# MULTICRITERIA ANALYSIS OF L1 All-ADAPTIVE FLIGHT CONTROL SYSTEM

The paper provides an architectural overview of the multi-parametric design of the L1 adaptive control system in a multiple dimensional criteria space. In particular, the study addresses the task of the all-adaptive flight control system design of the dynamically-scaled GTM AirSTAR aircraft. The study starts with a detailed justification of the choice of the L1 approach for the all-adaptive flight control system. The discussion addresses the systematic design procedures given by the L1 adaptive control theory and outlines its main challenge. While the nominal solution given by the L1 adaptive control theory accounts for a number of design parameters and desired control performance specifications it still cannot predict how this nominal design fits into an extended space of desired performance criteria which might be critical in adverse flight conditions. Addressing this issue, the paper presents results of the application of the Parameter Space Investigation method for the design of the L1 flight control system. On the one hand, these results highlight good inherent accuracy of the design procedures of the L1 adaptive control theory. On the other hand, they illustrate how an application of the multi-parametric search method achieves deeper understanding of system performance in a wider control criteria space. The results and conclusions of this paper have contributed to the improvement of the flying qualities and the robustness margins of the all-adaptive L1-augmented AirSTAR aircraft during its flight test deployment.

### Introduction

The initial impulse for this research was given by the Integrated Resilient Aircraft Control (IRAC) project led by the NASA Aviation Safety Program which primary objective is to advance state of the art in the adaptive control technology. Special interest of the IRAC program is in piloted flight under adverse flight conditions such as unusual attitudes, control surface failures, icing, and structural damages. Besides theoretical advancement, the subscale flight-testing and pilot evaluation of experimental adaptive control laws, especially in conditions beyond the normal flight envelop, are seen as critical directions of the program. Advancing control theory, developing new adaptive control algorithms, and then integrating them onboard of subscale aircrafts or full-scale research prototypes and rigorously validating the theoretical claims is what takes to bring a new control methodology to life.

Inner-loop adaptive flight control systems (FCS) are seen as an appealing technology [1] to improve aircraft performance with reduced pilot compensation in adverse flight conditions or in the event of control surface failures and vehicle damage. Under these conditions, which are characterized by a high degree of uncertainty with respect to a nominal aircraft, the achievable levels of performance and flying qualities (FQ) that a nonadaptive FCS can provide might be limited. However, in applications with stringent performance and robustness specifications [2], several limitations of conventional adaptive control architectures have been identified which render the design of these conventional adaptive controllers overly challenging. Among these limitations [2], [3], are of special relevance: (*i*) the lack of transient characterization of the closed-loop response; (*ii*) the limited analysis framework for robustness and performance guarantees for closed-loop adaptive systems, and *(iii)* the lack of systematic design guidelines to solve the trade-off between adaptation, performance, and robustness.

The L1 adaptive control theory [4] addresses precisely these limitations. The key feature of this novel approach is the decoupling of adaptation and robustness. In fact, in L1 adaptive control architectures, the speed of adaptation is limited only by the available hardware (computational power and high-frequency sensor noise), while the trade-off between performance and robustness can be addressed via conventional methods from classical and robust control. Fast adaptation, which enables compensation of the undesirable effects of rapidly varying uncertainties and significant changes in system dynamics, is critical towards achieving guaranteed transient performance without enforcing persistency of excitation, applying gain scheduling of the control parameters, or resorting to control reconfiguration or high-gain feedback. Moreover, starting with the formulation of the desired closed-loop performance specifications the L1 adaptive control theory provides the systematic design procedures that significantly reduce the tuning effort. In turn, this leads to reduction in both the design cycle time and the development costs.

The main challenge, however, for the design of the L1 FCS is the optimal tuning of its elements to provide desired flying qualities with satisfactory robustness margins. While the theory of L1 adaptive control provides systematic design guidelines to address the trade-off between performance and robustness, the optimization of the design of the L1 adaptive controller is still largely open and hard to address. The main difficulty is the non-convex and non-smooth nature of the underlying optimization problem that involves the L1-norm of cascaded linear systems. Randomized parametric algorithms have been proven to be effective in control-related non-convex optimization problems, and therefore they seem attractive for the optimal design of L1 adaptive controllers [4],[5]. Parameter Space Investigation (PSI) method [6], [7], is one of those approaches that explain our interest in employing its flexible powers to the task of L1 all-adaptive FCS design optimization. Combining advances of the L1 adaptive control with the ability of multidimensional search not only confirms the benefits of L1 theory as verifiable adaptive control architecture but also presents the benefits of higher level optimization architecture suitable for formulating and solving a milticriteria control optimization problem.

The paper is organized as follows. The formulation of the design optimization of the L1 FCS of the NASA AirSTAR[8], [9], flight test vehicle is addressed in Section II. This section also provides a brief discussion of the PSI method and the workflow of the optimization process, including the identification of a nominal prototype design, the construction of the feasible solution set, and the improvement of the prototype. Section III provides solution of the optimization problem and summarizes the key results. The paper ends with a concluding discussion of the achieved result and its impact to the final flight test implementation.

### Formulation of the FCS Design Problem

#### II.A. AirSTAR – Airborne Research Platform

AirSTAR[8], [9] is an integrated flight test infrastructure, which utilizes remotely-piloted, jet-powered subscale models for experimental flight-testing. IRAC project utilizes the AirSTAR ability to rigorously evaluate research control laws in adverse flight conditions. Its current primary flight test vehicle, the GTM tail number T2, is shown in Fig. 1. The T2 is a twin-engine jet-powered and dynamically-scaled (5.5%) civil transport aircraft, was designed and instrumented to perform control law evaluation, experiment design and modeling research, in-flight failure emulation, and flight in upset conditions.



Figure 1. GTM AirSTAR unmanned aircraft and its full-scale prototype.

To supplement the AirSTAR development a high fidelity nonlinear aerodynamic models for the subscale turbine powered Generic Transport Model (GTM) aircraft have been developed. The GTM vehicle (Fig.1) has been extensively tested in the NASA Langley wind tunnels with particular emphasis on modeling nonlinear regions of the extended flight envelope well beyond nominal flight as well as developing a database for a number of structural damage scenarios [10],[11]. The high fidelity nonlinear simulation of the GTM aircraft, built up from the extensive wind tunnel data and flight experiments, has been incorporated into the AirSTAR facility. Thus, the high fidelity nonlinear flight dynamics model is the primary object of the L1 flight control system design.

#### II.B. L1 Flight Control System of AirSTAR

The research control law developed for the AirSTAR flight test vehicle has a primary objective of achieving reliable tracking for a variety of tasks with guaranteed stability and robustness in the presence of uncertain dynamics, such as changes due to rapidly varying flight conditions during standard maneuvers, and unexpected failures. Ideally, the flight control system should provide level 1 flying qualities[12],[13] under nominal and adverse flight conditions.

The L1 FCS designed for this application is a three axes angle of attack (*α*), roll rate (*p*)—sideslip angle (*β*) all--adaptive system. The FCS consists of two decoupled L1 controllers, one for the longitudinal channel, and another one for control of the lateral-directional dynamics. The longitudinal L1 controller is implemented as a Single-Input Single-Output system, and uses feedback in angle of attack and pitch rate to generate an elevator control signal in order to track angle of attack reference signals. The lateral/directional L1 controller is a Multiple-Input Multiple-Output architecture, and uses feedback in sideslip angle, roll rate, and yaw rate to generate aileron and rudder commands in order to track sideslip-angle and roll-rate reference signals with reduced coupling. In the current L1 FCS, the pilot adjusts directly the thrust level using the throttle lever. The main challenge in the design of the L1 FCS is in the optimal tuning of the state predictor and the low-pass filters to provide desired flying qualities with satisfactory robustness margins. The key difficulty in tuning is the non-convex and non-smooth nature of the underlying optimization problem that involves the L1 norm of cascaded linear systems. The reader is referred to [14–16] for a more detailed explanation of the L1 FCS implemented on the NASA AirSTAR flight test vehicle.

The design of the longitudinal L1 FCS is based on the linearized short-period dynamics of the GTM at the reference flight condition (80 kt, 1000 ft); its main elements and the design variables (*DV*) are represented in Figure 2. Since the airplane is level 1 FQ at this flight condition, the desired dynamics of the *state predictor* are chosen to be close to those of the actual aircraft. For the nominal prototype design, the natural frequency of the poles of the system is reduced from  to , while the damping ratio is increased from 0.47 to 0.85. A first-order low-pass filter with DC gain 1 and a bandwidth of  was used in the matched contribution to the elevator command, while two cascaded first-order low-pass filters were used in the unmatched channel, both having DC gain equal to 1 and bandwidths of and respectively. Finally, the adaptation sampling time was set to 600Hz, which corresponds to the fastest integration cycle allowed in the AirSTAR flight control computer. A first-order prefilter with of bandwidth was added to shape the pilot command. This prototype design of the state predictor, the low-pass filters, the adaptation sampling rate, and the prefilter, delivers the angle of attack (AOA) response similar to the desired one, see Figure 3 where  designates the desired angle-of-attack response. This nominal design ensures a time-delay margin of the inner-loop of approximately 85 *msec* and a gain margin of 7.2 *dB*, in wings-level flight at the reference flight condition. At this flight condition, the FQs are predicted to be level 1 and the FCS design has no predicted Pilot Induced Oscillation (PIO) tendencies (for an acquisition time of 1.5 *sec*). Naturally, these metrics are the performance criteria, their initial values provided above for the nominal design will serve as the guidelines for the motivated definition of the criteria constraints.



Figure 2. Longitudinal channel of the L1 flight control architecture.

|  |  |
| --- | --- |
|  |  |
| (a) Angle of attack, [17] | (b) Elevator deflection, |

Figure 3. Prototype Design. 3 deg-AOA step response for the prototype design

We next outline the key steps of the PSI method in the design optimization of the L1 FCS for the NASA GTM airplane[6], [7], [18] . In particular, the objective of the optimization task is to minimize the difference between the desired and actual responses, while ensuring satisfactory FQs and not overloading the actuators. The optimization methodology proposed in this work contains three sequential steps. First, we take advantage of the systematic design guidelines of the L1 adaptive control theory to find a nominal prototype solution satisfying a given set of control specifications. Then, taking the prototype solution as a reference design, the PSI method is used for the construction of the feasible solution set and for determining an initial direction of improvement for the design of the FCS [14–16]. Finally, the PSI method is again used to determine an optimal design that satisfies an extended set of performance and robustness constraints and improves the initial reference prototype.

Although both the PSI and the software package MOVI [18], that implements the method, were developed to address problems with high dimensionality of the design and the criteria spaces, for the sake of clarity, we keep the design problem within a reasonable complexity. Thus, the design procedure is applied to the design of the longitudinal channel only. Furthermore, the results included in this study are obtained by the MOVI software package combined with the MatLab[19] environment, and are based on the full nonlinear simulation of the two-engine-powered dynamically-scaled GTM AirSTAR system, which was released by NASA in December 2009 [9] .

### III. Statement of the Optimization Task

This section addresses the formulation of the design optimization of the L1 FCS. In particular, this section provides a brief discussion of the workflow of the optimization process, including the identification of a nominal prototype design, the construction of the feasible solution set, and the improvement of the prototype.

#### III.A. Criteria and Design Variables

##### Design Variables

Since the primary objective is to improve the FQ of the prototype design while guaranteeing satisfactory robustness margins, we include the natural frequency and the damping ratio of the poles of the state-predictor dynamics (which can speed up or slow down the response of the augmented aircraft), and the bandwidth of the low-pass filter in the matched channel (which can be used to adjust the time-delay margin of the inner-loop) as design variables (DV); see Figure 2 illustrating their place in the L1 flight control architecture. Furthermore, we also consider the optimization of the bandwidth of the prefilter, which can be used to shape the pilot command as to prevent elevator rate limiting and avoid structural mode flight interaction. The following list summarizes the set of optimization parameters that define the design variable space:

* *DV1*. *Natural frequency of the state-predictor poles* (*rad/sec*)
* *DV2*. *Damping ratio of the state-predictor poles*
* *DV3*. *Bandwidth of the “matched” low-pass filter* (*rad/sec*)
* *DV4*. *Bandwidth of the pilot-command prefilter* (*rad/sec*)

##### Criteria and Pseudo-Criteria

The set of design criteria considered in this study is chosen to evaluate performance and robustness properties of the GTM aircraft augmented with the L1 FCS. To provide an adequate assessment of the performance characteristics and flying qualities of the L1-augmented aircraft both pilot-off-the-loop and pilot-in-the-loop performance metrics are included in the design procedure. The metrics considered can thus be classified in three categories:

1. *Pilot-off-the-loop performance metrics;*
2. *Robustness metrics;*
3. *Flying qualities and PIO metrics.*

Because the present material addresses only the design of the longitudinal channel of the L1 FCS, the set of metrics used in this study are mainly based on the (time-domain) longitudinal response of the GTM with the L1 FCS closing the inner-loop. We note that some of the metrics used in this study were also proposed in [20] for the evaluation of aircraft augmented with an adaptive FCS.

***Pilot-off-the-Loop Performance Metrics.*** This first set of metrics evaluates the performance of the augmented aircraft by characterizing its response to step inputs. In particular, the pilot-off-the-loop performance metrics are based on the time-domain response to a step command of 3 *deg* held for 4 *sec* in AOA (see Figure 3), starting from a wings-level flight condition. The metrics capture the deviation of the actual response of the aircraft from a given desired response – which is defined to provide satisfactory flying qualities without reaching the physical limits of the platform –, as well as different measures of control activity, load factor, and cross-coupling.

Next we need to introduce some key notation to facilitate the definition of these metrics. Below, alone with the previously defined (AOA) and , the  is the angle-of-attack pilot command;  is the angle of sideslip; is the sideslip-angle desired response; *p* is the roll rate; is the roll-rate desired response; *A*z is the vertical acceleration; is the elevator deflection command. Let  be the time instant at which the step command is applied, and define  as the final time instance considered for the performance evaluation ( *sec*). With the above notations, the metrics are formally defined as follows:

*P1*. *Final deviation:* this metric captures the final deviation of the actual AOA response from the desired AOA response at 4 *sec* after the application of the step command. This metric is set to zero if the actual response reaches the AOA reference command before the end of the 4-second step:



This metric penalizes or excludes sluggish responses. In this study this metric along with the rest of the metrics below are normalized to the amplitude of the step command (3 *deg*).

*P2. Maximum deviation from desired AOA response:* this metric captures the maximum deviation (in absolute value) of the actual AOA response from the desired AOA response:



*P3*. *Integral deviation from desired AOA response:* this metric is defined as the (truncated) *L*2-norm of the deviation of the actual AOA response from the desired AOA response:



*P4*. *Overshoot in AOA response:* this metric captures possible overshoots and low-damping characteristics in the AOA response:



*P5*. *Maximum deviation from desired AOA rate response:* this metric captures the maximum rate deviation (in absolute value) of the actual AOA response from the desired AOA response:



*P6*. *Integral deviation from desired AOA rate response:* this metric is defined as the (truncated) L2-norm of the rate deviation of the actual AOA response from the desired AOA response:



The metrics *P1* to *P6* provide essential characterization of the transient response of the augmented aircraft when compared to a given desired response. Next, we present a set of metrics that can be extracted from the same step response experiment utilizing different flight dynamics characteristics which complement the AOA-based metrics defined above.

*P7*. *Maximum vertical acceleration:* Load factor (and passenger comfort) requirements can be captured by the maximum vertical acceleration during the step response:



*P8*. *Control effort:* this metric is defined as the (truncated) L2-norm of the elevator deflection command:



This metric penalizes the flight control designs that require a high control activity to achieve a desired control objective. It is important to note, however, that a high control effort might just be the result of a faster AOA response, and therefore a large *P8* might not always be an undesirable response characteristic.

*P9*. *Maximum elevator rate:* Excessive control rate can be identified by the following metric:



This metric penalizes designs with high elevator rates in order to prevent undesirable effects from rate limiting.

*P10*. *Maximum elevator acceleration:* High-order derivatives of the control commands are coupled to the flexible modes of the aircraft. The following metric, based on the second derivative of the elevator command, captures excessive accelerations and oscillations in the control command that could potentially lead to unwanted structural mode interactions:



*P11*. *Maximum of* L1 *prediction error:* this metric captures the maximum error between the actual system state and the state of the L1 state predictor, usually denoted by :



In L1 adaptive control architectures, the accurate estimation of system uncertainties and the performance guarantees rely on the (small) “size” of the prediction error. This metric is used to monitor the correct functioning of the L1 adaptive controller.

*P12*. *Maximum deviation in cross-coupling dynamics:* this metric captures the lateral-directional coupling induced by a command in the longitudinal channel:



This metric primarily provides valuable information for the design of the lateral-directional FCS.

*P13*. *Integral deviation in cross-coupling dynamics:* this metric is the integral version of the previous cross-coupling metric and is defined as follows:



Similar to *P12*, this metric would be more adequate for the design of the lateral-directional control system, and both are included in this study only to illustrate a set of additional metrics that can be derived from the response of the augmented aircraft to a command in the longitudinal channel.

***Robustness Margins.*** In this preliminary study, the only robustness metric considered for optimization is the *time-delay margin* of the closed-loop adaptive system. It is defined at the input of the aircraft (time delay inserted at the elevator deflection command), and it is derived from the time-domain response of the augmented aircraft. For a given wings-level flight condition and with the pilot-off-the-loop, a small perturbation in the trim (initial) condition is introduced. The time-delay margin is determined as the minimum time delay that produces sustained oscillations in the AOA response as the L1 FCS tries to stabilize the aircraft at the given trim condition. In this study, this robustness metric will be denoted by *R1*. Note that the time delay introduced in the elevator control channel is in addition to 25 *msec* that is already modeled in the AirSTAR simulation environment.

***Criteria addressing FQ and PIO characteristics.*** Finally, predictions for both flying qualities and PIO tendencies have also been included in order to complement the pilot-off-the-loop performance metrics presented above. For this study, we consider the Time-Domain Neal-Smith (TDNS) flying qualities and PIO criteria, which was specifically developed for nonlinear aircraft dynamics and nonlinear FCS. For a detailed description of this criterion, the reader is referred to [21]. The reader can also find in [21] a study on theprediction of flying qualities and adverse pilot interactions in the GTM augmented with the L1 FCS. We use four different metrics, extracting all of them from the TDNS criterion for an acquisition time of 1.5 *sec*, to characterize the FQ and PIO tendencies of the augmented aircraft:

*FQ1*. *Tracking performance:* In the TDNS criterion, the root-mean-squared tracking error is used to evaluate the closed-loop performance with the pilot-in-the-loop. A value of zero means that the pilot is able to perfectly track (with zero error) the reference command after the specified acquisition time.

*FQ2*. *Pilot workload:* In the TDNS criterion, the pilot workload is given by the pilot compensation phase angle (in degrees), which is derived from the optimal pilot model obtained from the criterion. A value of zero means that there is no need for either pilot lead or lag compensation.

*FQ3*. *FQ level:* The two metrics above, *FQ1* and *FQ2*, are used to determine the predicted FQ level based on the FQ boundaries proposed in the criterion. *FQ3* is a discrete metric, and it only admits the values 1, 2, and 3, which correspond to level 1, level 2, and level 3 flying qualities respectively.

*FQ4*. *PIO tendency:* The TDNS criterion also provides a prediction for the susceptibility of the augmented aircraft to PIO. This PIO-susceptibility metric is used to complement the flying qualities metrics discussed above. According to the TDNS criterion, a value above 100 implies that the augmented aircraft is PIO-prone, whereas a value below 100 indicates a PIO-immune configuration.

This set of FQ metrics is used at the second step when improving a prototype design of the longitudinal channel of the L1 FCS. For the *first step* of the prototype design and the *exploration* of the feasible set– only a subset of these metrics is utilized. The full set of metrics is used in the *last stage to optimize* the design of the adaptive control system. Based on the objectives of the task and previous flight control design expertise the following vectors of criteria {*P1, P2 , P3 , P4 , P5 , P6 , FQ1, FQ2, FQ3, FQ4, R1* } and pseudo-criteria {*P7, P8, P9, P10, P11, P12, P13*} were defined.

#### III.B. Criteria Constraints

Based on the metrics defined, the final design of the L1 FCS should *ideally* verify the set of control objectives at the reference flight condition of 80 kt of (equivalent) airspeed and 1000 ft of altitude. Corresponding to this flight conditions a set of three criteria constraints were defined a priory:

*P1≤*0.1, *FQ3*=1 and *P4*≤ 1.2.

The first and second conditions address directly the control specifications, namely the final value of the step response within 10% of the desired, and the predicted level 1 FQ. The third inequality imposes a 20% constraint on the overshoot in the step response, establishing thus a (loose) bound on the acceptable transient performance characteristics of the actual AOA response. Due to significant difficulty of defining all criteria constraints consistent with feasibility of the solution, the rest of the constraints were identified interactively while analyzing the test tables.

### IV. Solutions and Analysis

This section presents two steps of iterative application of the PSI method to the design improvement of the longitudinal channel of the L1 FCS. The first iteration utilizes the same set of the control metrics used by the L1 synthesis procedure. Numerical implementation of this first step is relatively efficient with the “computational price” of one solution measured in minutes. At the second step, while utilizing an extended set of criteria, the efficiency of numerical implementation becomes critical because the “computational price” of one solution is measured in tenth of minutes.

As it was mentioned above the PSI method does not alter the optimization task by “converging” a set of multiple criteria to just one scalar functional. To support this multicriteria approach, the MOVI package provides a rich set of analysis tools. Besides numerical results organized as a test table, it provides a number of visual tools. In particular, (i) the histograms of design variables, the (ii) criterion versus design variable, and (iii) the criterion versus criterion graphs are the most intuitive and effective tools used during the interactive analysis. A comprehensive introduction to the effective use of MOVI can be found in [6], [7].

#### IV.A. First iteration

Constructing the feasible solution set starts at the intervals of design variables, which have been identified with respect to a solution given by the L1 nominal synthesis (see Table 1).

Table 1. Initial intervals of design variables

|  |  |  |  |
| --- | --- | --- | --- |
| Design variable | L1 Prototype | Initial intervals of variation of design variables | |
| min | max |
| *DV1* | 5.50E+00 | 4.00E+00 | 8.00E+00 |
| *DV2* | 8.50E-01 | 5.00E-01 | 1.10E+00 |
| *DV3* | 2.00E+01 | 5.00E+00 | 3.00E+01 |
| *DV4* | 2.00E+01 | 1.00E+01 | 5.00E+01 |

The objective of the first step is to find a direction of improvement for the nominal design. More precisely, we aim here at determining tight intervals for the design variables characterizing the state predictor (*DV1* and *DV2*) that would provide level 1 FQ and would not deviate from the desired response defined previously. To this end, the design is to be minimized with respect to the following reduced number of criteria{*P1*, *P2*, *P3*, *P4*, *P5*, *P6*, *FQ1*, *FQ2*}.

The robustness metric *R1* and the PIO metric *FQ4* are not included in this first step because their evaluation is computationally expensive; these metrics will be considered in the next step of the optimization process when the domain of the design variables becomes significantly refined. The metrics *P7* through *P10* are not included in the set of criteria because improved flying qualities may require “high” values of these metrics. Nevertheless, they are included in the optimization process as pseudo-criteria[[1]](#footnote-1) thus providing useful inside onto the dynamics of the augmented aircraft. Similarly, the metric *P11*, which can be used to monitor the correct operation of the L1 adaptive controller, does not need to be minimized as long as it remains a couple of orders of magnitude below the system state (truncated) L∞-norm. Finally, the metrics *P12* and *P13* are included for the sake of completeness and should be considered only for the design of the lateral-directional control system.

The MOVI[18] package performs a predefined set of numerical trials and then forms a test table. Interactive work with the test table consists of sequential tightening of the DV constraints and is well supported by a number of graphical instruments implemented in MOVI.

The final results achieved in this first iteration of the optimization process, is based on 1024 tests. Out of these 1024 tests, 427 vectors did not satisfy the a priori given criteria constraints. The solutions that did not satisfy the constraints entered the *table of criteria failures;* every entry of this table is available for a detailed analysis. The remaining 597 vectors which did satisfy a priori given criteria constraints were used to construct the test table. While tightening the criteria constraints in the test table, the following new criteria constraints were formulated, see Table 2. Note, that while analyzing the test table, the constraint of *P4* was significantly tightened to the value of 1.02. Furthermore, the response on criteria *P1* is not presented in the table because all solutions provided identical response, *P1*=0. Only 20 solutions were found to be feasible according to these criteria constraints, all of them contributing to the Pareto optimal solutions. A fragment of the criteria table is given in Table 3.

Table 2. Criteria constraints

|  |  |  |  |
| --- | --- | --- | --- |
| *P2* ≤ 0.2 | (min) | *P9≤15* | (pseudo) |
| *P3* ≤ 0.2 | (min) | *P10≤300* | (pseudo) |
| *P4* ≤ 1.02 | (min) | *P11≤0.25* | (pseudo) |
| *P5* ≤ 1 | (min) | *P12≤0.01* | (pseudo) |
| *P6* ≤ 0.3 | (min) | *P13≤0.01* | (pseudo) |
| *P7* ≤ 0.25 | (pseudo) | *FQ1≤0.1* | (min) |
| *P8* ≤ 5 | (pseudo) | *FQ2≤45* | (min) |

Table 3. Fragment of Criteria Table

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Criteria | | Prototype | Pareto optimal solutions | | | | | | |
| #241 | #281 | #329 | #409 | #649 | #825 | #993 |
| *P2* | (min) | 1.30E-01 | 1.04E-01 | 8.40E-02 | 9.14E-02 | 8.97E-02 | 6.03E-02 | 7.44E-02 | 5.39E-02 |
| *P3* | (min) | 1.54E-01 | 1.16E-01 | 1.03E-01 | 1.06E-01 | 1.04E-01 | 8.72E-02 | 9.51E-02 | 8.84E-02 |
| *P4* | (min) | 1.0E+00 | 1.00E+00 | 1.00E+00 | 1.00E+00 | 1.00E+00 | 1.01E+00 | 1.00E+00 | 1.00E+00 |
| *P5* | (min) | 3.15E-01 | 5.37E-01 | 9.36E-01 | 9.68E-01 | 6.29E-01 | 8.63E-01 | 8.58E-01 | 6.89E-01 |
| *P6* | (min) | 1.49E-01 | 1.97E-01 | 2.21E-01 | 2.58E-01 | 2.03E-01 | 1.74E-01 | 1.94E-01 | 1.32E-01 |
| *P7* | (pseudo) | 1.51E-01 | 1.65E-01 | 1.74E-01 | 1.84E-01 | 1.72E-01 | 1.83E-01 | 1.78E-01 | 1.75E-01 |
| *P8* | (pseudo) | 3.24E+00 | 3.3E+00 | 3.29E+00 | 3.31E+00 | 3.30E+00 | 3.31E+00 | 3.30E+00 | 3.31E+00 |
| *P9* | (pseudo) | 5.96E+00 | 1.17E+01 | 7.55E+00 | 9.1E+00 | 1.09E+01 | 1.11E+01 | 9.36E+00 | 1.16E+01 |
| *P10* | (pseudo) | 1.07E+02 | 2.42E+02 | 1.34E+02 | 1.72E+02 | 2.16E+02 | 2.24E+02 | 1.76E+02 | 2.44E+02 |
| *P11* | (pseudo) | 7.45E-02 | 7.79E-02 | 6.02E-02 | 6.82E-02 | 8.10E-02 | 7.72E-02 | 7.22E-02 | 7.77E-02 |
| *P12* | (pseudo) | 1.01E-04 | 1.84E-04 | 1.87E-04 | 2.08E-04 | 2.06E-04 | 2.29E-04 | 2.14E-04 | 2.09E-04 |
| *P13* | (pseudo) | 3.16E-05 | 6.18E-05 | 6.87E-05 | 8.01E-05 | 7.00E-05 | 8.18E-05 | 7.58E-05 | 7.03E-05 |
| *FQ1* | (min) | 1.23E-02 | 6.73E-02 | 9.29E-02 | 9.74E-02 | 6.86E-02 | 9.05E-02 | 7.80E-02 | 8.85E-02 |
| *FQ2* | (min) | 5.36E+01 | 4.42E+01 | 4.35E+01 | 4.03E+01 | 4.22E+01 | 3.79E+01 | 4.10E+01 | 4.00E+01 |

Analysis of the criteria table shows that solution #993 is the most preferable one. This solution is equivalent to others with respect to criterion *P4*, it is superior to others over a set of 5 criteria {*P2, P3, P6, FQ1, FQ2*} and is weaker than the prototype only with respect to the criterion *P5*. Furthermore, remaining 19 solutions are better than the prototype with respect to four criteria {*P2, P3, FQ1, FQ2*}. However, none of the solutions are superior to the prototype with respect to criterion *P5.* In particular, this observation implies that if the prototype design vector was sampled by the system, then it would belong to the Pareto set. On the other hand this result confirms high accuracy of the nominal L1 synthesis procedure.

Analysis of the obtained solutions allows defining the direction of further search. In particular, the results have provided tight intervals for the design variables *DV1* and *DV2* characterizing the state-predictor dynamics, and have exposed the necessity of extending the initial intervals of variation of the design variables *DV3* and *DV4,* see for example Figure4. Based on these results and their analysis, a new experiment is carried out to (*i*) improve the feasible solution set, and to (*ii*) determine an optimal solution of the L1 FCS design that improves the prototype with respect to extended set of criteria.



Figure 4. A histogram of *DV4* distribution along with the prototype solution and the direction of improvement.

#### IV.B. Second iteration

Improvement of the feasible solution set is based on the analysis of the histograms and criteria table, see Table 3. The analysis results in adjusting the intervals of variation of the design variables that is given in Table 4.

Table 4. Refined intervals of design variables

|  |  |  |  |
| --- | --- | --- | --- |
| Design variable | Prototype | Initial intervals of variation of design variables | |
| min | max |
| *DV1* | 5.50E+00 | 5.50E+00 | 7.00E+00 |
| *DV2* | 8.50E-01 | 6.50E-01 | 0.90E+00 |
| *DV3* | 2.00E+01 | 9.80E+00 | 4.00E+01 |
| *DV4* | 2.00E+01 | 1.80E+01 | 6.50E+01 |

The criteria constraints remain unchanged, whereas the design is now to be optimized with respect to the extended set of criteria {*P2*, *P3*, *P4*, *P5*, *P6*, *FQ1*, *FQ2*, *FQ4*, *R1*}. All these criteria are to be minimized except for *R1*, which is to be maximized.

The results of second iteration are based on 512 tests producing 124 feasible solutions. All these solutions are Pareto optimal. The histograms in this second iteration have stronger distributions of the feasible solutions than in the first iteration. The top of Figure 5 represents the distribution of 124 solutions for *DV1*.



Figure 5. PSI Iteration 2. Distribution of feasible solutions of DV1 with the original (top) and with tightened criteria constraints (bottom).

Thus, analysis of the test table and histograms leads to a stronger set of criteria and pseudo-criteria constraints as presented in Table 5. According to these new constraints only 6 solutions are feasible, and all of them are Pareto optimal. The values of design variables and criteria of the Pareto optimal solutions are given in Tables 6 and 7 respectively. The new distribution of the feasible solutions for these criteria and pseudo-criteria constraints is significantly tighter as expected and is shown in Figure 5. These new histograms clearly identify tight intervals for all of the design variables in which the optimal solutions lie.

Table 5. Second iteration, refined criteria constraints

|  |  |  |  |
| --- | --- | --- | --- |
| *P2 ≤ 0.1* | (min) | *P10 ≤ 200* | (pseudo) |
| *P3 ≤ 0.15* | (min) | *P11 ≤ 0.1* | (pseudo) |
| *P4 ≤ 1.02* | (min) | *P12 ≤ 0.01* | (pseudo) |
| *P5 ≤ 1* | (min) | *P1 3≤ 0.01* | (pseudo) |
| *P6 ≤ 0.25* | (min) | *FQ1 ≤ 0.1* | (min) |
| *P7 ≤ 0.2* | (pseudo) | *FQ2 ≤ 45* | (min) |
| *P8 ≤ 5* | (pseudo) | *FQ4 ≤ 5* | (min) |
| *P9 ≤ 10* | (pseudo) | *R1 ≥ 80* | (max) |

Table 6. Second iteration. Table of design variables

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Design variable | Prototype | #993, First Iteration | Pareto optimal solutions | | | | | |
| #106 | #202 | #254 | #318 | #358 | #462 |
| *DV1*  *DV2*  *DV3*  *DV4* | 5.50E+00 | 6.12E+00 | 6.00E+00 | 5.99E+00 | 6.24E+00 | 6.23E+00 | 6.10E+00 | 6.18E+00 |
| 8.50E+00 | 7.09E-01 | 7.34E-01 | 7.49E-01 | 7.76E-01 | 7.33E-01 | 7.81E-01 | 7.18E-01 |
| 2.0E+01 | 2.70E+01 | 2.52E+01 | 1.67E+01 | 1.81E+01 | 2.18E+01 | 1.69E+01 | 1.58E+01 |
| 2.0E+01 | 4.93E+01 | 3.16E+01 | 3.20E+01 | 2.10E+01 | 2.72E+01 | 2.57E+01 | 3.11E+01 |

Table 7. Second iteration. Table of criteria

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Criteria | | Prototype | Pareto optimal solutions | | | | | |
| #106 | #202 | #254 | #318 | #358 | #462 | |
| *P2* | (min) | 1.30E-001 | 6.01E-01 | 8.45E-02 | 9.17E-02 | 7.04E-02 | 9.63E-02 | 8.28E-02 | |
| *P3* | (min) | 1.54E-01 | 9.60E-02 | 1.13E-01 | 1.11E-01 | 9.66E-02 | 1.18E-01 | 1.04E-01 | |
| *P4* | (min) | 1.0E+00 | 1.00E+00 | 1.00E+00 | 1.00E+00 | 1.00E+00 | 1.01E+00 | 1.00E+00 | |
| *P5* | (min) | 3.15E-01 | 6.13E-01 | 6.81E-01 | 8.85E-01 | 8.43E-01 | 7.71E-01 | 8.63E-01 | |
| *P6* | (min) | 1.49E-01 | 1.28E-01 | 1.67E-01 | 2.16E-01 | 1.78E-01 | 1.94E-01 | 2.11E-01 | |
| *P7* | (pseudo) | 1.51E-01 | 1.68E-01 | 1.67E-01 | 1.72E-01 | 1.76E-01 | 1.68E-01 | 1.78E-01 | |
| *P8* | (pseudo) | 3.24E+00 | 3.29E+00 | 3.29E+00 | 3.29E+00 | 3.30E+00 | 3.29E+00 | 3.31E+00 | |
| *P9* | (pseudo) | 5.96E+00 | 9.09E+00 | 9.1E+00 | 7.9E+00 | 9.05E+00 | 8.43E+00 | 9.54E+00 | |
| *P10* | (pseudo) | 1.07E+02 | 1.77E+02 | 1.78E+02 | 1.43E+02 | 1.72E+02 | 1.59E+02 | 1.85E+02 | |
| *P11* | (pseudo) | 7.45E-02 | 6.62E-02 | 6.60E-02 | 6.09E-02 | 6.74E-02 | 6.31E-02 | 6.95E-02 | |
| *P12* | (pseudo) | 1.01E-04 | 1.81E-04 | 1.70E-04 | 1.80E-04 | 2.01E-04 | 1.69E-04 | 1.98E-04 | |
| *P13* | (pseudo) | 3.16E-05 | 6.02E-05 | 5.89E-05 | 6.58E-05 | 7.13E-05 | 6.01E-05 | 7.27E-05 | |
| *FQ1* | (min) | 1.23E-02 | 9.93E-02 | 9.81E-02 | 9.34E-02 | 9.26E-02 | 9.20E-02 | 9.64E-02 | |
| *FQ2* | (min) | 5.36E+01 | 4.33E+01 | 4.41E+01 | 4.38E+01 | 4.14E+01 | 4.46E+01 | 4.10E+01 | |
| *FQ4* | (min) | 4.68E+00 | 4.08E+00 | 4.19E+00 | 3.87E+00 | 3.88E+00 | 3.84E+00 | 3.97E+00 | |
| *R1* | (max) | 8.50E+01 | 8.50E+01 | 1.05E+02 | 8.00E+01 | 8.50E+01 | 9.00E+01 | 1.00E+02 | |

Further analysis of Table 7 shows that all solutions of second iteration as well as the #993 from the first iteration belong to the very tight intervals of the first and second design variables. First three parameters (*DV1-DV3*) of #993 and #106 are almost identical. Observe, that #993, while providing good response of many criteria, however does not satisfy new constraints on criteria *P9* and *P10* (the elevator workload). Moreover, analysis of the #993 also shows that it fails to satisfy constraint of the flying qualities criteria *FQ3.*



1. Criterion *P2* (max AOA deviation) vs. design variable *DV3.*



(b) Criterion *R1* (time delay margin) vs. design variable *DV3*

Figure 6. PSI Iteration 2. Dependencies of criteria *P2* and *R1* on the design variable *DV3.*

The analysis of test tables, dependencies of criteria on design variables, dependencies between criteria, allows to determine the most preferable solutions, in particular:

Figure 6 shows the influence of the bandwidth of the “matched” low-pass filter (*DV3*) on the (pilot-off-the-loop) trade-off between performance criterion *P2* (*P3* shows the same trend) and robustness (*R1*) of the augmented aircraft. From this observation we conclude that criteria *P2* (*P3*) and *R1* are contradictory with respect to the design variable *DV3*. This means that improvement of the tracking performance requires an increase in the bandwidth of the low-pass filter, which in turn results in degradation of the time delay margin of the augmented aircraft, as predicted by theory.

Figure 7 shows the dependencies of the flying qualities criterion *FQ1* (*FQ2* is similar) on the design variable *DV2*. While in the first PSI iteration the dependency of the criterion *FQ2* on the design variable *DV2* was not obvious, now it becomes apparent that a smaller damping ratio results in reduced (lead) pilot compensation.

The dependency of flying qualities criteria *FQ1* and *FQ2* obtained in this iteration is also similar to those obtained in the first iteration, thus demonstrating significant improvement of predicted flying qualities over the prototype design but now in the extended criteria space.



Figure 7. PSI Iteration 2*.* Dependencies of criteria *FQ1* (tracking performance) on the design variable *DV2.*

Finally, Figure 8 shows the dependency between criteria *P3* and *R1*, which illustrates the fundamental trade-off between performance and robustness of the closed-loop adaptive system with the pilot-off-the-loop. While all of the optimal solutions reduce the deviations from the desired response with respect to the prototype design, only three of these solutions exhibit a better time delay margin than the prototype design (#202, #462), and two exhibit a similar margin (#106, #318).



Figure 8. PSI Iteration 2. Relation between criteria *R1* (time delay margin) and *P3* (integral deviation of AOA).

As a result of iterative two-step correction of initial constraints the six Pareto optimal solutions have been found. Their analysis shows that four solutions # 106, # 202, #358, # 462 surpassed the prototype by six criteria simultaneously.

Although all 6 solutions are practically equivalent, special advantage is given to the design vector #202 since it provides better tradeoff between the (predicted) flying qualities (*FQ1,FQ2*) and the time-delay margin (*R1*), while minimizing the difference with the desired response, see Fig. 9. The comparative analysis of the AOA and the elevator workload demonstrates that the optimal solution provides faster response (rise time) to the commanded AOA command while minimally increasing the elevator workload that is naturally expected.



|  |  |
| --- | --- |
| (a) Angle of attack, | (b) Elevator deflection, |

Figure 9. Optimal Design#202. 3 deg-AOA step response

### V. Conclusion.

The construction of the feasible solutions of the L1 adaptive controller satisfying desired performance and robustness specifications is a critical step for the optimal design of the L1 FCS. In particular, the eigenstructure of the state-predictor state matrix and the bandwidth of the low-pass filters are the key elements that characterize the performance of the L1 FCS. To optimize the design of these elements, an 18-criteria problem of improving a prototype solution was formulated and solved. The results presented demonstrate the application of PSI method to the multicriteria design optimization of the L1 FCS implemented on the GTM AirSTAR aircraft. The study has addressed both the construction of the feasible solution set, and the improvement of a nominal prototype design initially synthesized by the L1 theory.

On the one hand, the results have demonstrated that the consistent application of the systematic design guidelines of L1 adaptive control becomes particularly beneficial for the construction of the feasible solution set.Moreover, the results of this study are consistent with the theoretical claims of the Theory of L1 Adaptive Control in terms of robustness and performance. On the other hand, the developed procedure and the obtained results confirm the suitability of the PSI method for the multicriteria optimization of an adaptive FCS subject to desired control specifications.

The optimal design of the L1 FCS significantly improved understanding of the design tradeoffs between adaptation and robustness. Finally, the design guidelines learned during the interaction with GTM model utilizing MOVI software contributed to successful flight verification and validation of the designed all-adaptive control law at NASA LARC.

# Bibliography

[1] S. A. Jacklin et al., “Verification, Validation, and Certification Challenges for Adaptive Flight-Critical Control System Software,” in *AIAA Guidance Navigation and Control Conference and Exhibit*, 2004, no. August, pp. AIAA-2004-5258.

[2] K. A. Wise, E. Lavretsky, and N. Hovakimyan, “Adaptive Control of Flight: Theory, Applications, and Open Problems,” in *American Control Conference*, pp. 8-10.

[3] S. A. Jacklin, “Closing the Certification Gaps in Adaptive Flight Control Software,” in *AIAA Guidance Navigation and Control Conference*, 2008, p. 6988.

[4] N. Hovakimyan and C. Cao, *L1 Adaptive Control Theory*. Philadelphia, PA: Society for Industrial and Applied Mathematics, 2010.

[5] K. Ki, K. Kim, and N. Hovakimyan, “Development of Verification and Validation Approaches for L1 Adaptive Control : Multi-Criteria Optimization for Filter Design,” in *AIAA Guidance Navigation and Control Conference 2 5 August 2010 Toronto Ontario Canada*, 2010, no. August.

[6] R. B. Statnikov and J. B. Matusov, *Multicriteria Analysis in Engineering*. Dordrecht/Boston/London: Kluwer Academic Publishers, 2002.

[7] I. M. Sobolí and Statnikov R.B., *Selecting Optimal Parameters in Multicriteria Problems*, 2nd ed. Moscow: Drofa, 2006.

[8] T. L. Jordan, W. M. Langford, and J. S. Hill, “Airborne Subscale Transport Aircraft Research Testbed - Aircraft Model Development,” *AIAA*, vol. Paper 205-, no. AIAA-2005-6432, 2005.

[9] T. L. Jordan et al., “AirSTAR: A UAV Platform for Flight Dynamics and Control System Testing,” *Systems Engineering*, no. June, pp. 1-15, 2006.

[10] J. V. Foster et al., “Dynamics Modeling and Simulation of Large Transport Airplanes in Upset Conditions,” *AIAA*, no. AIAA-2005-5933, pp. 1-13, 2005.

[11] G. H. Shah, “Aerodynamic Effects and Modeling of Damage to Transport Aircraft,” pp. 1-13.

[12] R. Stengel and A. F. Dynamics, “Aircraft Flying Qualities,” *Control*, 2008.

[13] M. Standard, “MIL-HDBK-1797 Flying qualities of piloted aircraft,” *US Department of Defense pg*, 1797.

[14] C. Cao and N. Hovakimyan, *L1 adaptive controller for multi-input multi-output systems in the presence of unmatched disturbances*, no. 6. IEEE, 2008, pp. 4105-4110.

[15] I. M. Gregory, “L 1 Adaptive Control Design for NASA AirSTAR Flight Test Vehicle,” *System*, no. August, pp. 1-27, 2009.

[16] I. M. Gregory, “Flight Test of an L 1 Adaptive Controller on the NASA AirSTAR Flight Test Vehicle,” *Science*, no. August, pp. 1-31, 2010.

[17] E. Xargay, N. Hovakimyan, and I. M. Gregory, “L 1 Adaptive Flight Control System : Systematic Design and Verification and Validation of Control Metrics,” *Optimization*, no. August, pp. 1-34, 2010.

[18] R. B. Statnikov and A. R. Statnikov, “Software Package MOVI 1.4 for Windows: Userís Manual, Certificate of Registration.” .

[19] MathWorks, “MathWorks - MATLAB and Simulink for Technical Computing,” *MathWorks*, 2011. [Online]. Available: http://www.mathworks.com/.

[20] V. Stepanyan, K. Krishnakumar, N. Nguyen, and L. Van Eykeren, “Stability and Performance Metrics for Adaptive Flight Control,” *Control*, no. August, pp. 1-19, 2009.

[21] R. E. Bailey and T. J. Bidlack, “A quantitative criterion for pilot-induced oscillations - Time domain Neal-Smith criterion,” 1996.

[22] R. Choe, E. Xargay, N. Hovakimyan, and I. M. Gregory, “L 1 Adaptive Control under Anomaly : Flying Qualities and Adverse Pilot Interaction,” *AIAA Guidance Navigation and Control Conference 2 5 August 2010 Toronto Ontario Canada*, no. August, pp. 1-29, 2010.

1. Pseudo-criterion differs from a criterion by the fact that it is not included in the calculation of the Pareto front, for more details see [6]. [↑](#footnote-ref-1)