numerous publications describing the kinematics of moving frames. Most of the publications originate in the area ~~of~~ classical mechanics and rigid body dynamics. The publications in the area of flight dynamics and control always contain material addressing the attitude representation techniques and differential rotations, and thus can be a good source of reference information. The most recent and thorough presentation of these topics can be found in (Beard and McLain 2012) where the authors specifically address the kinematics and dynamics of small UAVs.

Rigid Body Dynamics

This section addresses the development of the dynamics of a rigid body. The discussion is based on the application of the Newton's laws ~~for~~ in? the cases of linear and angular motion. In particular, the second law of motion states that the sum of all external forces acting on a body in an inertial frame must be equal to the time rate of in? change of its linear momentum. On the other hand, the sum of the external moments acting on a body must be equal to the time rate of change of its angular momentum. The application of Applying these laws is the objective of this chapter.

We consider a fixed wing UAV as the rigid body and define its dynamics with respect to the body fixed coordinate system. Relations necessary to translate the inertial forces to the body fixed frame are also presented. Before proceeding to the derivation, it is necessary to present some assumptions typical ~~for~~ of the fixed wing UAVs:

- The mass of the UAV remains constant during ~~the~~ flight.
- The UAV is a rigid body.
- An Earth fixed frame can be considered ~~as~~ an inertial frame.

The relations derived in this chapter are general and can be applied to any rigid body; however, the treatment of the aerodynamic forces and moments acting on the body will be specific to the aerodynamically controlled fixed wing UAVs.

Conservation of Linear momentum

First, assume that a rigid body consists of a set of i –"isolated" elementary particles with mass m_i exposed to the external force $\mathbf{F}_i$ while being connected together by the internal forces $\mathbf{R}_i$. Since the set of N particles comprises a rigid body structure, the net force exerted by all the particles is $\sum_i^N \mathbf{R}_i = 0$. The set of external forces acting on the body is a combination of the gravity force acting in an inertial frame and the aerodynamic and propulsion forces defined with respect to the body fixed frame but expressed in the inertial frame. Thus, the linear momentum of a single particle expressed in an inertial frame obeys the equality

$$\mathbf{F}_i + \mathbf{R}_i = \frac{d}{dt}(m_i \mathbf{V}_i).$$

It is worth noting that the time derivative is taken in an inertial frame as well, thus calling for the results in and . Summing up all the N particles comprising the body gives the linear momentum equation of the entire body

$$\sum_{i=1}^{N}\mathbf{F}_i=\sum_{i=1}^{N}\frac{d}{dt}(m_i\mathbf{V}_i)$$

The left part of this equation represents the sum of all forces (gravitational, propulsion and aerodynamic) expressed in an inertial frame, with the right part depending on the velocity of the body defined in an inertial frame. Observing that (i) the individual inertial velocities are not independent (they comprise a rigid body), assuming ~~(ii)~~ that (ii) the mass is constant, and utilizing the result in for the total velocity of the i–th particle in an inertial frame, allows calculating the absolute time derivatives in an inertial frame in the following form for the calculation of

$$\sum_{i=1}^{N}\mathbf{F}_i=\sum_{i=1}^{N}\frac{d}{dt}(m_i\mathbf{V}_i)=\sum_{i=1}^{N}\frac{d}{dt}(m_i(\mathbf{V}_g^b+\ \times\ _i))=\sum_{i=1}^{N}m_i\frac{d\mathbf{V}_g^b}{dt}+\sum_{i=1}^{N}m_i\frac{d}{dt}(\ \times\ _i)=$$

$$\sum_{i=1}^{N}m_i\frac{d\mathbf{V}_g^b}{dt}+\frac{d}{dt}\left[\mathbf{\grave{u}}\times\sum_{i=1}^{N}m_i\mathbf{r}_i\right]$$

Here $\mathbf{\grave{u}}$ represents the angular velocity of the UAV body defined with respect to the

?

inertial frame, see. Defining $\mathbf{r}_{cg}$ –the vector of CG location as $m\mathbf{r}_{cg}=\sum_{i=1}^{N}m_i\mathbf{r}_i$, where $m=\sum_{i=1}^{N}m_i$ is the total mass of the body, simplifies the linear momentum equation.

$$\sum_{i=1}^{N}\mathbf{F}_i=\sum_{i=1}^{N}m_i\frac{d\mathbf{V}_g^b}{dt}+m\frac{d}{dt}[\ \times\ _{cg}]$$

Assuming that the location of CG does not change with time and applying the result in to the absolute derivatives of vectors $\mathbf{V}_g$ and $\mathbf{\grave{u}}$ results in

$$\mathbf{F}^b=\sum_{i=1}^{N}\mathbf{F}_i^b=m(\dot{\mathbf{V}}_g^b+\&\times\ _{cg}+\ \times\ _g^b+\ \times[\ \times\ _{cg}])$$

where

$\mathbf{F}^b=[X,Y,Z]$ – the externally applied forces expressed in the body frame;

$\mathbf{V}_g^b=[u,\upsilon,\omega]$ – the inertial velocity components defined in the body frame;

$\mathbf{\grave{u}}=[p,q,r]$ – the body angular rates defined in the body frame;

$\mathbf{r}_{cg}=[x_{cg},y_{cg},z_{cg}]$ – the body referenced location of the center of gravity~~,~~.

me ↑ Translation of the inertial forces to the body frame is justified by the ~~convenience~~ convenience of calculating the local body frame derivatives of the $\mathbf{V}_g$ and $\mathbf{\grave{u}}$ expressed in the body frame; the first one results in $\dot{\mathbf{V}}_g$, while the derivative of $\mathbf{\grave{u}}$ is independent on the coordinate frame ($\frac{d\mathbf{\grave{u}}}{dt}=\frac{\delta\mathbf{\grave{u}}}{\delta t}+\mathbf{\grave{u}}\times\mathbf{\grave{u}}=\frac{\delta\mathbf{\grave{u}}}{\delta t}$).

~~Utilizing the~~ double vector product identity $\mathbf{\grave{u}}\times[\mathbf{\grave{u}}\times\mathbf{r}_{cg}]=(\mathbf{\grave{u}}\cdot\mathbf{r}_{cg})\cdot\mathbf{\grave{u}}-(\mathbf{\grave{u}}\cdot\mathbf{\grave{u}})\cdot\mathbf{r}_{cg}$ allows, for the ~~expanding~~ the linear momentum equation in the most general scalar form as follows:

expansion of

$$X = m\left[\dot{u} + qw - rv + \dot{q}z_{cg} - \dot{r}y_{cg} + (qy_{cg} + rz_{cg})p - (q^2 + r^2)x_{cg}\right]$$
$$Y = m\left[\dot{v} + ru - pw + \dot{r}x_{cg} - \dot{p}z_{cg} + (rz_{cg} + px_{cg})q - (r^2 + p^2)y_{cg}\right]$$
$$Z = m\left[\dot{w} + pv - qu + \dot{p}y_{cg} - \dot{q}x_{cg} + (px_{cg} + qy_{cg})r - (p^2 + q^2)z_{cg}\right].$$

The last set of equations allows for the most general mass distribution inside the body. This set of equations might be useful when there is a need to model the placement of the body frame origin away from its CG. If the origin of the body-fixed frame is chosen at the CG, the last set of equations can be significantly simplified by substituting $\mathbf{r}_{cg} = [0,0,0]$, thus leading to

$$\mathbf{F}^b = m(\dot{\mathbf{V}}^b_g + \quad \times \quad {}^b_g).$$

Expanding the cross product ~~,~~ results in the following form of the linear momentum equations:

$$X = m\left[\dot{u} + qw - rv\right]$$
$$Y = m\left[\dot{v} + ru - pw\right]$$
$$Z = m\left[\dot{w} + pv - qu\right].$$

space

Resolving equations with respect to the derivatives (accelerations in the body frame) leads to the standard form of differential equations suitable for immediate mathematical modeling.

$$\dot{u} = \frac{X}{m} + \left[rv - qw\right]$$
$$\dot{v} = \frac{Y}{m} + \left[pw - ru\right]$$
$$\dot{w} = \frac{Z}{m} + \left[qu - pv\right]$$

Conservation of Angular momentum

Applying the law of conservation of angular momentum to an i-th particle in a moving frame is very similar to the approach used above. Consider a particle subjected to the internal and external moments. Similarly to the linear momentum case, the sum of internal moments acting on the particle should be equal to zero ($\sum_{i=1}^{N}\mathbf{M}_i = 0$), while the external moments arise from the inertial gravity and the body attached forces, such as aerodynamic and propulsion. Thus, the conservation of angular momentum calculated across the entire rigid body results in

$$\sum_{i=1}^{N}(\mathbf{M}_i + \mathbf{r}_i \times \mathbf{F}_i) = \sum_{i=1}^{N}\mathbf{r}_i \times \frac{d}{dt}(m_i\mathbf{V}_i).$$

Since the sum of internal moments cancel ~~, then~~ applying the Coriolis theorem leads to

each other out

$$\sum_{i=1}^{N}\mathbf{r}_i\times\mathbf{F}_i=\sum_{i=1}^{N}m_i\mathbf{r}_i\times\frac{d}{dt}(\mathbf{V}_i)=$$

$$\sum_{i=1}^{N}m_i\mathbf{r}_i\times(\frac{\delta\mathbf{V}_g^b}{\delta t}+\ \times\ {}_g^b+\frac{\delta ù\&}{\delta t}\times\ {}_i+\ \times[\frac{\delta ù\&}{\delta t}\times\ {}_i])=$$

$$\sum_{i=1}^{N}m_i\mathbf{r}_i\times(\frac{\delta\mathbf{V}_g^b}{\delta t}+\ \times\ {}_g^b)+\sum_{i=1}^{N}m_i\ {}_i\times[\frac{\delta ù\&}{\delta t}\times\ {}_i]+\sum_{i=1}^{N}m_i\ {}_i\times\left[\ \times[\frac{\delta ù\&}{\delta t}\times\ {}_i]\right]$$

The first term can be expanded by utilizing the definition of the CG.

$$\sum_{i=1}^{N}m_i\mathbf{r}_i\times(\frac{\delta\mathbf{V}_g^b}{\delta t}+\ \times\ {}_g^b)=m\ {}_{cg}\times(\frac{\delta\mathbf{V}_g^b}{\delta t}+\ \times\ {}_g^b)=\begin{bmatrix}m\left[y_{cg}(ò\&+p\upsilon-qu)-z_{cg}(ù\&+ru-p\omega)\right]\\ m\left[z_{cg}(ù\&+q\omega-r\upsilon)-x_{cg}(ò\&+p\upsilon-qu)\right]\\ m\left[x_{cg}(ù\&+ru-p\omega)-y_{cg}(ù\&+q\omega-r\upsilon)\right]\end{bmatrix}$$

Utilizing the double vector product identity allows for the expanding of the second term as follows:

$$\sum_{i=1}^{N}m_i\mathbf{r}_i\times(\frac{dù}{dt}\times\mathbf{r}_i)=\sum_{i=1}^{N}m_i(\frac{dù}{dt}(\mathbf{r}_i\cdot\mathbf{r}_i)-\mathbf{r}_i(\frac{dù}{dt}\cdot\mathbf{r}_i))=$$

$$=\begin{bmatrix}\sum_{i=1}^{N}m_i((y_i^2+z_i^2)p\&-(y_iq\&+z_ir\&)x_i)\\ \sum_{i=1}^{N}m_i((z_i^2+x_i^2)q\&-(z_ir\&+x_ip\&)y_i)\\ \sum_{i=1}^{N}m_i((x_i^2+y_i^2)r\&-(x_ip\&+y_iq\&)z_i)\end{bmatrix}=\begin{bmatrix}I_{xx}p\&+I_{xy}q\&+I_{xz}r\&\\ I_{yx}p\&+I_{yy}q\&+I_{yz}r\&\\ I_{zx}p\&+I_{zy}q\&+I_{zz}r\&\end{bmatrix}=\begin{bmatrix}I_{xx}&I_{xy}&I_{xz}\\ I_{yx}&I_{yy}&I_{yz}\\ I_{zx}&I_{zy}&I_{zz}\end{bmatrix}\begin{bmatrix}p\&\\ q\&\\ r\&\end{bmatrix}=\mathbf{I}\times\&$$

Space

The equation is obtained by recognizing the moments of inertia, their symmetrical properties, and combining them into a matrix form defines the inertia tensor $\mathbf{I}$ that allows converting the entire double vector product into a very compact form.
the conversion of

$$I_{xx}=\sum_{i=1}^{N}m_i(y_i^2+z_i^2)\qquad I_{yy}=\sum_{i=1}^{N}m_i(z_i^2+x_i^2)\qquad I_{yy}=\sum_{i=1}^{N}m_i(x_i^2+y_i^2)$$

$$I_{xy}=I_{yx}=-\sum_{i=1}^{N}m_ix_iy_i\quad I_{xz}=I_{zx}=-\sum_{i=1}^{N}m_ix_iz_i\quad I_{yz}=I_{zy}=-\sum_{i=1}^{N}m_iy_iz_i$$

The diagonal terms of $\mathbf{I}$ are called the moments of inertia. The off-diagonal terms are called the products of inertia, and they define the inertia cross coupling. The moments of inertia are directly proportional to the UAV's tendency to resist angular acceleration about a specific axis of rotation. For a body with axes of symmetry, the inertia tensor has zero off-diagonal terms that significantly simplify its form and the final equations of angular momentum.

The last term in utilizes twice the same double cross product expansion, thus leading to

?

$$\sum_{i=1}^{N} m_i \mathbf{r}_i \times [\ \ \times [\ \ \times\ _i]] = \sum_{i=1}^{N} m_i\ _i \times ((\ \ \cdot\ _i)\ \ -(\ \ \cdot\ \ )\ _i) =$$

$$= \begin{bmatrix} I_{yz}(q^2 - r^2) + I_{xz}pq - I_{xy}pr \\ I_{xz}(r^2 - p^2) + I_{xy}rq - I_{yz}pq \\ I_{xy}(p^2 - q^2) + I_{yz}pr - I_{xz}qr \end{bmatrix} + \begin{bmatrix} (I_{zz} - I_{yy})rq \\ (I_{xx} - I_{zz})rp \\ (I_{yy} - I_{xx})qp \end{bmatrix}.$$

Denoting the body components of the total moment acting on the UAV as $\mathbf{M} = [L, M, N]$, and combining the results in – lead to the following complete angular momentum equations in an expanded form:

$$\begin{aligned} L = I_{xx}\dot{p} + I_{xy}\dot{q} + I_{xz}\dot{r} + \\ I_{yz}(q^2 - r^2) + I_{xz}pq - I_{xy}pr + (I_{zz} - I_{yy})rq + \\ m\left[y_{cg}(\dot{w} + pv - qu) - z_{cg}(\dot{v} + ru - pw)\right] \\ M = I_{yx}\dot{p} + I_{yy}\dot{q} + I_{yz}\dot{r} + \\ I_{xz}(r^2 - p^2) + I_{xy}rq - I_{yz}pq + (I_{xx} - I_{zz})rp + \\ m\left[z_{cg}(\dot{u} + qw - rv) - x_{cg}(\dot{w} + pv - qu)\right] \\ N = I_{zx}\dot{p} + I_{zy}\dot{q} + I_{zz}\dot{r} + \\ I_{xy}(p^2 - q^2) + I_{yz}pr - I_{xz}qr + (I_{yy} - I_{xx})qp + \\ m\left[x_{cg}(\dot{v} + ru - pw) - y_{cg}(\dot{u} + qw - rv)\right]. \end{aligned}$$

In the case of a typical UAV with a vertical plane of symmetry spanned by body fixed axes x_b, z_b, the two pairs of the off-diagonal terms of $\mathbf{I}$ matrix become zero, namely $I_{xy} = I_{yx} = 0$ and $I_{yz} = I_{zy} = 0$. This significantly simplifies the above equations:

$$\begin{aligned} L &= I_{xx}\dot{p} + (I_{zz} - I_{yy})rq + I_{xz}(\dot{r} + pq) \\ M &= I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + I_{xz}(r^2 - p^2) \\ N &= I_{zz}\dot{r} + (I_{yy} - I_{xx})qp + I_{zx}(\dot{p} - qr). \end{aligned}$$

These equations represent the complete rotational dynamics of a typical fixed wing UAV with a longitudinal plane of symmetry.

Complete set of 6DOF Equations of Motion

The final set of 6DOF equations of motion describing the kinematics and dynamics of a generic UAV with a longitudinal plane of symmetry modeled as a rigid body can be summarized as follows:

$$\begin{aligned} X &= m\left[\dot{u} + qw - rv\right] \\ Y &= m\left[\dot{v} + ru - pw\right] \\ Z &= m\left[\dot{w} + pv - qu\right] \end{aligned}$$

$$\begin{aligned} L &= I_{xx}\dot{p} + (I_{zz} - I_{yy})rq + I_{xz}(\dot{r} + pq) \\ M &= I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + I_{xz}(r^2 - p^2) \\ N &= I_{zz}\dot{r} + (I_{yy} - I_{xx})qp + I_{zx}(\dot{p} - qr) \end{aligned}$$

Gravitation

in comparison

Assuming that the flight altitude is negligible ~~comparing~~ to the radius of the Earth, ~~it is~~ sufficient to consider the gravity's magnitude constant. Then, the effect of the Earth's gravitation can be naturally modeled in the body-carried frame by the force applied to the CG of the UAV; the gravitational force is proportional to the gravitational constant g and is called the weight of the UAV.

$$\mathbf{F}_{gr} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

Before substituting this force expression into the equations of motion it needs to be transformed into the body frame. The inertial to body rotation R_u^b enables this transformation;

$$\mathbf{F}_{gr}^b = R_u^b \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = mg \begin{bmatrix} -\sin\theta \\ \sin\phi\cos\theta \\ \cos\theta\cos\phi \end{bmatrix}$$

Since the gravitational force acts through the CG of the airplane, the corresponding moment contribution is zero, $\mathbf{M}_{gr} = [0,0,0]^T$.

Propulsion

The configuration of the propulsion system of modern fixed wing UAVs varies greatly. The existing architectures can be categorized by the number of engines, their type, and their installation arrangement in the airframe. A thorough review of the existing architectures along with some future projections and trends in the modern and future UAV systems can be found in (OSD 2001). However, what is common across all possible configurations is that the vector of thrust in all systems is set parallel to the existing axes of symmetry; the thrust vectoring is not a common feature of fixed wing UAVs.

The thrust is naturally represented in the body fixed reference system. The direction of the thrust vector $\mathbf{F}_{tr}$ is usually fixed and lies in the plane of symmetry or is parallel to it; however it may not be aligned with the longitudinal x_b-axis. If the orientation of the thrust vector $\mathbf{F}_{tr}$ varies in its reference to the airframe then a separate coordinate system analogous to the wind axes should be defined, thus introducing the required rotation of the thrust vector to the body fixed coordinate system. It is a common design requirement that the installation of multiple engines should not introduce any unbalanced moments, thus not inducing any loss of control efforts for the UAV stabilization. For the analysis of a nominal flight regime the thrust vector $\mathbf{F}_{tr}$ is considered fixed with respect to the body fixed frame.

For the sake of simplicity, consider a typical fixed wing UAV architecture where the installation of one or multiple engines results in the cumulative thrust $\mathbf{F}_{tr}$ vector passing through the CG, and the only moment being the torque generated by the reactive force from the rotating propeller. Thus, the net force X_{tr} of thrust in x_b direction and the moment L around x_b axis can be considered proportional to the thrust control command δ_{tr}. Moreover, thrust characteristics of most conventional engines are always functions of the air density and the airspeed. Thus, the contributing force and moment resulting from the propulsion system can be presented as follows:

$$\mathbf{F}_{tr} = \begin{bmatrix} F_{tr}(V_a, h, \delta_{tr}) \\ 0 \\ 0 \end{bmatrix}; \mathbf{M}_{tr} = \begin{bmatrix} M_{tr}(V_a, h, \delta_{tr}) \\ 0 \\ 0 \end{bmatrix} ?, ; .$$

A particular example of modeling the propulsion force for the case of a micro UAV can be found in (Beard and McLain 2012).

Unsteady atmosphere

Turbulent atmosphere enters the equations of motion by changing the vector $\mathbf{V}_a$ resolved with ~~respect~~ ect to the air. In the previous discussion of the wind frame, ~~it~~ was assumed that the wind $\mathbf{V}_w$ defined in the LTP frame is constant, thus the velocities are related by the "wind triangle" equation:

$$\mathbf{V}_a = \mathbf{V}_g - \mathbf{V}_w .$$

The most common approach (McRuer, Ashkenas and Graham 1999) in wind modeling is to consider the two components contributing to the wind. The first one $V^u_{wsteady}$ defines the steady wind in the Earth fixed frame, and therefore it can be presented ~~by the~~ measurements in the LTP frame. The second component V^b_{wgust} is stochastic, which ~~that~~ represents the short period disturbances or gusts expressed in the body fixed frame. Since the equations of motion are written in the body fixed frame, the LTP to body rotation R^b_u serves the purpose of this transformation.

$$V^b_w = R^b_u V^u_{wsteady} + V^b_{wgust}$$

From the components of the wind and the UAV velocity, both resolved in the body frame, it is therefore possible to find the body frame components of the air velocity as

$$\mathbf{V}^b_a = \begin{bmatrix} u_w \\ \upsilon_w \\ \omega_w \end{bmatrix} = \begin{bmatrix} u \\ \upsilon \\ \omega \end{bmatrix} - R^b_u \begin{bmatrix} u_{wsteady} \\ \upsilon_{wsteady} \\ \omega_{wsteady} \end{bmatrix} - \begin{bmatrix} u_{wgust} \\ \upsilon_{wgust} \\ \omega_{wgust} \end{bmatrix} .$$

These body frame components of the air velocity enable straightforward calculation of the airspeed and the angles of attack and sideslip as

$$V_a = \sqrt{u_w^2 + \upsilon_w^2 + \omega_w^2}; \quad \alpha = a\tan(\frac{\omega_w}{u_w}); \quad \beta = a\sin(\frac{\upsilon_w}{\sqrt{u_w^2 + \upsilon_w^2 + \omega_w^2}}).$$

Modeling of the stochastic and steady components of wind is primarily based on experimental observations expressed using linear filters. The most widely used techniques are represented by von Karman and Dryden wind turbulence models (Hoblit 2001). Both methods are well supported with their numerical implementations.

Aerodynamics

Aerodynamic forces and moments depend on the interaction of an aircraft with the airflow, which may be also in motion relative to the Earth. However, for the purpose of representing the nominal aerodynamic effects, the large-scale motion of the atmosphere is not critical and therefore will be considered constant; in fact it will only affect the navigation of the UAV.

The small perturbation theory (Ashley and Landahl 1985) is one of the approaches used in describing the aerodynamic interaction of a given aerodynamic shape with airflow. The perturbation in aerodynamic forces and moments are functions of variations in state variables and control inputs. The control inputs here are the deflections of the control surfaces of an airplane that modify the airflow around the body, thus generating the desired aerodynamic effects. The nomenclature of the control surfaces and their control mechanization depends on the particular aerodynamic composition of the airplane. Nevertheless, the principles describing the effects of the control surface deflection on the generated forces and moments are the same. Consider the following control effectors of a classical aerodynamic configuration: the elevator, the aileron, and the rudder (see Figure 9). In this configuration the ailerons are used to control the roll angle ϕ, the elevator is used to control the pitch angle θ, the rudder controls the yaw angle ψ.

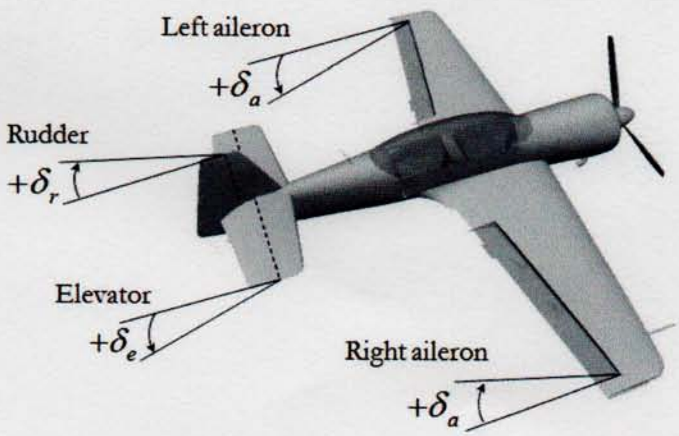

Figure 9. Control surfaces of a classical aerodynamic configuration.

Their deflections are denoted as δ_a – for the aileron, δ_e – for the elevator, and δ_r – for the rudder. The positive deflection of a control surface is defined by applying the right-hand rule to the rotation axis of the surface. The positive direction of the aileron, elevator and rudder deflections is also depicted in ~~the~~ Figure 9.

Deflection of the control surfaces results ~~in~~ modification of the pressure distribution around the body. For example, deflecting the elevator primarily changes the pitching moment acting on the airplane. In turn, ~~this~~ results in changing the angle of attack of the wing that increases the lifting power of the airplane. The calculation of aerodynamic characteristics of one or more lifting surfaces with variable deflections of the control surfaces at various attitudes with respect to the airflow can be accomplished by utilizing well-developed linear panel methods (Hess 1990, Henne 1990) conveniently implemented in various software packages (Fearn 2008, Kroo 2012).

? find different second verb

The key result presented by the panel methods captures the effect of pressure distribution in the form of parameterized forces and moments versus the angles of attack and side-slip, and airspeed; they play a role of states here. For example, considering the longitudinal plane, the effect of pressure action acting on a fixed wing can be modeled using a total force F_Σ and pitching moment M_w acting on the wing. It is common to project the total force to the wind axes, thus resulting in the lift F_{lift} and drag F_{drag} force components. Figure 10 demonstrates the approach to modeling ~~of~~ aerodynamic effects in the wind and body fixed frames with respect to the vector of the free airstream $\mathbf{V}_\infty$.

pe... or "act...

= effect of pressure acting on...

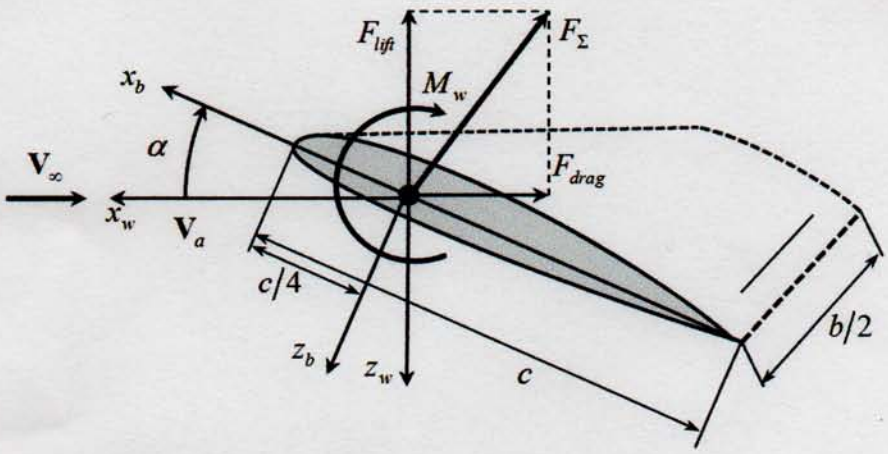

Figure 10. Definition of lift, drag and pitching moment in the wind frame.

As shown in the figure, the lift F_{lift} and drag F_{drag} forces act in the wind frame and are applied at the aerodynamic center of the lifting surface that is located at the quarter-chord point (c – is the length of the mean aerodynamic chord). The pitching moment M_w acts around the aerodynamic center. Then, the forces and moment are represented in a form connecting a number of surface specific parameters and the states in the following form :

$$F_{lift} = \tfrac{1}{2}\rho V_a^2 S C_L$$
$$F_{drag} = \tfrac{1}{2}\rho V_a^2 S C_D$$
$$M_w = \tfrac{1}{2}\rho V_a^2 S c C_m,$$

where C_L, C_D, C_m are the nondimensional aerodynamic coefficients (to be parameterized), S is the planform area of the wing surface, c and b are the mean aerodynamic chord and the wing span. The same approach is applied to each of the aerodynamic surfaces comprising the airplane.

It is common practice to consider the total aerodynamic forces and moments in projections to the longitudinal and lateral planes of the airplane. The benefit of this approach is in the simplicity of representing the aerodynamic effects and in providing a natural ground for the nonlinear model decomposition at the next step of the control system design. Thus, the longitudinal forces and moments consist of lift, drag and pitching moment acting in the vertical plane of symmetry. The lateral side force and yawing moment are caused by the asymmetric airflow around the airplane; the asymmetry can be caused by the side wind or intentional deflection of the rudder. For the majority of fixed wing UAVs, the key states that define the parameterization of the aerodynamic coefficient are the angle of attack α, the side slip β, body rates $[p,q,r]$, and

?

the controls, which are the surface deflections $[\delta_e, \delta_r, \delta_a]$. The most general functional form of

or ; the controls are the surface...

the longitudinal and lateral aerodynamics can be presented as follows:

Table 2. Parameterization of longitudinal and lateral aerodynamics

Longitudinal channel	Lateral channel
$F_{drag} = \frac{1}{2}\rho V_a^2 S C_D(\alpha, q, \delta_e)$	$F_{side} = \frac{1}{2}\rho V_a^2 S C_Y(\beta, p, r, \delta_r, \delta_a)$
$F_{lift} = \frac{1}{2}\rho V_a^2 S C_L(\alpha, q, \delta_e)$	$L_w = \frac{1}{2}\rho V_a^2 S b C_l(\beta, p, r, \delta_r, \delta_a)$
$M_w = \frac{1}{2}\rho V_a^2 S c C_M(\alpha, q, \delta_e)$	$N_w = \frac{1}{2}\rho V_a^2 S b C_n(\beta, p, r, \delta_r, \delta_a)$

Without delving deeper into the intricacies of aerodynamic parameterization, it is sufficient to demonstrate the final form of forces and moments defined in the wind coordinate frame:

- longitudinal plane

$$F_{drag} = \frac{1}{2}\rho V_a^2 S\left(C_{D_0} + C_D^{\alpha}\alpha + C_D^q \frac{c}{2V_a}\cdot q + C_D^{\delta_e}\cdot\delta_e\right)$$

$$F_{lift} = \frac{1}{2}\rho V_a^2 S\left(C_{L_0} + C_L^{\alpha}\alpha + C_L^q \frac{c}{2V_a}\cdot q + C_L^{\delta_e}\cdot\delta_e\right)$$

$$M_q = \frac{1}{2}\rho V_a^2 S c\left(C_{M_0} + C_M^{\alpha}\alpha + C_M^q \frac{c}{2V_a}\cdot q + C_M^{\delta_e}\cdot\delta_e\right)$$

- lateral plane

$$F_{side} = \frac{1}{2}\rho V_a^2 S\left(C_{Y_0} + C_Y^{\beta}\beta + C_Y^p \frac{b}{2V_a}\cdot p + C_Y^r \frac{b}{2V_a}\cdot r + C_Y^{\delta_r}\cdot\delta_r + C_Y^{\delta_a}\cdot\delta_a\right)$$

$$L_w = \frac{1}{2}\rho V_a^2 S b\left(C_{l_0} + C_Y^{\beta}\beta + C_l^p \frac{b}{2V_a}\cdot p + C_l^r \frac{b}{2V_a}\cdot r + C_l^{\delta_r}\cdot\delta_r + C_l^{\delta_a}\cdot\delta_a\right)$$

$$N_w = \frac{1}{2}\rho V_a^2 S b\left(C_{n_0} + C_n^{\beta}\beta + C_n^p \frac{b}{2V_a}\cdot p + C_n^r \frac{b}{2V_a}\cdot r + C_n^{\delta_r}\cdot\delta_r + C_n^{\delta_a}\cdot\delta_a\right)$$

The presented parameterization is a simple linear approximation of the aerodynamics given by the Taylor series expansion, ~~taken~~ with respect to the given trim conditions. The coefficients $C_{f/m}^{state}$ – are the nondimensional partial derivatives of the corresponding forces and moment (denoted in the subscript) ~~taken~~ with respect to the corresponding state or control (denoted in the superscript). The coefficients with zero in the subscript denote the forces and moments calculated when all states, including the control surface deflection, are zero; for example C_{l_0} denotes the roll moment coefficient estimated at $\beta = p = r = \delta_r = \delta_a = 0$. The common naming convention suggests that those derivatives ~~taken~~ with respect to states $[\alpha, \beta, p, q, r]$ are called the stability derivatives, and those with respect to controls $[\delta_e, \delta_r, \delta_a]$ are called the control derivatives. The Static stability of an aircraft with respect to disturbances in some variable is ~~directly reflected~~ in the sign of a particular derivative. For example, the sign ~~of~~ C_M^{α} should be negative to guarantee static stability in pitching motion, while the sign ~~of~~ C_N^{β} should be positive for the directional static stability.

Each of the presented coefficients is usually a function of states. The precision requirement of the linear parameterization greatly depends on the operational envelope ment ? of the UAV and its intended use; the higher the maneuverability of the UAV, the more terms ~~are~~ necessary to accurately represent the aerodynamics. Each of the coefficients has very intuitive physical meaning and is usually studied separately. An interested reader is referred to (Beard and McLain 2012) for a detailed discussion of the aerodynamic coefficients of small and micro fixed wing UAVs.

One last step needs to be performed before the aerodynamics- defined in the wind coordinates can be plugged into the equations of motion – defined in the body fixed frame. The transformation from the wind to the body frame serves this purpose.

Therefore, the total forces and moments acting on the fixed wing UAV can be presented as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R_u^b \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + \begin{bmatrix} F_{tr}(V_a, h, \delta_{tr}) \\ 0 \\ 0 \end{bmatrix} + \frac{1}{2}\rho V_a^2 S \times R_w^b \begin{bmatrix} C_D(\alpha, q, \delta_e) \\ C_Y(\beta, p, r, \delta_r, \delta_a) \\ C_L(\alpha, q, \delta_e) \end{bmatrix}$$

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} M_{tr}(V_a, h, \delta_{tr}) \\ 0 \\ 0 \end{bmatrix} + \frac{1}{2}\rho V_a^2 S \times \begin{bmatrix} bC_l(\beta, p, r, \delta_r, \delta_a) \\ cC_M(\alpha, q, \delta_e) \\ bC_n(\beta, p, r, \delta_r, \delta_a) \end{bmatrix} .$$

Conclusions

The objective of this chapter was to provide a review of the theoretical material required to enable accurate mathematical modelling of the free and controlled motion of a generic fixed wing UAV. The key building blocks presented were the coordinate frames and their transformations, kinematics of rotation, dynamics of motion, and the definition of forces and moments acting on the airplane. The Kinematics of spatial rotation ? is what connects the three building blocks of the "Kinematics–Dynamics–Actions" triad. In addition to ~~Besides~~ the 6DOF equations of motion describing the kinematics and dynamics of a rigid body motion, the tools and methods developed in this chapter contribute significantly into the UAV flight dynamics, system identification, control, guidance and navigation.

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I hope my "markings" were helpful!
Good luck,
Mary Anne