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This section introduces an approach that facilitates onboard mission planning and task execution by an autonomous UAV. The ultimate goal of this design is to enable near real-time mission planning or its rapid modification should the operational conditions change and need arise. As soon as the mission objectives are defined by the high level cognitive components, then the set of specific tasks targeting an application of the UAV in a priory given operational environment, needs to be designed. The task should specify the trajectory (both the path and the velocity profile) such that the mission objectives are achieved, the UAV and its payload do not exceed the flight dynamics constraints, and that the operational constraints such as radar detection, airspace deconfliction and collision avoidance conditions are met. Thus, this section outlines a path generation approach that is suitable for near real-time computation of feasible trajectories for a single UAV that accounts for the flight dynamics and operational constraints, and can be followed by resorting to the path following algorithm.[2]

Nomenclature

J	Jacobian Matrix
f	Residual value vector
x	Variable value vector
F	Force, N
m	Mass, kg
Δx	Variable displacement vector
α	Acceleration, m/s^2

Subscript

i	Variable number

I. Feasible Path Generation

This section introduces an approach that facilitates onboard mission planning and task execution by an autonomous UAV. The ultimate goal of this design is to enable near real-time mission planning or its rapid modification should the operational conditions change and need arise. As soon as the mission objectives are defined by the high level cognitive components, then the set of specific tasks targeting an application of the UAV in a priory given operational environment, needs to be designed. The task should specify the trajectory (both the path and the velocity profile) such that the mission objectives are achieved, the UAV and its payload do not exceed the flight dynamics constraints, and that the operational constraints such as radar detection, airspace deconfliction and collision avoidance conditions are met. Thus, this section

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primarily outlines a path generation approach that is suitable for near real-time computation of feasible trajectories for a single UAV that accounts for the flight dynamics and operational constraints, and can be followed by resorting to the path following algorithm.[2]

Consider a single UAV that is tasked to fly a typical mission while avoiding detection by an a priory known set of radars; the UAV performs its mission starting from its current state (initial conditions) and arriving to the final conditions specified by the mission planner. While the exact duration of the mission is not know a priory, it is quite typical that the duration needs to be minimized; this can be justified by the nature of the mission itself and/or the restrictions on energy expenditures imposed by the UAV platform. Furthermore, suppose the objective of the UAV task is not only to minimize the mission duration or the vehicle energy expenditures while meeting dynamical constraints (e.g. bounds on maximum accelerations), but also to minimize the probability of being detected by a number of radars.

The described problem belongs to the class of optimal control problems and it would be desirable to have an algorithm capable of solving it in near real time. Unfortunately, classical indirect approaches of calculus of variations (Bellmans dynamic programming, Pontrjagins maximum principle) can handle only very simple problems like a double integrator where even an analytical solution is possible. However, obtaining an optimal solution off-line for more realistic system is still quite difficult. That is why various simplifying approaches have been developed to provide a near-optimal solution in close to real-time rather than optimal solution (collocation method, method of differential inclusions, etc.) off-line.

This work adopts the key idea of the approach proposed in Ref.[1] The method is based on explicit separation of the path and the velocity associated with it. Both, the path and the velocity profile are represented by using some reference functions expressed via a limited number of variable parameters such as, for example, length of the path. The remaining states and controls can be determined using inverse dynamics of original non-linear equations driving the system. As soon as the values of variable parameters are fixed, then the path and the associated velocity profile are given, and the required controls can be quickly evaluated using the inverse dynamics. When done, a cost function representing the objective of the optimization task can be calculated. Implementing this procedure iteratively enables calculating the best solution (path and velocity profile) by varying a limited number of variable parameters. The following brief explanation outlines the key elements of this approach related to the tactical components of the cognitive mission planner. For detailed presentation of the approach and its historical evolution an interested reader is referred to the publication[1]and the references herein.

Thus, the approach to path generation exploits decoupling of spatial and temporal specifications. Let $p_c(\tau) = [x(\tau), y(\tau), z(\tau)]^\top$ denote a desired path to be followed by a single UAV in 3D space, parameterized by $\tau \in [0, \tau_f]$. For computational efficiency, assume each coordinate $x(\tau), y(\tau), z(\tau)$ is represented by an algebraic polynomial of degree N of the form

$$x_i(\tau) = \sum_{k=0}^{N} a_{ik}\tau^k, \qquad i = 1, 2, 3, \tag{1}$$

where we set $x_1 = x, x_2 = y, x_3 = z$ for notational convenience. The degree N of polynomials $x_i(\tau)$ is determined by the number of boundary conditions that must be satisfied. Notice that these conditions (that involve spatial derivatives) are computed with respect to the parameter τ; this parameter will be later related to actual temporal derivatives. Let d_0 and d_f be the highest-order of the spatial derivatives of $x_i(\tau)$ that must meet specified boundary constraints at the initial and final points of the path, respectively. Then, the minimum degree N^* of each polynomial in (1) is $N^* = d_0 + d_f + 1$. For example, if the desired path includes constraints on initial and final positions, velocities, and accelerations (second-order derivatives), then the degree of each polynomial is $N^* = 2 + 2 + 1 = 5$. Explicit formulae for computing boundary conditions $p_c'(0), p_c''(0)$ and $p_c'(\tau_f), p_c''(\tau_f)$ are given later in this section. Additional degrees of freedom may be included by making $N > N^*$. As an illustrative example, Table 1 shows how to compute the polynomial coefficients in (1) for polynomial trajectories of 5^{th} and 6^{th} degree. For 6^{th} degree polynomial trajectories, an additional constraint on the fictitious initial jerk (third-order derivatives) is included, which increases the order of the resulting polynomial and affords extra (design) parameters $x_i'''(0); i = 1, 2, 3$.

It is important to clarify how temporal constraints may be included in the feasible path computation process. A trivial solution would be to make $\tau = t$. However, very little control exists over the resulting speeds even with fifth and sixth order polynomials, because once $x_1(t), x_2(t), x_3(t)$ have been computed to satisfy the boundary constraints imposed, speed v is inevitably given by

$$v(t) \quad = \quad \sqrt{\dot{x}_1^2(t) + \dot{x}_2^2(t) + \dot{x}_3^2(t)}. \tag{2}$$

We therefore consider a different procedure that will enable meeting strict boundary conditions and constraints without increasing the complexity of the path generation process. To this effect, let $v_{\min}, v_{\max}$ and $a_{\max}$ denote predefined bounds on the vehicle's speed and acceleration, respectively. Let $\eta(\tau) = d\tau/dt$, yet to be determined, dictate how parameter τ evolves in time. A path $p_c(\tau)$ (with an underlying assignment $\eta(\tau)$) is said to constitute a *feasible* path if the resulting trajectory can be tracked by an UAV without exceeding pre-specified bounds on its velocity and total acceleration along that trajectory. With an obvious use of notation, we will later refer to a spatial path only, without the associated $\eta(\tau)$, as a feasible path.

From (2), and for a given choice of $\eta(\tau)$, the temporal speed $v_p(\tau(t))$ and acceleration $a_p(\tau(t))$ of the vehicle along the path (abbv. $v_p(\tau)$ and $a_p(\tau)$, respectively) are given by

$$\begin{aligned} v_p(\tau) &= \eta(\tau)\sqrt{x_1'^2(\tau) + x_2'^2(\tau) + x_3'^2(\tau)} \;=\; \eta(\tau)||p_c'(\tau)||, \\ a_p(\tau) &= = ||\ddot{p}_c(t)|| = ||p_c''(\tau)\eta^2(\tau) + p_c'(\tau)\eta'(\tau)\eta(\tau)||. \end{aligned} \tag{3}$$

Table 1. Examples of computation of the coefficients of 5$^{\text{th}}$ and 6$^{\text{th}}$ order polynomial paths.

5$^{\text{th}}$ order	
Boundary conditions	$x_i(0), x_i'(0), x_i''(0), x_i(\tau_f), x_i'(\tau_f), x_i''(\tau_f)$
d_0/d_f	2/2
N^*/N	5/5
Linear algebraic matrix equation to solve for the coefficients a_{ik}	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & \tau_f & \tau_f^2 & \tau_f^3 & \tau_f^4 & \tau_f^5 \\ 0 & 1 & 2\tau_f & 3\tau_f^2 & 4\tau_f^3 & 5\tau_f^4 \\ 0 & 0 & 2 & 6\tau_f & 12\tau_f^2 & 20\tau_f^3 \end{bmatrix} \begin{bmatrix} a_{i0} \\ a_{i1} \\ a_{i2} \\ a_{i3} \\ a_{i4} \\ a_{i5} \end{bmatrix} = \begin{bmatrix} x_i(0) \\ x_i'(0) \\ x_i''(0) \\ x_i(\tau_f) \\ x_i'(\tau_f) \\ x_i''(\tau_f) \end{bmatrix}$
6$^{\text{th}}$ order	
Boundary conditions	$x_i(0), x_i'(0), x_i''(0), x_i'''(0), x_i(\tau_f), x_i'(\tau_f), x_i''(\tau_f)$
d_0/d_f	3/2
N^*/N	5/6
Linear algebraic matrix equation to solve for the coefficients a_{ik}	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 1 & \tau_f & \tau_f^2 & \tau_f^3 & \tau_f^4 & \tau_f^5 & \tau_f^6 \\ 0 & 1 & 2\tau_f & 3\tau_f^2 & 4\tau_f^3 & 5\tau_f^4 & 6\tau_f^5 \\ 0 & 0 & 2 & 6\tau_f & 12\tau_f^2 & 20\tau_f^3 & 30\tau_f^4 \end{bmatrix} \begin{bmatrix} a_{i0} \\ a_{i1} \\ a_{i2} \\ a_{i3} \\ a_{i4} \\ a_{i5} \\ a_{i6} \end{bmatrix} = \begin{bmatrix} x_i(0) \\ x_i'(0) \\ x_i''(0) \\ x_i'''(0) \\ x_i(\tau_f) \\ x_i'(\tau_f) \\ x_i''(\tau_f) \end{bmatrix}$

In the scope of this work we are interested in small UAVs that operate essentially at constant speeds. Clearly, in this case speed constraints can be easily satisfied for any constant $v_p \in [v_{\min}, v_{\max}]$. This in turn defines

$$\eta(\tau) = \frac{v_p}{||p_c'(\tau)||} \tag{4}$$

$$\dot{p}_c(t) = v_p \frac{p_c'(\tau)}{||p_c'(\tau)||} \tag{5}$$

Now using (3, 4) we obtain

$$\ddot{p}_c(t) = \frac{v_p^2}{||p_c'(\tau)||^2}(I - \frac{p_c'(\tau)(p_c'(\tau))^T}{||p_c'(\tau)||^2})p''(\tau). \tag{6}$$

Therefore, we can choose

$$p_c'(0) = \frac{\dot{p}_c(0)}{||\dot{p}_c(0)||} \tag{7}$$

$$p_c'(\tau_f) = \frac{\dot{p}_c(t_f)}{||\dot{p}_c(t_f)||}. \tag{8}$$

to satisfy boundary conditions on $\dot{p}_c(t)$. Similarly, setting

$$\begin{aligned} p_c''(0) &= \ddot{p}_c(0) \\ p_c''(\tau_f) &= \ddot{p}_c(t_f), \end{aligned}$$

satisfies equation (6) at the boundaries.

On the other hand, the total acceleration a_p of a vehicle flying along the path $p_c(\tau)$ at a constant speed is the product of the curvature of the path with its velocity along the path squared. The curvature of the path $p_c(\tau)$ is given by

$$\kappa(\tau) = \frac{1}{||p'(\tau)||}||\frac{d}{d\tau}\frac{p_c'(\tau)}{||p_c'(\tau)||}||.$$

Thus, using simple algebra it can be shown that

$$\begin{aligned} a_p(\tau) &= v_p^2\kappa(\tau) \\ &= \frac{v_p^2}{||p_c'(\tau)||^2}||(I - \frac{p_c'(\tau)(p_c'(\tau))^T}{||p_c'(\tau)||^2})p''(\tau)||, \end{aligned}$$

which as expected is the norm of $\ddot{p}_c(t)$ (see equation (6). Therefore, for the case of constant velocities v_p a feasible path must satisfy the following set of constraints

$$v_{\min} \le v_p \ \le v_{\max}, \quad a_p(\tau) \le a_{\max}, \ \forall \tau \in [0, \tau_f]. \tag{9}$$

for a pre-specified acceleration bound $a_{\max}$.

In this paper, we use this simple definition of a feasible path to address the problem of a mission planning of a tactical UAV whereby the UAV must avoid detection by a radar and accomplish the mission in a minimum time. The approach proposed here finds a feasible path that makes the minimum time mission planning problem easily solvable by a single UAV flying at constant speeds along the path. Next, we make these ideas more precise.

Let l_f denote the total path length and v_p denote its velocity along this path. Then

$$l_f = \int_0^{\tau_f} ||p_c'(\tau)||\ d\tau.$$

It follows immediately that the time of flight t_f of UAV is given by

$$t_{f_{\min}} = \int_0^{\tau_f} \frac{||p_c'(\tau)||}{v_\mathrm{p}}\ d\tau.$$

Define a cost function $J = t_f$. Then, making J arbitrarily small over the set of feasible paths, feasible velocities and accelerations will result in the desired solution to the minimal time problem discussed above. Therefore, we propose to solve the following path generation problem

$$F : \begin{cases} \min\limits_{\tau_f, v_p,} \{J\} \\ \textit{subject to boundary conditions and limitations } (9) \\ \textit{while} \min\limits_{j=1,\dots,n,} ||P_{det_i}(\tau)||^2 \le E^2, \end{cases} \tag{10}$$

Solution to the optimization problem F includes an optimal paths and constant speed profile that together minimize J subject to boundary conditions and control limitations (9) and an additional penalty function Δ that besides penalizing the UAV states $\eta(\tau)$, controls $\xi(\tau)$ and their derivatives $\pi(\tau)$ defines the UAV's detection probability P_{det_i} by a set of $i = 1, \dots, n$ radars. The choice of penalty function is what makes the optimization task relevant to the tactical sense of the UAV mission. In application to the task at hand it can be represented as follows:

$$\begin{aligned} \Delta = \textstyle\sum_{j=1}^{k} w_j \cdot max(0; \eta - \eta_{bound})^2 + \sum_{j=1}^{l} w_{j+k} \cdot max(0; \xi - \xi_{bound})^2 \\ + \textstyle\sum_{j=1}^{m} w_{j+k+l} \cdot max(0; \pi - \pi_{bound})^2 + \sum_{i=1}^{n} w_{j+k+l+m+n} \cdot max(0; P_{det_i} - P_{bound})^2 \end{aligned} \tag{11}$$

where the values of η_{bound},ξ_{bound},π_{bound} are the design bounds defined a priory, and w_j are the weight coefficients ($\sum_{j=1}^{j+k+l+m+n} w_j = 1$) that are chosen heuristically to ensure specified accuracy of matching the constraints. As a result the optimization problem is reduced to a nonlinear programming problem with an objective of minimizing the scalar function of a limited number of variables.

The optimization problem F can be effectively solved in near real-time by constructing a penalty function G as discussed in works[1,3] and by using any zero-order optimization technique like the Hooke-Jeeves pattern direct-search algorithm[4] or Nelder-Mead downhill simplex algorithm.[5] In the task at hand the Hooke-Jeeves optimization algorithm was implemented online at $4Hz$ thus producing a near optimal solution not faster than 4 times a second. Constraining the update rate of the optimization code allowed balancing the computational load of the onboard CPU.

A particular example of tactical UAV mission solved by utilizing the path generation algorithm will be provide later in the experimental results section.

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