Chapter 2

Coordinate Systems

2.1 Background

The need to define appropriate coordinate systems arises from two considerations. First, there may be some particular coordinate system in which the position and velocity of the aircraft "make sense". For navigation we are concerned with position and velocity with respect to the Earth, whereas for aircraft performance we need position and velocity with respect to the atmosphere. Second, there are coordinate systems in which the phenomena of interest are most naturally expressed. The direction of a jet engine's propulsive force may often be considered fixed with respect to the body of the aircraft.

All coordinate systems will be right-handed and orthogonal. Coordinate systems will be designated by the symbol F with a subscript intended to be a mnemonic for the name of the system, such as F_I for the inertial reference frame. The origin of the system will be denoted by O and a subscript (e.g. O_I). If we speak of where a coordinate system is we mean where its origin is. Axes of the system are labled x, y, and z with the appropriate subscript. Unit vectors along x, y, and z will be denoted $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ respectively and subscripted appropriately.

It is customary in flight dynamics to omit subscripts when speaking of certain body-fixed coordinate systems. If this is not the case then the lack of subscripts will be taken to mean a generic system.

The definition of a coordinate system must state the location of its origin and the means of determining at least two of its axes, the third axis being determined by completing the right-hand system. The location of the origin and orientation of the axes may be arbitrary within certain restrictions, but once selected may not be changed. Following are the main coordinate systems of interest:

2.2 The Coordinate Systems

2.2.1 The Inertial reference frame, F_I

The location of the origin may be any point that is completely unaccelerated (inertial), and the orientation of the axes is usually irrelevant to most problems so long as they too are fixed with respect to inertial space. For purposes of this course the origin is at the Great Galactic Center.

2.2.2 The Earth-centered reference frame, F_{EC}

As its name suggests this coordinate system has its origin at the center of the Earth (figure 2.1). Its axes may be arbitrarily selected with respect to fixed positions on the surface of the Earth. We will take x_{EC} pointing from O_{EC} to the point of zero latitude and zero longitude on the Earth's surface, and z_{EC} in the direction of the spin vector of the Earth. This coordinate system obviously rotates with the Earth.

2.2.3 The Earth-fixed reference frame, F_E

This coordinate system (figure 2.2) has its origin fixed to an arbitrary point on the surface of the Earth (assumed to be a uniform sphere). x_E points due North, y_E points due East, and z_E points toward the center of the Earth.

2.2.4 The local-horizontal reference frame, F_H

This coordinate system (figure 2.3) has its origin fixed to any arbitrary point that may be free to move relative to the Earth (assumed to be a uniform sphere). For example, the origin may be fixed to the center of gravity (CG) of an aircraft and move with the CG. x_H points due North, y_H points due East, and z_H points toward the center of the Earth.

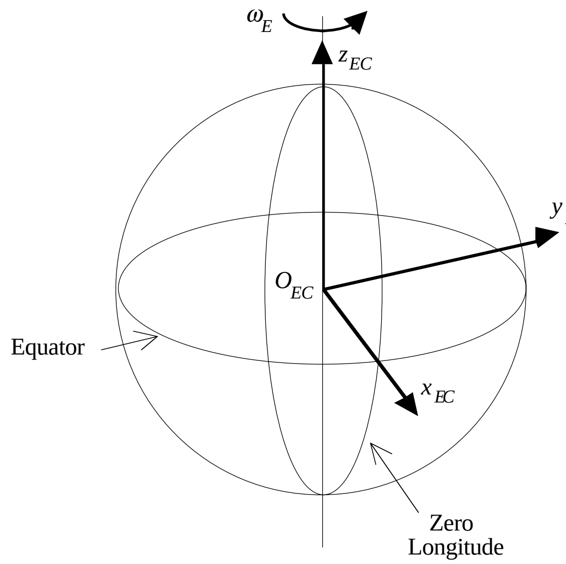

Figure 2.1: Earth-Centered Reference Frame

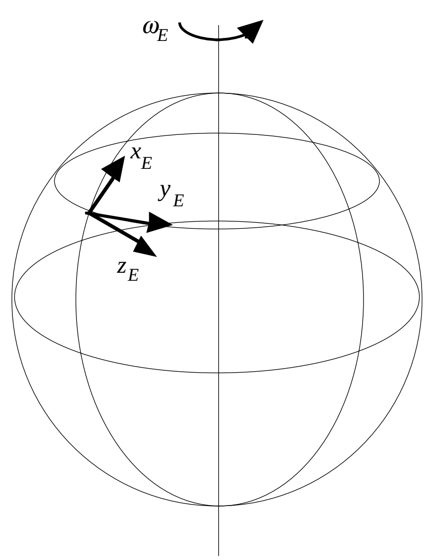

Figure 2.2: Earth-Fixed Reference Frame

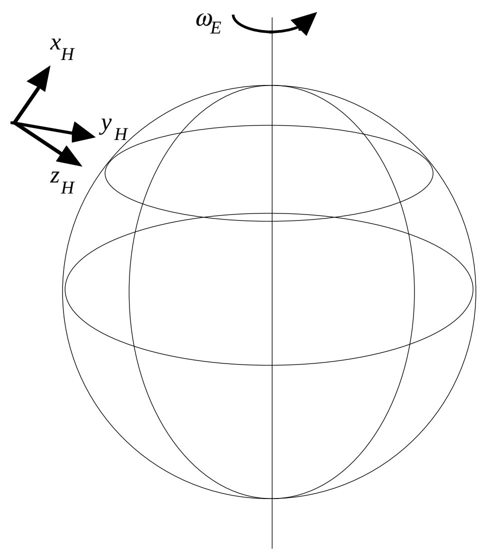

Figure 2.3: Local-Horizontal Reference Frame

2.2.5 Body-fixed reference frames, F_B

Body-fixed means the origin and axes of the coordinate system are fixed with respect to the (nominal) geometry of the aircraft. This must be distinguished from body-carried systems in which the origin is fixed with respect to the body but the axes are free to rotate relative to it. In flight dynamics body-fixed reference frames usually have their origin at the *CG*. In some applications in which the *CG* will change appreciably (e.g., flight simulation) the origin may be at some fixed fuselage reference point. (In ship stability and control, body-fixed coordinate systems are usually at the ship's center of buoyancy.) Determination of the orientation of the axes is as follows (see figure 2.4): if the aircraft has a plane of symmetry (and we will assume in this course that they all do) then x_B and z_B lie in that plane of symmetry. x_B is chosen to point forward and z_B is chosen to point downward. There are some obvious difficulties with this specification: Forward may mean "toward the pointy end" or "in the direction of flight". In high angle-of-attack flight this gets confusing, since the direction of flight may be more toward the underside of the aircraft than toward the nose. There are clearly an infinite number of reference frames that satisfy the definition of a body-fixed coordinate system. Figure 2.5 shows two possibilities.

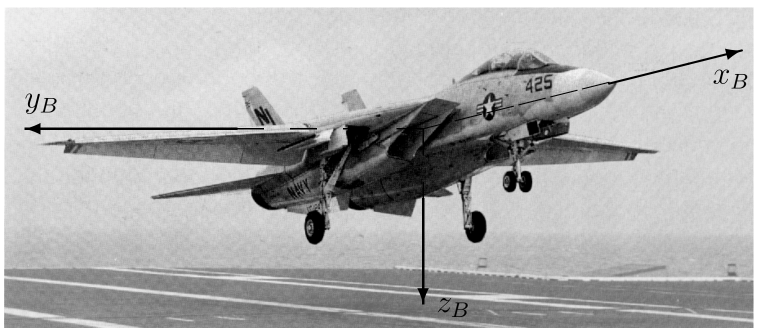

Figure 2.4: Body-Axis Reference Frame. x_B and z_B are in the aircraft plane of symmetry.

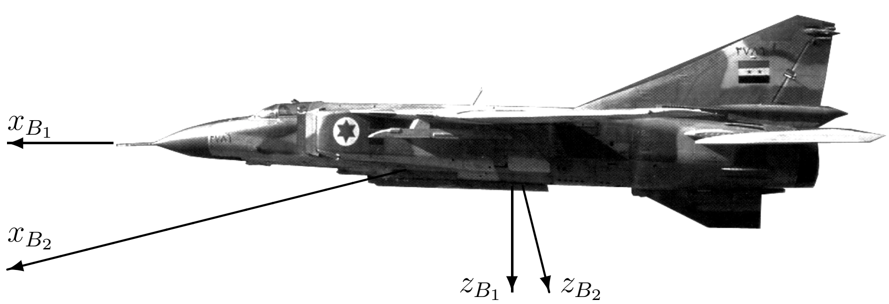

Figure 2.5: Two Different Body-Axis Reference Frames. y_B is common, and plane of page is the aircraft plane of symmetry.

Some of the more important body-fixed coordinate systems are:

Principal axes, F_P

For every rigid body an orthogonal coordinate system may be found in which cross-products of inertia are zero. By the assumption of a plane of symmetry (geometric and mass symmetry) two of these axes will lie in the plane of symmetry. These are named x_P and z_P , and the typical longish nature of aircraft will permit one of these axes to be selected to be toward the nose, and this is x_P .

Zero-lift body-axis system, F_Z

Assuming the relative wind lies in the plane of symmetry (no sideslip) then there is a direction of the wind in this plane for which the net contribution of all surfaces of the aircraft toward creating lift is zero. The x_Z axis is selected to be into the relative wind when lift is zero. This is usually toward the nose of the aircraft, permitting the other axes to be chosen as described.

Stability-axis system, F_S

This is a special system defined as follows (see figure 2.6): We consider the aircraft in some reference flight condition, usually steady flight so that the relative wind is seen from a constant direction by the aircraft. The x_S axis is taken as the projection of the velocity vector of the aircraft relative to the air mass into the aircraft plane of symmetry. Normally this is toward the nose of the aircraft, permitting the other axes to be chosen as for other body-axis systems. It is important to remember that this is a true body-axis system, and that once defined the orientation of the axes relative to the aircraft is fixed, even if the direction of the relative wind changes.

This definition of the stability-axis system is that of Etkin [9, 10]. Other authors, Stevens and Lewis [17] for instance, define the stability-axes as a body-carried (not fixed) coordinate system in which the x-axis is the projection of the time-varying velocity vector into the plane of symmetry. By this definition the stability axes are mid-way between the wind axes (below, section 2.7) and some body axis system.

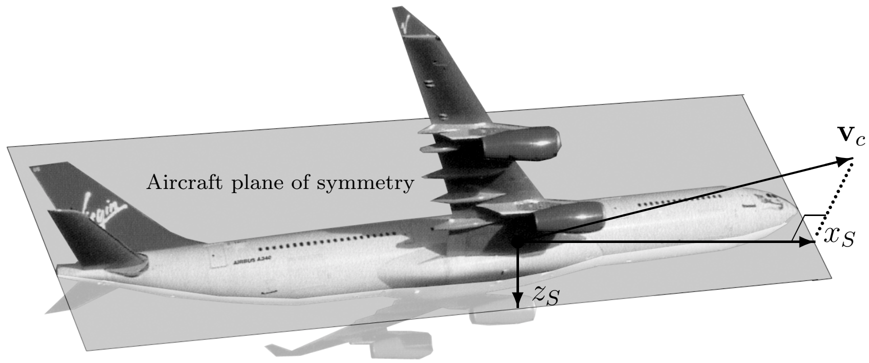

Figure 2.6: Stability-Axis Reference Frame. x_S is the orthogonal projection of a reference $\mathbf{v}_c$ onto the aircraft plane of symmetry.

2.2.6 Wind-axis system, F_W

Shown in figure 2.6, F_W is a body-carried (origin fixed to the body, normally the CG) coordinate system in which the x_W axis is in the direction of the velocity vector of the aircraft relative to the air mass. The z_W axis is chosen to lie in the plane of symmetry of the aircraft and the y_W axis to the right of the plane of symmetry. Note that x_W need not lie in the plane of symmetry. If the relative wind changes, the orientation of the wind-axes changes too, but z_W always lies in the plane of symmetry as defined.

2.2.7 Atmospheric reference frame

It is hard to define a reference frame that characterizes the motion of the atmosphere. What we usually think we know is the motion of the atmosphere relative to the Earth's surface, so a local horizontal reference frame might seem appropriate. But the atmosphere is not a rigid body with a meaningful center of gravity to which to affix the origin of such a system, and at any rate such a system would have to be characterized with rotational properties so that the atmosphere did not fly tangentially off into space. In flight dynamics problems (as opposed to navigation problems) it is the instantaneous interaction of the airframe with the air mass that is of interest. For these probvlems the atmosphere is typically thought of as a separate earth-fixed system to which appropriate extra components are added to account for winds, gusts, and turbulence.

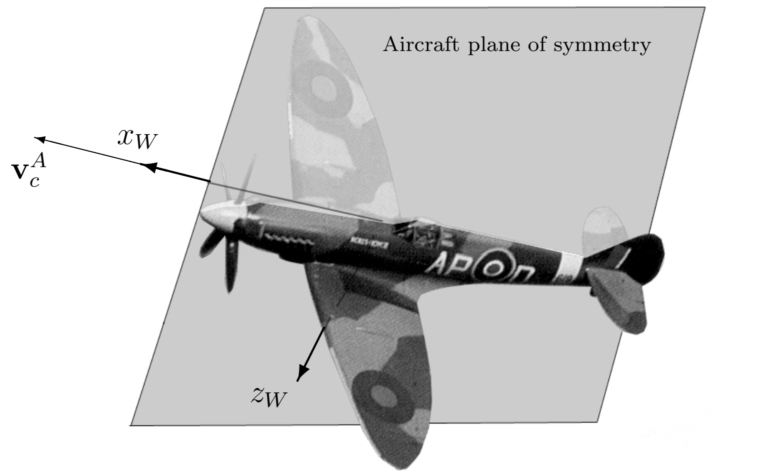

Figure 2.7: Wind-Axis Reference Frame. z_W lies in the aircraft plane of symmetry, but in general x_W does not.

2.3 Vector Notation

Vectors are denoted by **bold** symbols. Vectors are often defined as some measurement (position, linear velocity, angular velocity, linear acceleration, angular acceleration, etc.) of one point or body relative to another. These points and bodies will be referred to by some physical name (or a symbol such as c for CG) or by the name of a coordinate system whose origin is fixed to the point or body. We will use a subscript to denote the first such point and a superscript for the other. Omission of a superscript may usually be taken to mean we are referring to an inertial reference frame. Commonly used symbols are $\mathbf{r}$ for position, $\mathbf{v}$ for linear velocity, $\mathbf{a}$ for linear acceleration, $\boldsymbol{\omega}$ for angular velocity, and $\boldsymbol{\alpha}$ for angular acceleration. Examples:

$\mathbf{r}_p^E$:	Position vector of the point p relative to (the origin of) some F_E.
$\boldsymbol{\omega}_E^{EC}$:	Rotation of some F_E relative to the Earth-centered coordinate system.
$\mathbf{a}_c$:	Linear acceleration of the point c relative to inertial space.

Vectors can exist notionally as just described, but to quantify them they must be *represented* in some coordinate system. Once defined, the notional vector can be represented in any coordinate system by placing the vector at the origin of the coordinate system and finding its components in the x,

y, and z directions. In general these components will be different in two different coordinate systems, unless the two coordinate systems have parallel axes. We will denote the coordinate system in which a vector is represented by affixing a second subscript. In order to avoid multiple subscripts, curly braces $\{\cdot\}$ will be used to enclose the vector, and the subscript appended to the right brace will indicate the system in which the vector is represented. Examples:

$\left\{\mathbf{r}_p^E\right\}_E$: The vector is $\mathbf{r}_p^E$, the position vector of the point p relative to (the origin of) some F_E. The vector is represented in F_E, as $\mathbf{r}_p^E = r_x\mathbf{i}_E + r_y\mathbf{j}_E + r_z\mathbf{k}_E$ or, in vector notation

$$\mathbf{r}_p^E = \begin{Bmatrix} r_x \\ r_y \\ r_z \end{Bmatrix}$$

$\left\{\mathbf{v}_c^E\right\}_B$: The vector is $\mathbf{v}_c^E$, the velocity of point c relative to the origin of an Earth-fixed reference frame. The components of the vector are as projected into a body-fixed coordinate systemF_B

$\{\boldsymbol{\omega}_B\}_B$: The vector is $\boldsymbol{\omega}_B$, the inertial angular rotation rate of a body-fixed coordinate system. The omission of a superscript suggests the inertial reference frame. The vector is represented in a body-fixed coordinate system.

Once defined it may be convenient to omit the subscripts and superscripts of a vector, especially if the vector is referred to repeatedly, or if the notation becomes cumbersome.

2.4 Customs and Conventions

2.4.1 Latitude and Longitude

Position on the Earth is measured by *latitude* and *longitude*. Denote latitude by the symbol λ and longitude by the symbol μ. We will measure latitude positive north and negative south of the equator, $-90\,\text{deg} \leq \lambda \leq +90\,\text{deg}$; and longitude positive east and negative west of zero longitude, $-180\,\text{deg} < \mu \leq +180\,\text{deg}$. Latitude and longitude may refer to either an Earth-fixed coordinate system λ_E, μ_E or a local horizontal cooridinate system λ_H, μ_H.

2.4.2 Body Axes

The names of the axes in a body axis system are often not subscripted, and appear simply as *x, y,* and *z.*

2.4.3 "The" Body-axis System

This ill-defined term is frequently used at all levels of discussion of aircraft flight dynamics. Its usage often seems to imply that there is only one body-axis system, when in fact there are an infinite number. If it is unclear, then the user of such an expression should be asked to specify how it was chosen.

"The" body-axis system often refers to the fuselage reference system described by Liming [14]. In this system the y-axis is positive aft (the negative of our x_B-axis) and the x-axis is positive out the left wing (the negative of our y_B-axis). This leaves the z-axis positive up (the negative of our z_B-axis). Liming described the x-y plane as the *waterline plane*, taken as horizontal when the aircraft is in the "rigged" position, i.e., on the shop floor during assembly. Water lines are measured along the z-axis from this plane. The x-z plane is called the *fuselage station plane.* Fuselage stations (coordinates along the y-axis) are numbered beginning at the nose of the aircraft, not the origin. The y-z plane is the plane of symmetry, and coordinates along the x-axis (spanwise) are called buttock, or butt lines. Liming describes the origin as simply the intersection of the plane of symmetry (which is well defined) and the other two planes, which are not well defined.

2.4.4 Aerodynamic Angles

The angle between the velocity vector and the plane of symmetry, measured in the plane $x_W - y_W$, is called sideslip and is denoted by the symbol β. Sideslip is positive when the relative wind is from the right of the plane of symmetry, as shown in figure 2.8.

Consider the projection of the velocity vector $\mathbf{v}_c^A$ into the plane of symmetry, and assume some body-fixed coordinate system has been defined. The angle between this projection and the x_B axis is called the angle-of-attack and is given the symbol α. It is positive when the relative wind is from below the x_B axis as shown in figure 2.9.

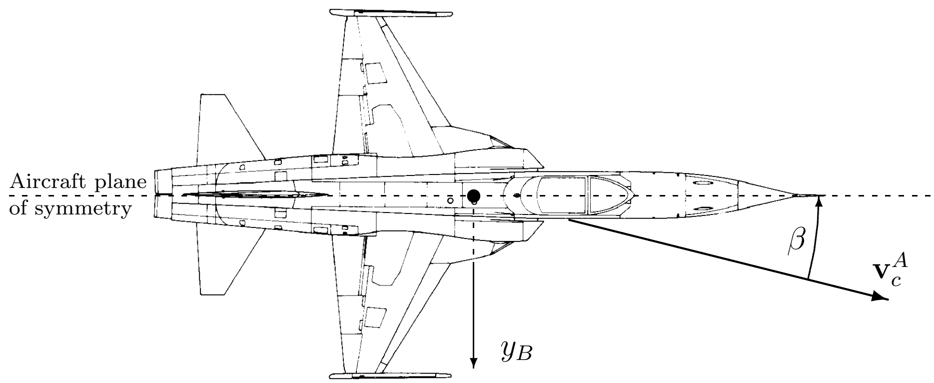

Figure 2.8: Sideslip angle, β. The axis y_B and velocity vector $\mathbf{v}_c^A$ lie in the plane of the page.

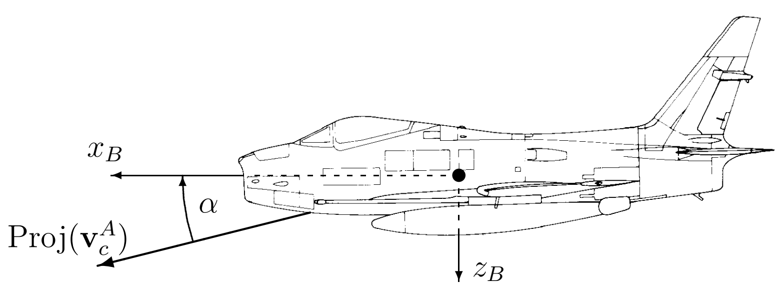

Figure 2.9: Angle-of-Attack, α. Plane of the page is the aircraft plane of symmetry, x_B-z_B. Proj($\mathbf{v}_c^A$) is the projection of $\mathbf{v}_c^A$ onto the plane of symmetry.