#### Earth-Centered and Geodetic Coordinate Frames

It is convenient to consider two coordinate frames connected to the Earth. The *Earth Centered Earth Fixed* (ECEF) coordinate system is fixed to the Earth and therefore it rotates at the Earth’s sidereal rate with respect to the *Earth Centered Inertial* (ECI) frame that represents non-rotating inertial frame. The ECI frame is usually denoted  while the ECEF frame is denoted. Both frames are right-handed orthogonal and have their origins at the center of the Earth. The ECI frame has its axis aligned with the direction of the Earth’s rotation vector,  and axes placed in the equatorial plane with fixed in some celestial reference direction; for example a line connecting the Sun’s center and the Earth’s position at vernal equinox. The ECEF has  and axes placed in the equatorial plane and axis aligned with axis, see Figure 1 where the Earth is modeled as a spheroid. The axis is usually attached to the intersection of the Greenwich meridian and the equator, and the axis completes the right hand system.

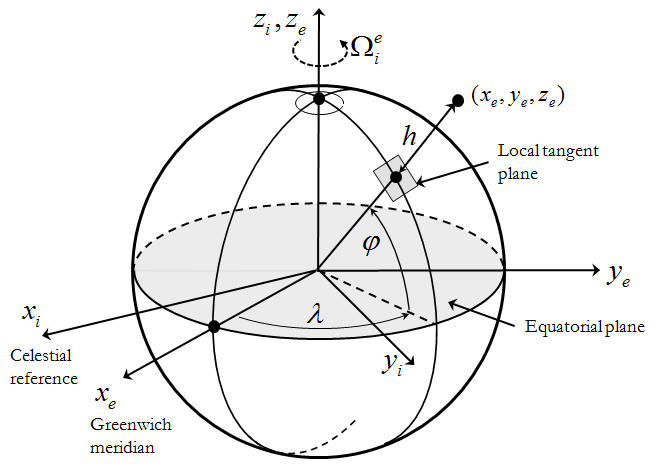


Figure 1. ECI, ECEF and geodetic coordinate frames.

It is worth noting that the ECEF axes definition may vary; however, the definition always states the attachment of two vectors to the direction of the Earth rotation and the Greenwich meridian as the inherent Earth properties. The sidereal rate  is the rate of ECEF rotation with respect to the ECI; the latter one is often call the true inertial frame. If necessary, for the purpose of UAV flight description, this rate can be approximated by one full rotation in 23h56’4.099”, thus resulting in 15.04106718 deg/h. Therefore, the transformation from ECI to ECEF frame is a plain rotation around the  axis defined by a single rotation by an angle , where **- is the time interval.

The *Local Geodetic* { } frame is usually associated with the ECEF frame, see Figure 1. It has the same origin at the center of the Earth. The frame defines the orientation of a line normal to Earth’s surface and passing through the point of interest. The orientation of the line is defined by two angles, – geodetic latitude and – geodetic longitude, with the height above the Earth’s surface; eventually these three parameters, along with the components of the UAV velocity vector, become the major navigation states. For most UAV applications it is sufficiently accurate to model Earth’s surface as an oblate spheroid with given -equatorial and -polar radiuses, or one of the radiuses and the -ellipticity . Last revisited in 2004, the datum of World Geodetic System (WGS-84) provides the following parameters for the oblate spheroid modeling: ,. The resulting transformation from the geodetic { } to the ECEF frame is as follows:



where - the eccentricity of oblate ellipsoid is defined as

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#### Accounting for the Earth Rotation rate

The complete set of 6DOF equations of motion presented above is an approximation of the rigid body kinematics and dynamics and is valid as long as the assumption of the flat Earth model satisfies the task at hand. During the high speed flight or in long duration and extended range missions the precision of the derived states will suffer from omitting the sidereal rate of the rotating Earth. The key reason for the error is in the accumulation over time of the Coriolis and centripetal accelerations induced by the rotating Earth. Thus, the following derivation outlines how the Earth rotation can be accounted for in the definition of the inertial velocity and acceleration vectors.

First, define the ECI as the true inertial frame. Next, by using the simplifying properties of defining the free motion of a rigid body with respect to the CG and utilizing the Coriolis theorem, resolve the absolute time derivative of the CG position vector in the true inertial frame as follows

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Taking the second time derivative and assuming that that the sidereal rate of the Earth rotation is constant () results in

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In the  denotes the vector of inertial angular velocity resolved in the body frame, and  is the same as ****. The equation updates the kinematic dead reckoning equation in [X], while the vector of inertial acceleration in should be used in the application of the second Newtonian law in [X] and [X].

Applying the angular velocities addition theorem, the can be represented as a sum of the angular velocity vector  of body frame {*b*} resolved in the body-carried frame {*n*}, the angular velocity vector  of body-carried frame resolved in ECEF frame , and the sidereal rate of the Earth rotation vector resolved in the true inertial frame . Thus the last equation can be also written as

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What remains is to define the elements of that enable calculation of the vector cross products.

The term is the Coriolis and the term is the centripetal accelerations.

The angular velocity vector can be obtained from the geodetic latitude () and longitude () rates, which in turn can be calculated from the NED components of. The transformation of rates of the geodetic system (,) to the body–carried stabilized frame  can be obtained similarly to [X] by a left-handed rotation around the East axis through the latitude angle 

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The rate of change of latitude and longitude (see [X]) can be calculated from the  northern and eastern components of the velocity vector as follows:

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where *h* is the height of CG above the reference oblate spheroid and

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are the estimates of the reference spheroid radius in the meridian and normal directions at given latitude and longitude. Substituting into the equation results in the estimate of  as follows:

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The Earth sidereal rotation vector has only one component in ECEF frame. Resolving for conveniencein the frame by a single rotation produces

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thus completing the definition of all terms in . Obviously, the result of substituting of all the vectors into is cumbersome however it demonstrates the point of how the Earth sidereal rate can be accounted for. To give a reader a sense of numerical significance of the resulting acceleration, the following numerical example compares the contribution of the Coriolis and the centripetal terms with an assumption that the UAV is at the constant low height in the wings level flight due East and is not maneuvering; and . In these conditions the centripetal term becomes equal to the Coriolis term at the speed of 914 m/s. In turn, when at the equator latitude, the third vertical component of the Coriolis acceleration is about 0.27 m/s2 that is 2.7% of the acceleration due to gravity (9.8 m/s2 ). Thus, the applicability of the simplifying flat Earth assumption becomes justified for a case of a short duration and relatively low speed flight of modern UAVs.

The corresponding linear and angular momentum equations can be obtained by applying the second Newtonian law; the procedure is similar to the simplified case presented above and resulted in [X]. Utilizing the same set of assumptions (**A.1-2**) and the resolving forces and moments with respect to the CG results in the linear momentum equation to remain the same however the kinematic and the linear momentum equations to be modified by utilizing . Applying the second Newtonian law to the linear motion of the CG and accounting for a new result in - gives



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where , as before, is the sum of all externally applied forces applied at CG resolved in the body frame. Equations -are the only new relations derived in a true inertial frame  thus accounting for the rotating Earth.

This new set of equations should be used when accurate modeling is required for a UAV moving faster than 600m/s over the Earth or when long distance and duration navigation is considered.