# Reference Number JSCE1691

Dear Editor,

On behalf of my colleagues I would like to thank you and the reviewers for the time they devoted to reading the manuscript and pointing out issues that needed clarification.

In response to your and their comments we have

shortened the proofs in the appendix and now each proof fits in one page. The current appendix includes four pages of derivations (rather than eight) and represents 15% of the manuscript (rather than 25%). We have also removed some of the equations in the main body, shortening the revised manuscript by one additional page. As a result, the manuscript is now 27 pages long (rather than 32), and has a total of 59 numbered equations (rather than 81).

It is our opinion that further cuts, either in the main body or in the appendices, would compromise the rigor and clarity of the exposition. In fact, the current proofs in the manuscript are already hard to follow, as many technical details are now missing. While we agree that experimentation is crucial to validate the developed algorithms, we also think that theoretical derivations are critical to understand both their advantages and limitations.

In what follows we attach our response to each of the reviewers. Thank you for your consideration of our work.

Sincerely,

Vladimir Dobrokhodov

# Response to the first reviewer

1. *The authors should give the control schemes and MatLab design diagrams in "IV. Solutions and Analysis".*

RE: We understand the motivation behind the suggestion. However it is impractical to present the multilayer complicated Simulink diagram of the designed controller without its sufficient explanation; it seems physically impossible (due to the page number limitations) to present and only briefly discuss a lot of relevant details of the implementation. The diagram in Fig.2 is in fact the simplified version of the controller implemented in Simulink that provides sufficient details of the longitudinal controller implementation. The paper provides references to the previous works where thorough details of the AirStar mathematical model are presented.

Nevertheless, we followed the suggestion and included a separate section “II.C. Multicriteria Optimization Framework.” which objective is to justify the choice of the optimization method and to present a high level architecture that illustrates interaction between the AirSTAR simulation environment, L1 adaptive controller and the PSI method. A short overview accompanies the diagram as follows:

The architecture of the optimization framework is presented next in . The framework integrates the AirSTAR model and the L1 adaptive controller both implemented in Simulink, the criteria calculating scripts implemented in MatLab, and the PSI method implemented by the MOVI software. Convenience of the environment enables utilizing the capabilities of the high-fidelity nonlinear simulation, ease of control design and implementation and the vast set of features of the MOVI package implementing the PSI method.



Figure 4. Optimization framework.

1. *In 21 papers of "Bibliography", there are 7 papers written by the authors of Dr. Hovakimyan and Dr. Cao even though they have been focusing on the development and the application of L1 adaptive control for many years.*

RE: We revised all references of the paper with the objective of keeping only the key publications relevant to the points of discussion along the entire text of the paper. As per the references to the works of Professors Hovakimyan and Cao the following considerations were applied:

Ref [2] - retained in the revised manuscript - the work provides a general overview of the safety critical control problems in aerospace engineering that specifically call for adaptive control approaches; in essence this work frames the scope of issues in the area.

Ref [4] - retained in the revised manuscript - is the monograph that represents the most comprehensive description of the L1 theory of adaptive control.

Ref [5] - retained in the revised manuscript – while the monograph Ref.[4] discussed the issues associated with the systematic choice of the controller design the work in Ref.[5] provided an approach to solving the problems associated with non-convex optimization in a multidimensional design space. Thus the reference not only acknowledged an existing work in the field but also build a smooth motivated transition to the employment of the PSI method where the same design optimization problem is solved in multiple criteria space by using higher performance sampling approach based on LP-tau sequences [1, 2, 3].

Ref [15] - retained in the revised manuscript - is the detailed description of the L1 adaptive controller which design was motivated by the control challenges (match and unmatched uncertainties of the MIMO system) of the AirStar platform.

Ref [16-17,21] – deleted and substituted by single most recent publication[4] containing the most up to date material.

# Response to the third reviewer

1. *This paper addresses an important aspect of adaptive control using L1 criterion and applies the results to a real world problem at NASA.*

RE: N/A

# Response to the forth reviewer

1. *This paper mainly focused on the application of the Parameter Space Investigation method for the multi-criteria design optimization of the L1 adaptive flight control system implemented on the two turbine powered dynamically-scaled GTM AirSTAR aircraft, experimental results are also given.*

*I believe the contents are not new, the innovative contents are not highlighted. The authors failed to compare the presented method with other current methods, and the feasibility and effectiveness, especially the advantages of the presented approach are not verified.*

RE:

The Introduction section is modified to address the criticism by explicitly formulating the work objectives:

The paper also illustrates the suitability of the PSI method (and the MOVI software package) as a tool for formulating and solving multi-criteria optimization problems for design of adaptive flight control systems. This work is not intended to compare the effectiveness of the various optimization methods; it rather provides a sensible perspective on a complexity of the multicriteria analysis and suggests a viable approach to the solution of safety critical control problems typical in aerospace engineering. A reader who is interested in explicit comparison of various multicriteria analysis methods is referred to the monograph [7] that provides an essential overview of modern approaches in multicriteria decision making.

Benefits of the PSI method are further justified in the Introduction section by adding the following details:

The availability of an initial feasible solution may narrow the design variable space over which the search for feasible solutions should be performed. Furthermore, considering the benefits of sampling the multidimensional design variables space by the LP-tau quasi-random sequences, the number of trials required for the construction of the feasible set may be significantly reduced.

This method explicitly addresses the issues associated with high dimensionality of the criteria and the functional constraint spaces. It takes into account the complexity and the computational expenses of sampling the design space of high dimensionality by employing the quasi-random sampling (LP-tau sequences, see [6], [7]) which yield converging results with by a factor of 4-8 smaller sample sizes compared to the other methods.

A new section “II.C. Multicriteria Optimization Framework.” is included to justify the choice of the optimization method and to present a high level overview of the optimization framework.

To further support the claim of innovative results the sensitivity analysis of the optimal design was added to section IV.B. The result addresses the problem of optimality of the final design and it robustness with respect to variation of the design variables in the neighborhood of the chosen solution.

As the last step in verifying the proximity to optimal design and to ensure robustness of the solution to small variations of the design variables in the vicinity of the design #202 the sensitivity analysis was performed. The idea of this step is to calculate a criterion response in the direction defined by a design variable in the neighborhood of the optimal solution (#202).



Figure 6. Sensitivity plots.

As an example, shows the dependency of flying qualities (*FQ1*) and desired-model tracking performance (*P2*) criteria on the design variables *DV1* and *DV3* ; each figure represents the case where only one design variable is changing while all the remaining design variables are fixed at the optimal value(#202). Compact distribution of the Pareto solutions, the smooth behavior of the criteria and their negligible gradient confirm the robustness and proximity to optimal design of the vector #202.

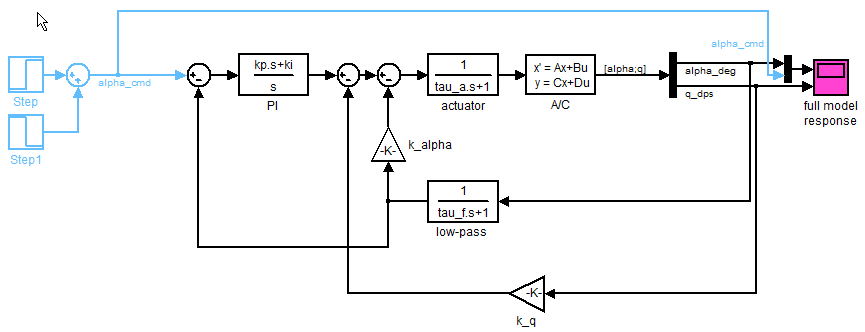
# Response to the fifth reviewer

1. *My largest concern is that the response of the prototype (shown page 8) and case #202 (page 25) to a 3deg change in AoA for 4 seconds appears to be divergent if held longer than 4 sec. This might be due to changes in flight condition of an increase in climb angle. Would this occur on a linearization of the system? Include an analysis of what is happening and why we should not be concerned.*

RE: The solution is not divergent; the deviation with respect to the desired constant response is due to the phugoid mode [5], which is stable and under-damped, and appears as unmatched dynamics in the short-period dynamics. The L1 controller is able to partially compensate for this mode, but not able to completely cancel it. The same behavior would occur in a linearized model of the aircraft that considered the states airspeed, AoA, pitch rate, and pitch angle. It would not appear in a simplified linear model of the short-period dynamics (AoA and pitch rate).

This is a standard behavior, which also occurs when using conventional flight control systems based on PID control. This slow oscillation can be easily compensated by the human pilot or an outer-loop autopilot, so it is of no concern. Below we attach the linearized model

and the results illustrating this behavior:





To avoid the ambiguity in reading the plot the following comment is added when the nominal AoA response is described first time (see page 7) and when the optimal solution is analyzed against the nominal design (see page 25):

Page 7: This prototype design of the state predictor, the low-pass filters, the adaptation sampling rate, and the prefilter, delivers an AOA response similar to the desired one (); see Figure 3 illustrating the well-damped phugoid dynamics of the AOA.

Page 25: Although all 6 solutions are practically equivalent, preference is given to the design vector #202, as it provides better tradeoff between the (predicted) flying qualities (*FQ1, FQ2*) and the time-delay margin (*R1*), while minimizing the difference with the desired response; see Figure. 9 illustrating a well-shaped phugoid response of the airplane.

1. *On page 6, the longitudinal system is described as SISO, that uses AoA and pitch rate feedback. I understand the single output is intended to be the control variable AoA, but the system has two outputs being used in the control.*

RE: The comment is partially correct. Thorough explanation would require reproducing a significant portion of the control system design that takes into account matched and unmatched uncertainties. Instead, the description of both the longitudinal and lateral control systems description is modified and an interested reader is referred to the theoretical material in references [15-17]. The resulting modification is as follows:

The implemented longitudinal L1 controller utilizes feedback in AOA and pitch rate to generate an elevator control signal in order to track AOA reference signals. The lateral/directional L1 controller uses feedback in AOSS, roll rate, and yaw rate to generate aileron and rudder commands in order to track sideslip-angle and roll-rate reference signals with reduced coupling.

1. *For P12 (page 13), it is unclear if the max includes the entire equation of just the delta e term. Although in P13 it is expressed more clearly that it should include the entire equation.*

RE: The equation defining the maximum deviation in cross-coupling dynamics is revised and presented in the following unambiguous form;



1. *On page 15, 'a priory' should read 'a priori'.*

RE: corrected

1. *On page 17, 'useful inside' should read 'useful insight'.*

RE: corrected

1. *On page 24, the statement is made that smaller damping ratio results in reduced pilot compensation, but Figure 7 appears to show the opposite.*

RE: In the revised version it is clarified that the flight qualities criteria *FQ1*(*FQ2*) is analyzed versus the design variable *DV2* corresponding to the damping ratio of the state predictor. As it follows from the definition of both criteria given on page 14, the objective of the optimization consists in minimizing the criteria. The revised conclusion is presented as follows:

While in the first PSI iteration the dependency of the criterion FQ2 on the design variable DV2 was not obvious, now it is clear that reducing the DV2 from 0.85 of the prototype design to about 0.75 of optimal solutions reduces the tracking error captured by the criterion FQ1 (similarly for the pilot workload - FQ2) by about 20%. Thus, it becomes apparent that a smaller damping ratio of the state predictor results in reduced (lead) pilot compensation.

# References

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| [1] | I. Sobol, "Uniformly Distributed Sequences with an Additional Uniform Property," *USSR Computational Mathematics and Mathematical Physics,* vol. 16, pp. 236-242, 1977. |
| [2] | H. Niederreiter, "Random Number Generation and quasi-Monte Carlo Methods," *SIAM,* 1992. |
| [3] | H. Niederreiter, "Low-Discrepancy Simulation," in *Handbook of Computational Finance*, Springer Berlin Heidelberg, 2012, pp. 703-729. |
| [4] | E. Xargay, N. Hovakimyan, V. Dobrokhodov, I. Kaminer, C. Cao and I. M. Gregory, "L1 Adaptive Control in Flight," in *Intelligent Systems, Progress in Aeronautics and Astronautics Series*, American Institute of Aeronautics and Astronautics, 2012. |

# List of key changes