). The desired numerical solution is obtained by enforcing (*a*) the zero-normal-velocity boundary condition at one point on each panel (called the control point) and (*b*) the Kutta condition at each trailing-edge segment. This gives a number of equations equal to the number of unknowns. Because of the linearity of the representation of Equation (1), those equations representing the normal-velocity condition are linear, whereas those representing the Kutta condition may or may not be, depending on what form is used. The coefficient matrix of the linear equations for the zero-normal-velocity boundary condition is the set of influences of the panels at each other's control points, which are obtained by integrating the corresponding point singularity formulas [Equation (1)] over each panel. This is called the matrix of influence coefficients. Calculation of this matrix and solution of the system of equations are the two major computational tasks of the panel-method procedure. Application of the Kutta condition does not add significantly to the computational effort because the number of trailing-edge segments is small compared with the number of control points. Once the singularities are known, the fluid-dynamic quantities, both on and off the body, may be calculated in a straightforward manner.

The decision to use panels having finite singularity strengths per unit area to effect the numerical integration, rather than a more widely known technique such as quadratures, was dictated by the fact that in practical cases the limit of small panel size cannot be approached in the sense that panel dimensions cannot be made small compared with all physical dimensions of the body. For example, the wing of a transport aircraft has a thickness of the order of 10% of its chord and a spanwise extent of perhaps 10 times the chord of a typical airfoil section. Reducing the panels' spanwise extents to make them small compared with the thickness would require several hundred lifting strips (Figure 2) and result in a totally impractical calculation. Typically 10–20 lifting strips are employed, and a panel's spanwise dimension often exceeds 50% chord. The use of panels ensures that certain important quantities are correct even if the small-panel limit is not approached. For example, as a point approaches the true curved body the normal velocity approaches 2π times the local source strength. The same thing happens with a panel, but it certainly would not happen with a quadrature, which uses point singularities.

The above does not imply lack of numerical convergence. For all but the most complex cases, convergence can be demonstrated by halving all panel dimensions and noting that the calculated results do not change significantly. This is an important advantage of the panel method over a field method. For the latter, in order to obtain a reasonable answer at all, the full capacity of the computer often must be used.

Singularity Mix

As discussed above, there is a nonuniqueness in the integral-equation formulation, and thus an arbitrary condition must be applied to the source density σ and/or the dipole density μ in order to remove it. Clearly this may be accomplished in many ways, but the methods, in general, have used either of two possible formulations. These may be designated the source method (Hess et al. 1985) and the Green's Identity method (Johnson 1980, Maskew 1982), which was first formulated by Morino (Morino & Luo 1974). This list is not intended to be complete but rather to include the methods in common use at the present time. A rather complete bibliography, including the historical development, is contained in the review by Hess (1986).

In the source method, each panel has an independent value of source density. The dipole distribution is taken as zero on the nonlifting portion of the configuration, and on the lifting portion it is replaced by a set of distributions, each of which has an independent value on one lifting strip of panels and is zero on the other lifting strips (Figure 2). Thus there is an unknown dipole strength for each trailing-edge segment, and in the final solution these represent, for example, the spanwise load distribution on the wing. The normal-velocity boundary condition is applied in a straightforward manner by setting the exterior normal velocity to zero at the panel control points. Thus the matrix of influence coefficients is a vector matrix of velocities whose order equals the total panel number. Advantages claimed for the source method are robustness, insensitivity to irregularities in panel distribution, and the extremely well-conditioned nature of the matrix of influence coefficients.

As the name indicates, the Green's Identity method uses the singularity mix suggested by that theorem, where the dipole strength equals the exterior surface value of perturbation potential, and the source strength its normal derivative. Accordingly, the dipole strength is taken as independent on each panel, and the source strength is set equal to $-\mathbf{n}\cdot\mathbf{U}$. It can be shown that under these circumstances the dipole distribution that makes the total exterior normal velocity on the surface zero also gives zero for the perturbation potential in the interior. Accordingly, the boundary condition is applied in this alternate fashion by setting the interior perturbation potential equal to zero at the panel control points. Thus, velocities due to the individual panels need not be calculated, and the matrix of influence coefficients is a scalar matrix, which requires only one-third the storage of a vector matrix. This is the main advantage claimed for this approach. However, for portions of the configuration where flow conditions are extreme, such as flaps and slats on wings, recent investigators (Tinoco et al. 1987) have concluded that for good results, *both* types of boundary conditions should be applied. Thus, not only are scalar and vector matrices required for portions of the configuration, but since two equations per panel must be satisfied in these regions, the order of the system of equations is increased. An obvious objection to this formulation is that since the solution consists only of values of potential at control points, the velocity and thus pressure must be obtained by the equivalent of numerical differentiation. However, no difficulties due to this cause have been reported. There is no doubt that the interior potential boundary condition leads to a greater sensitivity of the solution to panel irregularities, particularly geometrical mismatches between edges of adjacent panels. This approach definitely handles interior flows much better than the first-order source method but, it appears, not as well as the higher order source method (see below).

Kutta Condition

In lifting flows the circulation distributions on the lifting portions drive the entire solution. Accordingly, accurate determination of these circulations is crucial. Values of circulation are determined mainly from the Kutta condition along the trailing edge, and thus specification of the Kutta condition is more important than any other detail of the numerical implementation.

The theoretical form of the Kutta condition states that the velocity shall remain finite all along the sharp trailing edge of a lifting portion, i.e. a wing (Figure 1). Such a condition cannot be imposed in a numerical method, and an equivalent alternative condition must be found. Physically, it is evident that the limiting values of the pressure as the trailing edge is approached along the upper surface and along the lower surface must be equal. This equal pressure serves very well as a numerical form of the Kutta condition and is used in the source method (Hess 1974, Hess et al. 1985). Since pressure contains the square of velocity, which is linear in the source and dipole strengths, the equal-pressure Kutta condition is quadratic in these strengths. This nonlinearity has led to its rejection by most investigators, who have selected a linear alternative to the Kutta condition, so that the entire set of equations to be solved will be linear. A variety of linear conditions involving wake direction or dipole continuity have been used in the various Green's Identity methods. Some are clearly approximate, but all can lead to a nonphysical pressure mismatch at the trailing edge. Figure 3 (Margason et al. 1985) shows calculated chordwise

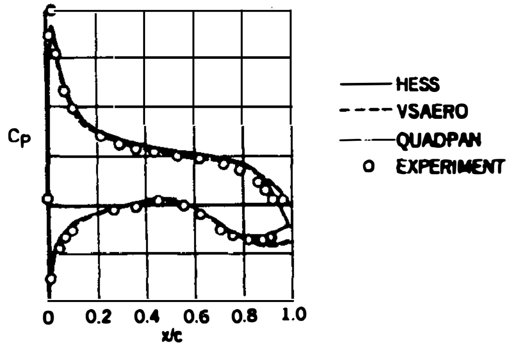

Figure 3 Mismatch in trailing-edge pressures due to alternate Kutta conditions.

pressure distributions on a wing—one using the source method with an equal-pressure Kutta condition (Hess 1974), and two using Green's Identity methods with a linear Kutta condition. The latter two calculations exhibit a pressure mismatch at the trailing edge of about one half of the freestream dynamic pressure. Remarkably, the pressure at upstream locations seems little affected, at least in this case.

Solution of the Equations

The matrix of influence coefficients representing the zero-normal-velocity boundary condition is very well conditioned. However, when the linear Kutta-condition equations are added, the resulting matrix is not well conditioned. If the full set of equations is solved by a direct-elimination method, there is no numerical difficulty, but the computing times become prohibitively long if the panel number is much greater than 2000. An itcrative mcthod makcs panel numbers in excess of 10,000 quite feasible, but it must be a block iterative method with the ill-conditioned Kutta-condition equations embedded in the blocks that are solved directly. Considerable investigation is necessary to find methods that converge rapidly for all geometries of interest, but most panel-method developers have done this, and the resulting procedures are quite efficient.

In the source method, the sources of a lifting strip are grouped into a block together with the dipole strength for that strip. The associated equations are the zero-normal-velocity equations for panels of that strip plus the Kutta condition at the trailing-edge segment. On nonlifting portions the division is into 50×50 diagonal blocks. The nonlinear equations representing the Kutta conditions are linearized by the well-known Newton-Raphson procedure at each iteration. Thus, the Kutta-condition equations may be thought of as linear equations whose coefficients vary as the iteration proceeds. To obtain fast convergence, the iterative procedure is block Gauss-Seidel with an acceleration scheme (Clark 1985). Convergence is usually obtained within 20 iterations.

Matrix of Influence Coefficients

As mentioned above, the influences of the panels at each other's control points, either potential or velocity, are obtained by integrating the corresponding quantities for a point source and a point dipole over the panel. If the panel is planar, or at least piecewise planar, this can be done analytically and specific formulas obtained, which depend on the various panel dimensions and the location of the control point with respect to the panel. The resulting formulas are rather elaborate because they contain the effects of every detail of the panel geometry. Such precision is necessary for points near the panel, but at faraway points the exact formulas are overaccurate and computationally expensive. Accordingly, various approximate formulas are used for calculational efficiency. Use of such formulas need not involve the loss of any accuracy at all in the final solution, because errors due to this source can be made small compared with the basic approximation of the body by panels and the singularities by various polynomials. Some methods employ several ranges and use different formulas in each, but all methods replace the panel by a point source and a point dipole of the same total strengths for faraway points. In the source method, the point-singularity formulas are used if the control point is farther than four times the maximum panel dimension from the panel's control point. In large-panel-number cases this results in at least 95% of the influence matrix being calculated by the point-singularity formulas, with a corresponding reduction in computation time by an order of magnitudc.

Higher Order Approach

It is generally agreed that the use of plane panels having constant values of source and dipole strength constitutes a first-order or low-order panel method. There is disagreement, however, as to what constitutes a higher order panel method. Some investigators take the position that any refinement of the first-order approach is a higher order method. In particular, if a method uses higher degree polynomials on a plane panel, it is commonly referred to as a higher order method. However, a formal mathematical consistency analysis (Hess et al. 1985) shows that effects due to source or vorticity derivatives (linear variation) have the same mathematical order as the effect of panel curvature. (As mentioned above, a linear vorticity variation is equivalent to a quadratic dipole variation.) This is not to say that the two give equal effects in all cases, but it is easy to produce cases where their effects are comparable (Hess 1973) or even cases for which the curvature effects dominate (Hess 1975). It is true that only for flat panels can the panel influences be obtained in closed form by analytic integration. However, if one uses the expansions implied by a mathematical consistency analysis, the curvature term can, in effect, be projected into a tangent-plane panel and the integrations performed analytically there.

Experience and a variety of published results have shown that first-order methods are perfectly satisfactory for the vast majority of flows, including practically all exterior flows. It appears that the only two situations where a higher order analysis might be called for are internal flows, where a channel or duct of decreasing cross section can greatly accelerate the fluid, and cases of strong lift interaction, where lifting bodies in close proximity significantly affect each other's lift. Both effects are present in the ring wing or nacelle shown in Figure 4 (Miranda 1985). It consists of a symmetric NACA airfoil section 10% thick, rotated about an axis parallel to its chord line and displaced from it by 10% chord. The onset flow is parallel to the axis, so the flow is axisymmetric. Thus, the minimum area is one-fourth the entrance and exit areas. In the absence of lift interaction, i.e. for small chord-to-diameter ratios, the lifting effect is zero, but it is large indeed for the case shown. This body was selected several years ago by D. R. Bristow to be a particularly difficult case for a first-order source method. Figure 4 shows four calculated internal pressure distributions: one obtained by a highly accurate axisymmetric solution (Hess 1975) that

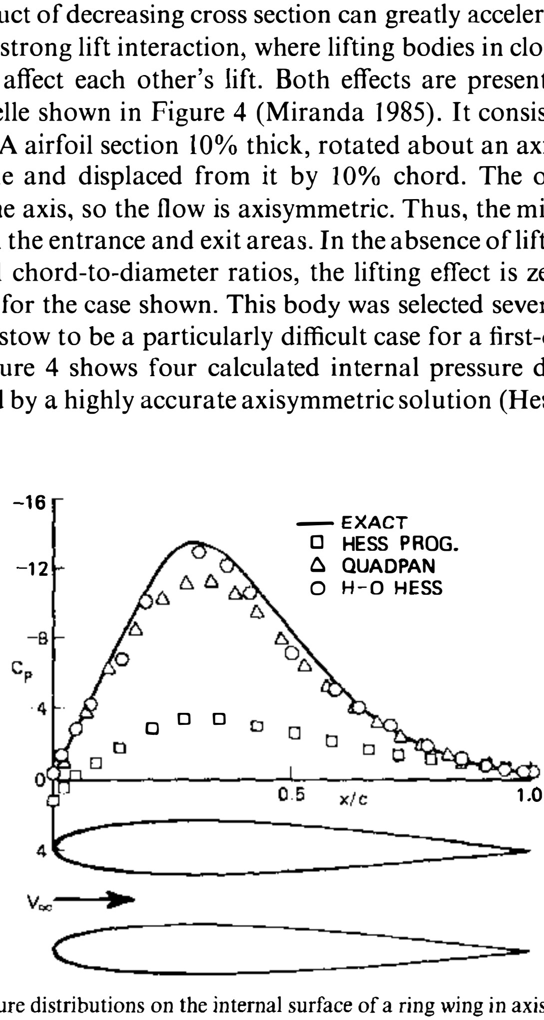

Figure 4 Pressure distributions on the internal surface of a ring wing in axisymmetric flow.

was refined until numerical convergence was obtained ("exact" in Figure 4), and three from three-dimensional panel methods [first-order source (Hess Prog.), first-order Green's Identity (QUADPAN), and higher order source (H-O Hess)]. The failure of the first-order source method is quite dramatic. The higher order source method is substantially exact and a significant improvement over the Green's Identity method.

APPLICATION OF THE STEADY LIFTING METHOD

As mentioned in the introduction, some major aerospace companies apply a panel method to large design cases about 10 times per day. Most of these cases are steady three-dimensional lifting problems. In the late 1970s, maximum panel numbers were typically about 1000. (In this section, panel number always refers to thosc used on half of the body on one side of the symmetry plane.) As a result, it was customary to calculate flow about a portion of the total configuration because the panel-number limit did not permit accurate representation of a complete airplane. Figure 5 shows a complete transport aircraft represented by 1000 panels; the inadequacy of the paneling is evident on the fuselage and even more evident on the vertical tail and aft nacelle. With the efficiency improvements that have been made over the years, particularly the iterative matrix solution, the panel-number limits have been raised until a complete aircraft can be modeled very adequately. Figure 6 shows a complete fighter aircraft (minus vertical tail) in high-lift configuration (i.e. 25° leading-edge droop and a detached trailing-edge flap deflected 20°) represented by 4000 panels. Similarly, Figure 7 shows a transport aircraft with 7000 panels. The latter

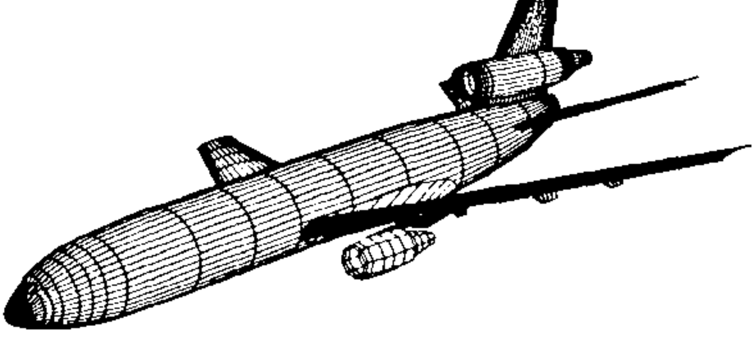

Figure 5 A 1000-panel representation of a transport aircraft.

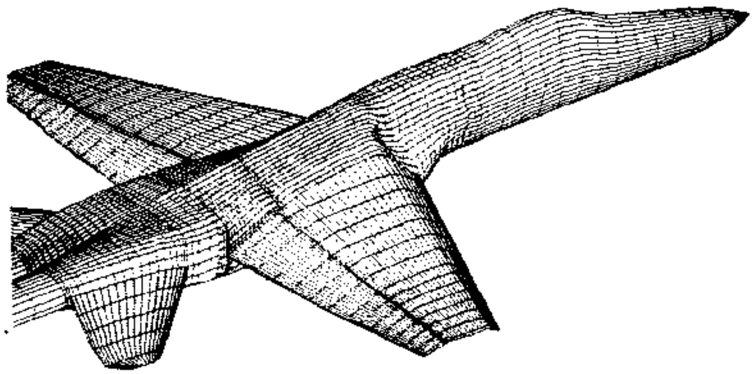

Figure 6 A 4000-panel representation of a fighter aircraft.

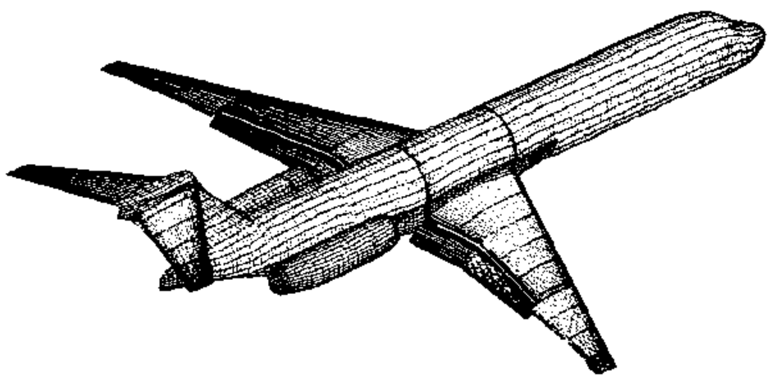

Figure 7 A 7000-panel representation of a transport aircraft.

panel number permits adequate representation of all aircraft components, including independent lifts, Kutta conditions, and wakes on the wing, detached flap, pylon, flow-through nacelle, and horizontal tail. Configurations such as these are routine overnight calculations. For example, at the author's facility the source panel method with 5000 panels requires about 40 minutes of CPU time on an IBM 3090.

The solutions obtained for configurations such as those shown are quite accurate, and any of them could display calculated pressure distributions indistinguishable from experimental data, as innumerable publications have done. In design work, many quantities of interest typically possess a lesser degree of accuracy, and it seems more appropriate to show such a comparison here. Figure 8 shows calculated and experimental lift increments due to the horizontal tail for the configuration of Figure 6. The

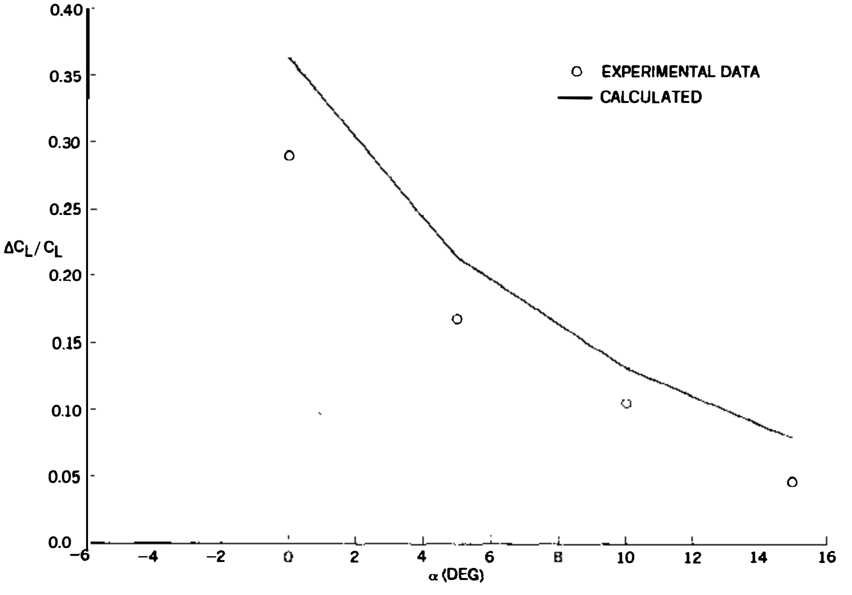

Figure 8 Increment in lift due to the horizontal tail for a fighter aircraft.

trend with angle of attack α is well predicted, but the level of the calculated results is too high. Perhaps this can be explained by the neglect of viscosity. Nevertheless, the calculated results shown in Figure 8 are perfectly adequate for most design purposes.

Many successful design applications involve flow conditions apparently beyond the range of validity of the panel method. E. N. Tinoco (unpublished communication) reports an effort to design the installation of a nacelle a very short distance beneath a wing in such a way that the shock losses were minimized at cruise conditions. Despite the transonic nature of the problem, it was analyzed using an incompressible panel method. The configuration that achieved the lowest local velocities incompressibly also had the weakest shock strengths.

Since the underlying formulation of the panel method is very general, it is customary for investigators—the author among them—to claim that their newly developed panel method can handle completely arbitrary configurations. This is true in principle but not in practice. When bodies having some new fluid-dynamic or geometric feature are encountered, a method will frequently fail because some implicit assumption in the numerical formulation is no longer valid. After a method has been applied to a large variety of problems, many difficulties will have arisen and one by one overcome by suitably improving the procedure. A panel method that has been heavily used in design for several years can fairly claim to have encountered almost every such difficulty and thus, indeed, be applicable to "arbitrary" bodies. A new method, or one that has been applied only occasionally, would have little or no chance of successfully calculating cases such as those of Figures 6 and 7.

GENERALIZATIONS OF THE PANEL METHOD

Panel methods are applicable to any fluid-dynamic problem governed by Laplace's equation. While the problem of steady, three-dimensional lift has received more attention, other situations are also of interest, and almost from the beginning, investigators have been engaged in modifying and generalizing panel methods to these problems. A selection of such methods is discussed in this section. No attempt at completeness has been made, partly because such an attempt would undoubtedly fail. Instead, the modifications discussed below are samples of what has been done. Some are rather mature efforts, whereas others merely show promise of what might be accomplished. Only three-dimensional problems are included.

Propellers

Propellers have proven a fruitful field of investigation by panel methods. Kerwin and his colleagues have extended their vortex-lattice code (Greeley & Kerwin 1982) to the case of shrouded propellers using a Green's Identity panel method (Kerwin et al. 1987). The source panel method was applied to isolated propellers (Hess & Valarezo 1985) and then to problems of propeller-airframe interference (Valarezo & Hess 1986). It has recently been extended to counterrotating propellers by W. O. Valarezo, and to wing-mounted tractor propellers by R. W. Clark (unpublished results). Figure 9 compares computed and experimental thrust loadings and efficiencies for a counterrotating propeller (CR). Also shown are the thrust results for a single rotor at the same advance ratio. Figure 10 shows an eight-bladed tractor propeller mounted on a wing of rectangular planform, and Figure 11 shows the distribution of section lift coefficient on the wing versus span location measured from the propeller axis, both with and without the effect of the propeller. The magnitude of the power effect is predicted satisfactorily.

Transonic Applications

The compressible full-potential equation may be written with a Laplacian operator on the left and various nonlinear terms on the right of the equal sign. It is natural to attempt to solve this equation iteratively by assuming

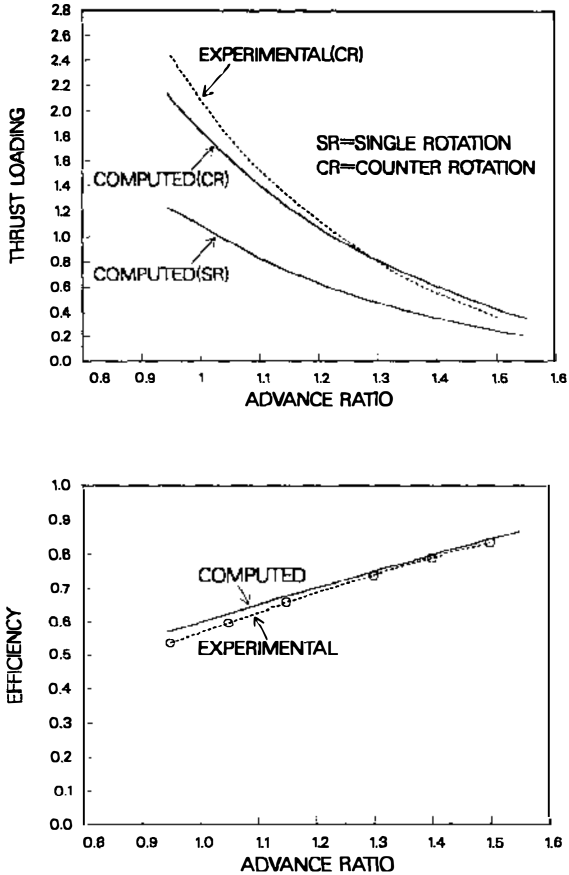

Figure 9 Performance of a counterrotating propeller.

the terms on the right from the previous iteration and solving the resulting Poisson equation by panel-method techniques. As is well known, the nonhomogeneous term in a Poisson equation may be interpreted as a source distribution over the field, and its effect may be computed by means of field panels whose dimension is greater by one than that of the boundary panels. In this approach there are no stringent requirements on the field panels. Any convenient distribution may be used. Thus, the very severe problems of grid generation that seem to be an inherent feature of finite-difference solutions are avoided, and in principle the method can be extended to arbitrarily complicated three-dimensional bodies. The two main problems of this approach are an efficient computation of the field panel influences and convergence of the iterations in cases with shocks.

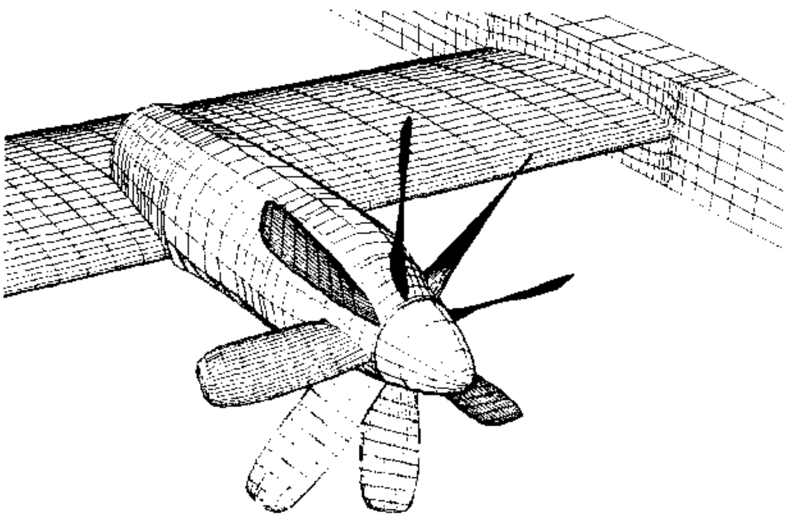

Figure 10 A tractor propeller on a straight wing.

Excellent results, including shock location and strength, have been obtained for symmetric two-dimensional airfoils at zero lift (Oskam 1985), and virtually all investigators have obtained good results for shock-free cases in both two and three dimensions. The case of lifting three-dimensional flows with shocks has proved much more difficult. Although scattered results of varying quality have been obtained, at present this is basically an unsolved problem.

Free-Surface Applications

Surface-ship and underwater-structure problems have been attacked successfully using panel methods in spite of the considerable complications caused by the presence of the air-water interface. On this so-called free surface the water pressure must be constant, and this is applied as a boundary condition, usually on the plane that represents the undisturbed free-surface location. There is also a radiation condition governing the wave direction at infinity. Two different problems have been addressed: (*a*) that of a surface ship at constant forward speed in an otherwise undisturbed ocean, and (*b*) that of a fixed structure on which a train of regular (simple harmonic) waves is incident.

Approaches to the ship translation problem follow that of Dawson (1977). Rankine-type $1/r$ sources are used, and both the underwater portion of the hull and the portion of the undisturbed free surface surrounding the hull must be paneled. The free-surface boundary condition is applied

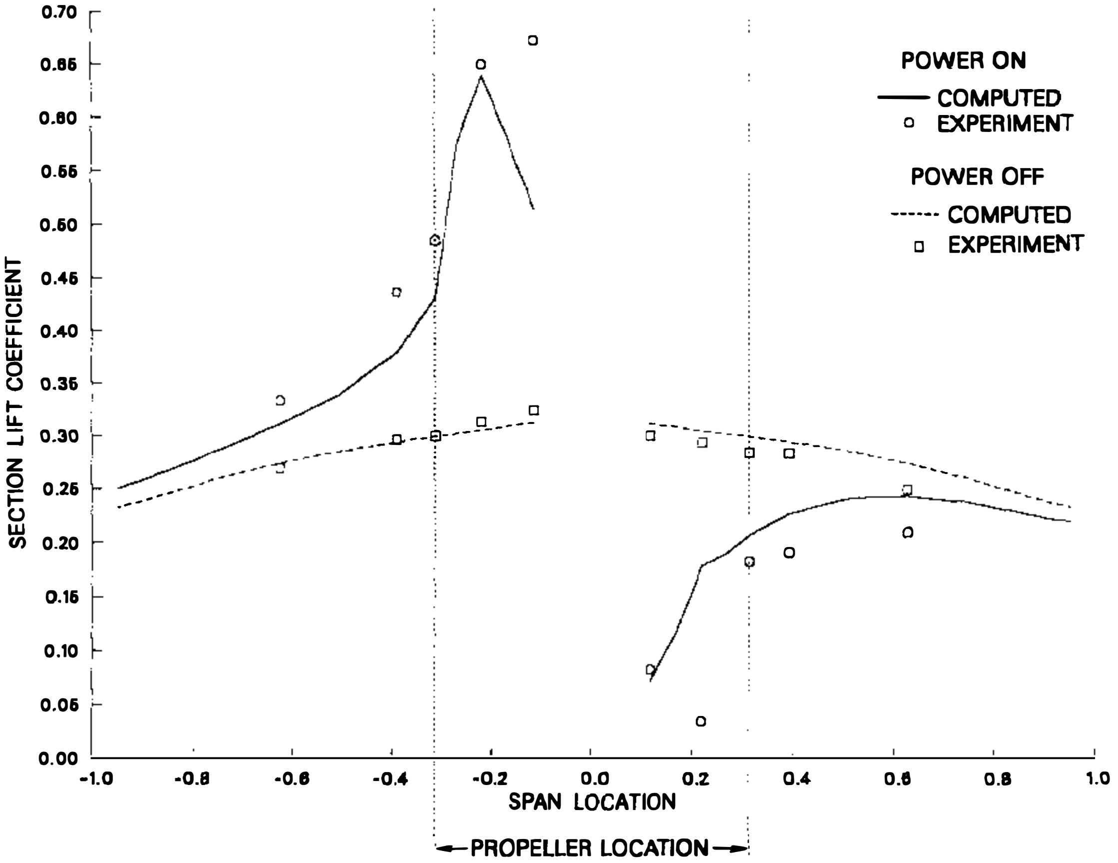

Figure 11 Spanwise distributions of section lift coefficient on a straight wing with and without the effect of a tractor propeller.

in a way that ensures that waves do not propagate ahead of the ship. Dawson considered the nonlifting case. Interest in the lifting case was sparked when Slooff (1984) applied the lifting source method to the design of the winged keel used on the yacht Australia II, which captured the America's Cup in 1983. In this analysis, the free surface was modeled as a solid boundary, but it was realized that improved accuracy could be obtained by using Dawson's free-surface model in a lifting panel method. Slooff (1984) outlines this approach, while Fei (1984) has incorporated lift into Dawson's approach.

The second problem, that of waves incident upon a fixed structure, which includes the related problem of a body oscillating in an otherwise undisturbed ocean, has been approached in a quite different way. Only the body, not the free surface, is paneled. The singularities used are not those of the simple Rankine type but are of a much more complicated type that identically satisfies the free-surface boundary condition. Computing the influence of a panel having such a singularity in a reasonable computing time is the key to this approach. A practical technique was developed by Newman (1985), and it has been incorporated into working codes by Breit et al. (1985).

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